Title: Bekenstein-Hawking entropy from strange metals

Date: Jun 16, 2015 12:00 PM

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Abstract: <p>The properties of a strange metal fermion model with infinite-range</p>

<p>interactions turn out to be closely related to those of charged black holes</p>

<p>with AdS2 horizons. I show that a microscopic computation of the ground</p>

<p>state entropy density of the fermion model yields precisely the Bekenstein-Hawking</p>

<p>entropy density of the black hole. The fermion model is UV finite and has no supersymmetry</p>

Bekenstein-Hawking entropy from strange metals

Perimeter Institute
June 16, 2015

Subir Sachdev

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

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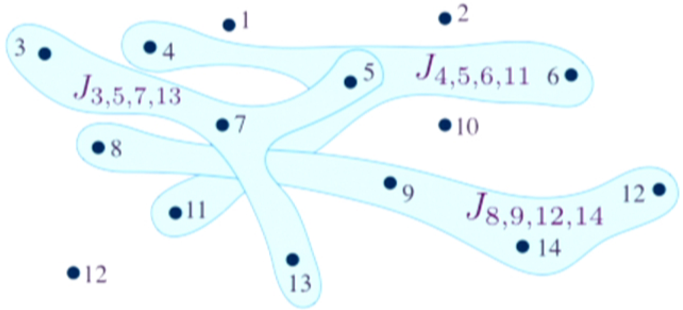
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

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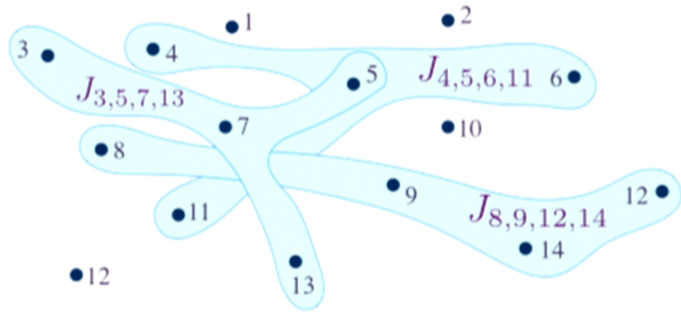

