Title: Entanglement entropy in two dimensional string theory

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Abstract:

Entanglement entropy in 2D string theory

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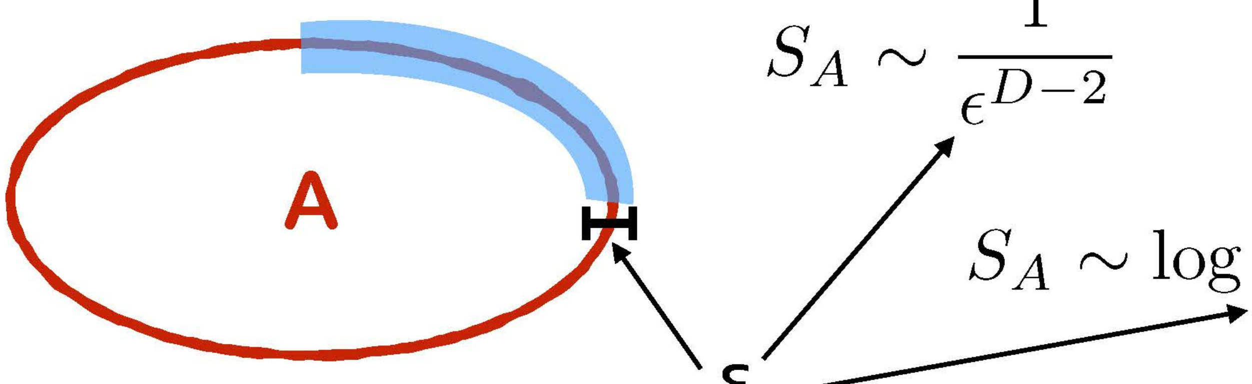
Emergent spacetime

- Suppose we are handed the ground state wavefunction of a quantum mechanical system that is supposed to describe an emergent semiclassical spacetime.
- Eg. ground state of N=4 SYM or the BFSS matrix quantum mechanics, perhaps.
- How would we 'see' the emergent spacetime in the wavefunction?

Entanglement + locality

- **Want to see that low energy excitations can be organized as local quantum fields in an emergent spacetime.**
- **Local QFTs are characterized by a large amount of short distance entanglement.**

[Bombelli et al., Srednicki, Callan-Wilczek,...]:



$$S_A \sim \frac{1}{\epsilon^{D-2}}$$

$$S_A \sim \log \epsilon$$

Entanglement + locality

- Can we find this accumulation of entanglement in the wavefunction?
- Looking for the entanglement of fields propagating in the bulk, not the entanglement of the microscopic d.o.f. that 'make up the spacetime' (cf. Bekenstein-Hawking and Ryu-Takayanagi entropies). However, these two types of entanglement are likely closely connected.

[Susskind-Uglum, Fiola-Preskill-Strominger-Trivedi, Bianchi-Myers, Faulkner-Lewkowycz-Maldacena, …].

2D string theory

- **The baby cousin of AdS/CFT.**

Worldsheet theory:
Liouville + c=1

Matrix Quantum Mechanics ⟷ **2D Einstein-Dilaton-'Tachyon'**

$$S = \beta N \int dt \, \mathrm{tr} \left[\frac{1}{2} \dot{M}^2 + V(M) \right]$$

$$S = \int dt dx \sqrt{-g} e^{-2\Phi} \left(R + 4(\nabla\Phi)^2 + 16 - (\nabla T)^2 + 4T^2 + \cdots \right)$$

