Title: AdS/CFT and the geometry of confinement

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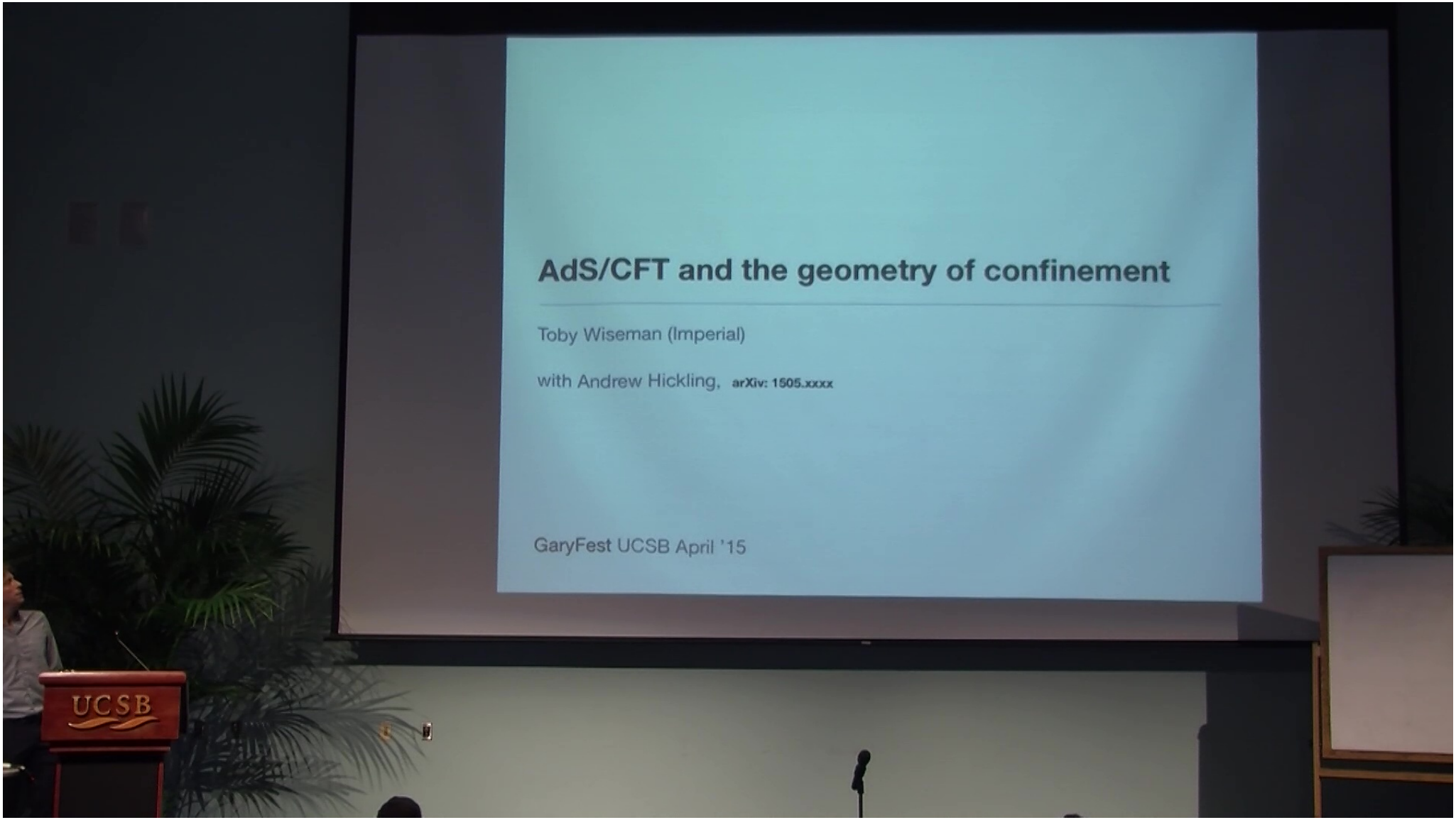
Abstract:

Toby Wiseman

AdS/CFT and the Geometry of Confinement


AdS/CFT and the geometry of confinement
Toby Wiseman (Imperial)
with Andrew Hickling, arXiv: 1505.xxxx
GaryFest UCSB April '15
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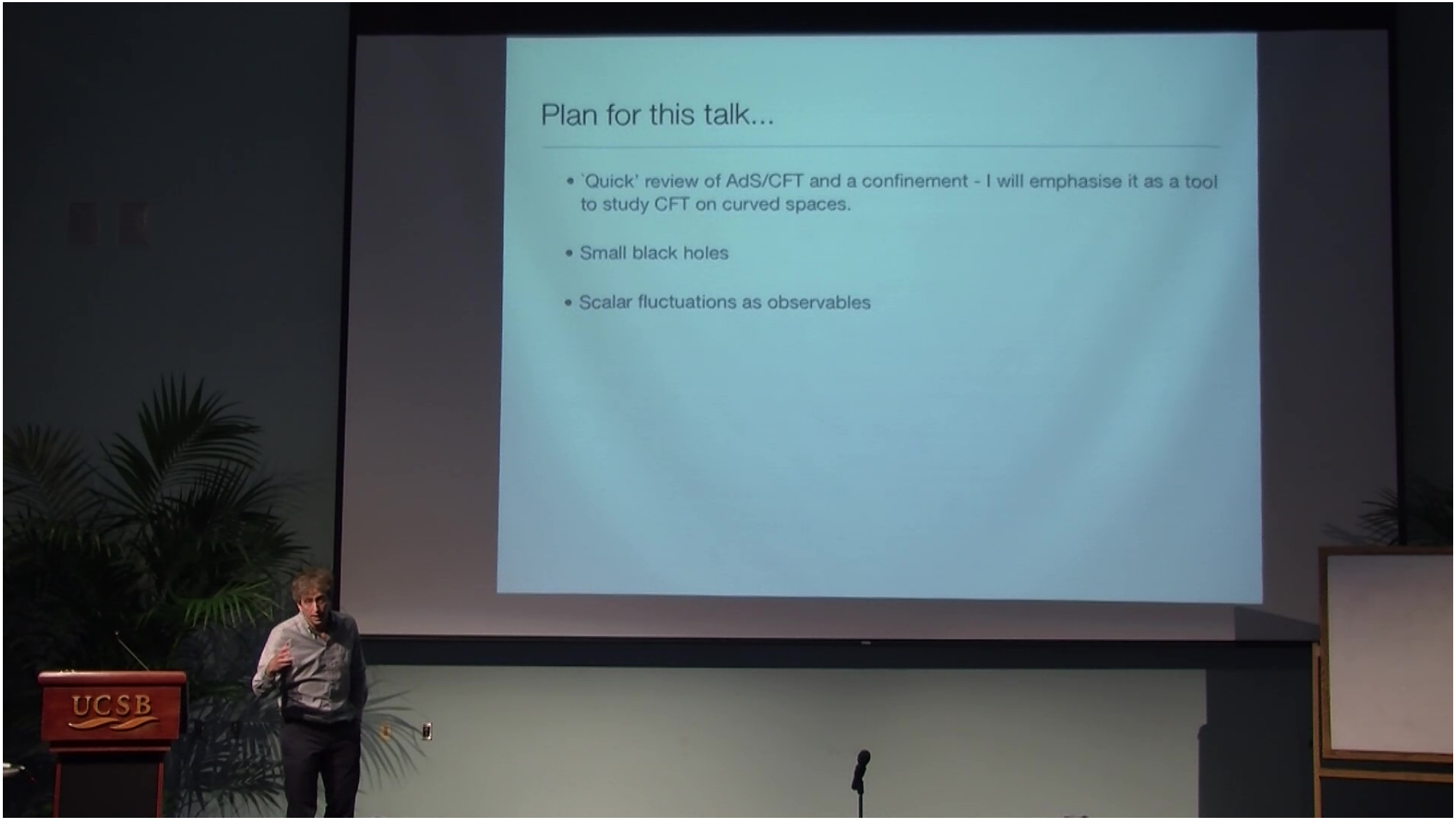