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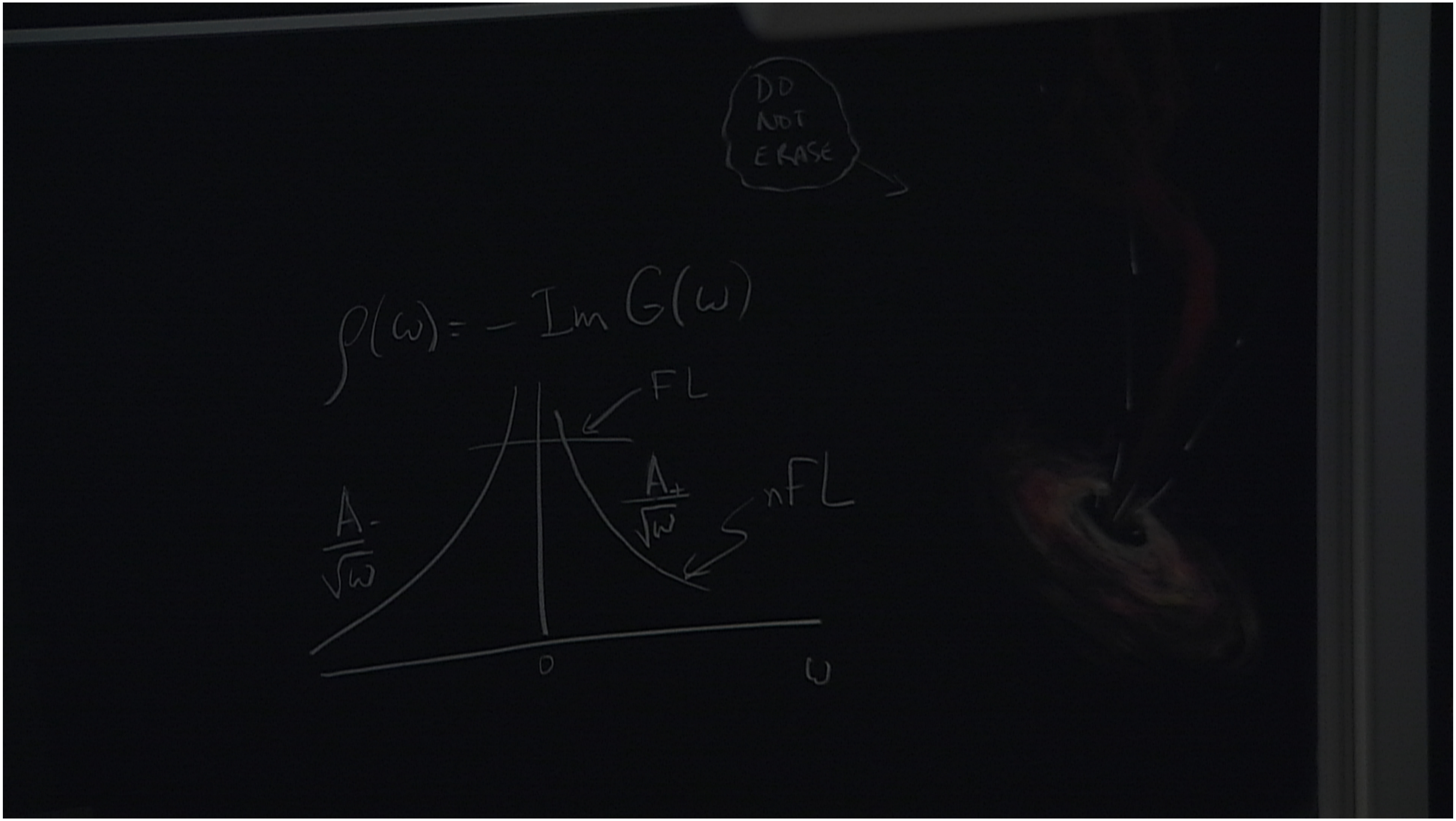
$$-\left\langle f_i(\tau) f_i^\dagger(\tau) \right\rangle \sim \begin{cases} -\tau^{-1/2}, & \tau > 0 \\ e^{-2\pi\mathcal{E}}\, |\tau|^{-1/2}, & \tau < 0. \end{cases}$$


DO NOT ERASE
ρ(ω) = − Im G(ω)
FL
nFL
A₋/√ω
A₊/√ω
0
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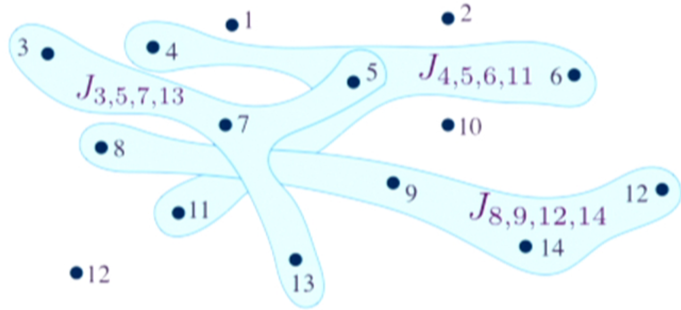

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Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi k_B \mathcal{E}$$

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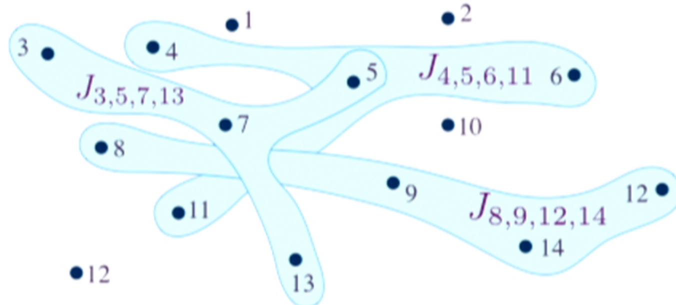

