Title: Economic inequality from statistical physics point of view - Victor Yakovenko

Date: Jun 10, 2015 11:00 AM

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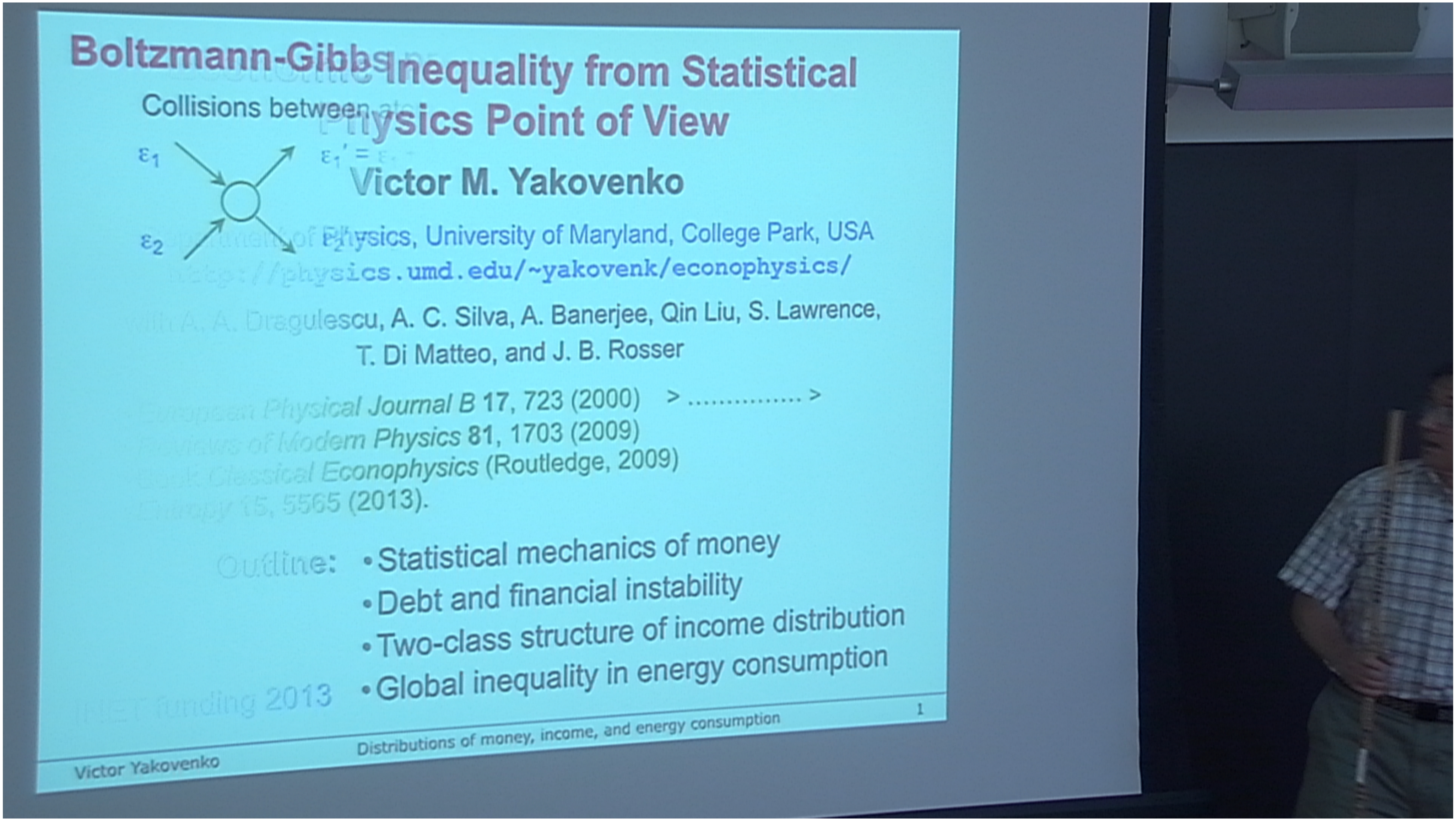
Abstract: <p>Similarly to the probability distribution of energy in physics, the probability distribution of money among the agents in a closed economic system is also expected to follow the exponential Boltzmann-Gibbs law, as a consequence of entropy maximization. Analysis of empirical data shows that income distributions in the USA, European Union, and other countries exhibit a well-defined two-class structure. The majority of the population (about 97%) belongs to the lower class characterized by the exponential ("thermal") distribution. The upper class (about 3% of the population) is characterized by the Pareto power-law ("superthermal") distribution, and its share of the total income expands and contracts dramatically during booms and busts in financial markets. Globally, data analysis of energy consumption per capita around the world shows decreasing inequality in the last 30 years and convergence toward the exponential probability distribution, in agreement with the maximal entropy principle. Similar results are found for the global probability distribution of CO2 emissions per capita. All papers are available at http://physics.umd.edu/~yakovenk/econophysics/. For recent coverage in Science magazine, see http://www.sciencemag.org/content/344/6186/828</p>


