Title: Quantum fluctuations as the seeds of cosmic structure.

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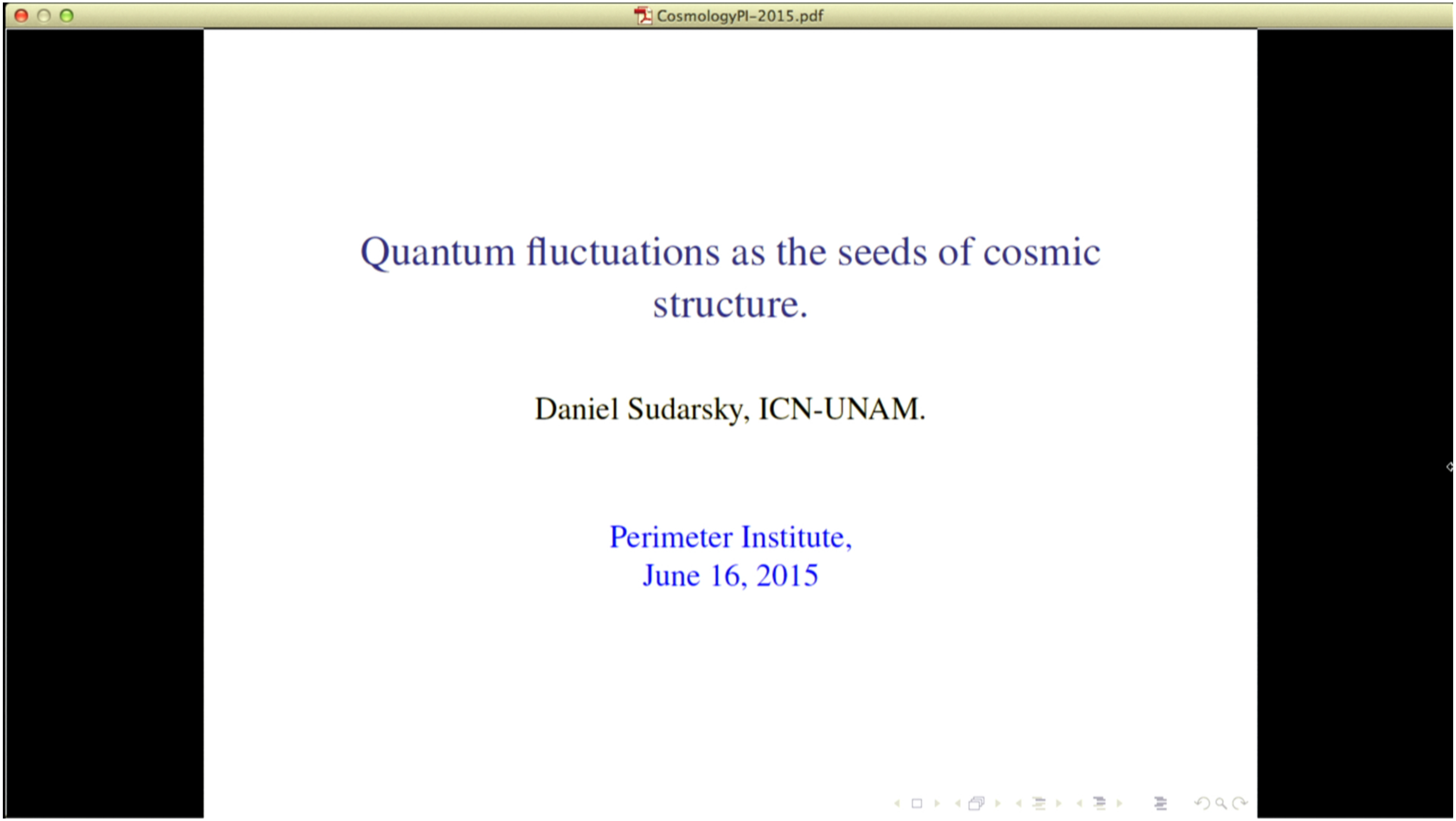
Abstract: <p>Although the inflationary predictions for the primordial power spectrum of density inhomogeneities seem very successful, there is an obscure part in our understanding of the emergence of the seeds of cosmic structure: How does a universe which at one pint in time is described by a state that is fully homogeneous and isotropic, evolve into a state that is not, given that the dynamics does not contain any source for the undoing of such symmetry? We will discuss an approach that offers a resolution of the issue in the context of proposals to deal with the measurement problem in quantum mechanics. We will see that the approach naturally resolves the difficulties raised by the lack of observation of tensor modes.</p>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Quantum fluctuations as the seeds of cosmic structure.
Daniel Sudarsky, ICN-UNAM.
Perimeter Institute,
June 16, 2015