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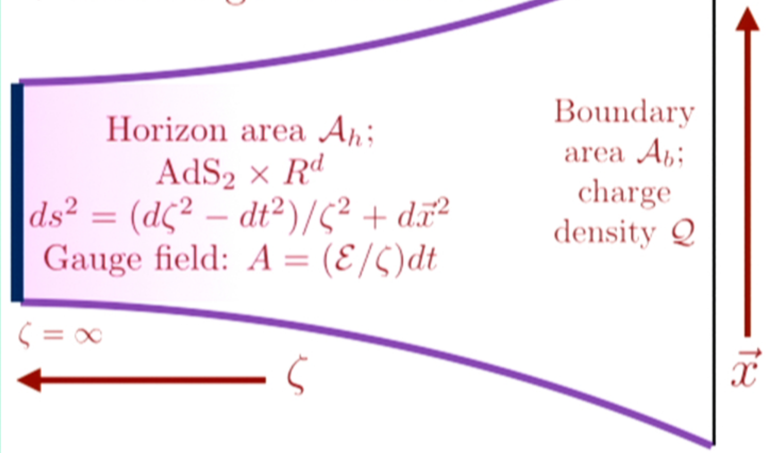
Einstein-Maxwell theory
+ cosmological constant



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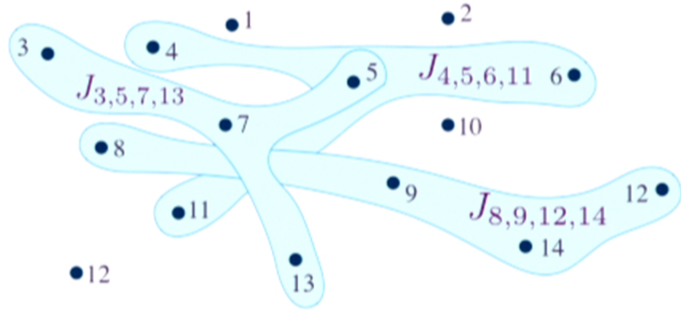

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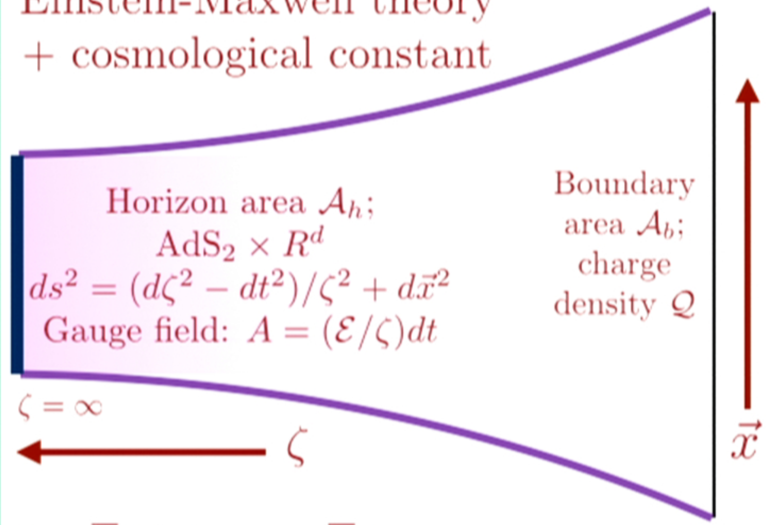

$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

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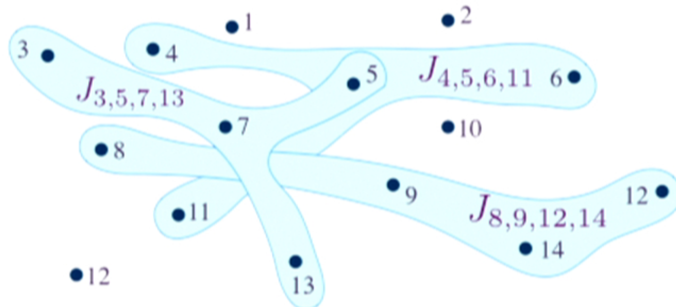

