Title: Topological Order Series

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Abstract:

Lectures on topological order: Fractional quatum Hall states

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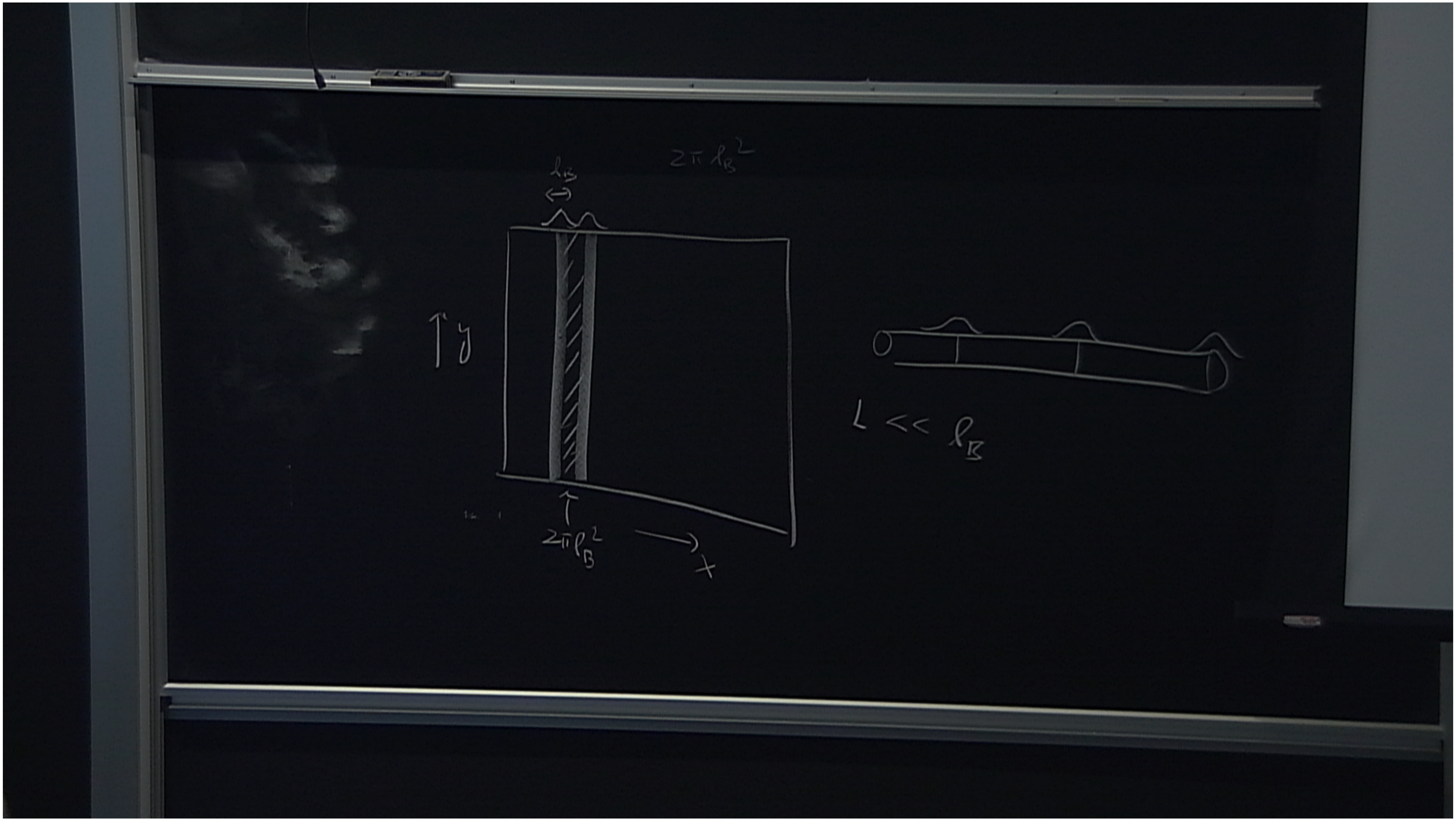
2015/6/11



