Title: A is for Axion (in the Alphabet of Bsm Curiosities)

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Abstract: <p>The QCD axion is a curious Dark Matter candidate, having a mass like the neutrino, but behaving as Cold Dark Matter. I will review how this occurs, and discuss the interesting question of whether WIMPs could be distinguished from axions with Large Scale Structure data.</p>

...in the Alphabet of Bsm Curiosities...

$\boldsymbol{A}$ is for $axion$

- particle from Beyond-the-Standard-Model, but *light*. (So forget usual EFT)
- very light ($m \sim 10^{-4}$ eV), very weakly coupled ($\lesssim 10^{-12}$), *theoretically beloved* (pseudo) scalar
- $m_a \sim m_\nu$, but $COLD$ Dark Matter
- one parameter model: couplings $\propto$ mass (for QCD axion Axion-Like-Particles = ALPS = same Lagrangian, couplings free)

Sacha Davidson *IPN de Lyon/CNRS*
arXiv:1405.1139 , 1307.8024 with M Elmer

Outline

1. why the QCD axion?
   - the strong CP problem
   - the axial anomaly in QFT and Peccei+Quinn's solution
   - astrophysical constraints
2. the story of the Universe (according to axions)
   - inflation and the birth of the axion: which first?
   - the QCD phase transition: the axion gets a mass
   - contributions to Cold Dark Matter: field, and particles from strings
3. structure formation with axion Dark Matter : distinguishing from WIMPs?
   - Sikivie's scenario and the Bose Einstein Condensate
   - D'après moi, principle is simple...the stress-energy tensor is different... practise is difficult : fluid DM in structure formation codes?

Why the axion : the strong CP problem of QCD

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But not observe electric dipole moment of neutron:

$$\Rightarrow \theta \lesssim 10^{-10} \text{...not} \sim 1?$$

Pich deRafael
Pospelov, Ritz

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How to get rid of θ?

1. add a symmetry, so that can "rotate" θ away?
2. add a dynamical mechanism so θ settles to zero?

Peccei-Quinn
Weinberg, Wilcek

...these are equivalent, but to see why, need the anomaly...

From the chiral anomaly to axion models

Peccei Quinn
Kim , ShifmanVainshteinZakharov
DineFischlerSrednicki,Zhitnitsky
Srednicki NPB85

1. the chiral anomaly says can remove it by a chiral phase rotn on massless quarks

$$q_L \to e^{-i\theta/4} q_L \quad , \quad q_R \to e^{i\theta/4} q_R \quad \Rightarrow \quad \theta \frac{g_s^2}{32\pi^2} G\widetilde{G} \to 0 \times \frac{g_s^2}{32\pi^2} G\widetilde{G}$$

In classical theory of massless quarks, chiral phase rotns are a sym:

$$\delta\mathcal{L} \propto \theta \partial_\mu J_5^\mu = 0$$

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2. but SM quarks are not massless :(

$$m\overline{q_L} q_R \to e^{i\theta/2} \overline{q_L} q_R$$

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2. but SM quarks are not massless :(
$$m\overline{q_L} q_R \to e^{i\theta/2} \overline{q_L} q_R$$

3. add ... quarks with a mass invariant under chiral rotns!
$\Rightarrow$ introduce new quarks, and new complex scalar $\Phi = |\Phi| e^{ia/f}$, such that $\Phi \to e^{-i\theta/2}\Phi$, whose vev ($\sim 10^{11}$ GeV) gives mass to new quarks
$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu \Phi^\dagger \partial^\mu \Phi + i\overline{\Psi} \not{D} \Psi + \{\lambda \Phi \overline{\Psi} \Psi + h.c.\} + V(\Phi)$$

4. θ is gone, $|\Phi|$ and new quarks are heavy...remains at low energy a, the axion.

Remains the axion at low energy

summary: traded CPV parameter θ for a dynamical field a (with potential min at 0),
who is phase of $\Phi \sim f e^{ia/f}$, $f \sim 10^{11}$ GeV.