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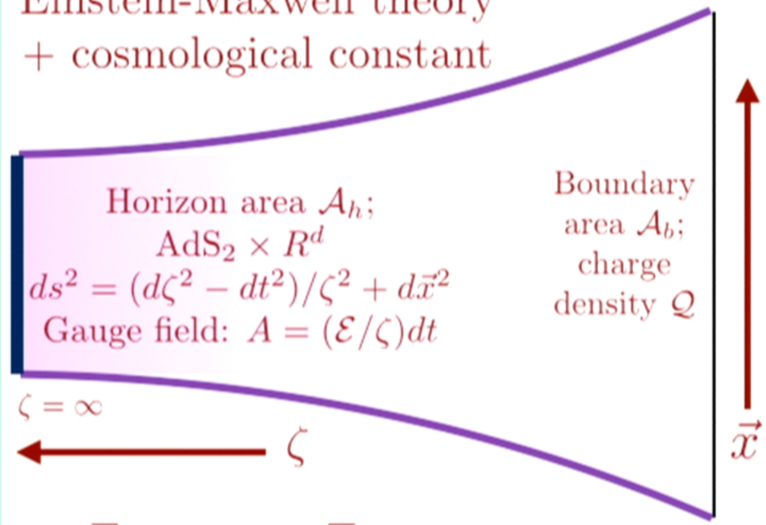

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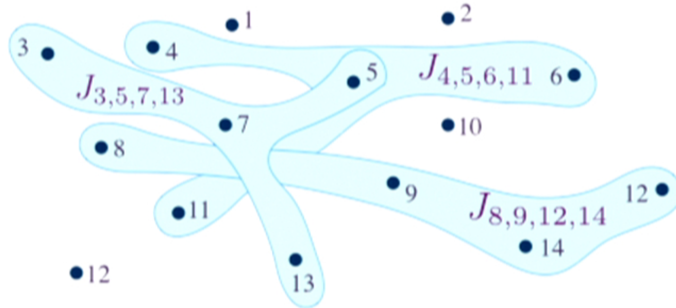

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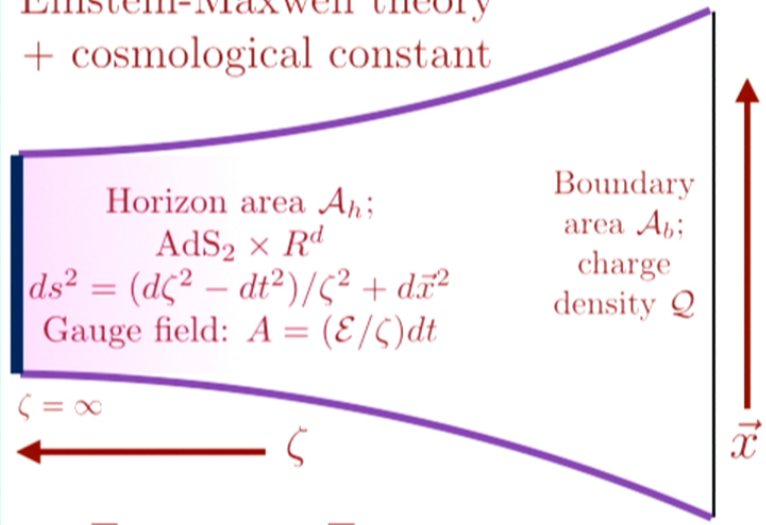

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Classical general relativity ('black hole mechanics') yields

$$\frac{1}{\mathcal{A}_b}\frac{\partial \mathcal{A}_h}{\partial \mathcal{Q}} = 8\pi G_N \mathcal{E}$$