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Plan:

1)The inflationary account for the emergence of the seeds of cosmic structure.... and the problem.

2) The usual answers and their shortcomings. (It is only if we are convinced that standard approaches are not satisfactory, that revolutionary attempts would be truly justified).

3) Our approach. Involve Spontaneous Collapse.

4) Brief description of a collapse theory known as Continuous Spontaneous Localization (CSL).

5) Adapting CSL to QFT and inflation. Comparing with observations.

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1) Cosmic Inflation:

Contemporary cosmology includes inflation as one of its most attractive components. Its biggest success is claimed to be the natural account for emergence of the seeds of cosmic structure and a correct estimate of the corresponding spectrum.

The staring point of the analysis is a a RW space-time background

$$dS^2 = a(\eta)^2\{-d\eta^2 + d\vec{x}^2\}$$

inflating under the influence of an inflaton background field $\phi = \phi_0(\eta)$ in a slow roll condition so that the scale factor behaves approximately as $a(\eta) = \frac{-1}{\eta H_I}$.

On top of this, one considers quantum fluctuations: $\delta\phi, \delta\psi, ..., \delta h_{ij}$ assumed to be characterized by the " vacuum state" (essentially the BD vacuum) $|0\rangle$.

From these 'fluctuations", one argues, the primordial inhomogeneities and anisotropies emerge.

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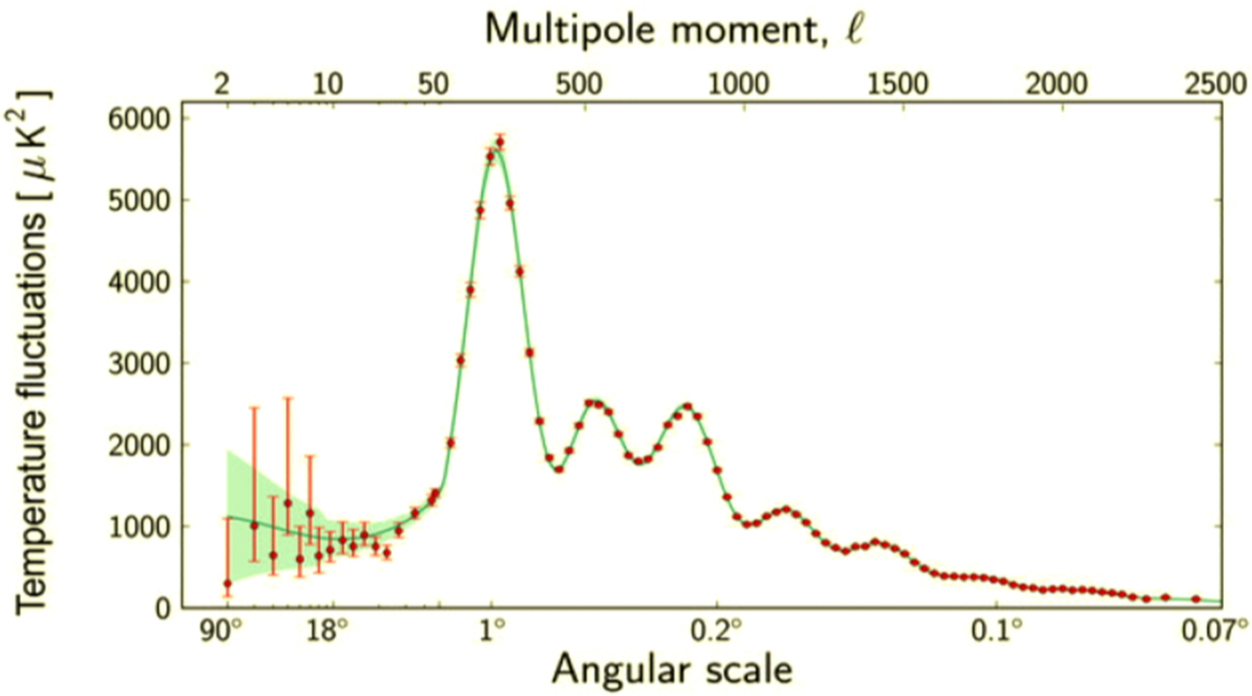
The analysis leads to a remarkable agreement with observations:
Note: The oscillations are related to late time plasma physics which is well understood and will be ignored in the rest of the talk.
(Image: ESA/ Planck Collaboration)