Boltzmann-Gibbs
Inequality from Statistical Physics Point of View
Collisions between
Victor M. Yakovenko
Physics, University of Maryland, College Park, USA
http://physics.umd.edu/~yakovenk/econophysics/
with A. A. Dragulescu, A. C. Silva, A. Banerjee, Qin Liu, S. Lawrence, T. Di Matteo, and J. B. Rosser
European Physical Journal B 17, 723 (2000)
Reviews of Modern Physics 81, 1703 (2009)
Book Classical Econophysics (Routledge, 2009)
Energy 15, 5565 (2013).
Outline:
• Statistical mechanics of money
• Debt and financial instability
• Two-class structure of income distribution
• Global inequality in energy consumption
Distributions of money, income, and energy consumption
Victor Yakovenko
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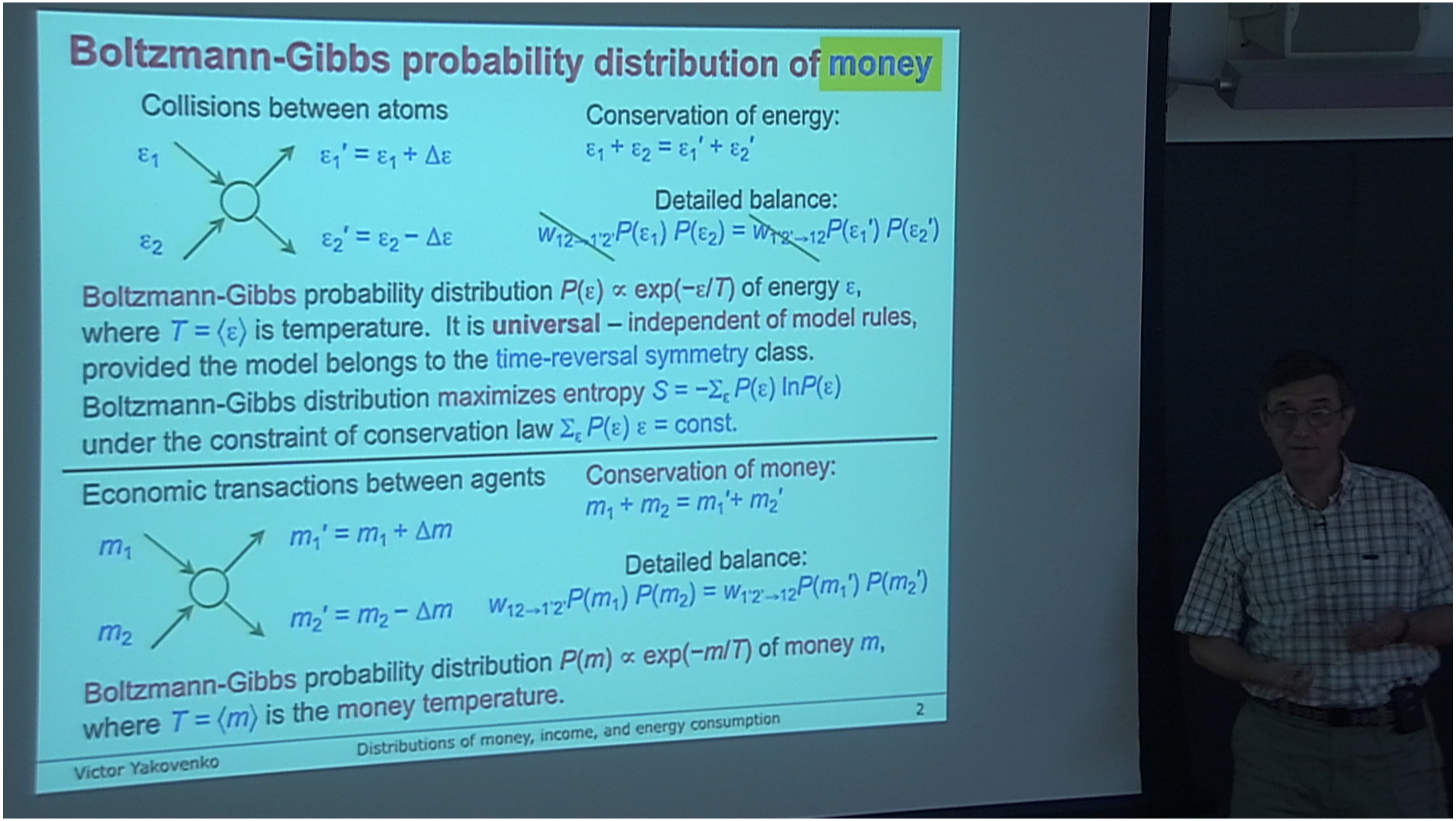
Boltzmann-Gibbs probability distribution of money
Collisions between atoms
ε_1
ε_2
$\varepsilon_1' = \varepsilon_1 + \Delta\varepsilon$
$\varepsilon_2' = \varepsilon_2 - \Delta\varepsilon$
Conservation of energy:
$\varepsilon_1 + \varepsilon_2 = \varepsilon_1' + \varepsilon_2'$
Detailed balance:
$w_{12\to1'2'} P(\varepsilon_1)\, P(\varepsilon_2) = w_{1'2'\to12} P(\varepsilon_1')\, P(\varepsilon_2')$
Boltzmann-Gibbs probability distribution $P(\varepsilon) \propto \exp(-\varepsilon/T)$ of energy ε,
where $T = \langle\varepsilon\rangle$ is temperature. It is universal – independent of model rules,
provided the model belongs to the time-reversal symmetry class.
Boltzmann-Gibbs distribution maximizes entropy $S = -\sum_\varepsilon P(\varepsilon) \ln P(\varepsilon)$
under the constraint of conservation law $\sum_\varepsilon P(\varepsilon)\, \varepsilon = \text{const}$.
Economic transactions between agents
m_1
m_2
$m_1' = m_1 + \Delta m$
$m_2' = m_2 - \Delta m$
Conservation of money:
$m_1 + m_2 = m_1' + m_2'$
Detailed balance:
$w_{12\to1'2'} P(m_1)\, P(m_2) = w_{1'2'\to12} P(m_1')\, P(m_2')$
Boltzmann-Gibbs probability distribution $P(m) \propto \exp(-m/T)$ of money m,
where $T = \langle m\rangle$ is the money temperature.
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Distributions of money, income, and energy consumption
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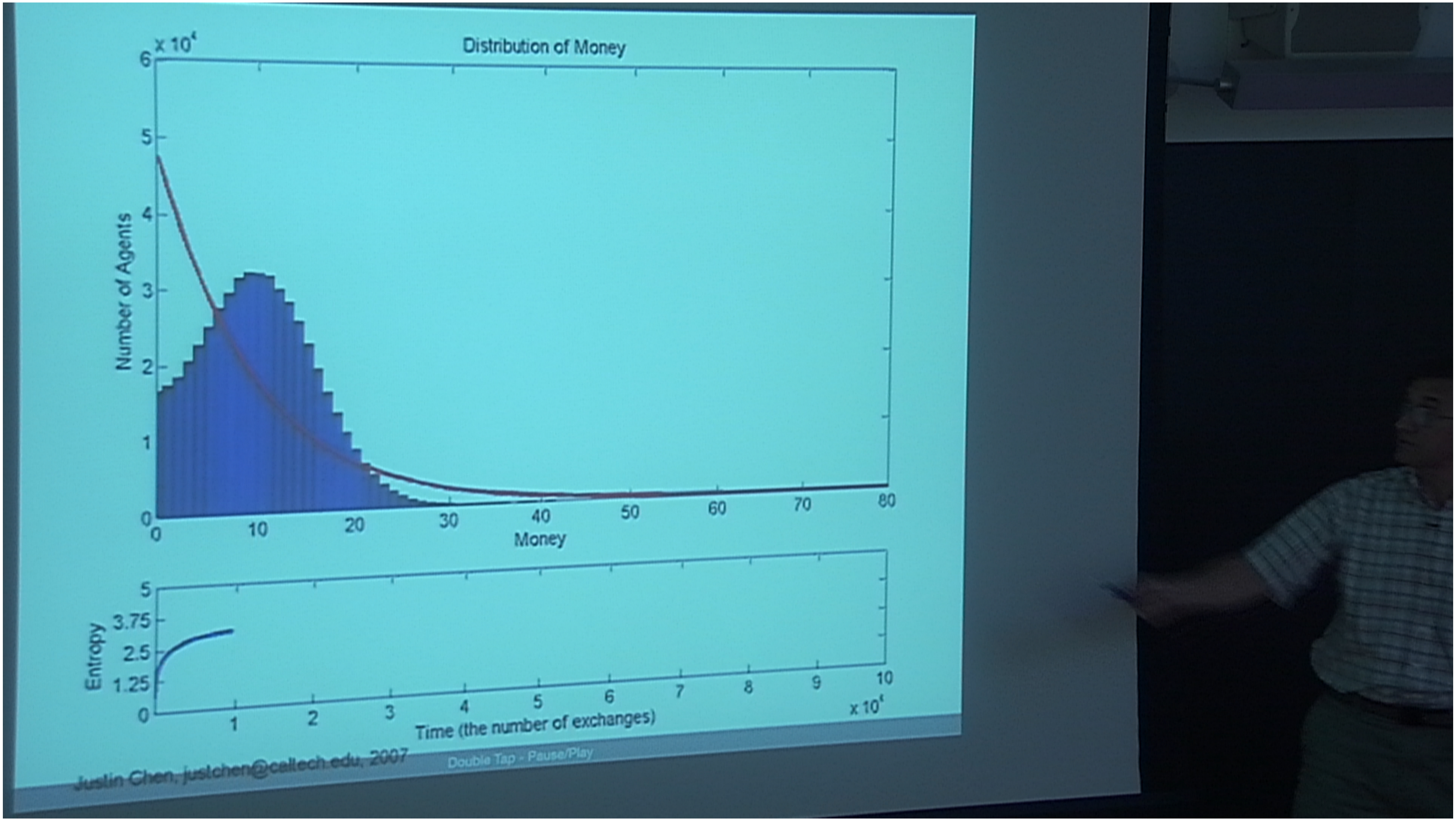
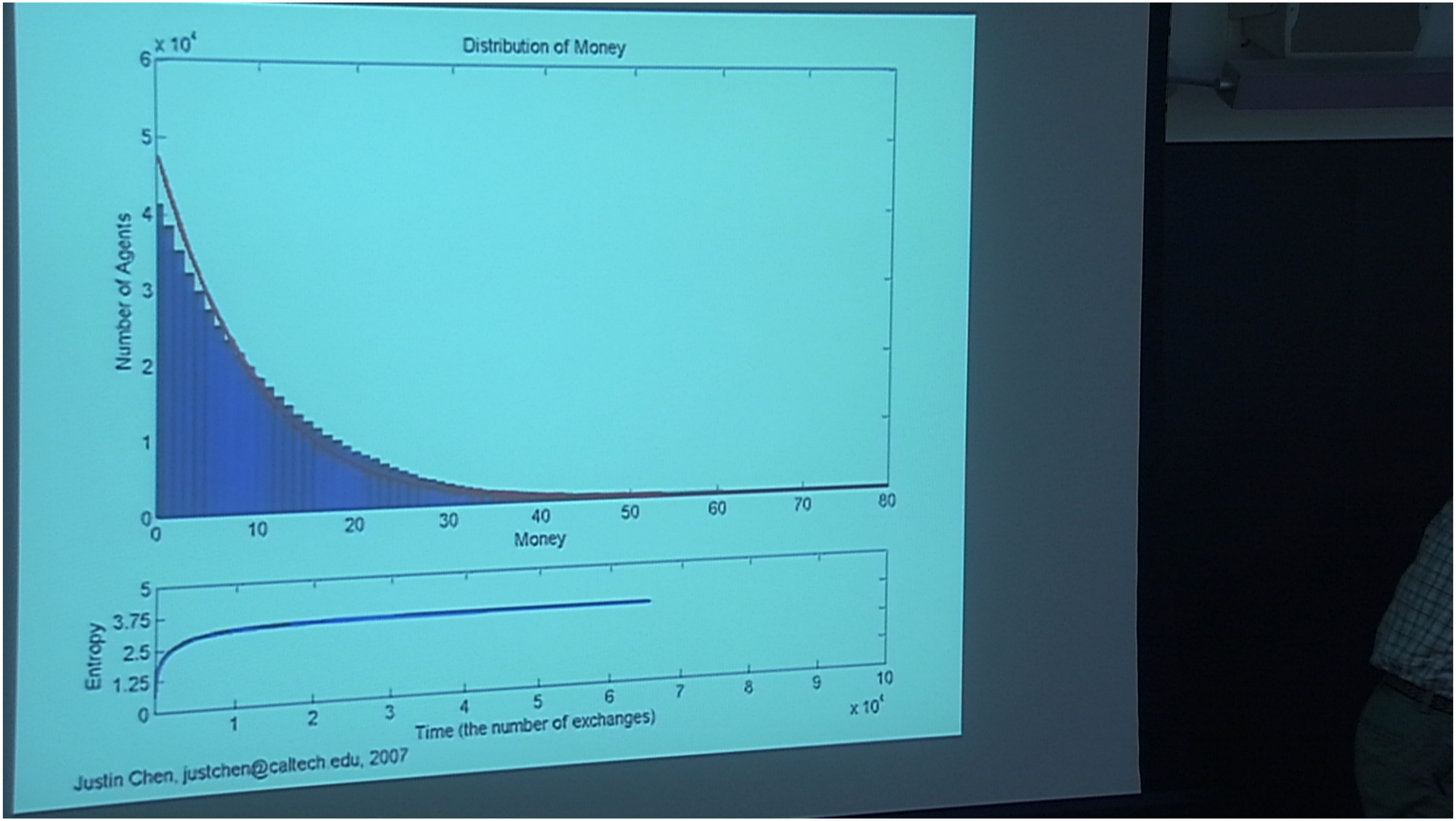


Distribution of Money
x 10^4
Number of Agents
Money
Entropy
Time (the number of exchanges)
x 10^4
Justin Chen, justchen@caltech.edu, 2007
Double Tap - Pause/Play