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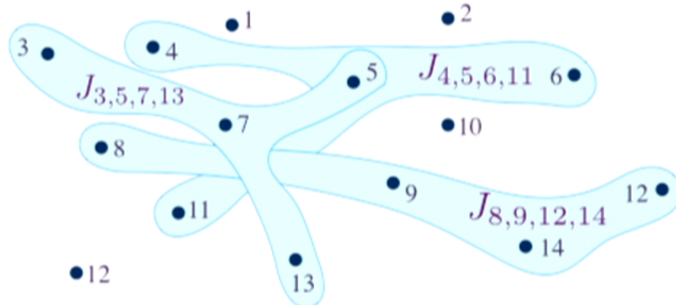

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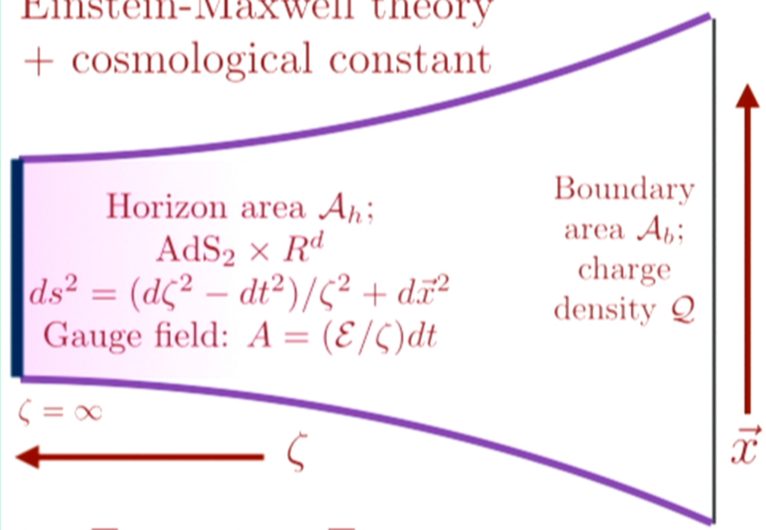

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Combination:

$$\mathcal{S} = \frac{\mathcal{A}_h}{4 G_N \mathcal{A}_b}$$

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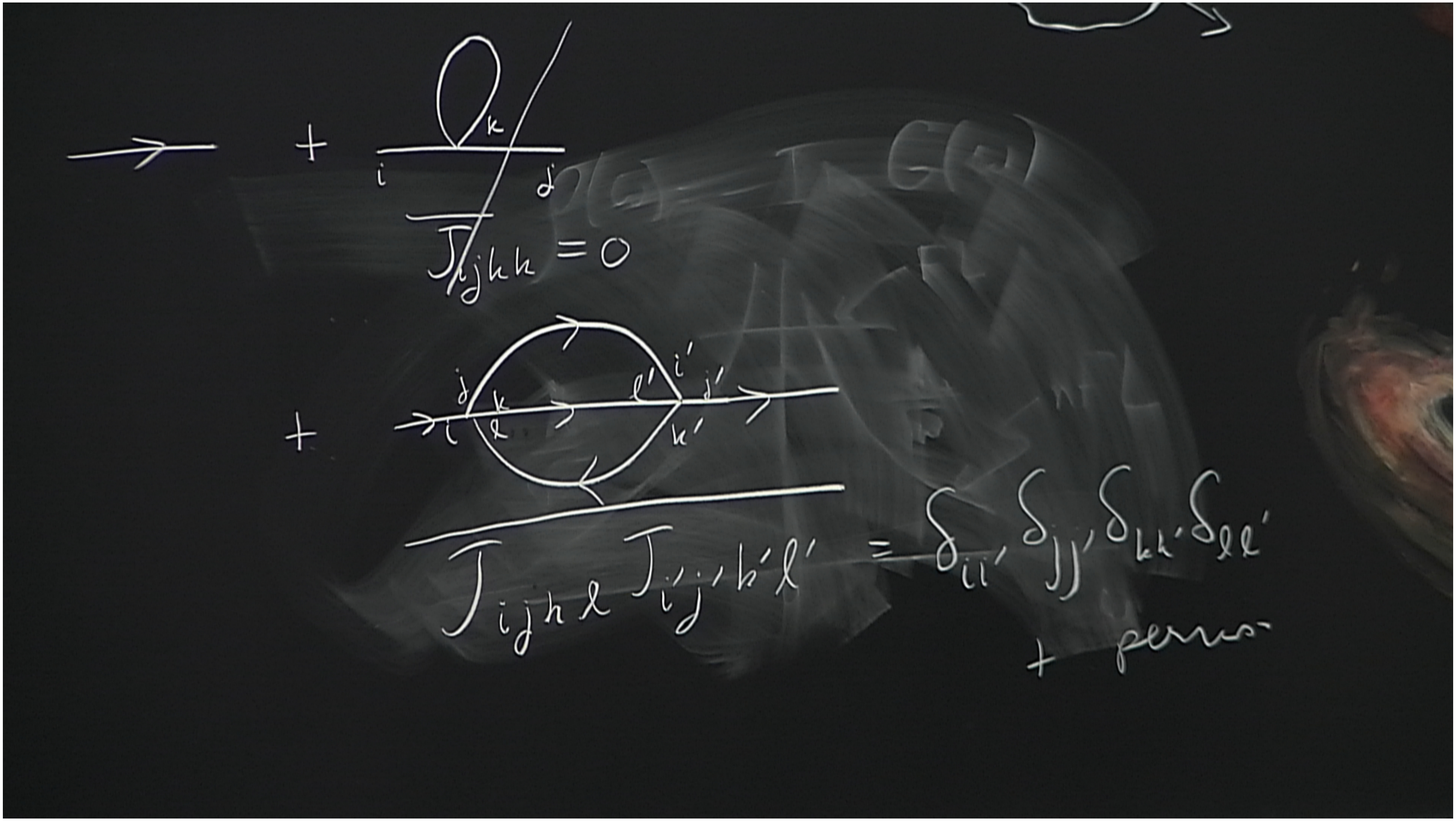
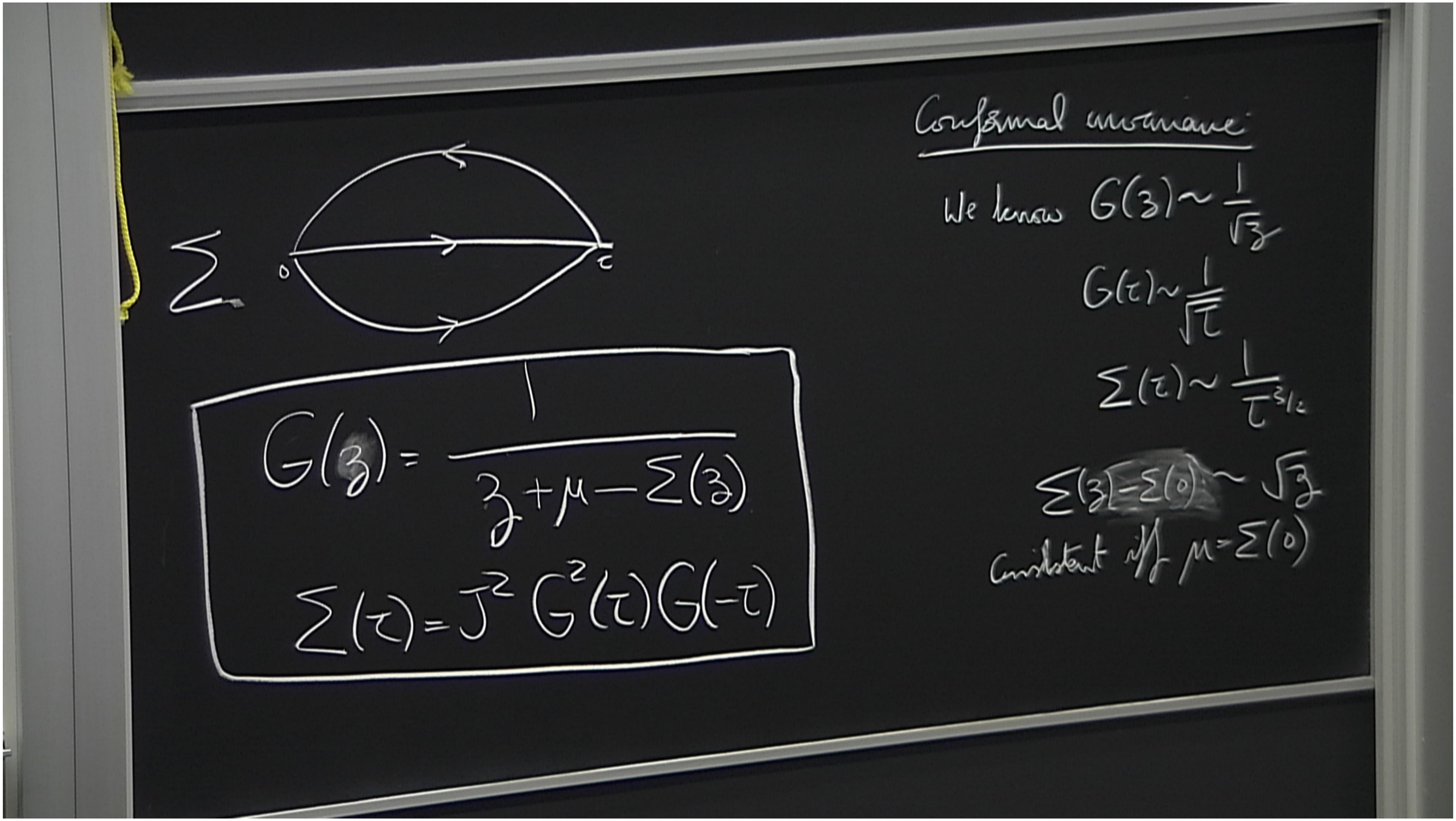
$$Z = \int \mathcal{D}c_i \, \exp\left(- \int_0^\beta d\tau \left[c_i^\dagger \frac{\partial c_i}{\partial \tau} - J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l \right] \right)$$