Plan for this talk...
• `Quick' review of AdS/CFT and a confinement - I will emphasise it as a tool to study CFT on curved spaces.
• Small black holes
• Scalar fluctuations as observables
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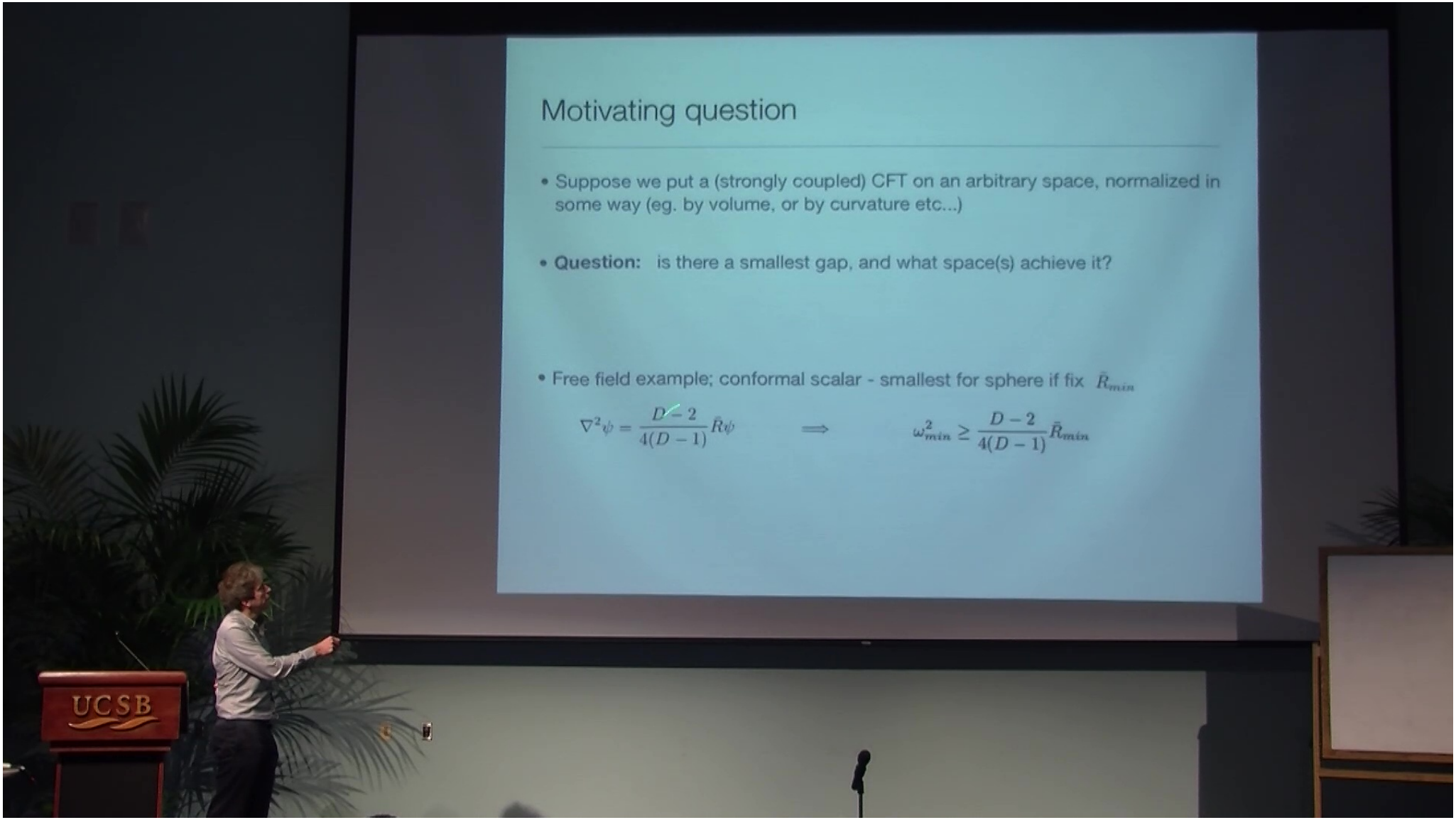
Motivating question
• Suppose we put a (strongly coupled) CFT on an arbitrary space, normalized in some way (eg. by volume, or by curvature etc...)