Distribution of Money
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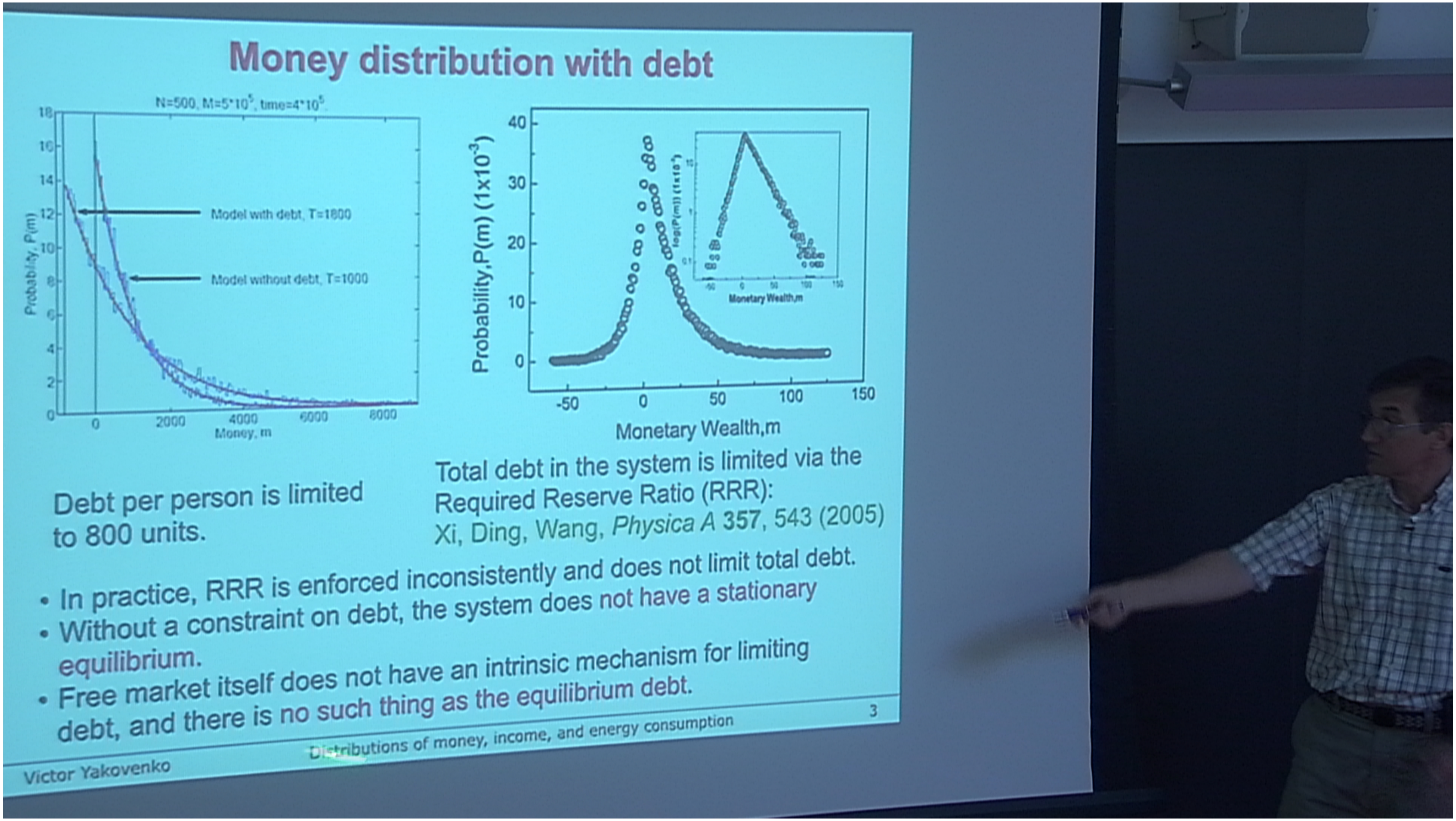
Money distribution with debt
N=500, M=5*10^5, time=4*10^5
Probability, P(m)
Model with debt, T=1800
Model without debt, T=1000
Money, m
Probability,P(m) (1x10^-3)
Monetary Wealth,m
Debt per person is limited to 800 units.
Total debt in the system is limited via the Required Reserve Ratio (RRR):
Xi, Ding, Wang, Physica A 357, 543 (2005)
• In practice, RRR is enforced inconsistently and does not limit total debt.
• Without a constraint on debt, the system does not have a stationary equilibrium.
• Free market itself does not have an intrinsic mechanism for limiting debt, and there is no such thing as the equilibrium debt.
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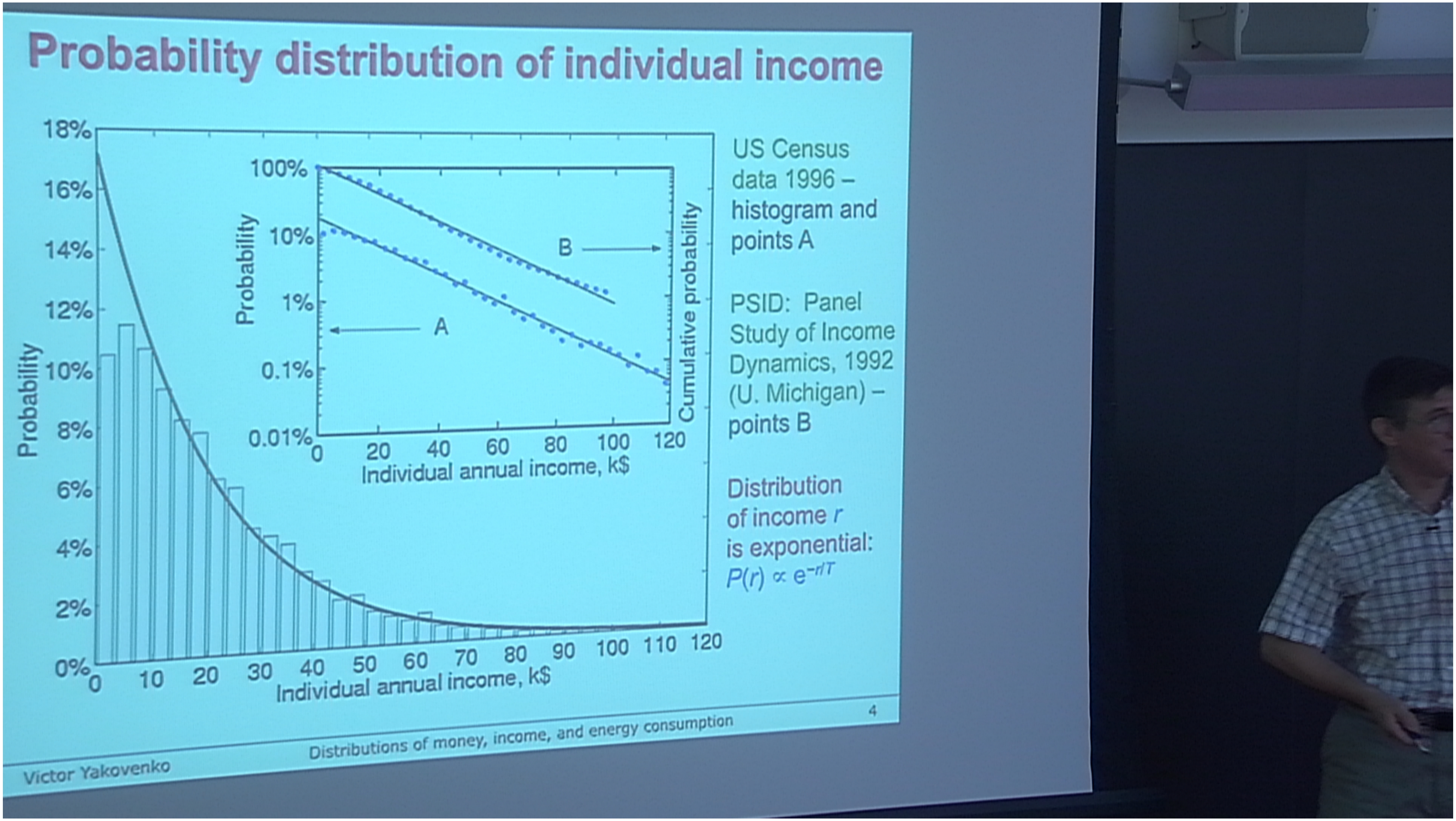

Probability distribution of individual income
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
Probability
0 10 20 30 40 50 60 70 80 90 100 110 120
Individual annual income, k$
100%
10%
1%
0.1%
0.01%
Probability
Cumulative probability
B
A
0 20 40 60 80 100 120
Individual annual income, k$
US Census data 1996 – histogram and points A
PSID: Panel Study of Income Dynamics, 1992 (U. Michigan) – points B
Distribution of income r is exponential: P(r) ∝ e^-r/T
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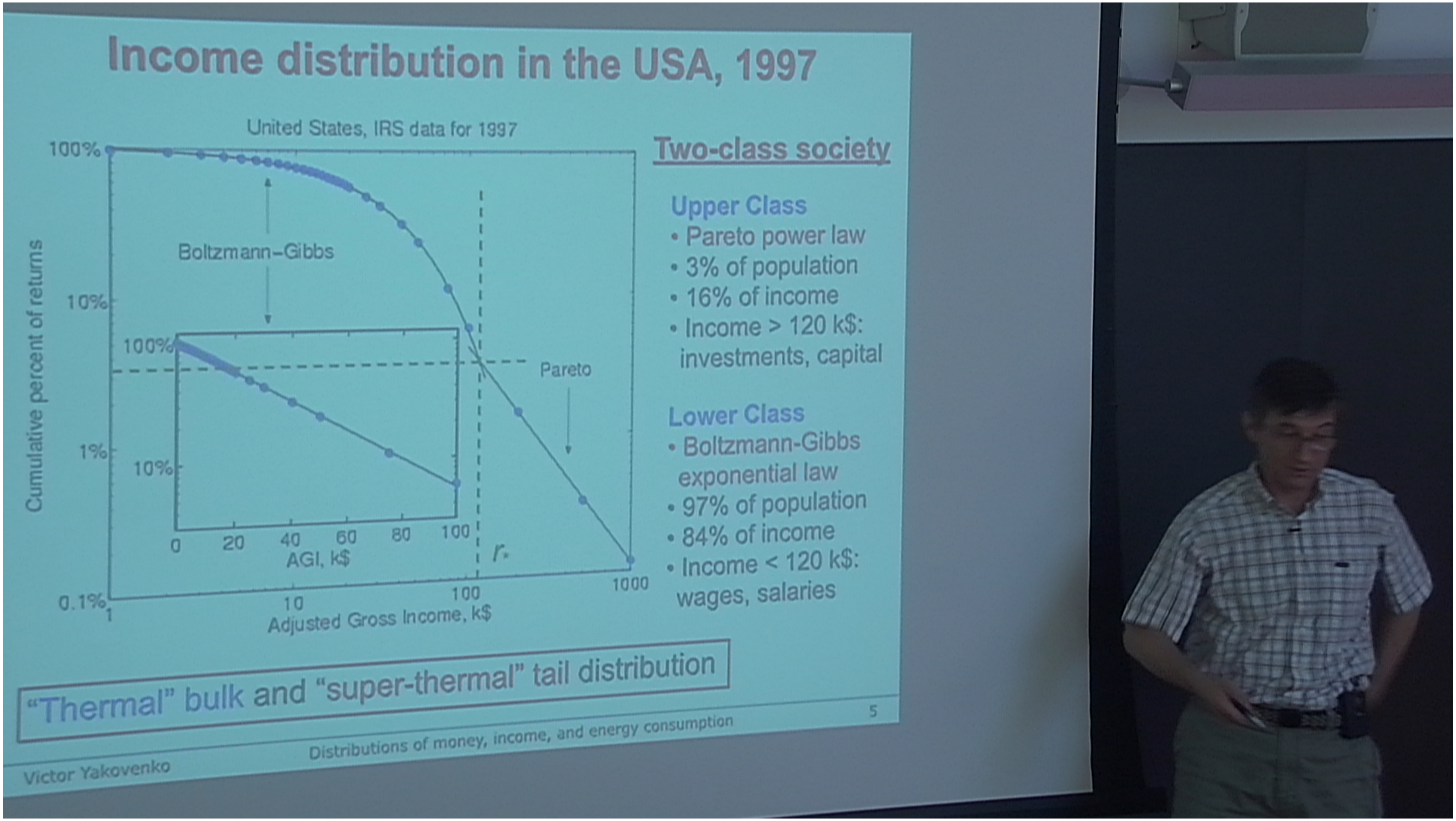

Income distribution in the USA, 1997
United States, IRS data for 1997
Cumulative percent of returns
100%
10%
1%
0.1%
Boltzmann–Gibbs
Pareto
100%
10%
0
20
40
60
80
100
AGI, k$
r_*
1
10
100
1000
Adjusted Gross Income, k$
Two-class society
Upper Class
• Pareto power law
• 3% of population
• 16% of income
• Income > 120 k$: investments, capital
Lower Class
• Boltzmann-Gibbs exponential law
• 97% of population
• 84% of income
• Income < 120 k$: wages, salaries
"Thermal" bulk and "super-thermal" tail distribution
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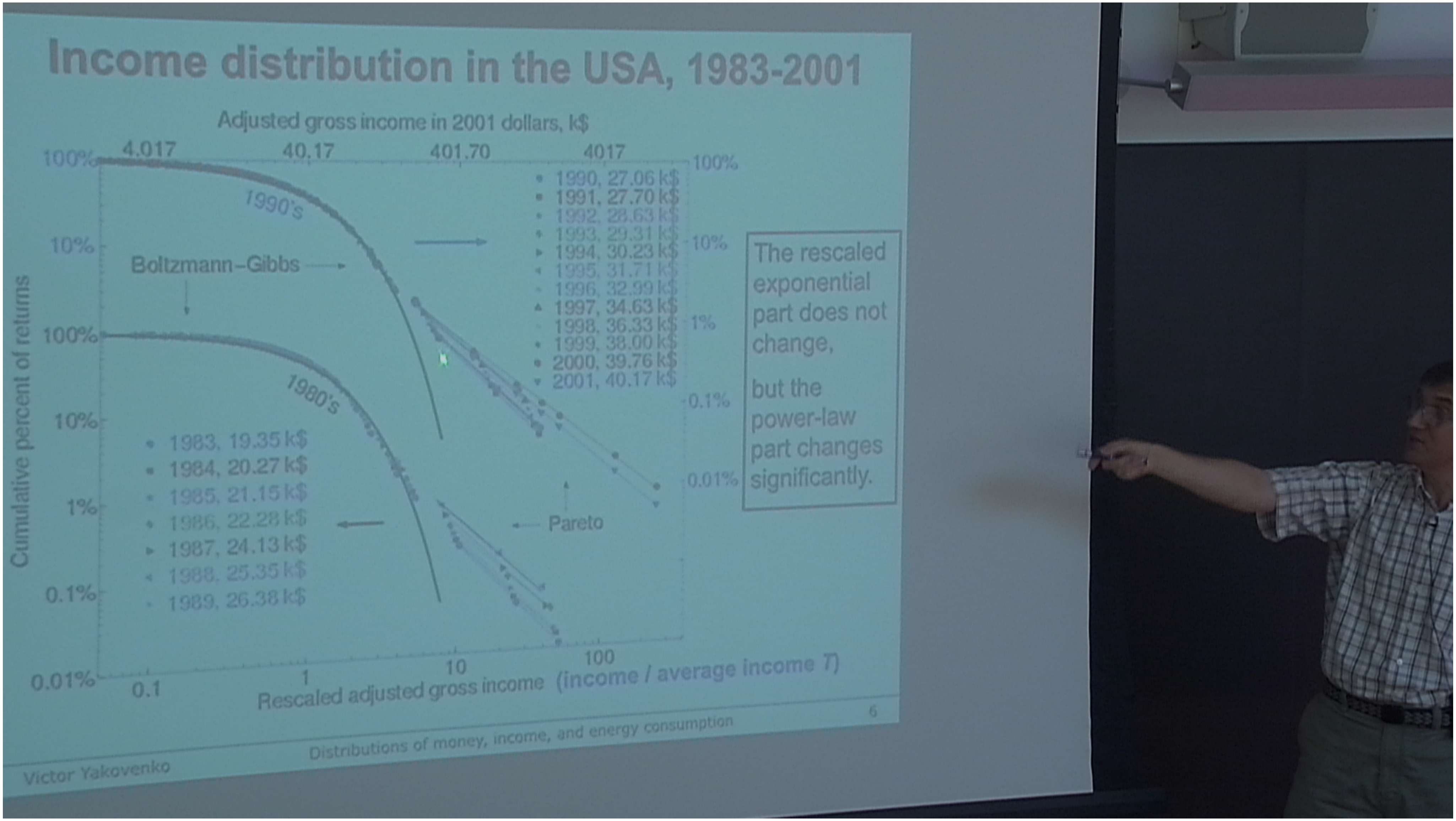