$\Rightarrow$ only new particle at low-energy is the (pseudo-) goldstone a

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f} \simeq 6 \times 10^{-5} \, \frac{10^{11}\ \text{GeV}}{f}\ \text{eV}$$

Srednicki NPB85

$$\text{couplings to SM} \quad \propto \quad \frac{1}{f} \propto m_a$$

always to gluons $\Leftrightarrow$ nucleon

model − dep to fermions (electrons) at tree

generically $\sim \frac{\alpha}{\pi f}$ to 2γ (triangle, and mixing with π)

$\Rightarrow$ one-parameter, one-particle model, couplings $\propto m_a$
self-interactions: $V(a) \approx f_{\rm PQ}^2 m^2 [1 - \cos(a/f_{\rm PQ})] \simeq \frac{1}{2} m^2 a^2 - \frac{m^2}{4! f_{\rm PQ}^2} a^4 + \frac{m^2}{6! f_{\rm PQ}^4} a^6 + \ldots$

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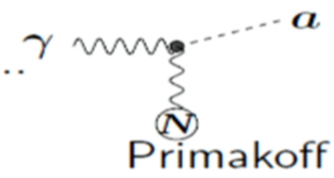
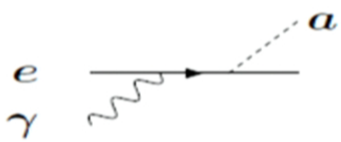
Astrophysical bounds

Raffelt...

axion light and (feebly) coupled to SM $\propto \frac{1}{f_{PQ}} \propto m_a$

$\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...



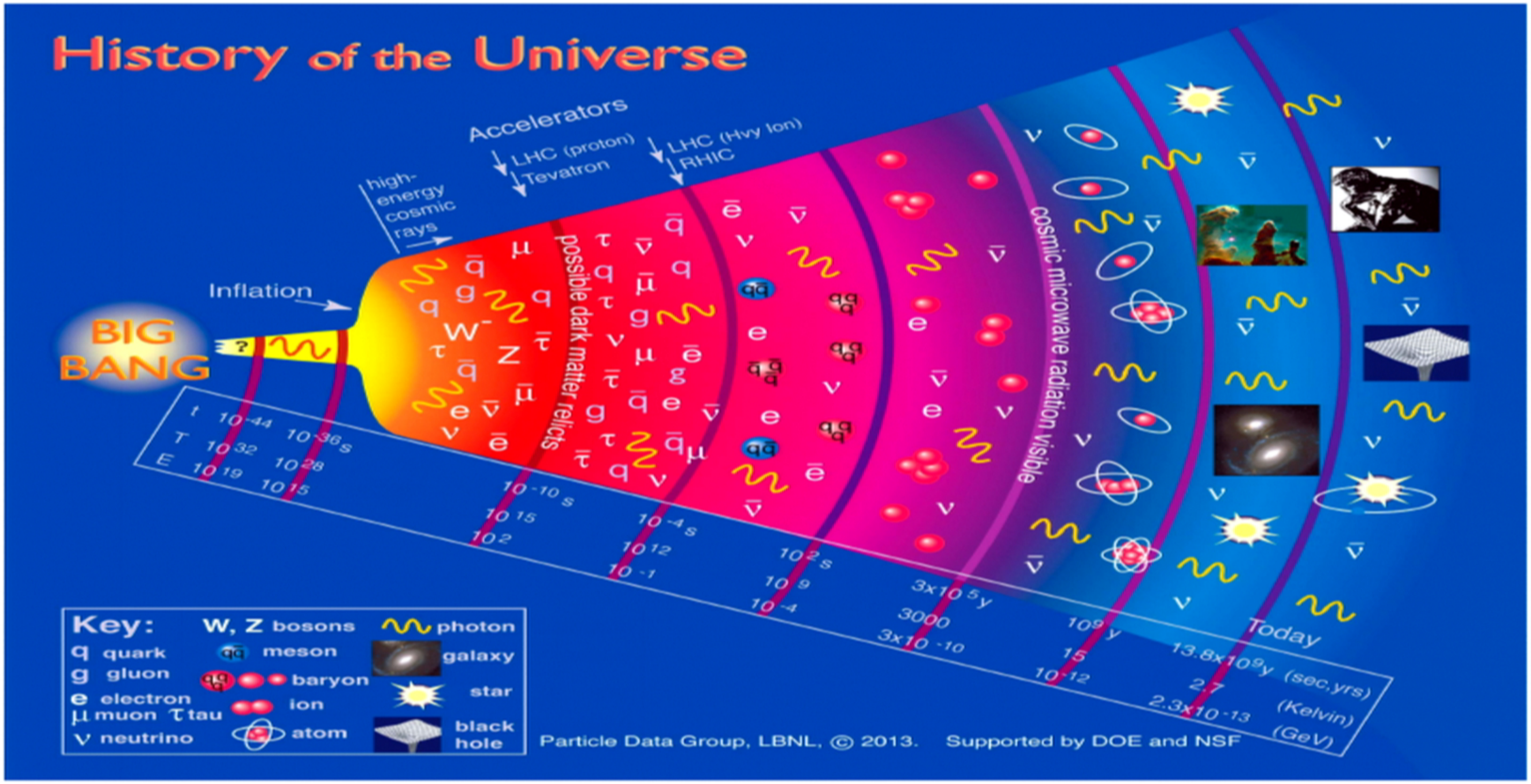


(axion couplings to e vs N vary across models by ~ 10)

upper bound on coupling to avoid rapid stellar energy loss:

$$m_a \lesssim 10^{-2} \text{ eV} \qquad\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$$