$$c_i \longrightarrow c_i^a \qquad a = 1 \ldots n \quad (n \to 0)$$

$$\overline{Z^n} = \int \mathcal{D}c_i^a \, \exp\left(- \int_0^\beta d\tau \, c^\dagger \frac{\partial c}{\partial \tau} - \frac{J^2}{N^3} \sum_{ab} \int_0^\beta d\tau \int_0^\beta d\tau' \left| \sum_i c^\dagger_{ia}(\tau) c_{ib}(\tau') \right|^4 \right)$$


$J_{ijkk} = 0$
$J_{ijkl} J_{i'j'k'l'} = \delta_{ii'}\delta_{jj'}\delta_{kk'}\delta_{ll'}$
+ perm.


Σ
0
τ
$G(z) = \frac{1}{z+\mu-\Sigma(z)}$
$\Sigma(\tau) = J^2 G^2(\tau) G(-\tau)$
Conformal invariance:
We know $G(z) \sim \frac{1}{\sqrt{z}}$
$G(\tau) \sim \frac{1}{\sqrt{\tau}}$
$\Sigma(\tau) \sim \frac{1}{\tau^{3/2}}$
$\Sigma(z) - \Sigma(0) \sim \sqrt{z}$
Consistent iff $\mu = \Sigma(0)$

$$\int d\tau_2 \, G(\tau_1, \tau_2) \Sigma(\tau_2 \tau_3) = \delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = J^2 G^2(\tau_1, \tau_2) G(\tau_2 - \tau_1)$$