Income distribution in the USA, 1983-2001
Adjusted gross income in 2001 dollars, k$
4.017
40.17
401.70
4017
100%
10%
1%
0.1%
0.01%
1990's
1980's
Boltzmann–Gibbs
Pareto
1990, 27.06 k$
1991, 27.70 k$
1992, 28.63 k$
1993, 29.31 k$
1994, 30.23 k$
1995, 31.71 k$
1996, 32.99 k$
1997, 34.63 k$
1998, 36.33 k$
1999, 38.00 k$
2000, 39.76 k$
2001, 40.17 k$
1983, 19.35 k$
1984, 20.27 k$
1985, 21.15 k$
1986, 22.28 k$
1987, 24.13 k$
1988, 25.35 k$
1989, 26.38 k$
Cumulative percent of returns
0.1
1
10
100
Rescaled adjusted gross income (income / average income T)
The rescaled exponential part does not change,
but the power-law part changes significantly.
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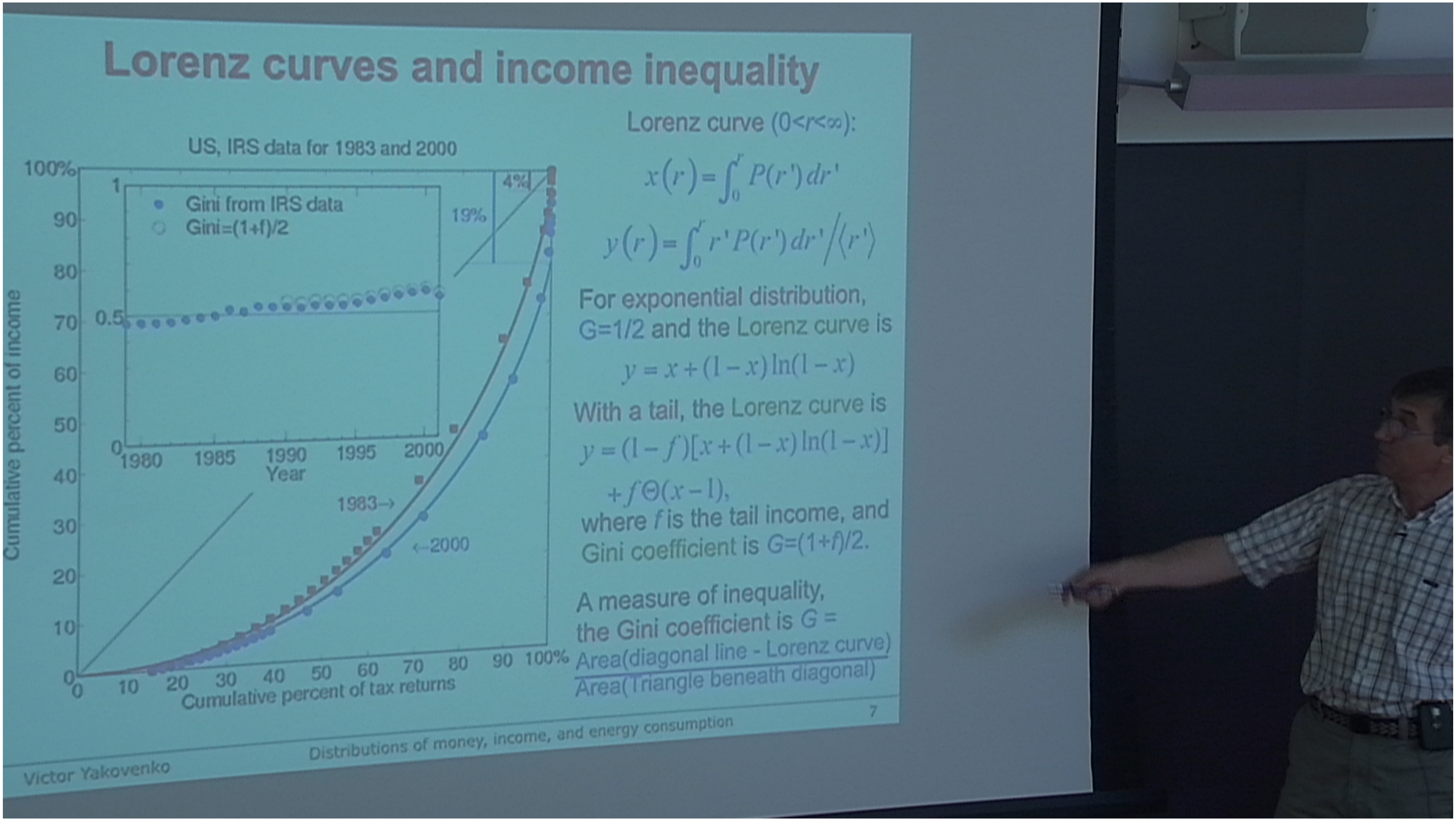
Lorenz curves and income inequality
US, IRS data for 1983 and 2000
Gini from IRS data
Gini=(1+f)/2
Year
1980
1985
1990
1995
2000
1983→
←2000
19%
4%
Cumulative percent of income
Cumulative percent of tax returns
Lorenz curve (0<r<∞):
$x(r)=\int_0^r P(r')\,dr'$
$y(r)=\int_0^r r'P(r')\,dr'/\langle r'\rangle$
For exponential distribution, G=1/2 and the Lorenz curve is
$y=x+(1-x)\ln(1-x)$
With a tail, the Lorenz curve is
$y=(1-f)[x+(1-x)\ln(1-x)]+f\Theta(x-1)$,
where f is the tail income, and Gini coefficient is G=(1+f)/2.
A measure of inequality, the Gini coefficient is G = Area(diagonal line - Lorenz curve) / Area(Triangle beneath diagonal)
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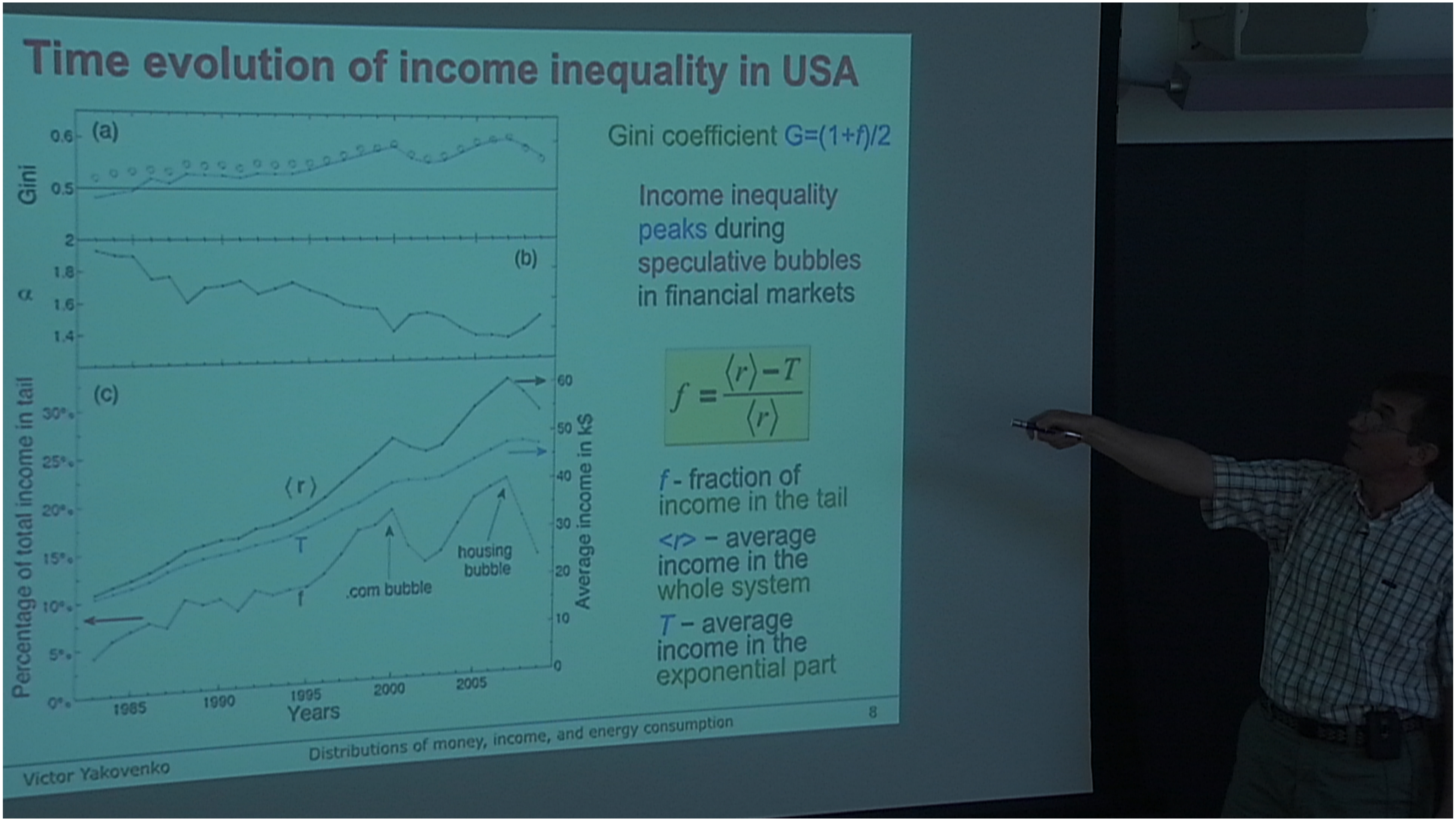