The story of the (QCD) axion Universe


History of the Universe
Accelerators
LHC (proton)
Tevatron
LHC (Hvy Ion)
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relics
cosmic microwave radiation visible
t 10-44 10-36s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-4 s
1012
10-1
102 s
109
10-4
3x105 y
3000
3x10-10
109 y
15
10-12
Today
13.8x109 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
e electron
ion
star
μ muon τ tau
ν neutrino
atom
black hole
Particle Data Group, LBNL, © 2013. Supported by DOE and NSF

Non-thermal axion production: *Cold* Dark Matter!

1. In the beginning, there was inflation
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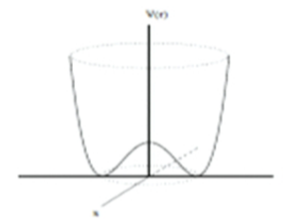
$$\Phi \rightarrow f e^{ia/f} \quad (f \sim 10^{12}\ \text{GeV})$$

   * a massless, random $-\pi f \leq a_0 \leq \pi f$ in each horizon

$$\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f^2/3$$

   * ...one string/horizon



PreskillWiseWilcek
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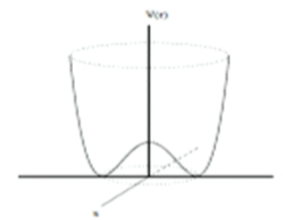

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3. Laaaater: QCD Phase Transition ($T \sim 200$ MeV): $\ldots m_\pi$ (tilt mexican hat)
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   $*$ energy density $m_a^2 \langle a_0 \rangle^2 / R^3(t)$ density today higher for smaller mass $\Rightarrow$ correct Ω for $m_a \gtrsim 10^{-5}$eV
   $*$ strings go away (radiate cold axion particles, $\vec{p} \sim H \lesssim 10^{-6} m_a$) Hiramatsu etal,etal+Saikawa

axion after inflation $\Rightarrow$ **oscillating axion field + cold particles** redshift like CDM

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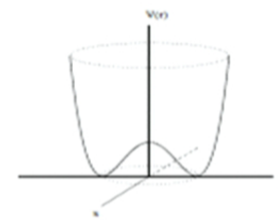

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Hiramatsu etal,etal+Saikawa

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Summary so far...

- QCD axion solves the strong CP problem
- for $m_a < 10^{-2}$ eV, stars live long enough (not cooled to fast)
- if born after inflation
  avoid isocurvature bound from PLANCK
  get correct Ω_{CDM} for $m_a \sim 10^{-4}$ eV

From the QCD Phase Transition to today

What does gravity do with axions?

(? distinguish from WIMPs in Large Scale Structure Data?)

Structure formation with axions: Sikivie's Scenario

Sikivie, Yang
Erken, Sikivie, Tam, Yang
Bannik, Sikivie

1. Consider DM axions... *HUGE* occupation number of low-$\vec{p}$ modes.
   a) This enhances interaction rates.
   b) In (thermal) equilibrium, would form a Bose Einstein Condensate.

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2. at $T_\gamma \sim$ keV, gravitational interaction rate $> H$, so "gravitational thermalisation" causes axions to form a "Bose-Einstein Condensate"

$$\Gamma \sim \frac{mG_N\rho_a R^3}{R} \sim \frac{G_N m_a^2 n_a}{H^2}$$

3. axion **BEC** can support vortices, which allow caustics in the galactic DM distribution. $\Leftrightarrow$ **axion DM signature?**

Rindler-Daller+Shapiro
Saikawa etal
SD+Elmer,SD
Berges+Jaeckel
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Guth etal

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I am confused...

1. what is a Bose Einstein Condensate?

   coherent scalar field carrying conserved particle number...

   but is it constant everywhere = coherent state of zero-mode particles?

   Or not neccessarily?

2. Are we talking about fields or particles? Does it matter?

3. What is thermalisation? How to quantify?

4. ...vortices in BECs...happen when? Why?

5. what observables are we trying to compute anyway?

$\Rightarrow$ ask the path integral! The path integral knows everything...

