Title: Euclidean Quantum Supergravity

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Abstract:

Euclidean Quantum Supergravity

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Based on work with Carl Turner arXiv:1408.3418

An Old Idea: Euclidean Quantum Gravity

$$\mathcal{Z} = \sum_{\text{topology}} \int \mathcal{D}g \;\exp\left(- \int d^4x \,\sqrt{g}\,\mathcal{R} \right)$$

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INSTANTONS IN CONFORMAL GRAVITY

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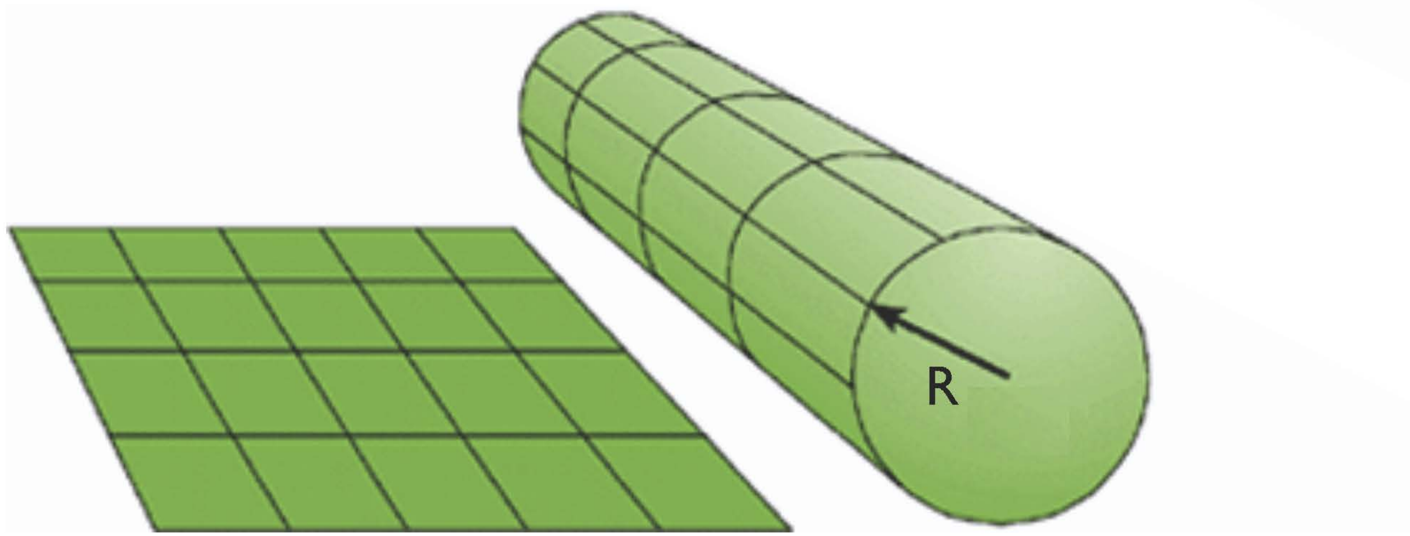
A Preview of the Main Result

Kaluza-Klein Theory: $\mathcal{M} = \mathbb{R}^{1,d-1} \times \mathbf{S}^1$



There is a long history of quantum instabilities of these backgrounds

- Casimir Forces
- Tunneling to "Nothing"

Gross, Perry and Yaffe '82
Appelquist and Chodos '83
Witten '82

The Main Result

Kaluza-Klein compactification of *N=1* Supergravity is unstable.

$$\boxed{\mathcal{W} \sim \exp\left(-\frac{\pi R^2}{4G_N} - i\sigma\right)}$$

Kaluza-Klein dual photon: $\partial_\mu \sigma \sim \frac{1}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}$

Grimm and Savelli 2011
(See also Harvey and Moore '99)