Time evolution of income inequality in USA
Gini coefficient G=(1+f)/2
Income inequality peaks during speculative bubbles in financial markets
f = (⟨r⟩ − T) / ⟨r⟩
f - fraction of income in the tail
<r> – average income in the whole system
T – average income in the exponential part
(a)
(b)
(c)
Gini
α
Percentage of total income in tail
Average income in k$
Years
.com bubble
housing bubble
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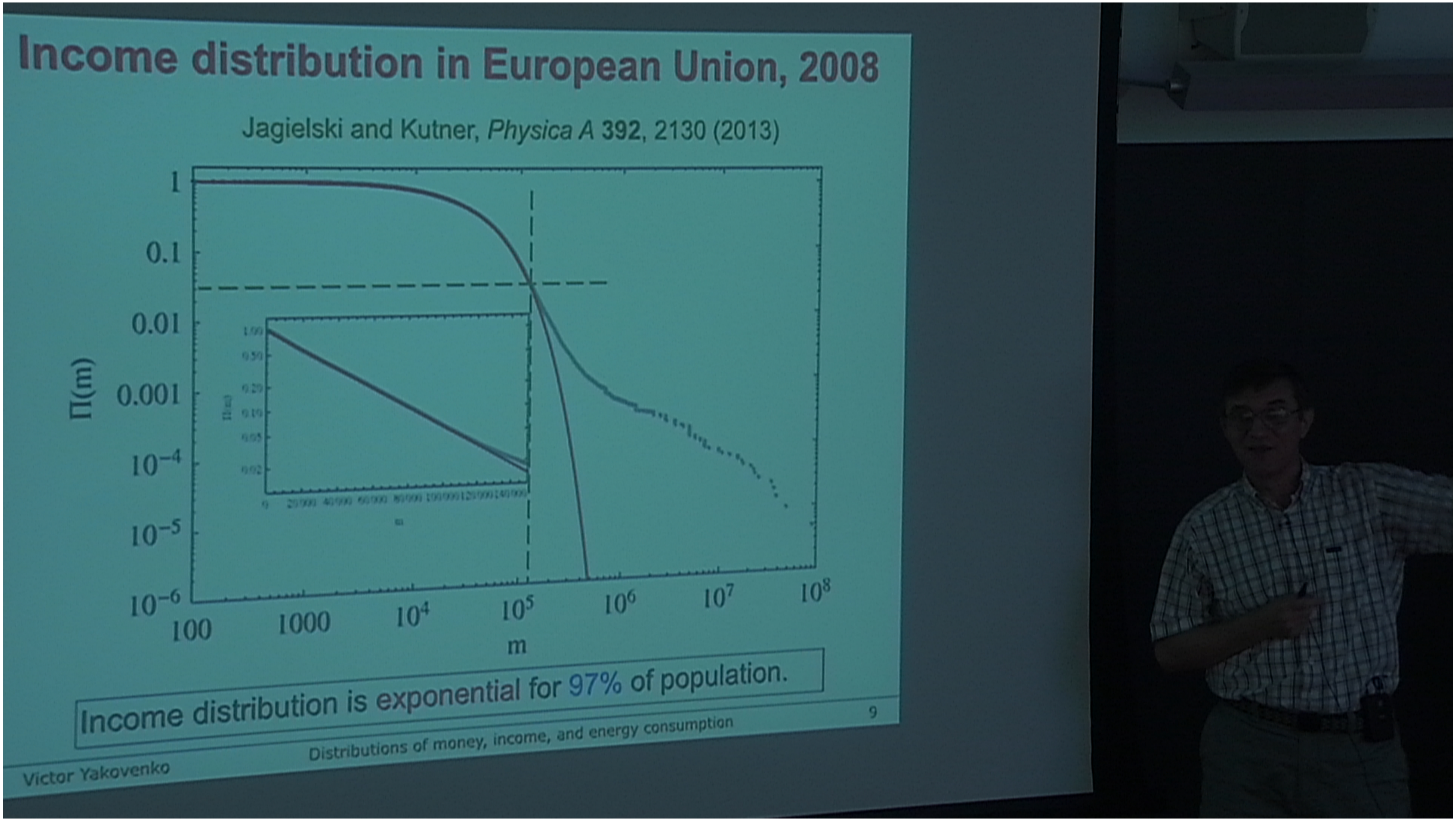
Income distribution in European Union, 2008
Jagielski and Kutner, Physica A 392, 2130 (2013)
Π(m)
1
0.1
0.01
0.001
10^-4
10^-5
10^-6
100
1000
10^4
10^5
10^6
10^7
10^8
m
Income distribution is exponential for 97% of population.
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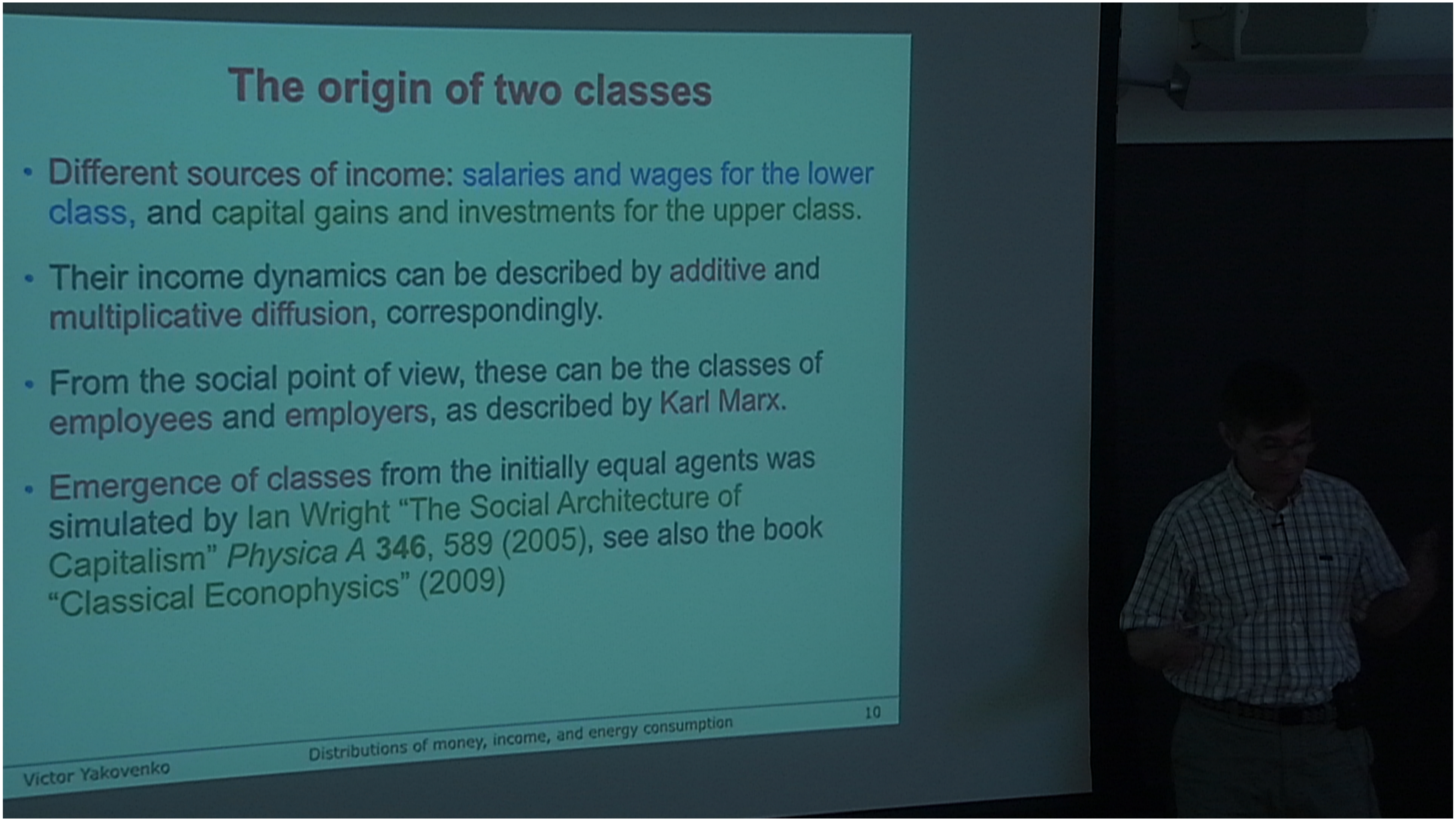
The origin of two classes
• Different sources of income: salaries and wages for the lower class, and capital gains and investments for the upper class.
• Their income dynamics can be described by additive and multiplicative diffusion, correspondingly.
• From the social point of view, these can be the classes of employees and employers, as described by Karl Marx.
• Emergence of classes from the initially equal agents was simulated by Ian Wright "The Social Architecture of Capitalism" Physica A 346, 589 (2005), see also the book "Classical Econophysics" (2009)
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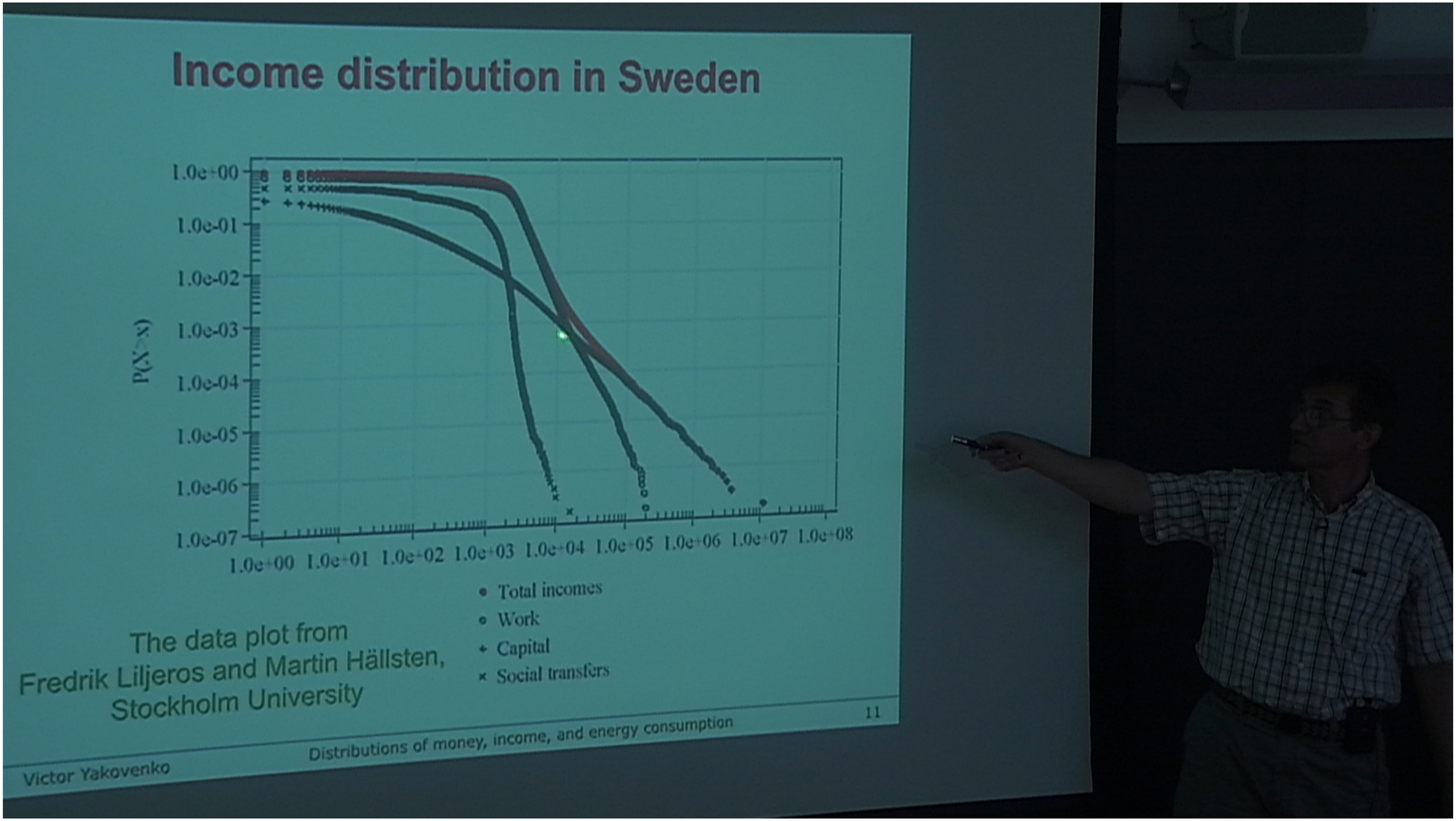