(usually tells you nothing because can't compute...

but axion most weakly coupled model I ever met, if perturbation theory works for QED, surely it works for axion?

Ask the Path Integral:

Suppose two CDM axion populations are classical field and distribution of cold particles (from strings).
What are relevant variables and equations to describe evolution?

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What are relevant variables and equations to describe evolution?

- variables = expectation values of n-pt functions ($a \equiv$ axion)

$$\langle a \rangle \leftrightarrow \text{ classical field} = \text{misalignment axions } a_{cl}$$
$$\langle a(x_1)a(x_2)\rangle \leftrightarrow \text{ (propagator)} + \text{distribution of particles } f(x,p)$$

- get Eqns of motion for expectation values in Closed Time Path formulation

$$\text{Einsteins Eqns with } T^{\mu\nu}(a_{cl}, f) + \text{quantum corrections}(\lambda, G_N)$$

⇒**leading order is simple:** Einsteins Eqns with $T^{\mu\nu}(a_{cl}, f)$.
In practise: compute $T^{\mu\nu}$ in usual 2nd quantised QFT, as expectation of the operator in a coherent state + bath of particles

Rediscovering...stress-energy tensors

non-rel axion particles are dust, like WIMPs:

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \end{bmatrix}$$

compare to perfect fluid: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}$. $P_{int} \propto \lambda^2 \to 0$, nonrel $\Rightarrow P \ll \rho, U = (1, \vec{v}), |\vec{v}| \ll 1$

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Classical field in non-relativistic limit

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix} \qquad \Delta T^i_j \sim \partial_i a \partial_j a \ , \ \lambda a^4$$

Sikivie

"extra" pressure with classical field... *not need Bose Einstein condensation!*

BE condensate described (at leading order) as a classical field. Misalignment axions already a classical field. No need to form a BE condensate ?

$\Rightarrow$ **is structure formation different?**

small fluctuations (linear eqns : QCDPT → mat-rad equality and beyond)

init. cdns: large-scale, "inflationary", adiabatic density fluctuations inherited at QCDPT: $\langle \frac{\delta M}{M} \rangle \sim cte$

Eqns of motion: Einsteins Eqns and $T^{\mu}{}_{\nu;\mu} = 0$. For linear adiabatic perturbations:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \overline{\rho}\delta + c_s^2 \frac{k^2}{R^2(t)}\delta = 0 \qquad \left(\delta \equiv \frac{\delta\rho(\vec{k},t)}{\overline{\rho}(t)} \right)$$

(H = Hubble rate, extra pressures in $c_s \simeq \partial P/\partial\rho$)
on LSS scales, $k^2 \to 0$, *same equation/dynamics as WIMPs*

Ratra, Hwang+Noh

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on LSS scales, $k^2 \rightarrow 0$, *same equation/dynamics as WIMPs* — Ratra, Hwang+Noh

?short distance differences:
* pressure and a Jeans length ($R_{Jeans} \sim \sqrt{\frac{H}{m}}\times$ horizon)
* a random from on horizon to next, so $\delta\rho_a/\rho_a \sim \mathcal{O}(1)$ on QCD horizon scale
(5km then, 10^9 km at T_{eq}) axion "miniclusters" $\langle\frac{\delta M}{M}\rangle \sim \sqrt{\frac{1}{M}}$ — Hogan, Rees

Distinguishing axions vs WIMPs in non-linear structure formation?

fluid eqns for non-relativistic axion field (black=eqns for dust) :

$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \rho \partial_t \vec{v} + \rho (\vec{v} \cdot \nabla) \vec{v} = -\rho \nabla V_N \pm \text{ extra pressures from field} \end{cases}$$

$\Rightarrow$ **hack a structure formation code to run fluid DM** (or field:Broadhurst etal)
compare to N-body (dust, phase-space) **code**

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- equivalent to non-relativistic eqns for axion field

$$a = \frac{1}{\sqrt{2m}} \left(\phi e^{-imt} + \phi^* e^{+imt} \right) \ , \ \phi(\vec{r}, t) = \sqrt{\frac{\rho}{m}} e^{-iS(\vec{r},t)} \ , \ \vec{v} = -\frac{1}{m} \nabla S \ , \ V_N = -\frac{GM(r)}{r} \ , \ g = -\frac{1}{(3! f^2)}$$

self-interaction pressure *inwards*: $\frac{\partial}{\partial r} r^{-n} < 0$

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$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \rho \partial_t \vec{v} + \rho(\vec{v} \cdot \nabla)\vec{v} = -\rho \nabla V_N + \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |g| \frac{\rho}{m^2} \right) \end{cases}$$