• Question: is there a smallest gap, and what space(s) achieve it?
• Free field example; conformal scalar - smallest for sphere if fix $\tilde{R}_{min}$
$\nabla^2\psi = \frac{D-2}{4(D-1)}\tilde{R}\psi \quad \Longrightarrow \quad \omega^2_{min} \geq \frac{D-2}{4(D-1)}\tilde{R}_{min}$
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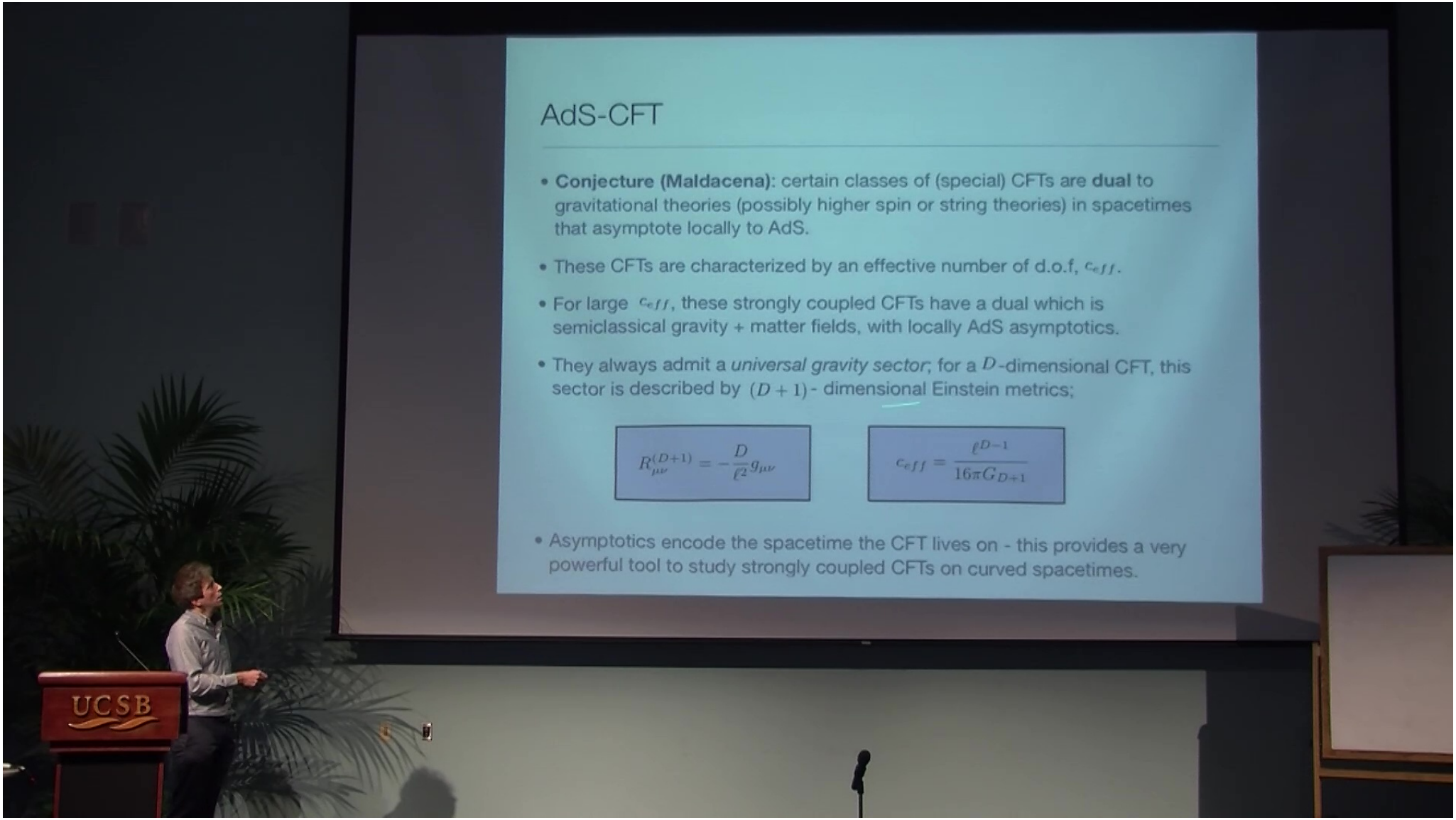
AdS-CFT
• Conjecture (Maldacena): certain classes of (special) CFTs are dual to gravitational theories (possibly higher spin or string theories) in spacetimes that asymptote locally to AdS.