Income distribution in Sweden
P(X>x)
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e+00 1.0e+01 1.0e+02 1.0e+03 1.0e+04 1.0e+05 1.0e+06 1.0e+07 1.0e+08
Total incomes
Work
Capital
Social transfers
The data plot from
Fredrik Liljeros and Martin Hällsten,
Stockholm University
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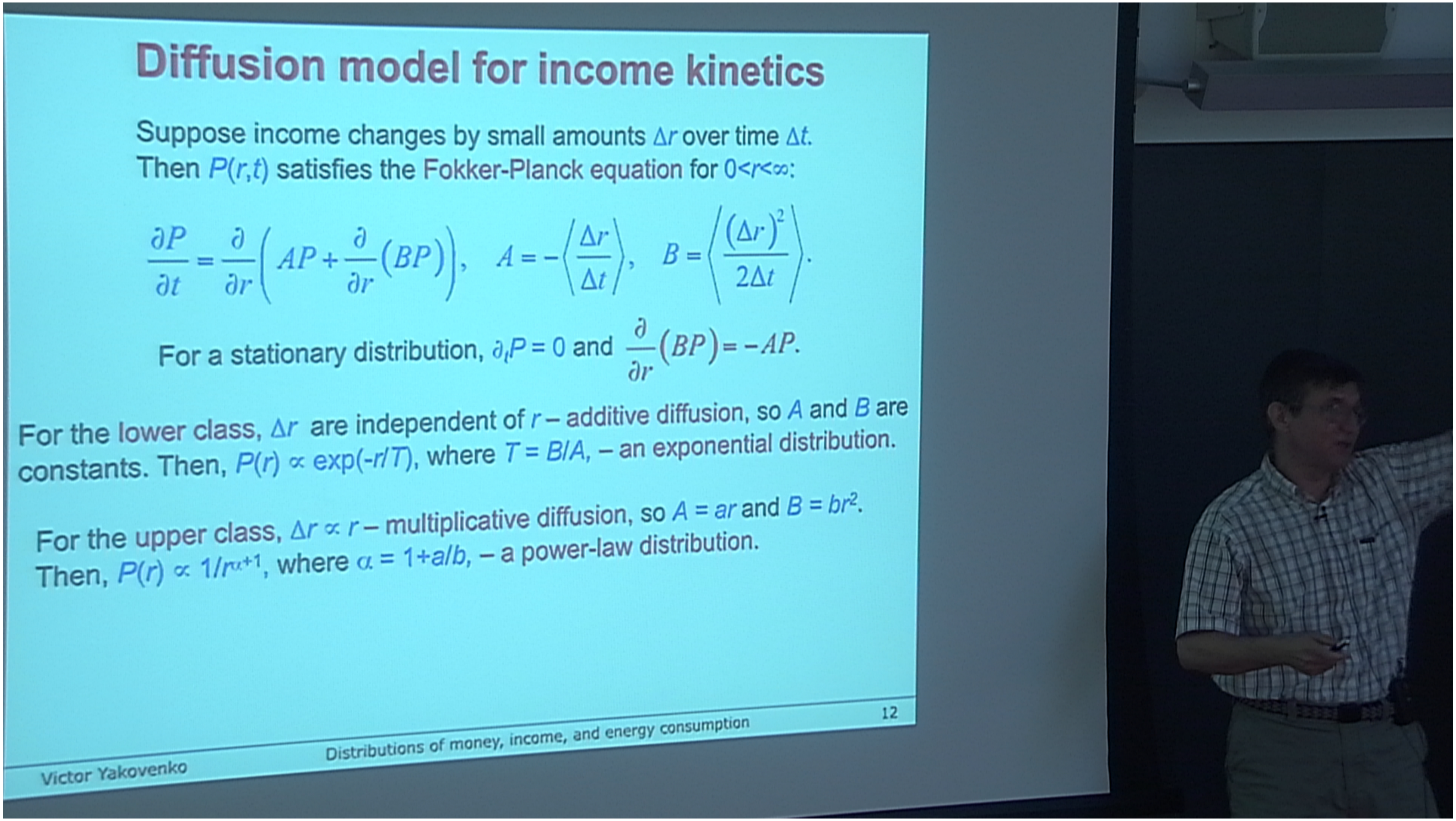
Diffusion model for income kinetics
Suppose income changes by small amounts Δr over time Δt.
Then $P(r,t)$ satisfies the Fokker-Planck equation for $0<r<\infty$:
$\frac{\partial P}{\partial t} = \frac{\partial}{\partial r}\left(AP + \frac{\partial}{\partial r}(BP)\right), \quad A = -\left\langle \frac{\Delta r}{\Delta t} \right\rangle, \quad B = \left\langle \frac{(\Delta r)^2}{2\Delta t} \right\rangle.$
For a stationary distribution, $\partial_t P = 0$ and $\frac{\partial}{\partial r}(BP) = -AP$.
For the lower class, Δr are independent of r – additive diffusion, so A and B are constants. Then, $P(r) \propto \exp(-r/T)$, where $T = B/A$, – an exponential distribution.
For the upper class, $\Delta r \propto r$ – multiplicative diffusion, so $A = ar$ and $B = br^2$.
Then, $P(r) \propto 1/r^{\alpha+1}$, where $\alpha = 1+a/b$, – a power-law distribution.
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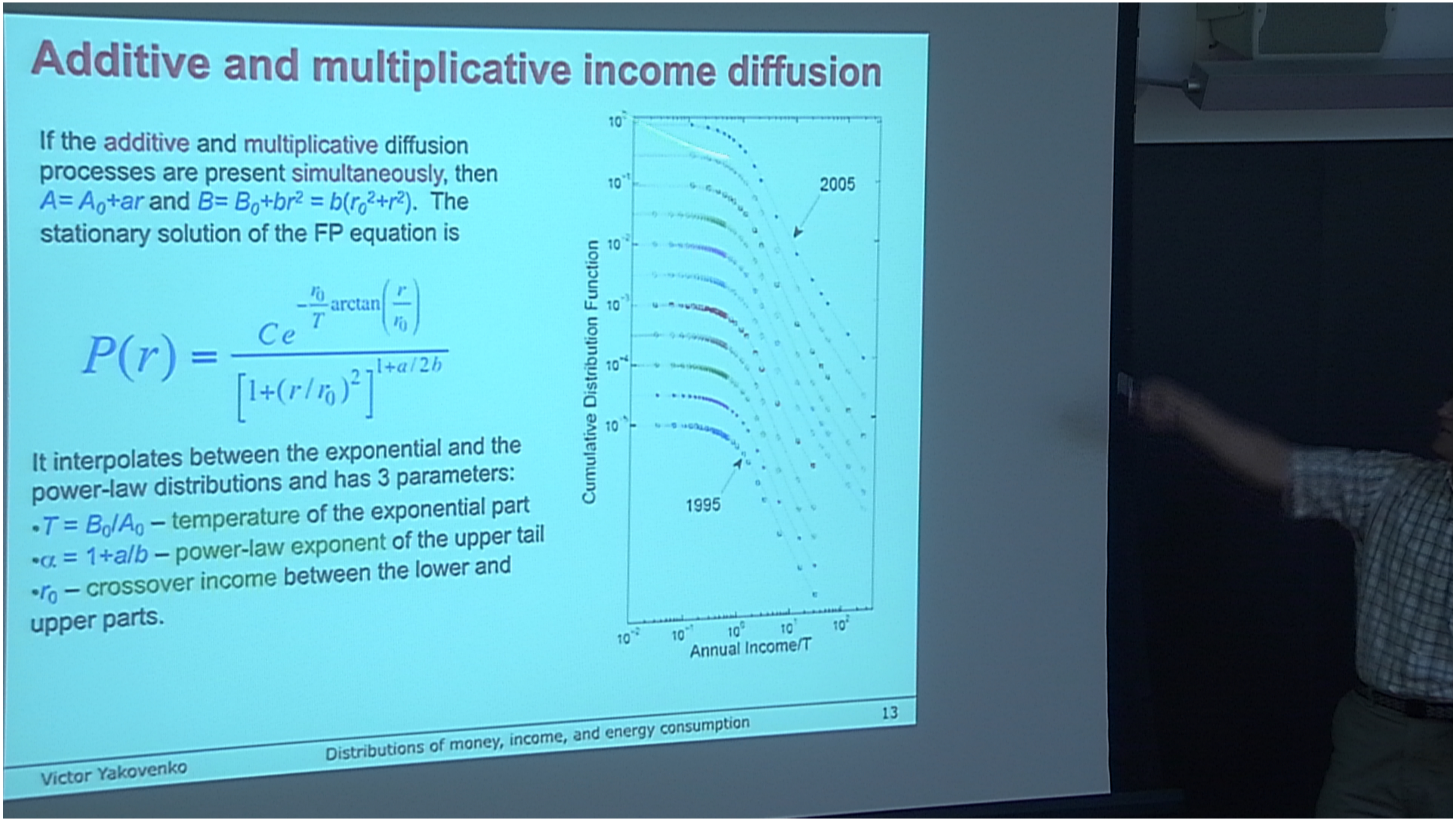
Additive and multiplicative income diffusion
If the additive and multiplicative diffusion processes are present simultaneously, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is
$$P(r) = \frac{C e^{-\frac{r_0}{T}\arctan\left(\frac{r}{r_0}\right)}}{\left[1 + (r/r_0)^2\right]^{1+a/2b}}$$
It interpolates between the exponential and the power-law distributions and has 3 parameters:
•$T = B_0/A_0$ – temperature of the exponential part
•$\alpha = 1 + a/b$ – power-law exponent of the upper tail
•r_0 – crossover income between the lower and upper parts.
Cumulative Distribution Function
Annual Income/T
2005
1995
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Additive and multiplicative income diffusion