$2\pi \ell_B^2$
ℓ_B
y
$2\pi \ell_B^2$
x
$L << \ell_B$

For the $\nu = 1$ Pfaffian state ($n = 2$ and $m = 2$)

$$S_1, S_2, \cdots : 0, 0, 2, 4, 8, 12, 18, 24, \cdots$$

$$l_1 l_2 l_3 \cdots : 0, 0, 2, 2, 4, 4, 6, 6, 8, 8, \cdots$$

$$n_0 n_1 n_2 \cdots : 20202020202020202 \cdots ,$$

- GPE rule: *(1) Three particles occupy* $\geq l_3 + 1 = 3$ *obitals or more* $(c = 1)$*. (2) The spread of two particles plus the spread of next two particles* $\geq (l_2 - l_1) + (l_3 - l_2) = 2$ $(c = 2)$*. (3) ...*
- the quasiparticle occupation patterns

$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 20202020202020202 \cdots \qquad Q_\gamma = 0$$

$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 02020202020202020 \cdots \qquad Q_\gamma = 1$$

$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 11111111111111111 \cdots \qquad Q_\gamma = 1/2$$

are the only **close-pacted** occupation patterns satisfying the above generalized Pauli exclusion rules.

- **close-pacted**:
  (1) add any electrons $\to$ violate the GPE rules.
  (2) shift any electrons $\to$ violate the GPE rules.

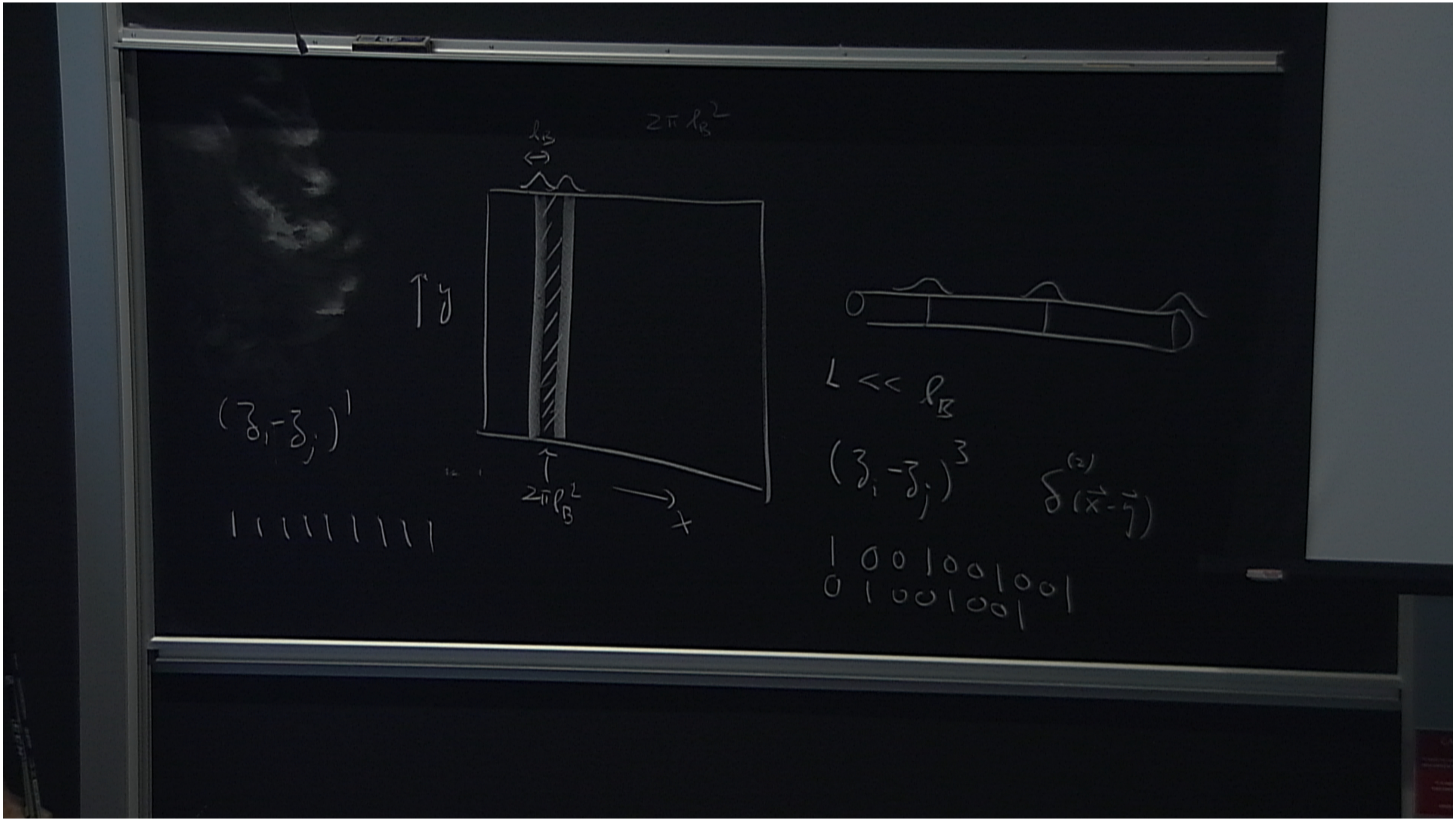
For the $\nu = 1$ Pfaffian state ($n = 2$ and $m = 2$)