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These are supposed to represent the primordial inhomogeneities which evolved into all the structure in our Universe: galaxies, stars planets, etc... AND THE THEORY FITS VERY WELL WITH THE

OBSERVATIONS. One is then very tempted to say "well that is it. What else do we want?".

However let us consider the following: The Universe was H&I, (both in the part that could be described at the "classical level", and the quantum level) as a result inflation (except from remnants from the pre inflationary regime suppressed by e^{-N}, to be ignored from now on). However we end with a situation which is not: Contains the primordial inhomogeneities which will result in our Universe structure and the conditions that permit our own existence.

How does this happen if the dynamics of the closed system does not break those symmetries.?

A similar issue was considered by N. F. Mott in 1929 concerning the α nuclear decay (The issue is what did he actually solve !). It is related to the "measurement problem" but in an aggravated form.

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CONCEPTUAL DIFFICULTIES IN QUANTUM THEORY

QM is extremely successful theory, however it still presents some serious shortcomings.

Let us illustrate the situation with a few quotes:

"Either the wave function as given by Schrödinger equation is not everything, or it is not right" Bell, J. S. , Are there quantum jumps?, in Speakable and unspeakable in quantum mechanics. Cambridge: Cambridge University Press, 201- 212 (1987).

Also, in a very recent paper entitled Collapse of the State Vector by S. Weinberg (arXiv:1109.6462v3 [quant-ph]).

"There is now in my opinion no entirely satisfactory interpretation of quantum mechanics".

I , personally, have been strongly influenced by R. Penrose writings in this regard.

A Simplified model: Mini-Mott   A test ground!

Consider a 2 level detector $|-\rangle$ (ground) y $|+\rangle$ (excited), and take two of them located at $x = x_1$ y $x = -x_1$. They are both initially in the ground state. Take a free particle with initial wave function $\psi(x, 0)$ given by a simple gaussian centered at $x = 0$ (so the whole set up is symmetric w.r.t $x \to -x$).

The particles's Hamiltonian: $\hat{H}_P = \hat{p}^2/2M$ while that of each detector is

$$\hat{H}_i = \epsilon \hat{I}_p \otimes \{|+\rangle^{(i)}\langle+|^{(i)} - |-\rangle^{(i)}\langle-|^{(i)}\}. \tag{1}$$

where $i = 1, 2$. The interaction of particle and detector 1 is

$$\hat{H}_{P1} = \frac{g}{\sqrt{2}}\delta(x - x_1\hat{I}_p) \otimes (|+\rangle^{(1)}\langle-|^{(1)} + |-\rangle^{(1)}\langle+|^{(1)}) \otimes I_2 \tag{2}$$

and similar expression for the particle's interaction with detector 2.