If the additive and multiplicative diffusion processes are present simultaneously, then $A = A_0 + ar$ and $B = B_0 + br^2 = b(r_0^2 + r^2)$. The stationary solution of the FP equation is

$$P(r) = \frac{C e^{-\frac{r_0}{T}\arctan\left(\frac{r}{r_0}\right)}}{\left[1+(r/r_0)^2\right]^{1+a/2b}}$$

It interpolates between the exponential and the power-law distributions and has 3 parameters:

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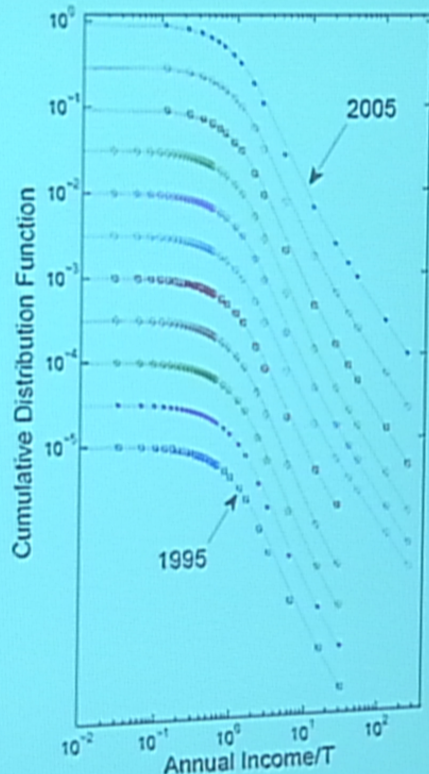
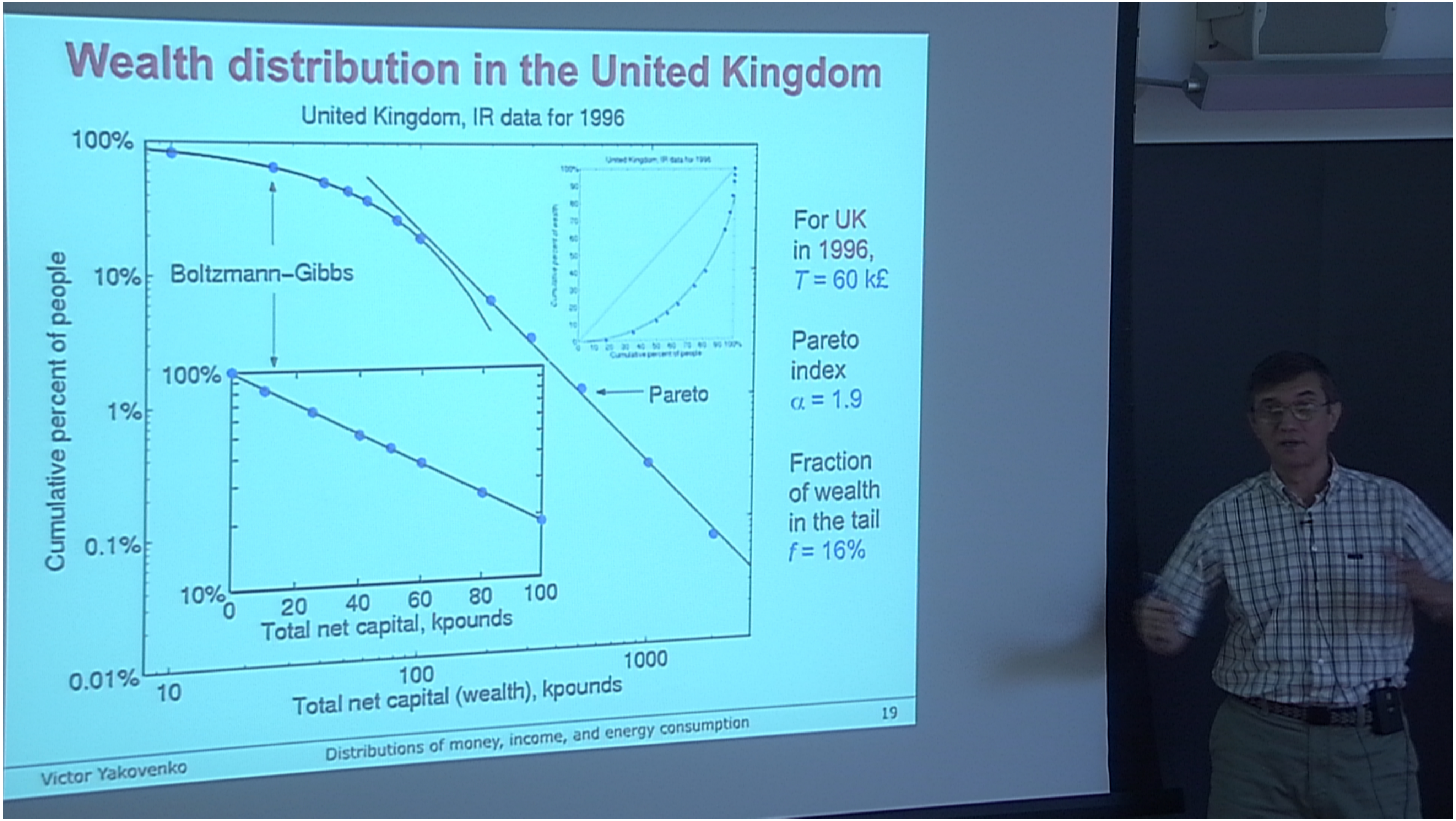
- r_0 – crossover income between the lower and upper parts.



Banerjee & Yakovenko, *RMP* (2009), NJP (2010)
Fiaschi & Marsili, *JEBO* (2012)
Karl Pearson, *Proc. Roy. Soc. London* (1895)

Victor Yakovenko — Distributions of money, income, and energy consumption — 13



Wealth distribution in the United Kingdom
United Kingdom, IR data for 1996
100%
10%
1%
0.1%
0.01%
Cumulative percent of people
Boltzmann–Gibbs
Pareto
100%
10%
0
20
40
60
80
100
Total net capital, kpounds
10
100
1000
Total net capital (wealth), kpounds
For UK
in 1996,
T = 60 k£
Pareto
index
α = 1.9
Fraction
of wealth
in the tail
f = 16%
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Distributions of money, income, and energy consumption
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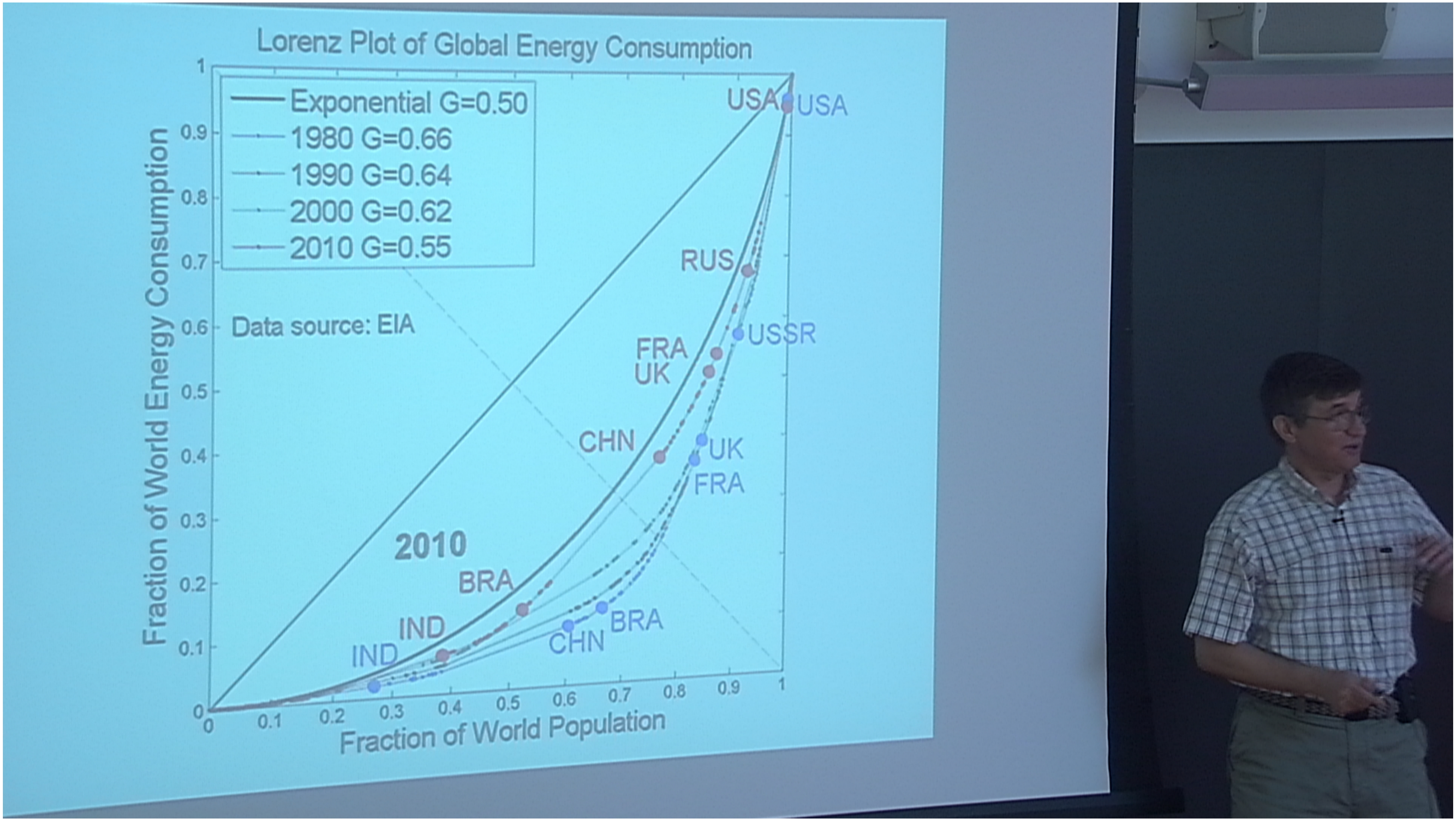
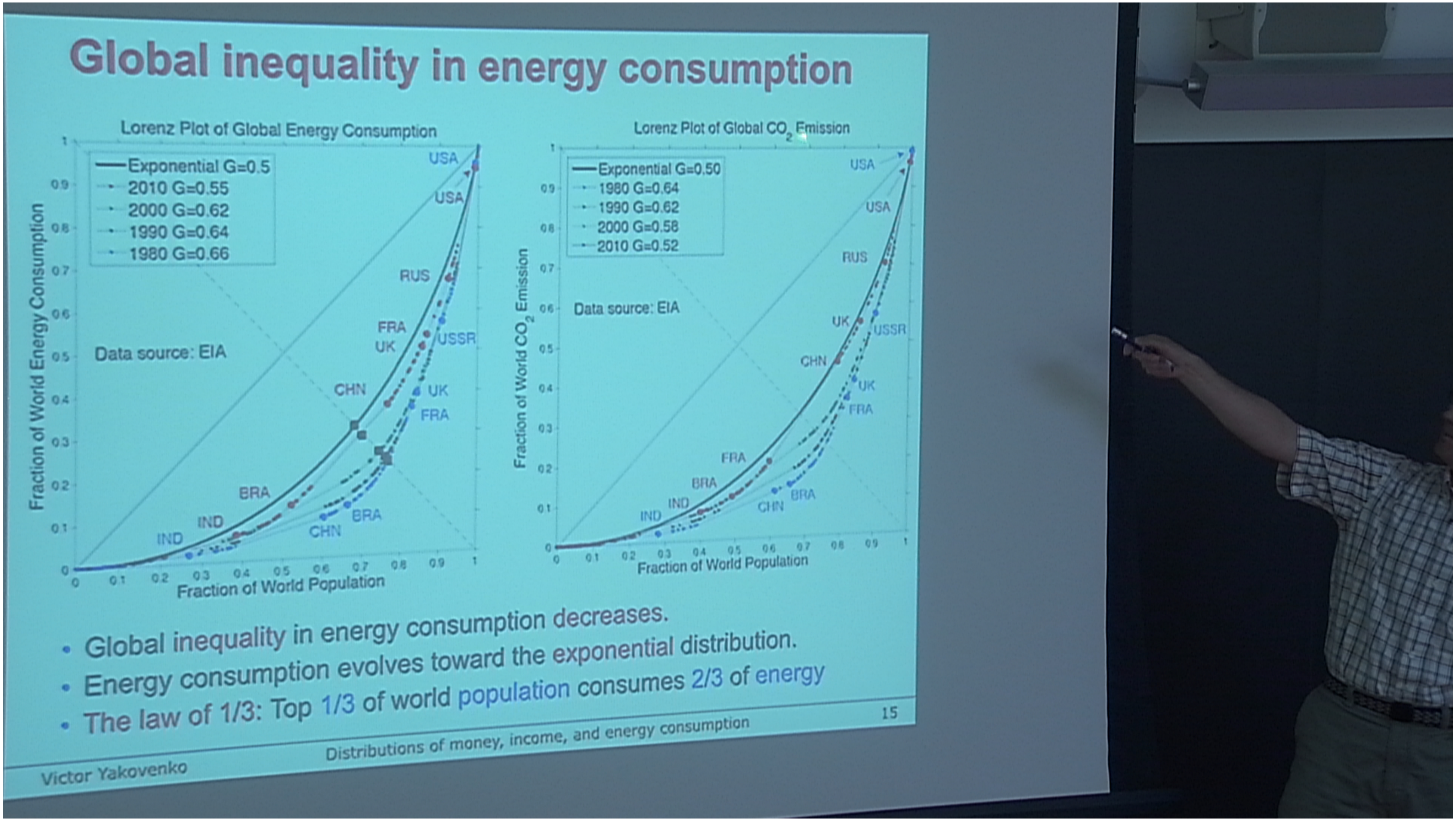