• These CFTs are characterized by an effective number of d.o.f, c_{eff}.
• For large c_{eff}, these strongly coupled CFTs have a dual which is semiclassical gravity + matter fields, with locally AdS asymptotics.
• They always admit a *universal gravity sector*; for a D-dimensional CFT, this sector is described by $(D+1)$- dimensional Einstein metrics;
$R_{\mu\nu}^{(D+1)} = -\frac{D}{\ell^2} g_{\mu\nu}$
$c_{eff} = \frac{\ell^{D-1}}{16\pi G_{D+1}}$
• Asymptotics encode the spacetime the CFT lives on - this provides a very powerful tool to study strongly coupled CFTs on curved spacetimes.
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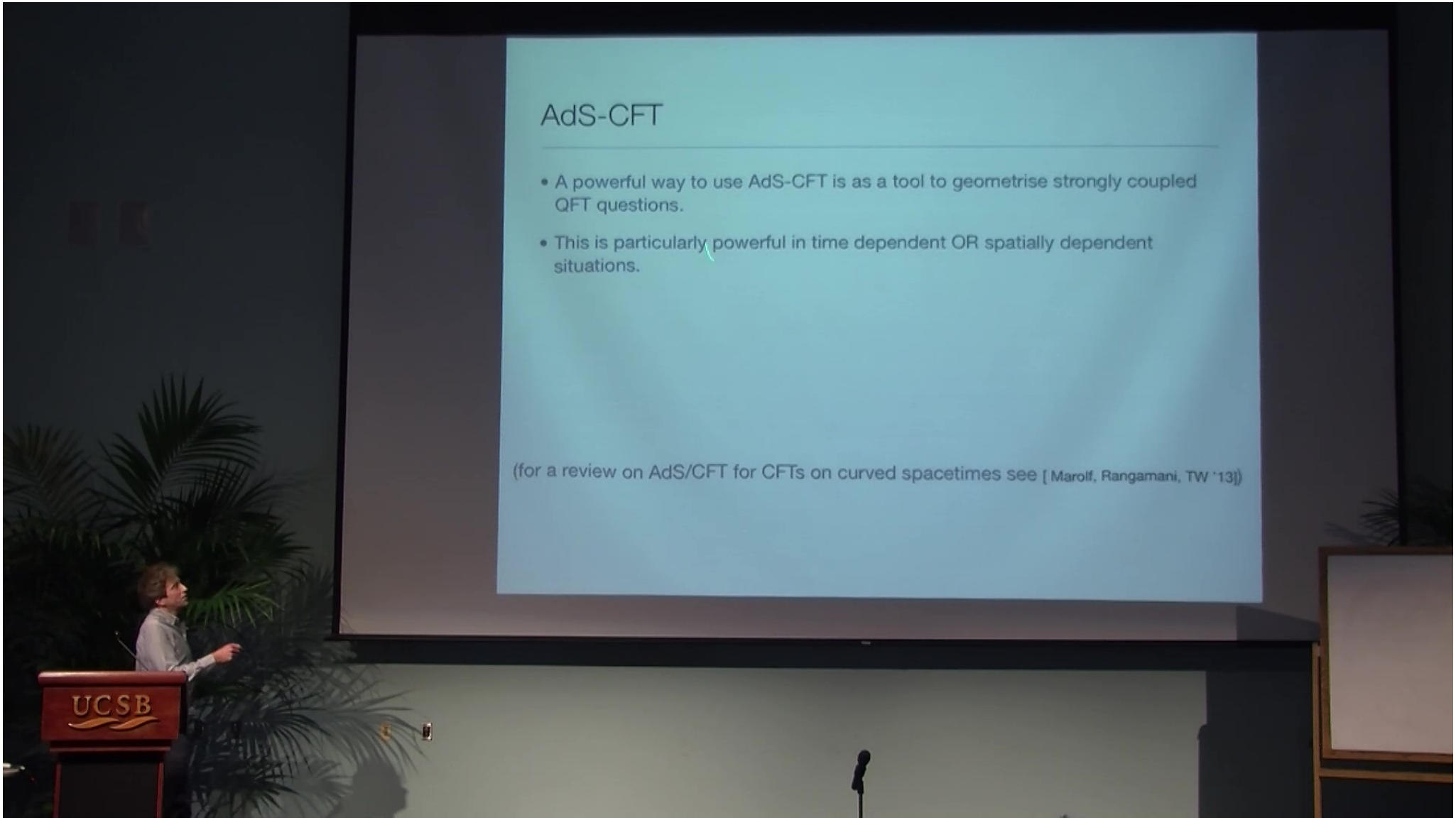
AdS-CFT
• A powerful way to use AdS-CFT is as a tool to geometrise strongly coupled QFT questions.
• This is particularly powerful in time dependent OR spatially dependent situations.