$$S_1, S_2, \cdots : 0, 0, 2, 4, 8, 12, 18, 24, \cdots$$
$$n_0 n_1 n_2 \cdots : 20202020202020202 \cdots$$

- Quasiparticle solutions ($S_{\gamma;a} \to l_{\gamma;a} \to n_{\gamma;l}$):

$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 20202020202020202 \cdots \qquad Q_\gamma = 0$$
$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 02020202020202020 \cdots \qquad Q_\gamma = 1$$
$$n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : 11111111111111111 \cdots \qquad Q_\gamma = 1/2$$

- All other quasiparticle solutions can be obtained from the above three by removing extra electrons $\to$ only 3 quasiparticle types.
- Ground state degeneracy on torus = number of quasiparticle types

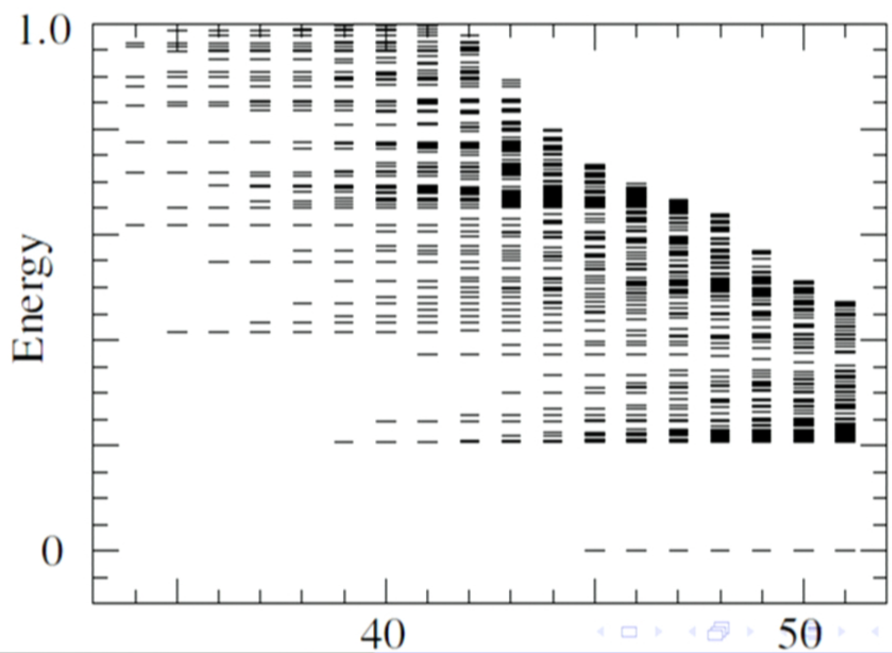


2π ℓ_B^2
ℓ_B
y
(z_i - z_j)^1
2π ℓ_B^2
x
L << ℓ_B
(z_i - z_j)^3
δ^(2)(x - y)

Edge states = zero-energy states of ideal Hamiltonian

- For ideal Hamiltonian $V_{1/3}(z_1, z_2) = -\partial_{z_1}\delta(z_1 - z_2)\partial_{z_1}$, the N electron state $P_{1/3} = \prod_{i<j}(z_i - z_j)^3$, is the zero-energy state with minimal angular momentum (the order of z_i's) $M_0 = N(N-1)$.
- Other zero-energy state has higher anglar momenta. Those zero energy states are the so called **edge states**:

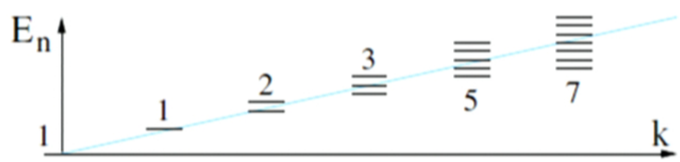
The energy spectrum of 100 lowest levels of the ideal Hamiltonian with 6 electrons