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Schrödinger's equation can be solved for the initial condition

$$\Psi(0) = \sum_x \psi(x,0)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)}$$

and it is clear that after some time t we have

$$\Psi(t) = \sum_x \psi_1(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_2(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |+\rangle^{(2)}$$

$$+ \sum_x \psi_0(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_D(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |+\rangle^{(2)}$$

One can interpret the last two terms easily: no detection and double detection (involving bounce) which is small $O(g^2)$.
Also we could think the first two terms indicate the initial symmetry was broken with high probability: Either detector 1 was excited or detector 2 was.
We just use some kind of Copenhagen interpretation and everything is fine, ...really?

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The problem can be seen by considering instead the:
alternative state basis for the detectors (or "context")

$$|U\rangle \equiv |+\rangle^{(1)} \otimes |+\rangle^{(2)} \quad (3)$$

$$|D\rangle \equiv |-\rangle^{(1)} \otimes |-\rangle^{(2)} \quad (4)$$

$$|S\rangle \equiv \frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} + |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \quad (5)$$

$$|A\rangle \equiv |\frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} - |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \quad (6)$$

In fact these are more convenient for describing issues related to symmetries of the problem.

It is then easy to see that the $x \to -x$ and $1 \to 2$ symmetry of the initial setting and of the dynamics prevents the excitation an asymmetric term.

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The issue is thus: can we or can we not describe things in this basis? And, if not, why not?

An experimental physicist in the Lab has no problem, he/she has many things that in practice (FAPP) indicate he should use the other basis (he knows that his detectors are always either exited or un-exited.. he never perceives them in superposition). The measurement problem is: exactly how does our theory account for that *experience* of our experimental colleague? Often we just don't care.

However, if we now have a situation where there is no experimentalist.... and nothing else in the universe (except perhaps for, say, a Maxwell field which is also in its vacuum state), we simply do not know what to do.

In that situation, why would we believe the conclusions drawn in the first context but not those of the second?. i.e. How do we account for the breakdown of the symmetry?

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2) THE USUAL ANSWERS and their shortcomings:

A) As in all QM situations, take into account that "we make measurement".

Even ignoring all the issues that come with the measurement problem in Quantum Theory, taking this view, amounts to saying that the conditions that made possible our own existence would be said to be the result of our own actions.

B)Environment-induced decoherence, possibly supplemented by a Many Worlds Interpretations (MWI).

i) Requires identification of D.O.F as an " environment" (and traced over). Entails using our limitations to "measure things", as part of the argument. We need a third person description; Dinosaurs!

ii) Does not tell us that the situation is now described by one element of the diagonal density matrix, but by all, and as such the situation is still symmetric. Need something like MWI.

iii) However, MWI (which seems to have other drawbacks) requires some criteria to determine the alternatives into which the word splits. What plays that role in the situation at hand?.

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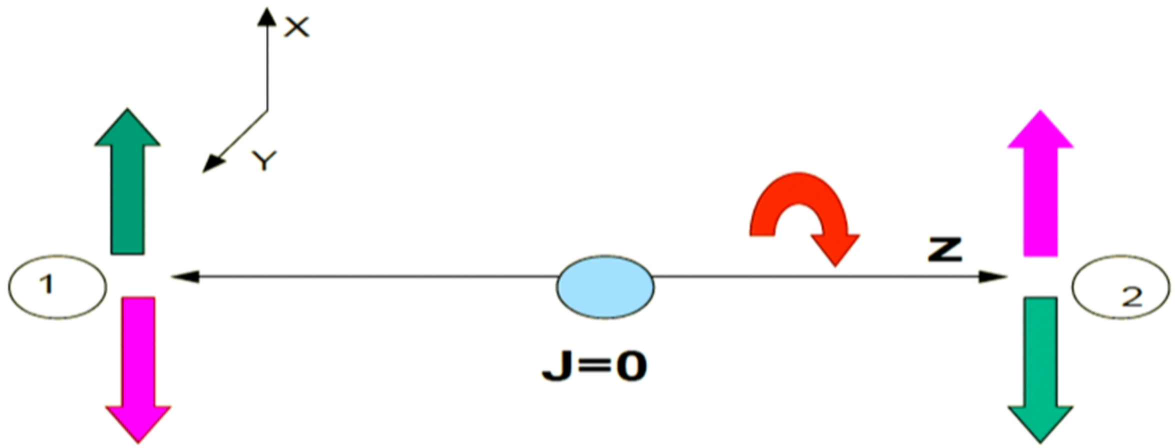
Even W. Zurek: "The interpretation based on the ideas of decoherence and ein-selection has not really been spelled out to date in any detail. I have made a few half-hearted attempts in this direction, but, frankly, I was hoping to postpone this task, since the ultimate questions tend to involve such anthropic attributes of the observership as perception, awareness, or consciousness, which, at present, cannot be modeled with a desirable degree of rigor." (quant-ph/9805065)