(for a review on AdS/CFT for CFTs on curved spacetimes see [Marolf, Rangamani, TW '13])
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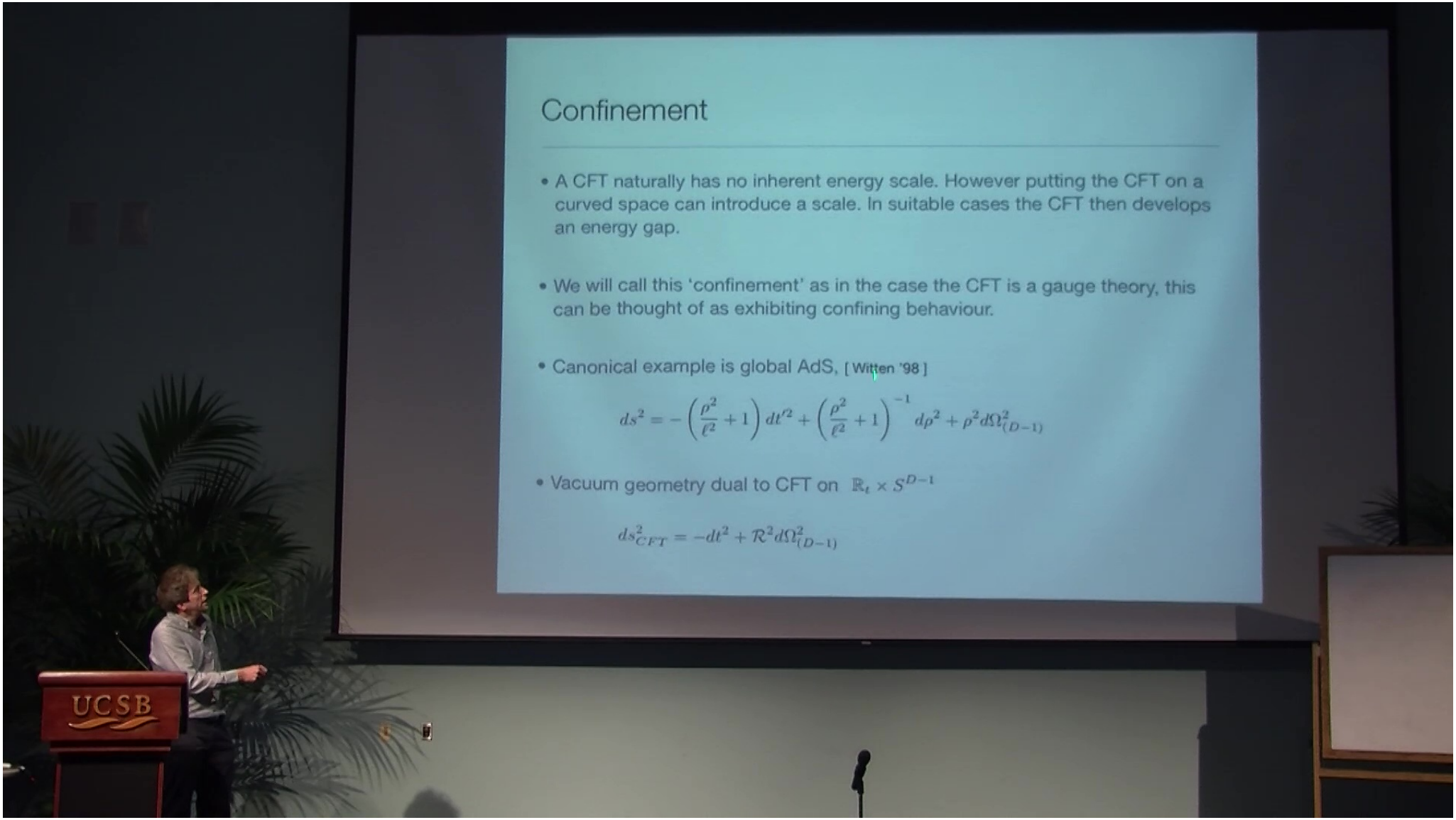
Confinement
• A CFT naturally has no inherent energy scale. However putting the CFT on a curved space can introduce a scale. In suitable cases the CFT then develops an energy gap.
• We will call this 'confinement' as in the case the CFT is a gauge theory, this can be thought of as exhibiting confining behaviour.
• Canonical example is global AdS, [Witten '98]
$$ds^2 = -\left(\frac{\rho^2}{\ell^2}+1\right)dt'^2 + \left(\frac{\rho^2}{\ell^2}+1\right)^{-1} d\rho^2 + \rho^2 d\Omega^2_{(D-1)}$$
• Vacuum geometry dual to CFT on $\mathbb{R}_t \times S^{D-1}$
$$ds^2_{CFT} = -dt^2 + \mathcal{R}^2 d\Omega^2_{(D-1)}$$
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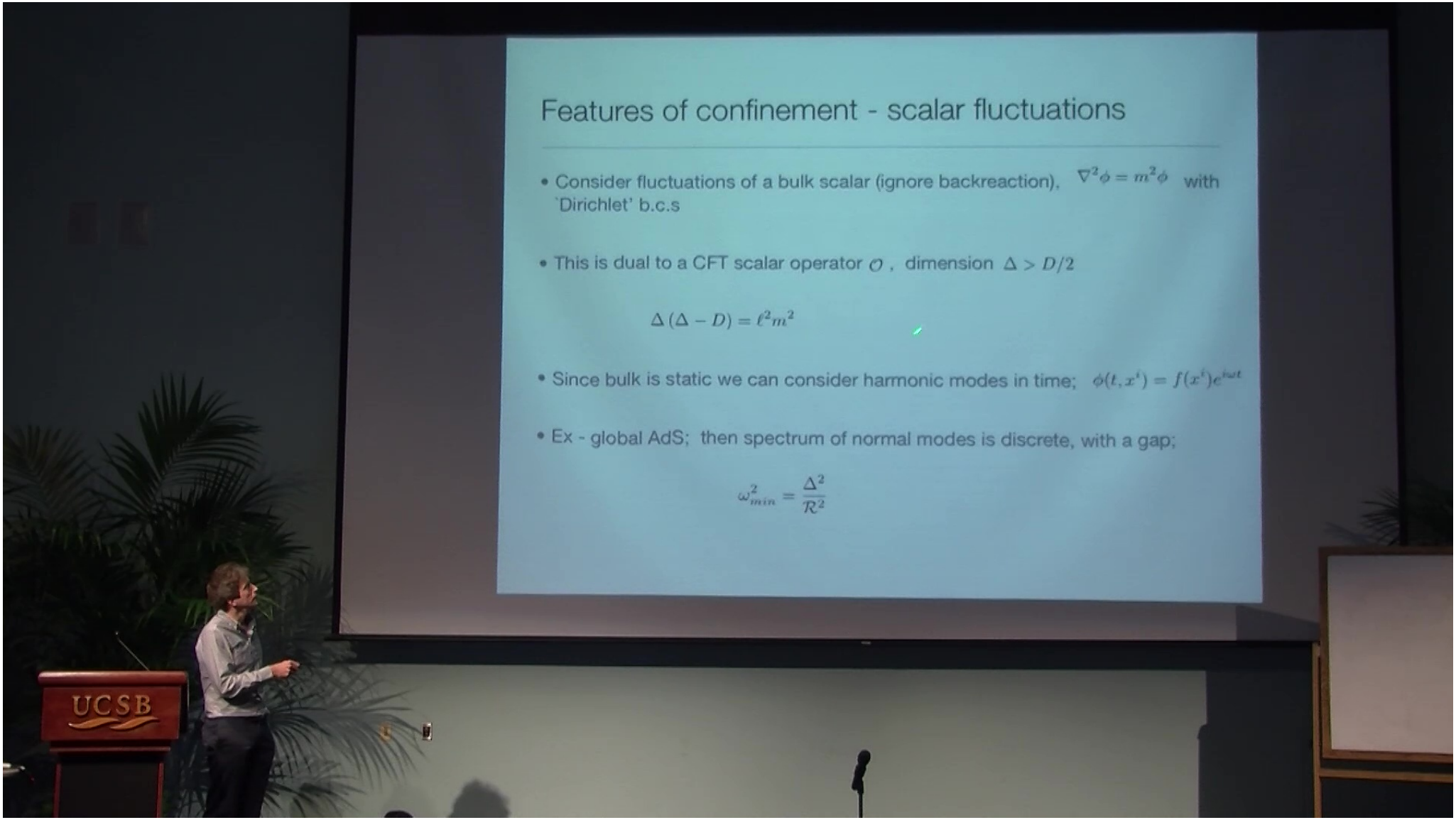