Edge spectrum of Laughlin state

For $\nu = 1/2$ **Laughlin state**, the edge states are obtained by deforming the Laughlin wave function without reducing the order of zeros:
$\Psi_{\text{edge}} = P_{\text{sym}}(\{z_i\})\Psi_{1/2}$ where P_{sym} is a symmetric polynomial, such as $\sum z_i$, $(\sum z_i)^2$, $\sum z_i^2, \dots$

M	M_0	M_0+1	M_0+2	M_0+3	M_0+4
# of states	1	1	2	3	5
P_{sym}	1	$\sum z_i$	$(\sum z_i)^2$	...	...
			$\sum z_i^2$	...	...

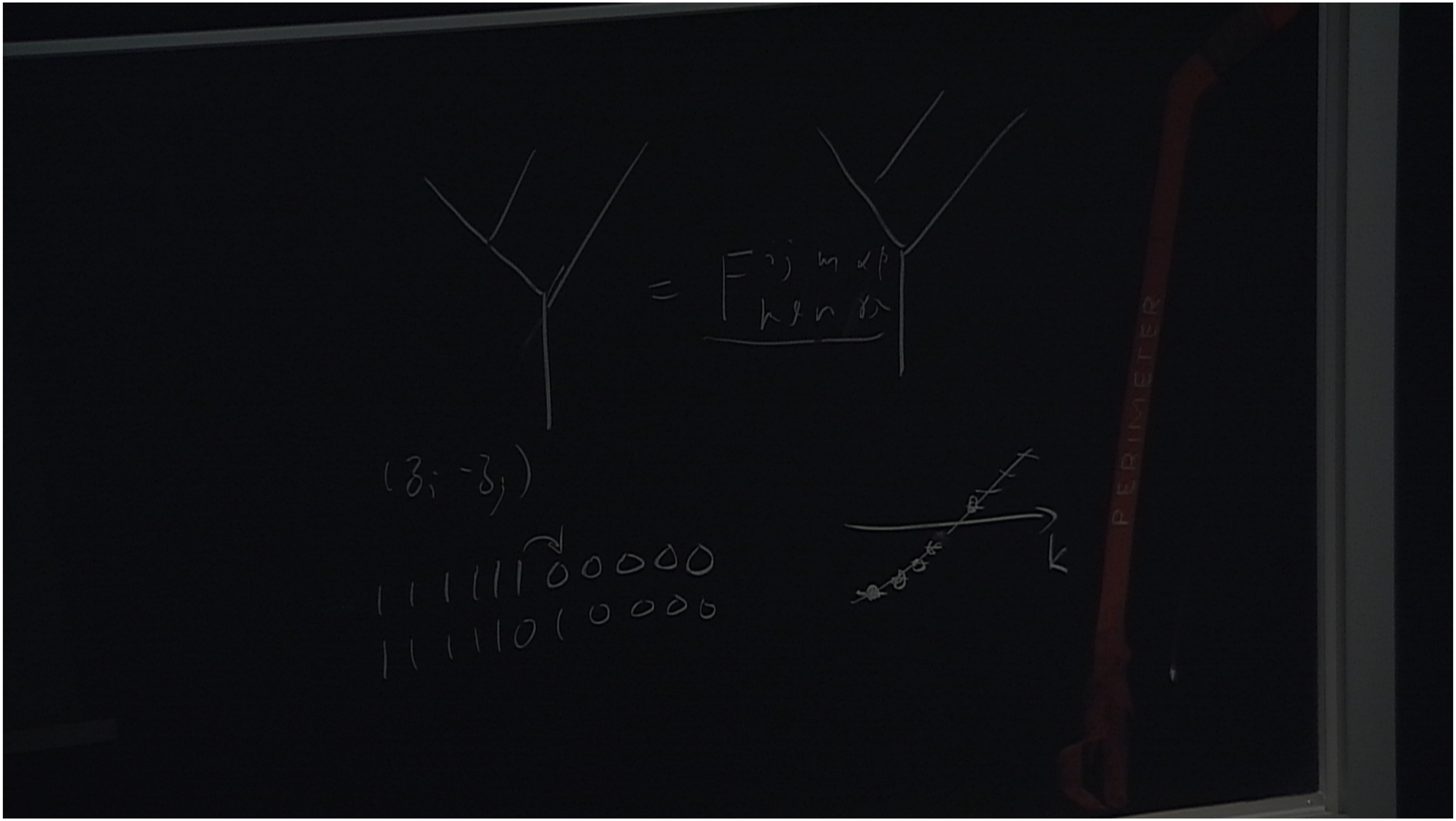


GPE rule and edge spectrum for the Pfaffian state

First kind of edge:

$$M_0 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020202|00000000\cdots$$
$$M_0+1 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020201|10000000\cdots$$
$$M_0+1 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020112|00000000\cdots \quad \text{not allowed}$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020201|01000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020200|20000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020111|10000000\cdots$$
$$M_0+3 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020201|00100000\cdots$$
$$M_0+3 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020200|11000000\cdots$$
$$M_0+3 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020111|01000000\cdots$$
$$M_0+3 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202020110|20000000\cdots$$
$$M_0+3 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 20202011111|10000000\cdots$$

M	M_0	M_0+1	M_0+2	M_0+3	M_0+4
# of states D_n	1	1	3	5	10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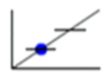
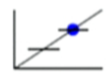
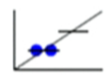
Low energy effective theory of edge excitations

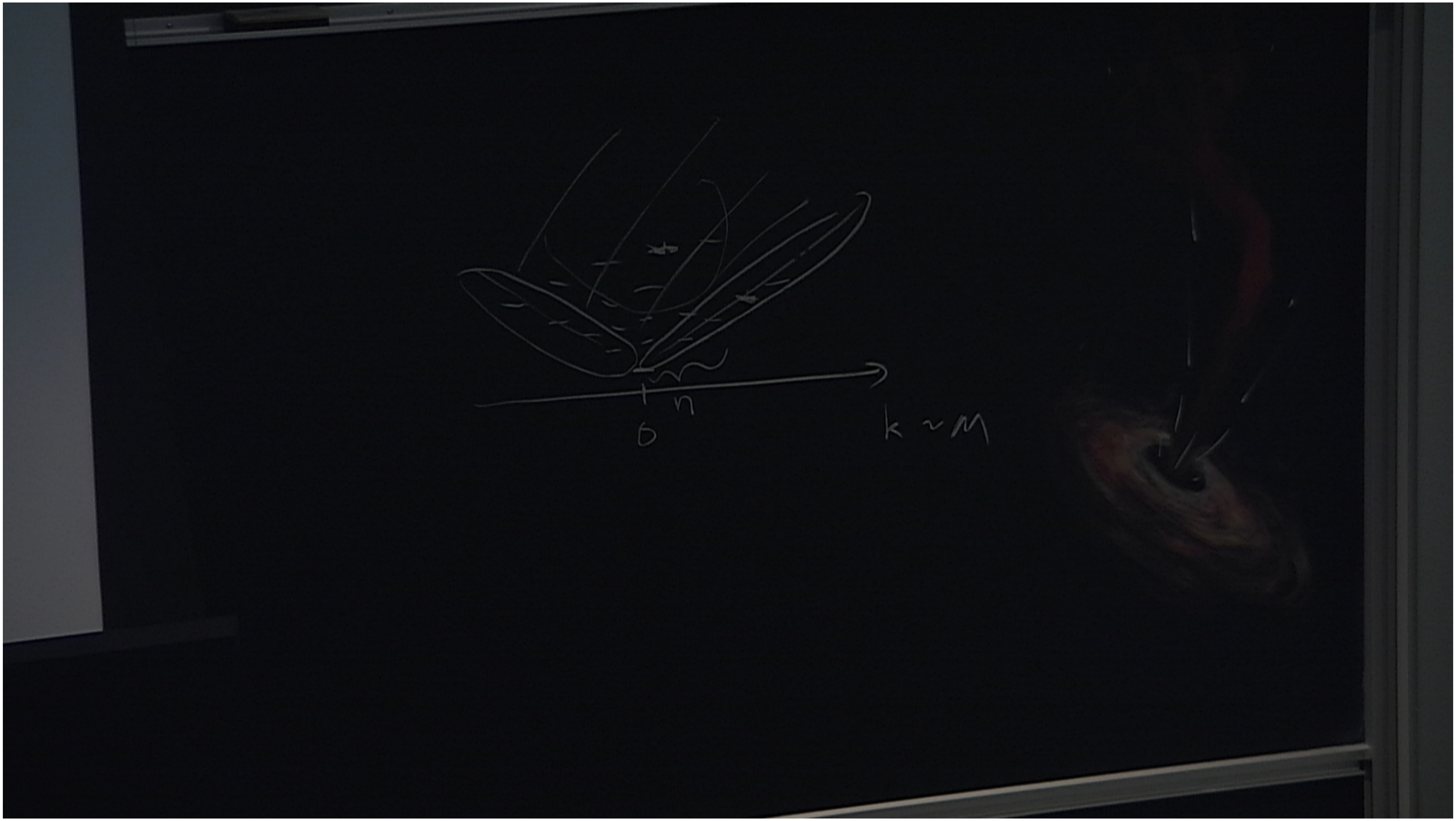
Quantization: view ρ_k and p_k as operators that satisfy $[\rho_k, p_{k'}] = \mathrm{i}\delta_{kk'}$. After quantization we have

