

Title: What if the universe accelerates because time is discrete?

Date: Jun 05, 2015 11:00 AM

URL: <http://pirsa.org/15060007>

Abstract: <p>In order to introduce the cosmological constant in a simplicial geometry, constant curvature should be introduced on simplex faces. This yields a compactification of the phase space and the finiteness of the Hilbert for each link. Not only the intrinsic, but also the extrinsic geometry turns out to be discrete, pointing to discreteness of time, in addition to space.</p>

arXiv:1502.00278

# *What if the universe accelerates because time is discrete?*

Francesca Vidotto

with Carlo Rovelli



Netherlands Organisation for Scientific Research

Institute for  
Mathematics, Astrophysics  
and Particle Physics

Radboud University Nijmegen



## THE PROBLEM

- Fact: our universe has a positive cosmological constant
- Does this affect the kinematics of LQG? (Borissov-Major-Smolín '96, Dupuis-Girelli '13)
- Replacing flat cells with uniformly curved cells (Bahr-Dittrich '09)
- Classical kinematics:  $\Gamma \equiv su(2) \times SU(2) \quad \forall \ell$
- Idea: replace the algebra with the group  $\rightarrow$  finiteness (Haggard-Han-Kaminski-Riello '14)
- Classically: compact phase space  $\rightarrow$  finite Liouville volume
- Quantum: finite # of Planck cells, finite # orthogonal states  $\rightarrow$  finite dim Hilbert space

$\Delta$  from discrete time

Francesca Vidotto

## U(1)xU(1)

- $h = e^{i\alpha}, k = e^{i\beta} \in \mathbb{C}$
- Symplectic 2-form:  $\omega = -h^{-1}dh \wedge k^{-1}dk$
- Poisson brackets:  $\{k, h\} = hk$

in the limit in which  
the radius of one of the two circles can  
be considered large we want to recover the  
symplectic form of the cotangent space

$$\omega = d\alpha \wedge d\beta$$

### ■ QUANTIZATION

- Hilbert space:  $|n\rangle \quad n = 1, \dots, N = \dim \mathcal{H}$
- Operators:  $k|n\rangle = e^{i\frac{2\pi}{N}n}|n\rangle$   
 $h|n\rangle = |n+1\rangle \quad \text{cyclic: } h|N\rangle = |1\rangle$

- Commutator:  $[h, k] = \left(e^{i\frac{2\pi}{N}} - 1\right) hk$

$$[\hat{a}, \hat{b}] = i\hbar \widehat{\{a, b\}}$$

$$\hbar = \frac{2\pi}{N} \rightsquigarrow \frac{[\text{Planck length}]}{[\text{cosmological constant}]}$$

$\Delta$  from discrete time

Francesca Vidotto

## U(1) x U(1)

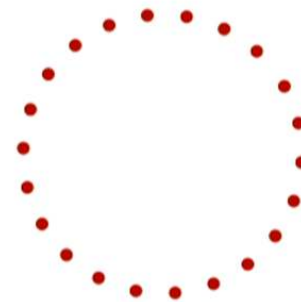
- $h = e^{i\alpha}, k = e^{i\beta} \in \mathbb{C}$
- Symplectic 2-form:  $\omega = -h^{-1}dh \wedge k^{-1}dk$
- Poisson brackets:  $\{k, h\} = hk$

in the limit in which  
the radius of one of the two circles can  
be considered large we want to recover the  
symplectic form of the cotangent space

$$\omega = d\alpha \wedge d\beta$$

### ■ QUANTIZATION

- Hilbert space:  $|n\rangle \quad n = 1, \dots, N = \dim \mathcal{H}$
- Operators:  $k|n\rangle = e^{i\frac{2\pi}{N}n}|n\rangle$   
 $h|n\rangle = |n+1\rangle \quad \text{cyclic: } h|N\rangle = |1\rangle$
- Commutator:  $[h, k] = \left(e^{i\frac{2\pi}{N}} - 1\right)hk$



discrete spectrum

$$[\hat{a}, \hat{b}] = i\hbar \widehat{\{a, b\}}$$

$$\hbar = \frac{2\pi}{N} \rightsquigarrow \frac{[\text{Planck length}]}{[\text{cosmological constant}]}$$

$\Delta$  from discrete time

Francesca Vidotto

## SU(2) x SU(2)

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form:  $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- ... but we are not going to use this!

limit: arc  $\ll R$   
where  
 $\theta = Tr[kh^{-1}dh]$

### ■ QUANTIZATION

- Hilbert space:  $L_2[SU(2)] \sim \oplus_{j=0}^{\infty} (\mathcal{H}_j \otimes \mathcal{H}_j)$