More recently he seems to have changed his mind.

In fact in the cases of symmetric situations, one faces an extra specific issue with the basis selection:



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The singlet state of the system is:

$$|\Psi\rangle = (1/\sqrt{2})\{|+;x\rangle^{(1)} \otimes |-;x\rangle^{(2)} - |-;x\rangle^{(1)} \otimes |+;x\rangle^{(2)}\} \qquad (7)$$

Considering the reduced density matrix for particle 1 (tracing over 2, considered as environment) one finds:

$$\rho^{(1)} = (1/2)\{|+;x\rangle\langle +;x| + |-;x\rangle\langle -;x|\} = (1/2)I \qquad (8)$$

So environmental decoherence does not offer a criteria for choosing the basis. There is a theorem that shows this problem is generic.

C) Consistent (or de-cohering) Histories. We believe the CH approach has some serious problems in general, but in any way, in the particular case at hand the answer we obtain depends on the questions we ask. In particular we can use the approach to conclude that, with probability 1 , our universe today is Homogeneoous and Isotropic.

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3) OUR APPROACH: The situation we face here is unique (Quantum + Gravity (GR) + Observations).
We want to be able to point to a physical process that occurs in time as the explaining the emergence of the seeds of structure. After all emergence means : Something that was not there at a time, is there at a latter time. We need to explain the breakdown of the symmetry of the initial state: Collapse can do this.

Collapse Theories: Important previous works in this direction: GRW, Pearle, Diosi, Penrose, Bassi (recent advances to make it compatible with relativity Tumulka, Bedningham) and now Weinberg.

We propose to add to the standard inflationary paradigm, a quantum collapse of the wave function as a self induced processes. Here we will illustrate the ideas with one rather well developed proposal formulated in the context of ordinary QM, known as CSL (P. Pearle) in an "adaptation" to the situation at hand.

NOTE HOWEVER THAT IN PRINCIPLE WE WOULD NEED A RELATIVISTIC VERSION.

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How would this fit with our current views regarding quantum gravity? This is a big question but let's recall some issues and conceptual difficulties still outstanding:

I) The Problem of Time. In Can Q.G. leads to timeless theory.

II) More generally how do we recover space-time from canonical approaches to QG ? (i.e. LQG).

Solutions to I) usually start by using some dynamical variable as a physical clock and then considering relative probabilities (and wave functions). It seems that in those considerations one recovers only an approx Schrödinger eq. with corrections that violated unitarity (Pullin & Gambini). Could something like this lie at the bottom of collapse theories?

Regarding II) we note that there are many suggestions indicating space-time might be an emergent phenomena... T. Jacobson, R Sorkin, N. Seiberg and many others....

In that case, at the level in which one can talk about space-time concepts is the classical description. However some quantum aspects might remain that would look like collapse. Hydrodynamic analogy.

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4) The Proposal:

The idea is that at the quantum level gravity is VERY different, and at large scales leaves something that looks like a collapse of the quantum wave function matter fields.

Thus the inflationary regime is one where gravity already has a good classical description but matter fields might still require a full quantum treatment.