$$[\rho_k, \rho_{k'}] = \frac{\nu}{2\pi} k \delta_{k+k'}, \qquad k, k' = \text{integer} \times \frac{2\pi}{L}$$

$$H = 2\pi \frac{v}{\nu} \sum_{k>0} \rho_{-k}\rho_k = \sum_{k>0} v k a_k^\dagger a_k, \quad a_k = \sqrt{\frac{2\pi}{\nu k}} \rho_k$$

M	M_0	M_0+1	M_0+2	M_0+3
# of states D_n	1	1	2	3
phonons				...
				...

$D_n \approx \frac{1}{4\sqrt{3n}} \mathrm{e}^{\pi\sqrt{c}\sqrt{\frac{2n}{3}}} \approx \mathrm{e}^{\pi\sqrt{c}\sqrt{\frac{2n}{3}}}, \quad c = 1,$
where c is the *central charge*.


n
0
k ~ M

GPE rule and edge spectrum for the Pfaffian state

Second kind of edge:

$$M_0 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111111|00000000\cdots$$
$$M_0+1 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111110|10000000\cdots$$
$$M_0+1 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111102|00000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111110|01000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111101|10000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111111020|10000000\cdots$$
$$M_0+2 : n_{\gamma;0} n_{\gamma;1} n_{\gamma;2} \cdots : \ 1111110202|00000000\cdots$$

M	M_0	M_0+1	M_0+2	M_0+3	M_0+4
# of states D_n	1	2	4	8	14

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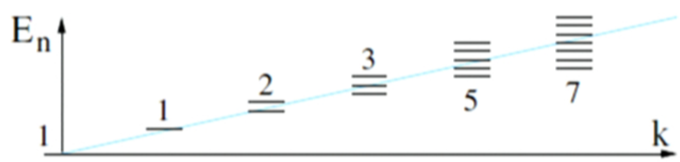

Low energy effective theory of edge excitations

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M	M_0	M_0+1	M_0+2	M_0+3
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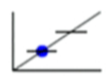
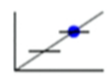
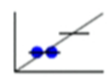
phonons				...
				...

$D_n \approx \frac{1}{4\sqrt{3}n} \mathrm{e}^{\pi\sqrt{c}\sqrt{\frac{2n}{3}}} \approx \mathrm{e}^{\pi\sqrt{c}\sqrt{\frac{2n}{3}}}, \quad c = 1,$
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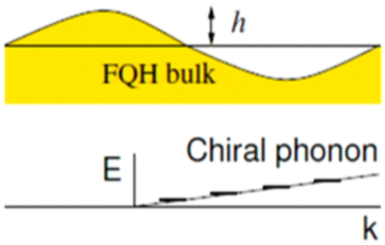


- The $\nu = 1/m$ state $\Psi_{1/m} = \prod(z_i - z_j)^m$.
  $\Rightarrow$ edge excitations = edge waves
  $\Rightarrow$ edge phonons after quantization.
  Displacement $h \propto$ 1D edge density $\rho = h\rho_{2D}$.
- Confining electric field induces edge current

$$\mathbf{j} = \sigma_{xy}\hat{z} \times \mathbf{E}, \qquad \sigma_{xy} = \nu\frac{e^2}{2\pi\hbar}$$