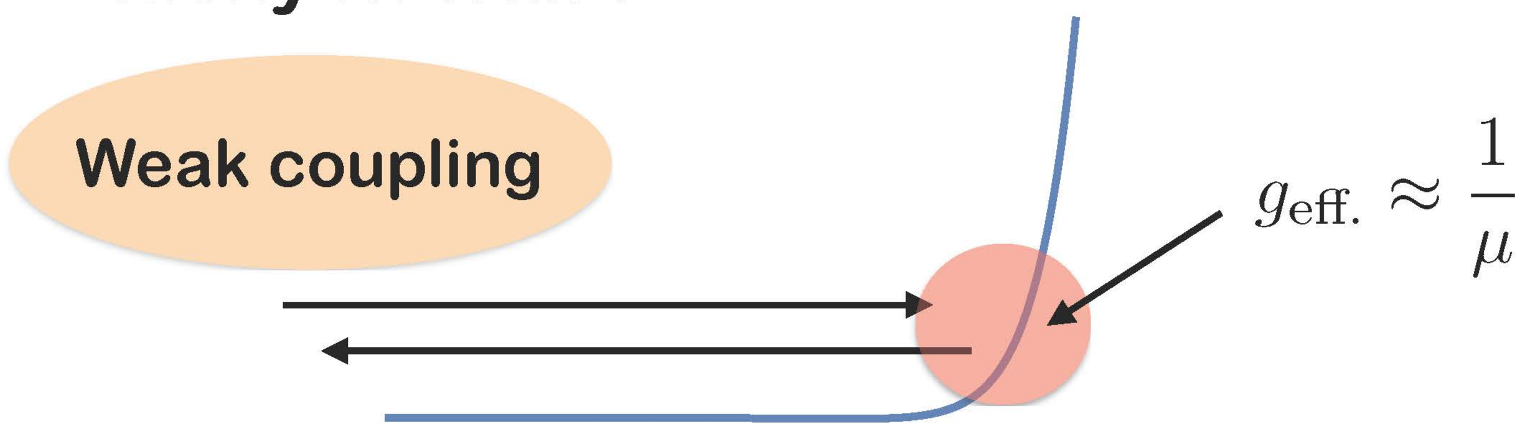
[Reviews: Klebanov, Ginsparg-Moore, Polchinski]

Spacetime physics

- **Linear dilaton background (as x→−∞):**

$$g_{\mu\nu} = \eta_{\mu\nu}\,, \quad \Phi = 2x + \cdots\,, \quad T = \mu\left(x + \frac{\log\mu}{2}\right)e^{2x} + \cdots\,.$$

- **String coupling:** $g_s = e^{\Phi} \sim e^{2x}$
- **Basic physics: scattering off the 'tachyon wall'.**

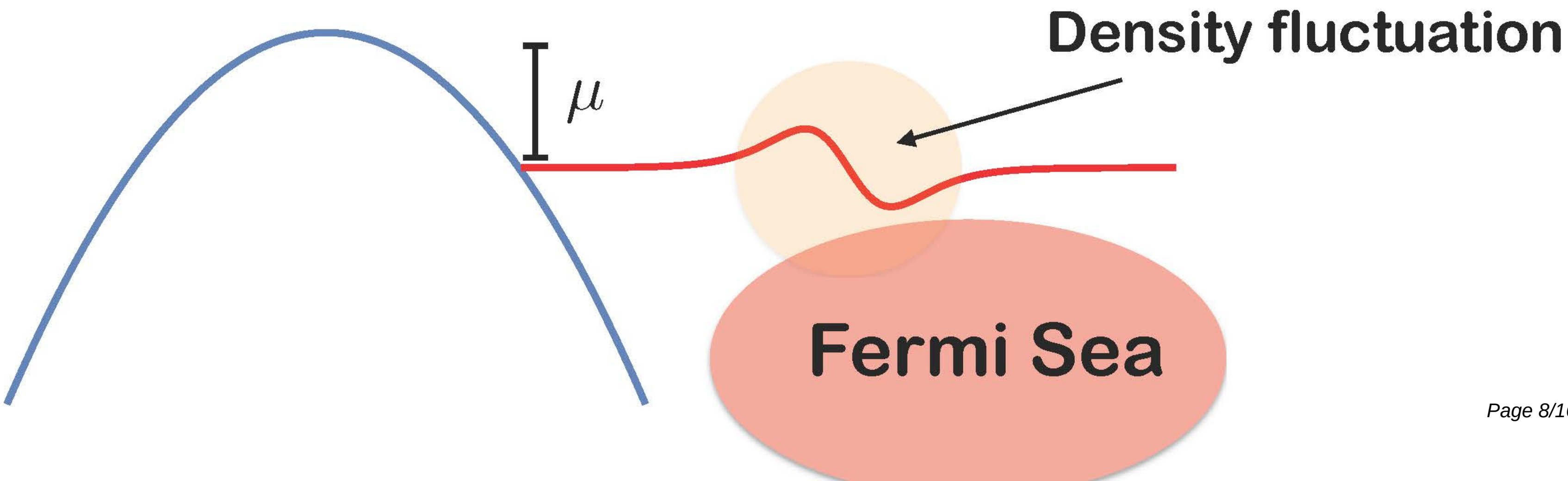


Eigenvalue physics

- **Diagonalize the matrix**
  $\Rightarrow$ **N non-interacting fermions in V(λ).**

- **Scaling limit:**

$$N \to \infty\,, \quad \mu = \beta N(\epsilon_c - \epsilon_F) \quad \text{fixed}\,.$$



Duality map

- Morally: spacetime tachyon
  $\longleftrightarrow$ collective density fluctuation

  [<u>Not quite</u>: non-dispersing string states mean actual map is nonlocal. ‘Leg poles’ etc.]