The setting will thus naturally be semiclassical Einstein's gravity to which we will incorporate an extra element: THE COLLAPSE) (This can be done precisely and rigorously based on an approach we call *Semiclassical Self-consistent Configurations (SSC)* developed in (*JCAP*. **045**, 1207, (2012)) :

Thus the view is that there is an underlying Quantum Theory of Gravity, (probably with no notion of time). By the "*time*" we recover space-time concepts, the semiclassical treatment is a very good one, its regime of validity includes the inflationary regime as long as $R << 1/l^2_{Plank}$.

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5) PRACTICAL TREATMENT:

We have checked that this is equivalent at the lowest order in perturbation theory with the one based on SSC.

We again split the treatment into that of a classical homogeneous ('background') part and an in-homogeneous part ('fluctuation'), i.e.

$$g = g_0 + \delta g, \ \phi = \phi_0 + \delta\phi.$$

The background is taken again to be Friedmann-Robertson universe, and the homogeneous scalar field $\phi_0(\eta)$ (in the SSC treatment this corresponds to the zero mode of the quantum filed).

Recall that our approach requires that we quantize the scalar filed but not the metric perturbation.

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Continuous Spontaneous Localization. The theory is defined by two equations:
i) A modified Schrödinger equation, whose solution is:

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2 \right]} |\psi, 0\rangle. \qquad (9)$$

($\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \qquad (10)$$

The processes U and R (corresponding to the observable $\hat{A}$) are unified. For non-relativistc QM the proposal assumes : $\hat{A} = \hat{\vec{X}}$.
Here λ must be small enough not to conflict with tests of QM in the domain of subatomic physics and big enough to result in rapid localization of "macroscopic objects". GRW suggested range: $\lambda \sim 10^{-16} sec^{-1}$. (Likely depends on particle mass).

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The point is that collapse theories (such as CSL) can account for the breakdown of symmetries in the Mini Mott example and in cosmological setting. So lets consider the latter.
Fixing a specific gauge and ignoring tensor perturbations:

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\right], \Psi(\eta, \vec{x}) \ll 1$$

Set $a = 1$ at the "present cosmological time", and assume that the inflationary regime ends at a value of $\eta = \eta_0$, (negative and very small in absolute terms). So,

$$a(\eta) = \frac{-1}{\eta H_I} \text{ with } \eta \in (-\mathcal{T}, \eta_0), \eta_0 < 0.$$

The scalar field must be treated using QFT in curved space-time (using SSC). The quantum state of the scalar field and the space-time metric satisfy Einstein's semiclassical eq.

$$G_{\mu\nu} = 8\pi G \langle \xi | \hat{T}_{\mu\nu} | \xi \rangle.$$

The delicate issue of the self-consistency is dealt with using SSC.

We will be concentrating on the modes other than the zero mode which we treat classically as an effective approximation.

$$G_{\mu\nu} = \langle 0|T_{\mu\nu}|0\rangle + \xi_{\mu\nu}$$

$$\langle \xi|T_{\mu\nu}|\xi\rangle - \langle 0|T_{\mu\nu}|0\rangle$$

$\boxed{\text{DO NOT ERASE}}$

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At the early stages of inflation which we denote by $\eta = -\mathcal{T}$, the state of the scalar field perturbation is described by the Bunch-Davies vacuum, and the space-time is 100 % homogeneous and isotropic.

In fact in the vacuum state the operators $\hat{\delta}\phi_k$ $\hat{\pi}_k$ are characterized by gaussian wave functions centered on 0 with uncertainties $\Delta\delta\phi_k$ and $\Delta\pi_k$.

The collapse modifies the quantum state, and generically the expectation values of $\widehat{\delta\phi_k}(\eta)$ and $\hat{\pi}_k(\eta)$.

We must now specify the rules governing the collapse. This is the result of some unknown aspect of physics, which we will here encode into a adapted vernon CSL theory.

The approach is based on making an "educated guess", which can later be contrasted with observations. The collapse will be controlled mode by mode by a stochastic function.