Lorenz Plot of Global Energy Consumption
Exponential G=0.50
1980 G=0.66
1990 G=0.64
2000 G=0.62
2010 G=0.55
Data source: EIA
2010
Fraction of World Energy Consumption
Fraction of World Population
USA
USA
RUS
USSR
FRA
UK
CHN
UK
FRA
BRA
IND
IND
BRA
CHN
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Global inequality in energy consumption
Lorenz Plot of Global Energy Consumption
Exponential G=0.5
2010 G=0.55
2000 G=0.62
1990 G=0.64
1980 G=0.66
Data source: EIA
Fraction of World Energy Consumption
Fraction of World Population
USA
RUS
FRA
UK
USSR
CHN
BRA
IND
Lorenz Plot of Global CO2 Emission
Exponential G=0.50
1980 G=0.64
1990 G=0.62
2000 G=0.58
2010 G=0.52
Data source: EIA
Fraction of World CO2 Emission
Fraction of World Population
• Global inequality in energy consumption decreases.
• Energy consumption evolves toward the exponential distribution.
• The law of 1/3: Top 1/3 of world population consumes 2/3 of energy
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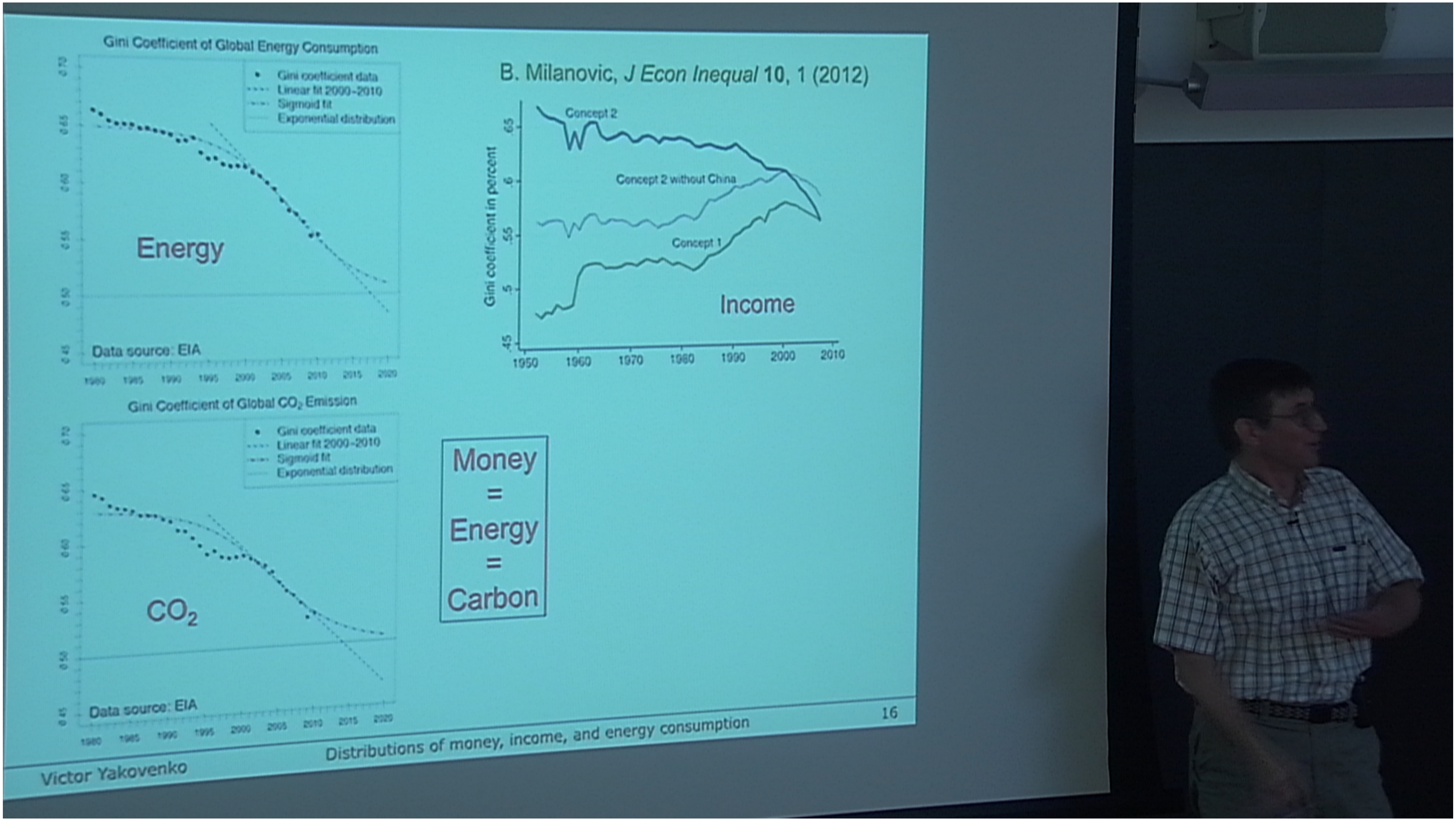
Distributions of money, income, and energy consumption


Gini Coefficient of Global Energy Consumption
Gini coefficient data
Linear fit 2000-2010
Sigmoid fit
Exponential distribution
Energy
Data source: EIA
B. Milanovic, J Econ Inequal 10, 1 (2012)
Concept 2
Concept 2 without China
Concept 1
Gini coefficient in percent
Income
Gini Coefficient of Global CO2 Emission
CO2
Money
=
Energy
=
Carbon
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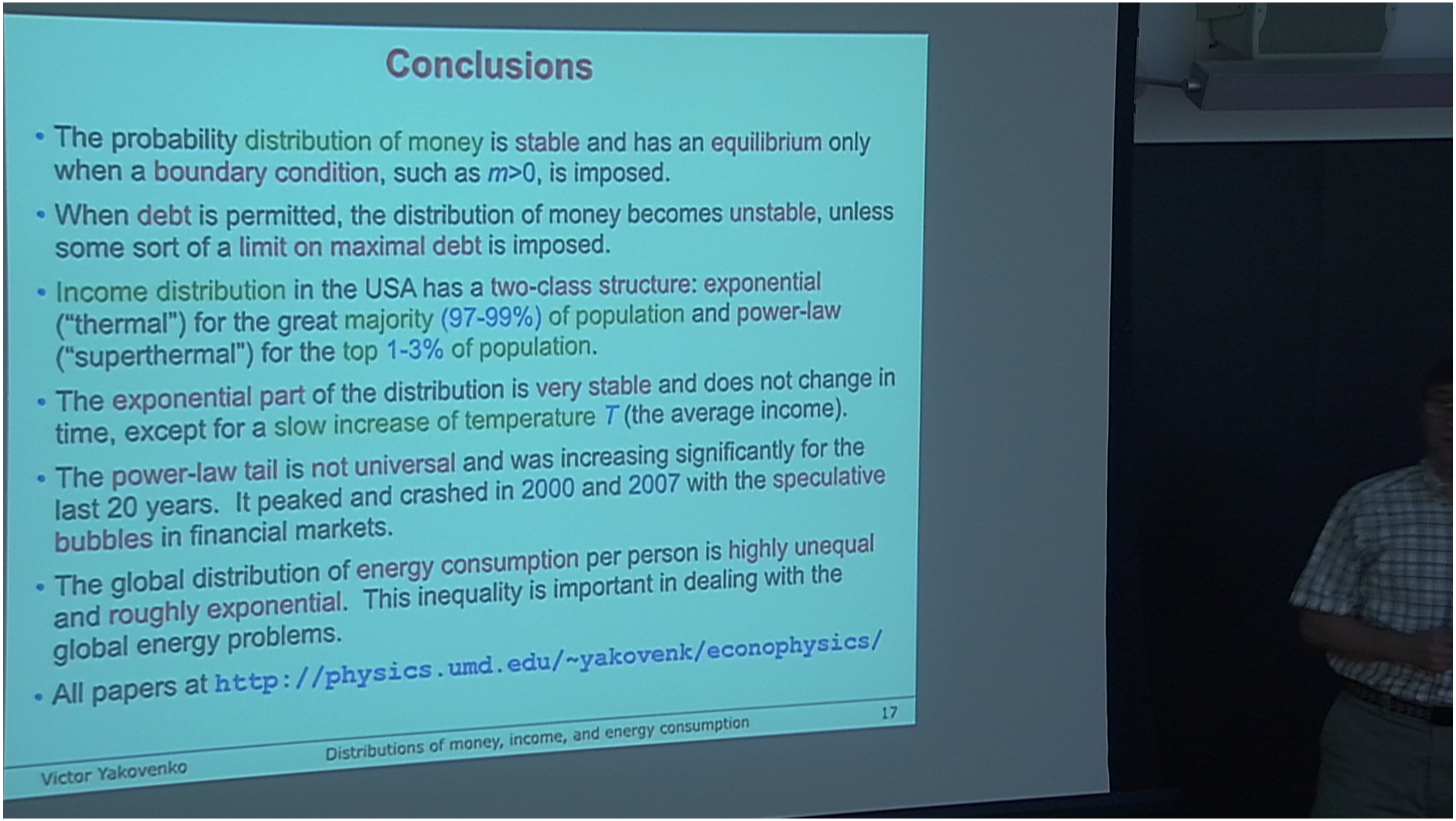

Conclusions
• The probability distribution of money is stable and has an equilibrium only when a boundary condition, such as m>0, is imposed.
• When debt is permitted, the distribution of money becomes unstable, unless some sort of a limit on maximal debt is imposed.
• Income distribution in the USA has a two-class structure: exponential ("thermal") for the great majority (97-99%) of population and power-law ("superthermal") for the top 1-3% of population.
• The exponential part of the distribution is very stable and does not change in time, except for a slow increase of temperature T (the average income).
• The power-law tail is not universal and was increasing significantly for the last 20 years. It peaked and crashed in 2000 and 2007 with the speculative bubbles in financial markets.
• The global distribution of energy consumption per person is highly unequal and roughly exponential. This inequality is important in dealing with the global energy problems.
• All papers at http://physics.umd.edu/~yakovenk/econophysics/
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