- Operators:  $\langle U | jmn \rangle = D_{mn}^j(U)$

- $h\psi(U) = U\psi(U)$

$$h_{AB} | jmn \rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix} \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix} | j' m' n' \rangle$$

- $J^i \psi(U) = L^i \psi(U)$

$\Delta$  from discrete time

Francesca Vidotto

# KNOTS

$$h_{AB}|jmn\rangle = \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ A & m & m' \end{array} \right)_q \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ B & n & n' \end{array} \right)_q |j'm'n'\rangle \text{ does not commute any more}$$

*∄ h reps*

- Wigner symbols as trivalent nodes

$$(h_{AB})_{m'n'}^{mn} = \begin{array}{c} A \\ | \\ \text{---} m \text{---} \\ | \\ n \text{---} \\ | \\ B \end{array}$$

- Acting with two operators:

$$h_{AB}h_{CD} = \begin{array}{c} A \quad C \\ | \quad | \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

- Inverse order:

$$h_{CD}h_{AB} = \begin{array}{c} A \quad C \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

*A* from discrete time

Francesca Vidotto

## SU(2) x SU(2)

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form:  $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- ... but we are not going to use this!

limit: arc  $\ll R$   
 where  
 $\theta = Tr[kh^{-1}dh]$

### ■ QUANTIZATION

### QUANTUM GROUPS

- Hilbert space:

$$\mathcal{H} = \bigoplus_{j=0}^{j_{max}} (\mathcal{H}_j \otimes \mathcal{H}_j)$$

- Operators:

$$\langle U | jmn \rangle = D_{mn}^j(U)$$

- $h\psi(U) = U\psi(U)$

$$h_{AB} | jmn \rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}_q \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}_q | j' m' n' \rangle$$

- $J^i \psi(U) = L^i \psi(U)$

$$J^i | jmn \rangle = \left( \tau_{mk}^{i(j)} \right)_q | jkn \rangle$$

$$q^r = -1 \quad j_{max} = \frac{r-2}{2}$$

$\Delta$  from discrete time

Francesca Vidotto

# KNOTS

$$h_{AB}|jmn\rangle = \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ A & m & m' \end{array} \right)_q \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ B & n & n' \end{array} \right)_q |j'm'n'\rangle \text{ does not commute any more}$$

$\nexists \hbar \text{ reps}$

- Wigner symbols as trivalent nodes

$$(h_{AB})_{m'n'}^{mn} = \begin{array}{c} A \\ | \\ \text{---} m \text{---} \\ | \\ n \text{---} \\ | \\ B \end{array}$$

- Acting with two operators:

$$h_{AB}h_{CD} = \begin{array}{c} A \quad C \\ | \quad | \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

*crossing operators*

$$h_{AB}h_{CD} = R_{AC}^{A'C'} R_{BD}^{B'D'} h_{C'D'} h_{A'B'}$$

Expanding in  $\hbar$  so that  $R \sim 1 + r$  :

$$\{h_{AB}, h_{CD}\} = r_{BD}^{B'D'} h_{CD'} h_{AB'} + r_{AC}^{A'C'} h_{C'D} h_{A'B}$$

- Inverse order:

$$h_{CD}h_{AB} = \begin{array}{c} A \quad C \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

$\Delta$  from discrete time

Francesca Vidotto

# KNOTS

$$h_{AB}|jmn\rangle = \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ A & m & m' \end{array} \right)_q \left( \begin{array}{ccc} \frac{1}{2} & j & j' \\ B & n & n' \end{array} \right)_q |j'm'n'\rangle \text{ does not commute any more}$$

$\nexists \hbar \text{ reps}$

- Wigner symbols as trivalent nodes

$$(h_{AB})_{m'n'}^{mn} = \begin{array}{c} A \\ | \\ \text{---} m \text{---} \\ | \\ n \text{---} \\ | \\ B \end{array}$$

- Acting with two operators:

$$h_{AB}h_{CD} = \begin{array}{c} A \quad C \\ | \quad | \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

*crossing operators*

$$h_{AB}h_{CD} = R_{AC}^{A'C'} R_{BD}^{B'D'} h_{C'D'} h_{A'B'}$$

Expanding in  $\hbar$  so that  $R \sim 1 + r$  :

$$\{h_{AB}, h_{CD}\} = r_{BD}^{B'D'} h_{CD'} h_{AB'} + r_{AC}^{A'C'} h_{C'D} h_{A'B}$$