$f(\sigma)$ and $g(\sigma)$ arbitrary

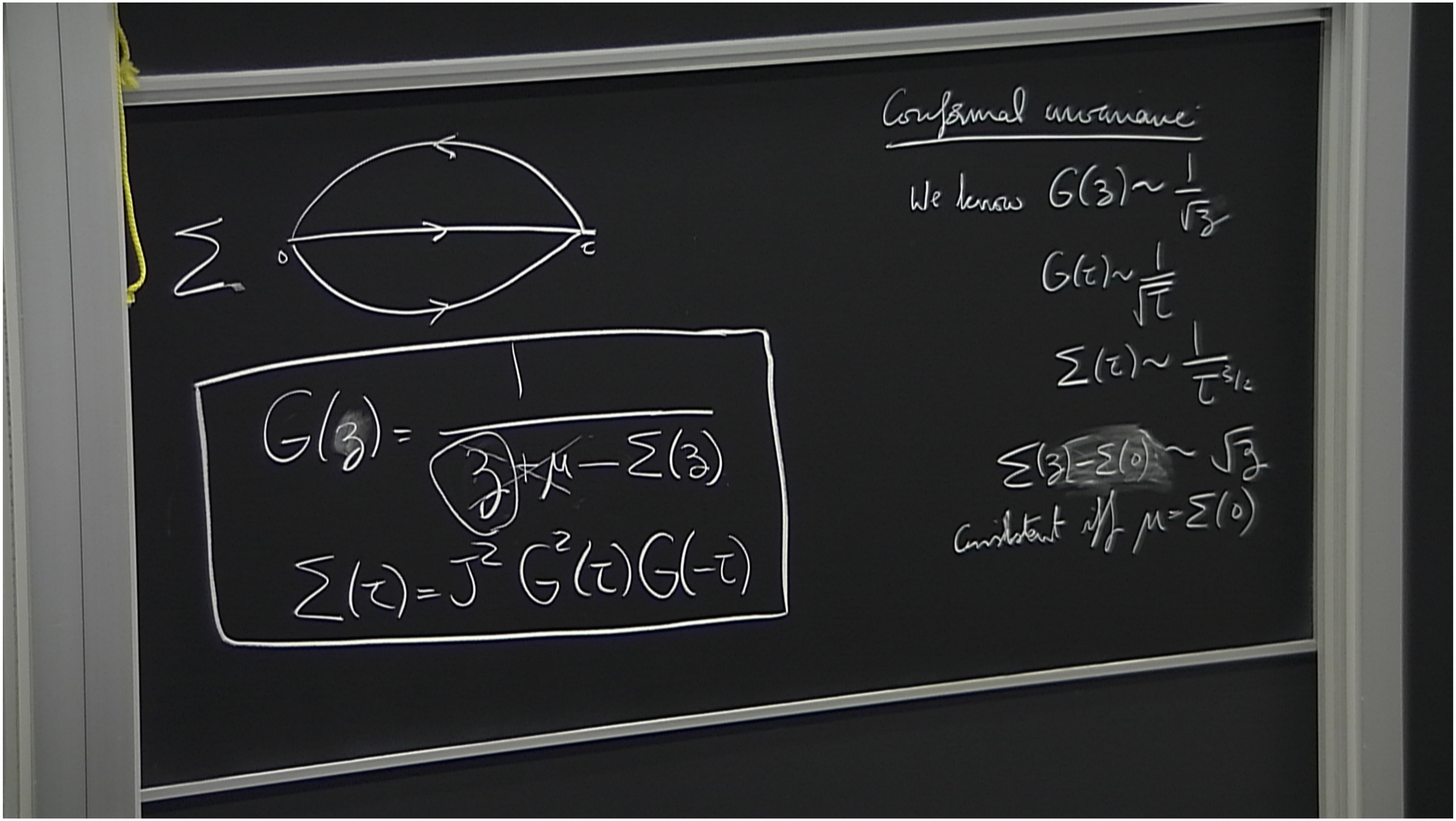
$$\tau = f(\sigma)$$

conformal

$$G(\tau_1, \tau_2) = \frac{1}{[f'(\sigma_1) f'(\sigma_2)]^{1/4}} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

gauge invariance

$$\Sigma(\tau_1, \tau_2) = \frac{1}{[f'(\sigma_1) f'(\sigma_2)]^{3/4}} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$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