Note: Our universe would correspond to one specific realization of these stochastic functions (one for each $\vec{k}$).

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The semi classical Einstein Equation we must focus on is:

$$-k^2\Psi(\eta,\vec{k}) = 4\pi G\phi_0'(\eta)\langle\hat{\delta\phi}'(\vec{k},\eta)\rangle = \frac{4\pi G\phi_0'(\eta)}{a}\langle\hat{\pi}(\vec{k},\eta)\rangle \quad (11)$$

($\langle\hat{\pi}(\vec{k},\eta)\rangle \equiv \langle\psi,\eta|\hat{\pi}(\vec{k})|\psi,\eta\rangle$). As we said at the start of inflation($\eta = -\mathcal{T}$) state is described by the Bunch-Davies vacuum, so $\langle\psi,-\mathcal{T}|\hat{\pi}(\vec{k})|\psi,-\mathcal{T}\rangle = 0$, and THUS as long as the state of the field is that vacuum, the space-time WILL BE 100% homogeneous and isotropic. The quantity of interest is:

$$\frac{\Delta T(\theta,\varphi)}{\bar{T}} = c\int d^3k e^{ik\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta_D)\rangle, \text{ where } c \equiv -\frac{4\pi G\phi_0'(\eta)}{3a}. \quad (12)$$

Here, $\vec{x}$ is a point on the intersection of our past light cone with the last scattering surface ($\eta = \eta_D$) and corresponds to the direction on the sky specified by θ,φ.Thus:

$$\alpha_{lm} = c\int d^2\Omega Y^*_{lm}(\theta,\varphi)\int d^3k e^{i\vec{k}\cdot\vec{x}}\frac{1}{k^2}\langle\hat{\pi}(\vec{k},\eta)\rangle. \quad (13)$$

There is no analogous to this expression in the standard approaches!

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The eq. above shows that the quantity of interest can be thought of as a result of a "random walk" on the complex plane. One can't predict the end point of such "walk" but can focus on the magnitude of the total displacement:

$$|\alpha_{lm}|^2 = (4\pi c)^2 \int d^3k d^3k' j_l(kR_D) j_l(k'R_D) Y_{lm}(\hat{k}) Y^*_{lm}(\hat{k}') \tag{14}$$

$$\frac{1}{k^2 k'^2} \langle \hat{\pi}(\vec{k}, \eta) \rangle \langle \hat{\pi}(\vec{k}', \eta) \rangle^*. \tag{15}$$

(Note: we need the product of expectation values and not the expectation value of the product !!) and estimate such value by an ensemble average. Thus we compute the ensemble average at "late times"

$$\overline{(\langle \hat{\pi}(\mathbf{k}, \eta) \rangle \langle \hat{\pi}(\mathbf{k}', \eta) \rangle^*)} = f(k) \delta(\mathbf{k} - \mathbf{k}').$$

Then,

$$\overline{|\alpha_{lm}|^2} = (4\pi c)^2 \int_0^\infty dk j_l(kR_D)^2 \frac{1}{k^2} f(k). \tag{16}$$

Now we need to use the theory controlling the Collapse.

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As we said we will here consider CSL. We still need to chose the operator $\hat{A}$ driving the collapse and the parameter λ.

We work with a rescaled field $y(\eta,\vec{x}) \equiv a\delta\phi(\eta,\vec{x})$ and its momentum conjugate $\pi_y(\eta,\vec{x}) = a\delta\phi'(\eta,\vec{x})$.

For simplicity, put everything in a Box of size L (to be removed at the end), and focus on a single mode $\vec{k}$, so we write:

$$Y \equiv (2\pi/L)^{3/2} y(\eta,\vec{k}), \qquad \Pi \equiv (2\pi/L)^{3/2} \pi_y(\eta,\vec{k}). \qquad (17)$$

As we saw, in order to compare with the observations, we need to evaluate the ensemble average $\overline{\langle\hat{\Pi}\rangle^2}$, and determine under what circumstances, if any, this is $\sim k$.