Features of confinement - scalar fluctuations
• Consider fluctuations of a bulk scalar (ignore backreaction), $\nabla^2\phi = m^2\phi$ with `Dirichlet' b.c.s
• This is dual to a CFT scalar operator $\mathcal{O}$, dimension $\Delta > D/2$
$\Delta(\Delta - D) = \ell^2 m^2$
• Since bulk is static we can consider harmonic modes in time; $\phi(t, x^i) = f(x^i)e^{i\omega t}$
• Ex - global AdS; then spectrum of normal modes is discrete, with a gap;
$\omega^2_{min} = \frac{\Delta^2}{\mathcal{R}^2}$
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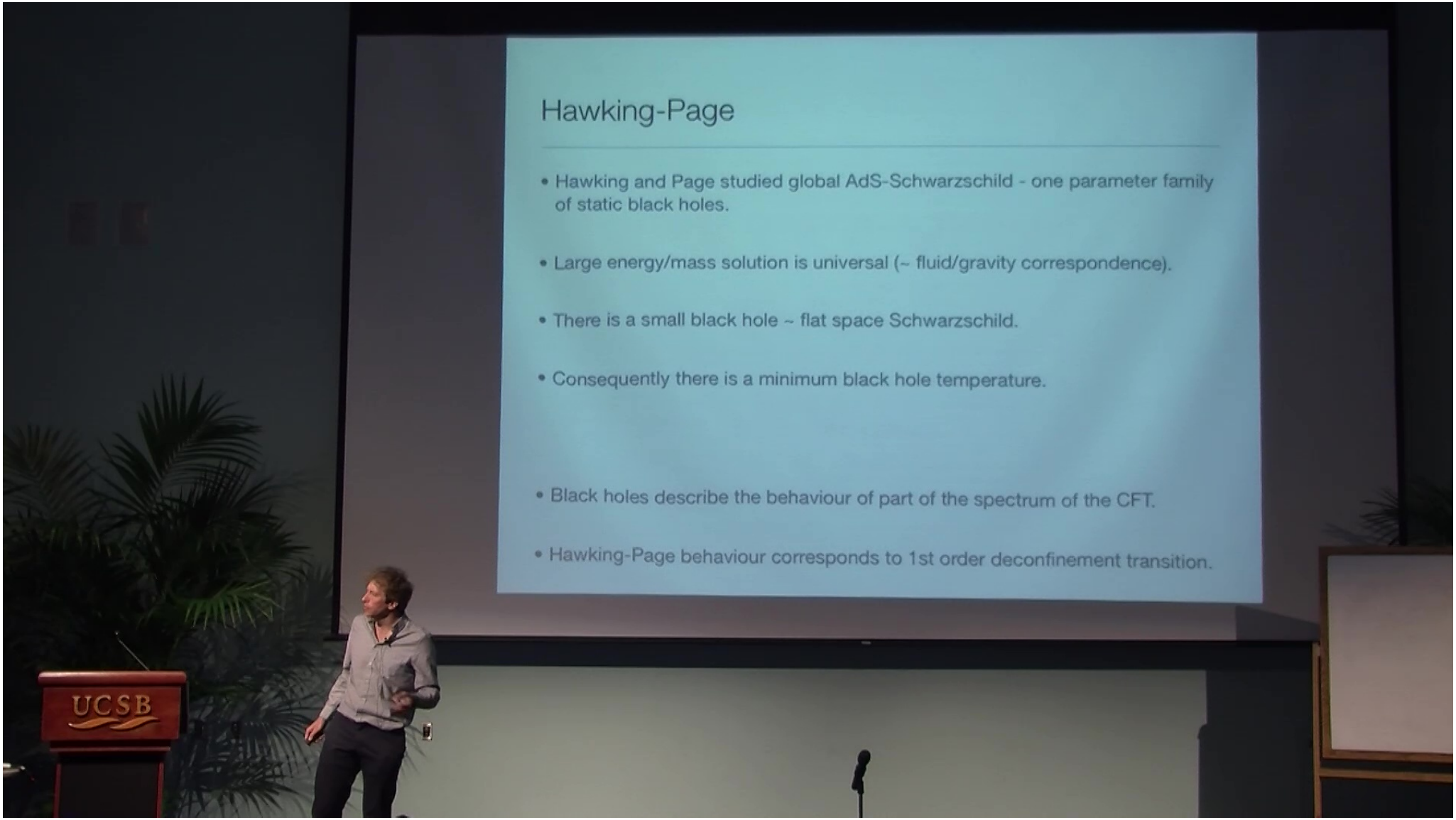
Hawking-Page
• Hawking and Page studied global AdS-Schwarzschild - one parameter family of static black holes.
• Large energy/mass solution is universal (~ fluid/gravity correspondence).
• There is a small black hole ~ flat space Schwarzschild.
• Consequently there is a minimum black hole temperature.
