Title: What if the universe accelerates because time is discrete?

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Abstract: <p>In order to introduce the cosmological constant in a simplicial geometry, constant curvature should be introduced on simplex faces. This yields a compactification of the phase space and the finiteness of the Hilbert for each link. Not only the intrinsic, but also the extrinsic geometry turns out to be discrete, pointing to discreetness of time, in addition to space.</p>

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What if the universe accelerates because time is discrete?

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THE PROBLEM

- Fact: our universe has a positive cosmological constant
- Does this affect the kinematics of LQG? (Borissov-Major-Smolin '96, Dupuis-Girelli '13)
- Replacing flat cells with uniformly curved cells (Bahr-Dittrich '09)
- Classical kinematics: $\Gamma \equiv su(2) \times SU(2) \quad \forall \ell$
- Idea: replace the algebra with the group —> finiteness (Haggard-Han-Kaminski-Riello '14)
- Classically: compact phase space —> finite Liouville volume
- Quantum: finite # of Planck cells, finite # orthogonal states —> finite dim Hilbert space

U(1)xU(1)

- $h = e^{i\alpha}, k = e^{i\beta} \in \mathbb{C}$
- Symplectic 2-form: $\omega = -h^{-1}dh \wedge k^{-1}dk$
- Poisson brakets: $\{k, h\} = hk$

in the limit in which
the radius of one of the two circles can
be considered large we want to recover the
symplectic form of the cotangent space
$\omega = d\alpha \wedge d\beta$

- **QUANTIZATION**
  - Hilbert space: $|n\rangle \qquad n = 1, ..., N = \dim \mathcal{H}$
  - Operators: $k|n\rangle = e^{i\frac{2\pi}{N}n}|n\rangle$

    $h|n\rangle = |n+1\rangle \qquad$ cyclic: $h|N\rangle = |1\rangle$
  - Commutator: $[h, k] = \left(e^{i\frac{2\pi}{N}} - 1\right) hk$

    $[\hat{a}, \hat{b}] = i\hbar\widehat{\{a, b\}}$ $\qquad \hbar = \dfrac{2\pi}{N} \rightsquigarrow \dfrac{\text{[Planck length]}}{\text{[cosmological constant]}}$

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discrete spectrum

SU(2)xSU(2)

limit: arc ≪ R
where
$\theta = Tr[kh^{-1}dh]$

- $(k = e^J, h) \in SU(2) \times SU(2)$
- Symplectic 2-form: $\omega = Tr[dk \wedge h^{-1}dh - kh^{-1}dh \wedge h^{-1}dh]$
- … but we are not going to use this!

- **QUANTIZATION**
  - Hilbert space: $L_2[SU(2)] \sim \oplus_{j=0}^{\infty}(\mathcal{H}_j \otimes \mathcal{H}_j)$
  - Operators: $\langle U|jmn\rangle = D^j_{mn}(U)$
    - $h\psi(U) = U\psi(U)$ $\quad h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}\begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}|j'm'n'\rangle$
    - $J^i\psi(U) = L^i\psi(U)$

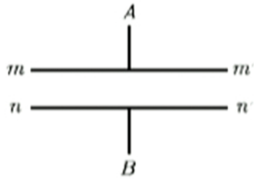
KNOTS

$$h_{AB}|jmn\rangle = \begin{pmatrix} \frac{1}{2} & j & j' \\ A & m & m' \end{pmatrix}_q \begin{pmatrix} \frac{1}{2} & j & j' \\ B & n & n' \end{pmatrix}_q |j'm'n'\rangle$$ does not commute any more

$\nexists$ ***h reps***

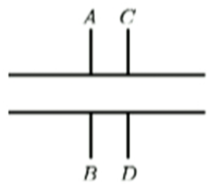
- Wigner symbols as trivalent nodes

$$(h_{AB})^{mn}_{m'n'} =$$



- Acting with two operators:

$$h_{AB}h_{CD} =$$



- Inverse order:

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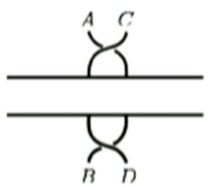

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    - $J^i\psi(U) = L^i\psi(U)$ $\quad J^i\,|jmn\rangle = \left(\tau^{i\,(j)}_{mk}\right)_q |jkn\rangle$

$q^r = -1 \qquad j_{max} = \frac{r-2}{2}$

Λ from discrete time

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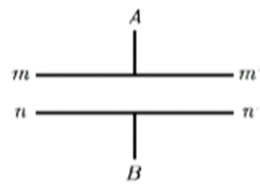

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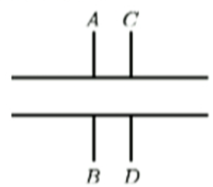

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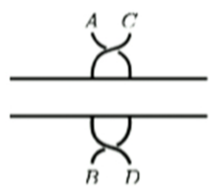

crossing operators

$$h_{AB}h_{CD} = R^{A'C'}_{AC} R^{B'D'}_{BD}\ h_{C'D'}h_{A'B'}$$

Expanding in $\hbar$ so that $R \sim 1 + r$:

$$\{h_{AB}, h_{CD}\} = r^{B'D'}_{BD} h_{CD'}h_{AB'} + r^{A'C'}_{AC} h_{C'D}h_{A'B}$$

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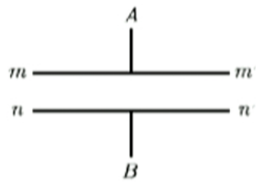
$(h_{AB})^{mn}_{m'n'} =$ 

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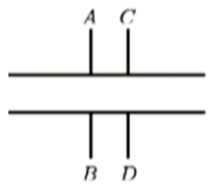
$h_{AB}h_{CD} =$ 

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quasi Poisson-Lie groups

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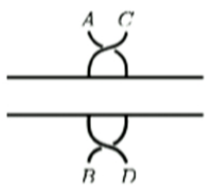
$h_{CD}h_{AB} =$ 

PHYSICS AND CONCLUSIONS

- We have introduced a modification of LQG kinematics
  - compact phase space
  - allows to introduce a positive cosmological constant

- finite dimensional Hilbert space dim determined by the ratio between the two constants:
  - quantization (physically: Planck constant scale)
  - simplex curvature / deformation of Poisson algebra (physically: cosmological constant)

- Hilbert space reduces to usual LQG one for triangles small compared to curvature radius

- A q-deformation of the dynamics:
  - renders quantum gravity finite (Turaev-Viro '92, Han '10)
  - amount to introduce the cosmological constant (Mizoguchi-Tada '91, Han '10) $q = e^{i\sqrt{\Lambda}\hbar G}$

- Compactness: discretization of the intrinsic and extrinsic geometry

- Time discreteness: $K_{ab} \sim dq_{ab}/dt$ where $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt\ \Delta t$
  minimum proper time Planckian, full discrete spectrum depends on cosmological constant

- Euclidean 2+1 $\rightarrow$ to do: Lorentzian 3+1 !!!

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COMMENTS ON THE 4D LORENTZIAN CASE

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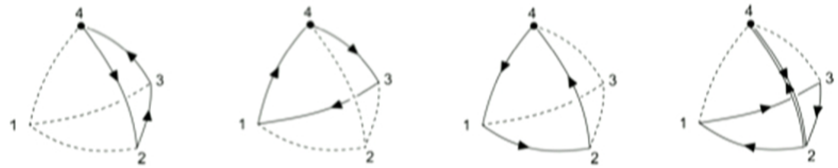

- Associate an SU(2) element for each face

(Haggard-Han-Kaminski-Riello '14)
(Charles-Livine '15)

- QUANTIZATION QUANTUM GROUPS

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