- Inverse order:

$$h_{CD}h_{AB} = \begin{array}{c} A \quad C \\ \text{---} \text{---} \\ | \quad | \\ B \quad D \end{array}$$

quasi Poisson-Lie groups

$\Delta$  from discrete time

Francesca Vidotto

## PHYSICS AND CONCLUSIONS

- We have introduced a modification of LQG kinematics
  - compact phase space
  - allows to introduce a positive cosmological constant
- finite dimensional Hilbert space dim determined by the ratio between the two constants:
  - quantization (physically: Planck constant scale)
  - simplex curvature / deformation of Poisson algebra (physically: cosmological constant)
- Hilbert space reduces to usual LQG one for triangles small compared to curvature radius
- A q-deformation of the dynamics:
  - renders quantum gravity finite (Turaev-Viro '92, Han '10)
  - amount to introduce the cosmological constant (Mizoguchi-Tada '91, Han '10)  $q = e^{i\sqrt{\Lambda}hG}$
- Compactness: discretization of the intrinsic and extrinsic geometry
- Time discreteness:  $K_{ab} \sim dq_{ab}/dt$  where  $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt \Delta t$   
minimum proper time Planckian, full discrete spectrum depends on cosmological constant
- Euclidean 2+1  $\rightarrow$  to do: Lorentzian 3+1 !!!

$\Delta$  from discrete time

Francesca Vidotto

## PHYSICS AND CONCLUSIONS

- We have introduced a modification of LQG kinematics
  - compact phase space
  - allows to introduce a positive cosmological constant
- finite dimensional Hilbert space dim determined by the ratio between the two constants:
  - quantization (physically: Planck constant scale)
  - simplex curvature / deformation of Poisson algebra (physically: cosmological constant)
- Hilbert space reduces to usual LQG one for triangles small compared to curvature radius
- A q-deformation of the dynamics:
  - renders quantum gravity finite (Turaev-Viro '92, Han '10)
  - amount to introduce the cosmological constant (Mizoguchi-Tada '91, Han '10)  $q = e^{i\sqrt{\Lambda}hG}$
- Compactness: discretization of the intrinsic and extrinsic geometry
- Time discreteness:  $K_{ab} \sim dq_{ab}/dt$  where  $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt \Delta t$   
minimum proper time Planckian, full discrete spectrum depends on cosmological constant
- Euclidean 2+1  $\rightarrow$  to do: Lorentzian 3+1 !!!



$\Delta$  from discrete time

Francesca Vidotto

## PHYSICS AND CONCLUSIONS

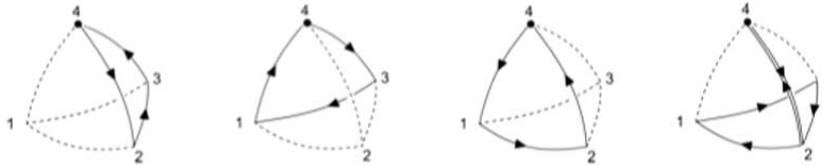
- We have introduced a modification of LQG kinematics
  - compact phase space
  - allows to introduce a positive cosmological constant
- finite dimensional Hilbert space dim determined by the ratio between the two constants:
  - quantization (physically: Planck constant scale)
  - simplex curvature / deformation of Poisson algebra (physically: cosmological constant)
- Hilbert space reduces to usual LQG one for triangles small compared to curvature radius
- A q-deformation of the dynamics:
  - renders quantum gravity finite (Turaev-Viro '92, Han '10)
  - amount to introduce the cosmological constant (Mizoguchi-Tada '91, Han '10)  $q = e^{i\sqrt{\Lambda}hG}$
- Compactness: discretization of the intrinsic and extrinsic geometry
- Time discreteness:  $K_{ab} \sim dq_{ab}/dt$  where  $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt \Delta t$   
minimum proper time Planckian, full discrete spectrum depends on cosmological constant
- Euclidean 2+1  $\rightarrow$  to do: Lorentzian 3+1 !!!

$\Delta$  from discrete time

Francesca Vidotto

## COMMENTS ON THE 4D LORENTZIAN CASE

- $(k = e^J, h) \in SU(2) \times SU(2)$



- Associate an  $SU(2)$  element for each face

(Haggard-Han-Kaminski-Riello '14)  
(Charles-Livine '15)

### ■ QUANTIZATION      QUANTUM GROUPS

- $$h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}_q \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}_q |j'm'n'\rangle$$

$A$  from discrete time

Francesca Vidotto