- Electron drift velocity at the edge

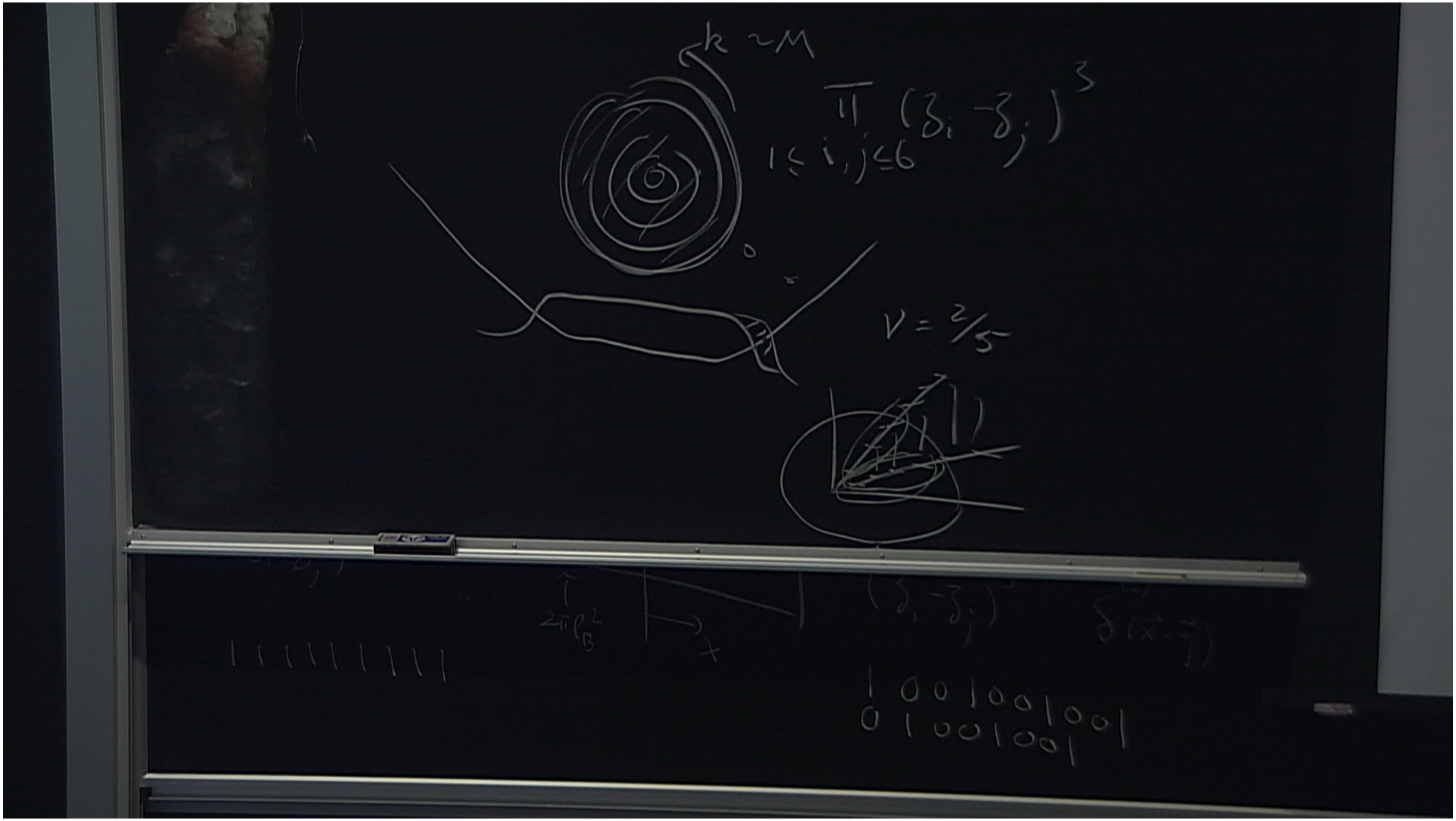
$$v = \frac{E}{B}c$$

- The wave equation for the propagating edge wave:

$$\partial_t\rho + v\partial_x\rho = 0, \qquad \rho(x) = nh(x)$$

- The Hamiltonian (i.e. the energy) of the edge waves:

$$H = \int \mathrm{d}x\ \frac{1}{2}e\rho E h = \int \mathrm{d}x\ \pi\frac{v}{\nu}\rho^2$$


k ~ M
Π (z_i - z_j)^3
1 ≤ i, j ≤ 6
ν = 2/5