  [Moore-Seiberg]
- Coordinate map:

$$x = \tau(\lambda) \equiv -\frac{1}{\sqrt{2}} \int_{\lambda_\star}^{\lambda} \frac{d\lambda'}{\sqrt{\frac{1}{2}\lambda'^2 - \mu}} \sim -\log\lambda$$

- Large μ limit:
  (1) Weak string coupling.
  (2) WKB limit of fermion dynamics.

Main result

- Spacetime expectation. Tachyon is a massless field, expect:

Interval

$$S_{\Delta x} = \frac{1}{3} \log \frac{\Delta x}{\epsilon}$$

Cutoff?

- We have computed the entanglement of the eigenvalues in an interval $[\lambda_1, \lambda_2]$:

$$\boxed{S_{[\lambda_1,\lambda_2]} = \frac{1}{3} \log \frac{\tau(\lambda_2) - \tau(\lambda_1)}{\sqrt{g_s(\lambda_1) g_s(\lambda_2)/\mu}}}$$

[Cf. Das '95]

[Cf. McGreevy-Verlinde]

Free fermion entanglement

- **Second quantized description:**

$$\Psi(t,\lambda) = \int_{-\infty}^{\infty} d\nu\, e^{i\nu t} a(\nu)\psi_\nu(\lambda)\,.$$

- **Reduced density matrix known explicitly:**

$$\rho_{[\lambda_1,\lambda_2]} = \exp\left(-\int_{\lambda_1}^{\lambda_2} dx_1 dx_2 \Psi^\dagger(x_1)\mathcal{H}(x_1,x_2)\Psi(x_2)\right)\,.$$

$$\mathcal{H} = \log\frac{1-C}{C}\,, \qquad C(x_1,x_2) \equiv \langle \Psi^\dagger(x_1)\Psi(x_2)\rangle$$

[Peschel, Klich]

Free fermion entanglement

- Expansion in terms of density fluctuations

$$S_A = \frac{\pi^2}{3} V_A^{(2)} + \frac{\pi^4}{45} V_A^{(4)} + \frac{2\pi^6}{945} V_A^{(6)} + \cdots$$

[Klich-Levitov]

- Eg:

$$V_A^{(2)} = \int_A d\lambda d\lambda' \Big(\langle n(\lambda) n(\lambda') \rangle - \langle n(\lambda) \rangle \langle n(\lambda') \rangle \Big)$$

$$n(\lambda) = \Psi^\dagger(\lambda) \Psi(\lambda)$$

Free fermion entanglement

- In WKB limit, known that leading log behavior captured by $V^{(2)}$ alone!

[Calabrese-Mintchev-Vicari]

- Take the WKB wavefunctions and do the integrals!

$$\psi_\nu(\lambda) = \frac{\sqrt{2}}{\sqrt{\pi p}} \sin\left(\int_{\sqrt{2\nu}}^{\lambda} p\, d\lambda - \frac{\pi}{4}\right)$$

$$p = \sqrt{\lambda^2 - 2\nu}$$

Summary

- Entanglement entropy allows us to 'see' the emergence of local physics from the Matrix Quantum Mechanics.

- Locality down to a scale set by the (weak) string coupling constant.

- Technical tool: reduced density matrices for free fermions + integrals of WKB wavefuntions.

Comments

- Singlet sector of single matrix QM too simple to access the ‘deeper’ questions of e.g. sub-AdS scale locality.
- Closely related: no genuine black holes.
- Hopefully, a first step and proof of concept towards unravelling the entanglement in more complicated MQM.
- Approaches such as MERA, that describe AdS-scale locality, need to be fleshed out with this kind of large N locality.


HAPPY
BIRTHDAY