• Black holes describe the behaviour of part of the spectrum of the CFT.
• Hawking-Page behaviour corresponds to 1st order deconfinement transition.
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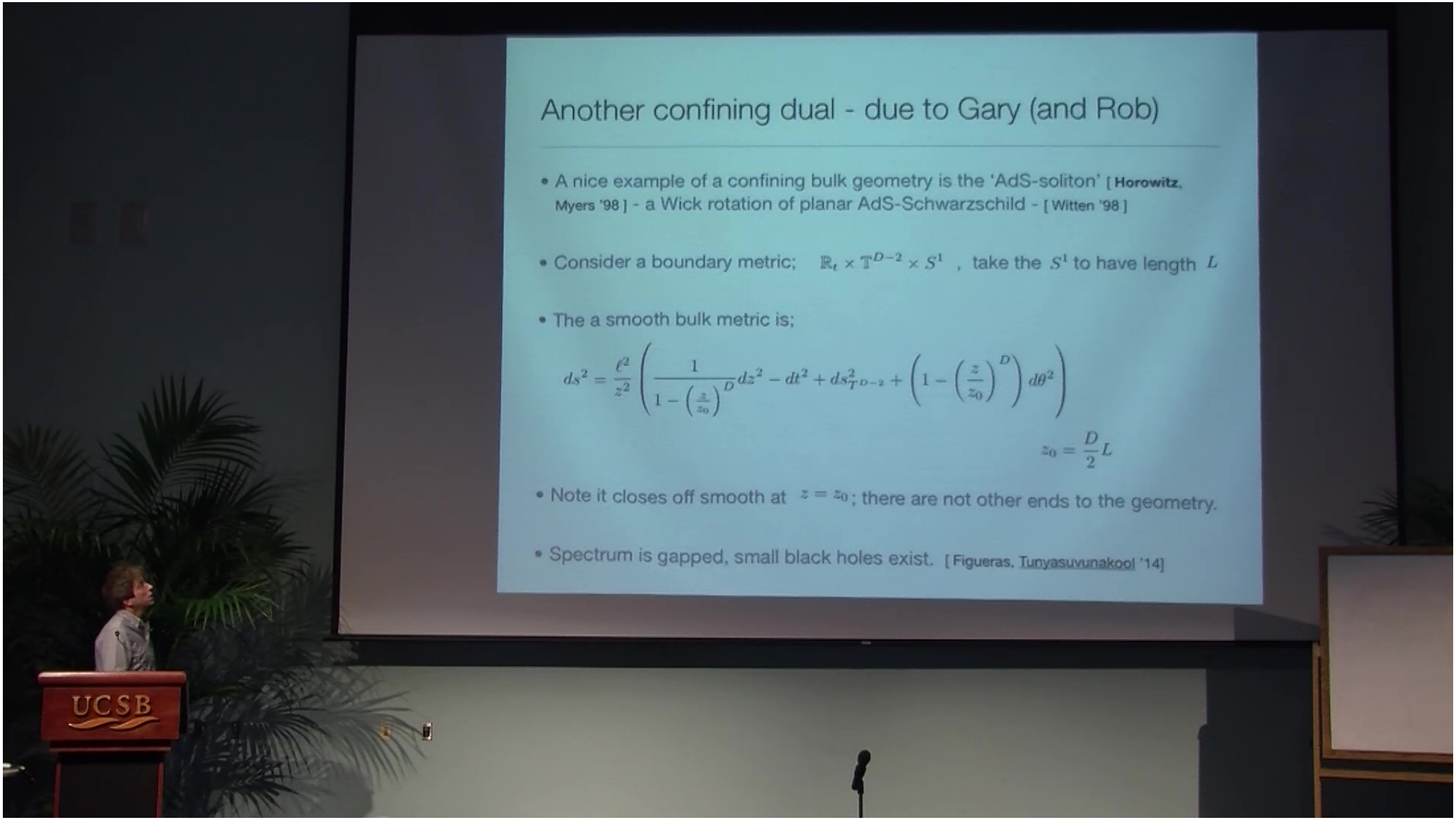
Another confining dual - due to Gary (and Rob)
• A nice example of a confining bulk geometry is the 'AdS-soliton' [Horowitz, Myers '98] - a Wick rotation of planar AdS-Schwarzschild - [Witten '98]
• Consider a boundary metric; $\mathbb{R}_t \times \mathbb{T}^{D-2} \times S^1$, take the S^1 to have length L
• The a smooth bulk metric is;
$$ds^2 = \frac{\ell^2}{z^2}\left(\frac{1}{1-\left(\frac{z}{z_0}\right)^D}dz^2 - dt^2 + ds^2_{T^{D-2}} + \left(1-\left(\frac{z}{z_0}\right)^D\right)d\theta^2\right)$$
$$z_0 = \frac{D}{2}L$$
• Note it closes off smooth at $z = z_0$; there are not other ends to the geometry.