- equivalent to non-relativistic eqns for axion field

$$a = \frac{1}{\sqrt{2m}} \left(\phi e^{-imt} + \phi^* e^{+imt} \right) \;,\; \phi(\vec{r}, t) = \sqrt{\frac{\rho}{m}} e^{-iS(\vec{r},t)} \;,\; \vec{v} = -\frac{1}{m} \nabla S \;,\; V_N = -\frac{GM(r)}{r} \;,\; g = -\frac{1}{(3! f^2)}$$

self-interaction pressure *inwards*: $\frac{\partial}{\partial r} r^{-n} < 0$

$*$ fluid parameters single-valued ($\Rightarrow$ shocks, etc.) ... different from $f(x, p)$

"Bose Stars" in GR (*eg* Liebling, Palenzuela): solns for classical field coupled to GR
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Broadhurt etal: numerics for the $m \sim 10^{-22}$ eV case

I fix m, g for QCD axion($m \sim 10^{-4}$ eV, $f \sim 10^{11}$ GeV); what sized solution?

To make Andromeda with an axion field?

Euler Eqn for the non-relativistic axion field:

$$\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} = \nabla\Big(- V_N + \frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |g| \frac{\rho}{m^2}\Big)$$

$V_N = -\frac{GM(r)}{r}$
$g \simeq \frac{1}{3! f^2}$

Neglect LHS (v constant?):

To make Andromeda with an axion field?

Euler Eqn for the non-relativistic axion field:

$$0 \;=\; \nabla\Big(- m^2 V_N + \frac{\nabla^2\sqrt{\rho}}{2\sqrt{\rho}} + |g|\rho \Big)$$

$V_N = -\frac{GM(r)}{r}$
$g \simeq \frac{1}{3! f^2}$

Neglect LHS (v constant?):

1. balance gravity with gradient pressure in object of mass M:
   $\frac{1}{R^2} \simeq \frac{m^2 M}{m_{pl}^2 R} \quad \Rightarrow \quad \frac{m_{pl}^2}{m^2}\frac{1}{M} \sim R_{Jeans}$

2. impose self-interactions $<$ gravity :
   $\frac{m^2 R^2}{m_{pl}^2} \gtrsim \frac{1}{f^2} \quad \Rightarrow \quad R \gtrsim \frac{m_{pl}}{fm}$

$$R \sim 10^7 \text{ cm} \sim 10^{-3} R_\odot \quad , \quad \rho \sim 0.2 \,\frac{\text{g}}{\text{cm}^3} \Leftrightarrow M \sim 10^{21}\text{g} \sim 10^{-12} M_\odot$$

Andromeda : $M \sim 10^{12} M_\odot$, flat rotn curves to 100s kpc

Speculations...

Back to "miniclusters": suppose the $\mathcal{O}(1)$ fluctuations in axion density are "frozen" 'til $\rho_a \sim \rho_{rad}$.

at matter-radiation equality, these "miniclusters" ($M \sim 10^{-8} M_\odot$, $R \sim 10^9$ km), decouple from Hubble flow and collapse. (recall: stable clumps were $10^{-12} M_\odot$)

gravitational binding energy has to go somewhere = gradients...

...??? axion field configuration "fragments" into $\sim 10^{-12} M_\odot$ lumps ?

???...??? if DM is axion field, the halo of our galaxy is a phase space distribution of lumps, number density $\sim 10^{-5}/\text{au}^3$ (plus the particles from strings?).
So in LSS data, axions look like WIMPs? ?? ????

Summary

The QCD axion solves the strong CP problem, and is consistent with astrophysics and laboratory constraints for $m_a \lesssim 10^{-2}$ eV.

Non-thermal production mechanisms in cosmology can generate the observed relic density of cold dark matter. If the axion is born after inflation, two populations can arise at the QCD Phase Transition: the classical "misalignment" field, and cold particles from the decay of strings. They give Ω_{CDM} for $m_a \sim 10^{-4}$ eV.

To understand what gravity does with axions from the QCD Phase Transition til today, the Path Integral suggests to calculate the axion stress-energy tensor, and solve Einsteins Eqns:
$\Rightarrow$ The classical axion field has extra pressures and viscosities with respect to WIMPs, which could affect non-linear structure formation.

$\Rightarrow$ *numerical galaxy formation*?

To distinguish axion from WIMP CDM:
direct detection, axion effects on γ propagation? ...
maybe Large Scale Structure data? (or maybe not, since analytic estimates suggest that the axion field fragments into lumps which might behave like WIMP CDM)

Trying to learn something analytically...(confusion in progress)

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