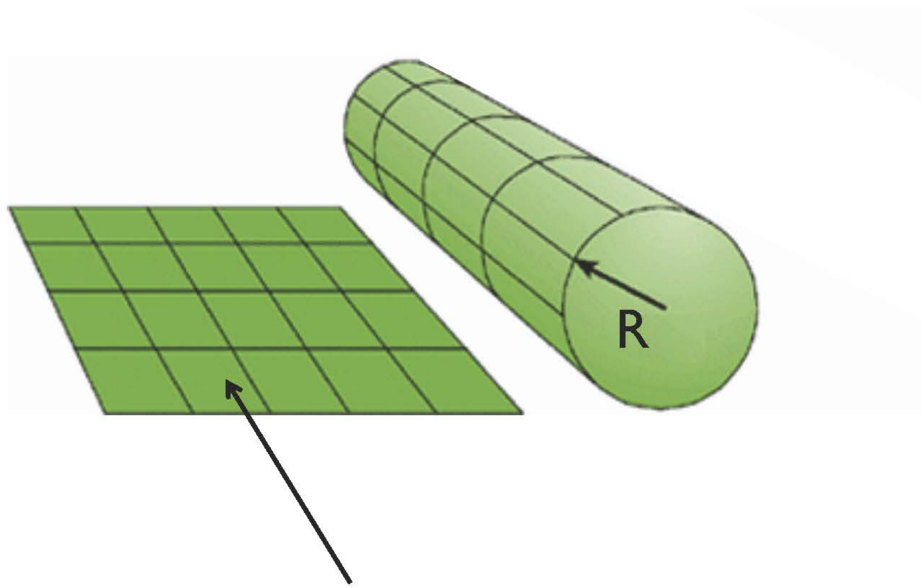
The Theory: *N=1* Supergravity

$$S = \frac{M_{\mathrm{pl}}^2}{2} \int d^4x \sqrt{-g} \left(\mathcal{R}_{(4)} + \bar{\psi}_\mu \gamma^{\mu\nu\rho} \mathcal{D}_\nu \psi_\rho \right)$$

Compactify on a Circle

$$ds^2_{(4)} = \frac{L^2}{R^2} ds^2_{(3)} + \frac{R^2}{L^2} \left(dz^2 + A_i dx^i\right)^2 \qquad z \in [0, 2\pi L)$$

$$\mathcal{M} = \mathbb{R}^{1,2} \times \mathbf{S}^1$$



Fields $R(x^i)$ and $A_i(x^i)$ live here

L is fiducial scale

Classical Low-Energy Physics

$$S_{\text{eff}} = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g}\, \mathcal{R}_{(4)}$$
$$= \frac{M_3}{2} \int d^3x \sqrt{-g_{(3)}} \left[\mathcal{R}_{(3)} - 2\left(\frac{\partial R}{R}\right)^2 - \frac{1}{4}\frac{R^4}{L^4} F_{ij}F^{ij} \right]$$

$$M_3 = 2\pi L M_{\text{pl}}^2$$

Or, if we work with the dual photon $\partial_\mu \sigma \sim \frac{1}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}$

$$S_{\text{eff}} = \int d^3x \sqrt{-g_{(3)}} \left[\frac{M_3}{2}\mathcal{R}_{(3)} - M_3 \left(\frac{\partial R}{R}\right)^2 - \frac{1}{M_3}\frac{L^2}{R^4}\left(\frac{\partial \sigma}{2\pi}\right)^2 \right]$$

Goal: Understand quantum corrections to this action.

Gravitational Instantons

Taub-NUT

$$ds^2 = U(\mathbf{x})d\mathbf{x}\cdot d\mathbf{x} + U(\mathbf{x})^{-1}\left(dz + \mathbf{A}\cdot d\mathbf{x}\right)^2$$

with $$U(\mathbf{x}) = 1 + \frac{L}{2(\mathbf{x}-\mathbf{X})} \quad \text{and} \quad \nabla\times\mathbf{A} = \nabla U$$

From the low-energy 3d perspective, these look like Dirac monopoles.

This is the gravitational verson of Polyakov's famous calculation.

Gross '84
Hartnoll and Ramirez '13

Doing the Computation

Action, Zero Modes, Jacobians, Determinants, Propagators….