Note: We must consider $\overline{\langle\hat{\Pi}\rangle^2}$ and NOT $\overline{\langle\hat{\Pi}^2\rangle}$!!

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$\hat{\Pi}$ as Generator of Collapse. Setting $\hat{A} = \hat{\Pi}$ we obtain:

$$\overline{\langle\hat{\Pi}\rangle^2} = \frac{\lambda k^2 \mathcal{T}}{2} + \frac{k}{2} - \frac{k}{\sqrt{2}\sqrt{1+\sqrt{1+4\lambda^2}}}. \tag{18}$$

Note that if we set $\lambda = 0$ (turn off CSL), we would have the standard quantum mechanics result $\overline{\langle\hat{\Pi}\rangle^2} = 0$ since $\langle\hat{\Pi}\rangle = 0$.

We see that agreement with the observed scale-invariant spectrum can be achieved if we assume the first term is dominant and we set

$$\lambda = \tilde{\lambda}/k \text{ where } \tilde{\lambda} \text{ is a constant (indep. of } k). \tag{19}$$

We note that this replaces the dimensionless collapse rate parameter λ with parameter $\tilde{\lambda}$ of dimension time^{-1}.

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In that case we obtain:

$$\overline{\langle\hat{\Pi}\rangle^2} = \frac{\tilde{\lambda}k\mathcal{T}}{2} + \frac{k}{2} - \frac{k}{\sqrt{2}\sqrt{1+\sqrt{1+4(\tilde{\lambda}/k)^2}}}. \quad (20)$$

Analogously, we considered: $\hat{Y}$ as Generator of Collapse and we obtained slightly different results.

Finally, comparisons with observations, using GUT scale inflation potential value, and standard values for the slow-roll parameter (order a few percent), leads to the estimate:

$$\tilde{\lambda} \sim 10^{-5} MpC^{-1} \approx 10^{-19} sec^{-1}.$$

Not very different from GRW suggestion !.

Collapse on Field Operators

We would like to understand how the collapse looks when described in terms of the space-time field operators. In one case we can start by defining

$$\tilde{y}(\vec{x}) \equiv \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i\vec{k}\cdot\vec{x}} k^{1/2} y(\vec{k}) = (-\nabla^2)^{1/4} \hat{y}(\vec{x}), \qquad (21)$$

The state vector evolution given by

$$|\psi, t\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\tilde{\lambda}} \int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x [w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{y}(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \qquad (22)$$

This is just the standard CSL state-vector evolution, where the collapse-generating operators (toward whose joint eigenstates collapse tends) are $\tilde{y}(\vec{x})$ for all $\vec{x}$.

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Similarly, in the case where we take $\hat{\Pi}$ as Generator of Collapse we have.

$$|\psi,\eta\rangle = \mathcal{T} e^{-i\int_{-\mathcal{T}}^{\eta} d\eta' \hat{H} - \frac{1}{4\lambda}\int_{-\mathcal{T}}^{\eta} d\eta' \int d^3x [w(\vec{x},\eta') - 2\tilde{\lambda}\tilde{\pi}(\vec{x})]^2} |\psi, -\mathcal{T}\rangle. \quad (23)$$

where $\tilde{\pi}(\vec{x}) \equiv (-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$.

This is just the standard CSL state-vector evolution, where the collapse-generating operators (toward whose joint eigenstates collapse drives all states) are $\tilde{\pi}(\vec{x})$ for all $\vec{x}$.

What are the fundamental reasons determining the appearance of the operators $(-\nabla^2)^{-1/4}\hat{\pi}(\vec{x})$ (or $(-\nabla^2)^{1/4}\hat{y}(\vec{x})$)?

A satisfactory answer will have to wait for a general theory expressing, in all situations, from particle physics, to cosmology, the exact form of the CSL-type of modification to the evolution of quantum states. Such generic theory would likely involve gravitation playing a fundamental role. The research must continue.