The One-Loop Determinants

Gravitinos
Graviton ghosts

$$\text{dets} = \frac{\det(\text{Fermions})}{\det(\text{Bosons})}$$

Gravitons
Gravitino ghosts

Supersymmetry $\Longrightarrow$ $\text{dets} = 1\ ?$

Hawking and Pope '78

(Using the Gibbons-Hawking-Perry prescription for rotating the conformal factor)

The One-Loop Determinants

A somewhat detailed calculation gives

$$\text{dets} = A\,(\mu^2)^{41/48}\left(\frac{1}{R^2}\right)^{7/48}$$

An ugly number

UV cut-off scale

Radius of the circle

The Superpotential

Putting all the pieces together gives

$$\mathcal{W} = C\left(\frac{\mu^2}{M_{\rm pl}^2}\right)^{41/48}\left(\frac{1}{M_{\rm pl}^2 R^2}\right)^{7/48} e^{-2\pi^2 M_{\rm pl}^2 R^2 - i\sigma}$$

$$C = \frac{\left(4e^{24\zeta'(-1)-1}\right)^{7/48}}{2(4\pi)^{3/2}}$$

Action of Taub-NUT

How to make sense of this?

- It's not holomorphic (in the naïve complex structure)
- It's UV divergent

Perturbative Quantum Corrections

Finite Quantum Corrections

The classical low-energy effective action is

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} M_3 \mathcal{R}_{(3)} - M_3 \left(\frac{\partial R}{R}\right)^2 - \frac{1}{M_3} \frac{L^2}{R^4} \left(\frac{\partial \sigma}{2\pi}\right)^2$$

One loop corrections to the kinetic terms give

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \frac{1}{2} \left(M_3 + \frac{5}{16\pi} \frac{L}{R^2}\right) \mathcal{R}_{(3)} - \left(M_3 - \frac{1}{6\pi} \frac{L}{R^2}\right) \left(\frac{\partial R}{R}\right)^2 \\ - \left(M_3 + \frac{11}{24\pi} \frac{L}{R^2}\right)^{-1} \frac{L^2}{R^4} \left(\frac{\partial \sigma}{2\pi}\right)^2\end{aligned}$$

Finite Quantum Corrections

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$$\mathcal{L}_{\text{eff}} = \frac{1}{2} M_3 \mathcal{R}_{(3)} - M_3 \left(\frac{\partial R}{R}\right)^2 - \frac{1}{M_3}\frac{L^2}{R^4}\left(\frac{\partial \sigma}{2\pi}\right)^2$$

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Something important: these two numbers are different!

The Complex Structure

The two fields *R* and σ must combine in a complex number

At one-loop

$$\mathcal{L}_{\rm eff} = \left(1 - \frac{1}{6\pi}\frac{L}{M_3 R^2}\right)\left(\frac{\partial R}{R}\right)^2 + \left(1 + \frac{11}{24\pi}\frac{L}{M_3 R^2}\right)^{-1}\frac{L^2}{R^4}\left(\frac{\partial\sigma}{2\pi}\right)^2$$

We want to write this in the form

$$\mathcal{L}_{\rm eff} = \frac{\partial^2 K(\mathcal{S},\mathcal{S}^\dagger)}{\partial S \partial S^\dagger}\,\partial\mathcal{S}\partial\mathcal{S}^\dagger$$

$$\Longrightarrow \quad \boxed{\mathcal{S} = 2\pi^2 M_{\rm pl}^2 R^2 + \frac{7}{48}\log(M_{\rm pl}^2 R^2) + i\sigma}$$

The Superpotential is now Holomorphic

$$\mathcal{W} = C\left(\frac{\mu^2}{M_{\rm pl}^2}\right)^{41/48}\left(\frac{1}{M_{\rm pl}^2 R^2}\right)^{7/48} e^{-2\pi^2 M_{\rm pl}^2 R^2 - i\sigma}$$

$$\Longrightarrow \qquad \mathcal{W} = C\left(\frac{\mu^2}{M_{\rm pl}^2}\right)^{41/48} e^{-\mathcal{S}}$$

With the one-loop corrected complex structure

$$\mathcal{S} = 2\pi^2 M_{\rm pl}^2 R^2 + \frac{7}{48}\log(M_{\rm pl}^2 R^2) + i\sigma$$

Divergences at One-Loop

't Hooft and Veltman '74
Deser, Kay and Stelle '77

At one-loop in pure gravity, there are three logarithmic divergences

$$\mathcal{R}^2 \quad , \quad \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} \quad , \quad \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

These two can be absorbed by a field redefinition of the metric

The Riemann[2] term can be massaged into Gauss-Bonnet.

$$\chi = \frac{1}{8\pi^2}\int d^4x\ \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$$

This is purely topological. It doesn't affect perturbative physics around flat space.

The Gauss-Bonnet Term

$$S_\alpha = \frac{\alpha}{8\pi^2} \int d^4x \; \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2 = \alpha\chi$$

The coupling runs logarithmically

$$\alpha(\mu') = \alpha(\mu) - \alpha_1 \log\left(\frac{\mu^2}{\mu'^2}\right)$$

where the *beta function* is given by

Christensen and Duff '78
Perry '78; Yoneya '78

$$\alpha_1 = \frac{1}{48 \cdot 15}\left(848 N_2 - 233 N_{3/2} - 52 N_1 + 7 N_{1/2} + 4 N_0\right)$$

For us…

$$\boxed{\alpha_1 = 41/48}$$

The Superpotential is now Finite

If we include the running of the Gauss-Bonnet term

$$\mathcal{W} = C\left(\frac{\mu^2}{M_{\rm pl}^2}\right)^{41/48} e^{-\mathcal{S}} e^{-\alpha(\mu)} = C\left(\frac{\Lambda_{\rm grav}^2}{M_{\rm pl}^2}\right)^{41/48} e^{-\mathcal{S}}$$

With the RG invariant scale

$$\Lambda_{\rm grav} = \mu \exp\left(-\frac{\alpha(\mu)}{2\alpha_1}\right)$$

Note: in supergravity $\Lambda_{\rm grav}$ is naturally complexified by the gravitational theta angle.

Conclusions

Kaluza-Klein compactification of N=1 supergravity is unstable

$$\mathcal{W} = C \left(\frac{\Lambda_{\rm grav}^2}{M_{\rm pl}^2} \right)^{41/48} e^{-\mathcal{S}}$$

The superpotential depends on a "hidden" scale $\Lambda_{\rm grav}$

Where else does this scale show up?

Happy Birthday Gary!

Extra Material

The Boundary of the Space

The boundary of Taub-NUT is not the same as the boundary of flat space.

$$\partial(\mathbb{R}^3 \times \mathbf{S}^1) = \mathbf{S}^2 \times \mathbf{S}^1 \quad \text{but} \quad \partial(\mathrm{TN}_k) = \mathbf{S}^3/\mathbf{Z}_k$$

Should we include such geometries in the path integral?

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Should we include such geometries in the path integral?

Yes!

c.f. Atiyah-Hitchin with boundary a circle fibre over $\boldsymbol{RP}^2$ for which the answer is no!

The Determinants

In a self-dual background, you can write the determinants as

$$\text{dets} = \left.\frac{\det' \not{D}^\dagger \not{D}}{\det \not{D} \not{D}^\dagger}\right|^{1/4}_{\text{spin}-3/2} \left.\frac{\det' \not{D}^\dagger \not{D}}{\det \not{D} \not{D}^\dagger}\right|^{-1/2}_{\text{spin}-1/2}$$

The Superpotential

The calculation gives

$$\mathcal{W} = C\left(\frac{\mu^2}{M_{\rm pl}^2}\right)^{41/48}\left(\frac{1}{M_{\rm pl}^2 R^2}\right)^{7/48} e^{-S_{\rm TN}-i\sigma} e^{-\tau_{\rm grav}}$$

$$C = \frac{\left(4e^{24\zeta'(-1)-1}\right)^{7/48}}{2(4\pi)^{3/2}}$$

$$S_{\rm TN} = 2\pi^2 M_{\rm pl}^2 R^2$$

Topological terms

All the pieces now fit together

$$\boxed{\mathcal{W} = C\left(\frac{\Lambda_{\rm grav}^2}{M_{\rm pl}^2}\right)^{41/48} e^{-\mathcal{S}}}$$

with $\mathcal{S} = 2\pi^2 M_{\rm pl}^2 R^2 + \frac{7}{48}\log(M_{\rm pl}^2 R^2) + i\sigma$

First…to

Henriette Elvang
Veronika Hubeny
Don Marolf

Thank you!