• Spectrum is gapped, small black holes exist. [Figueras, Tunyasuvunakool '14]
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'Gary the Kid'
$\theta \to it$
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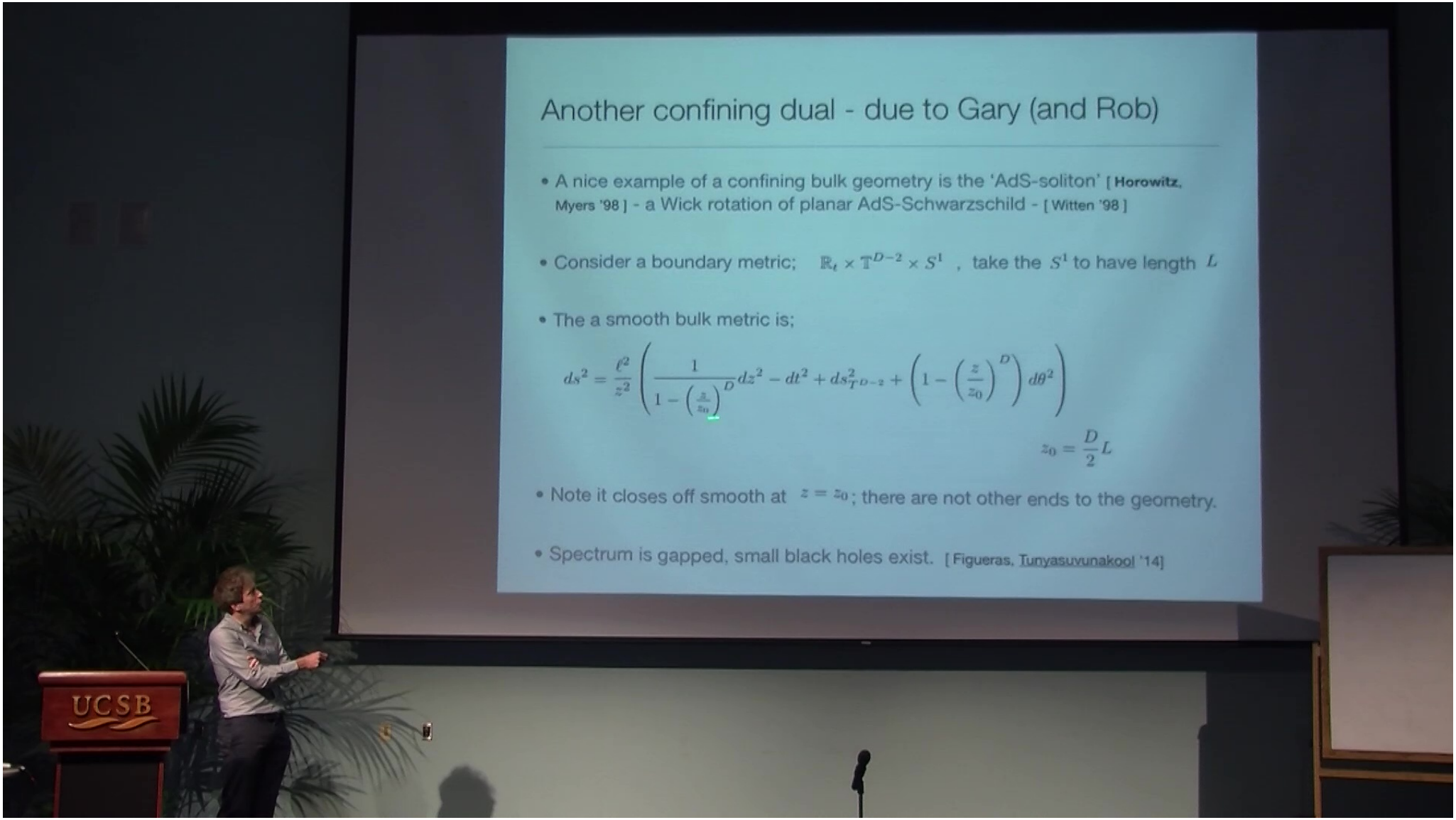
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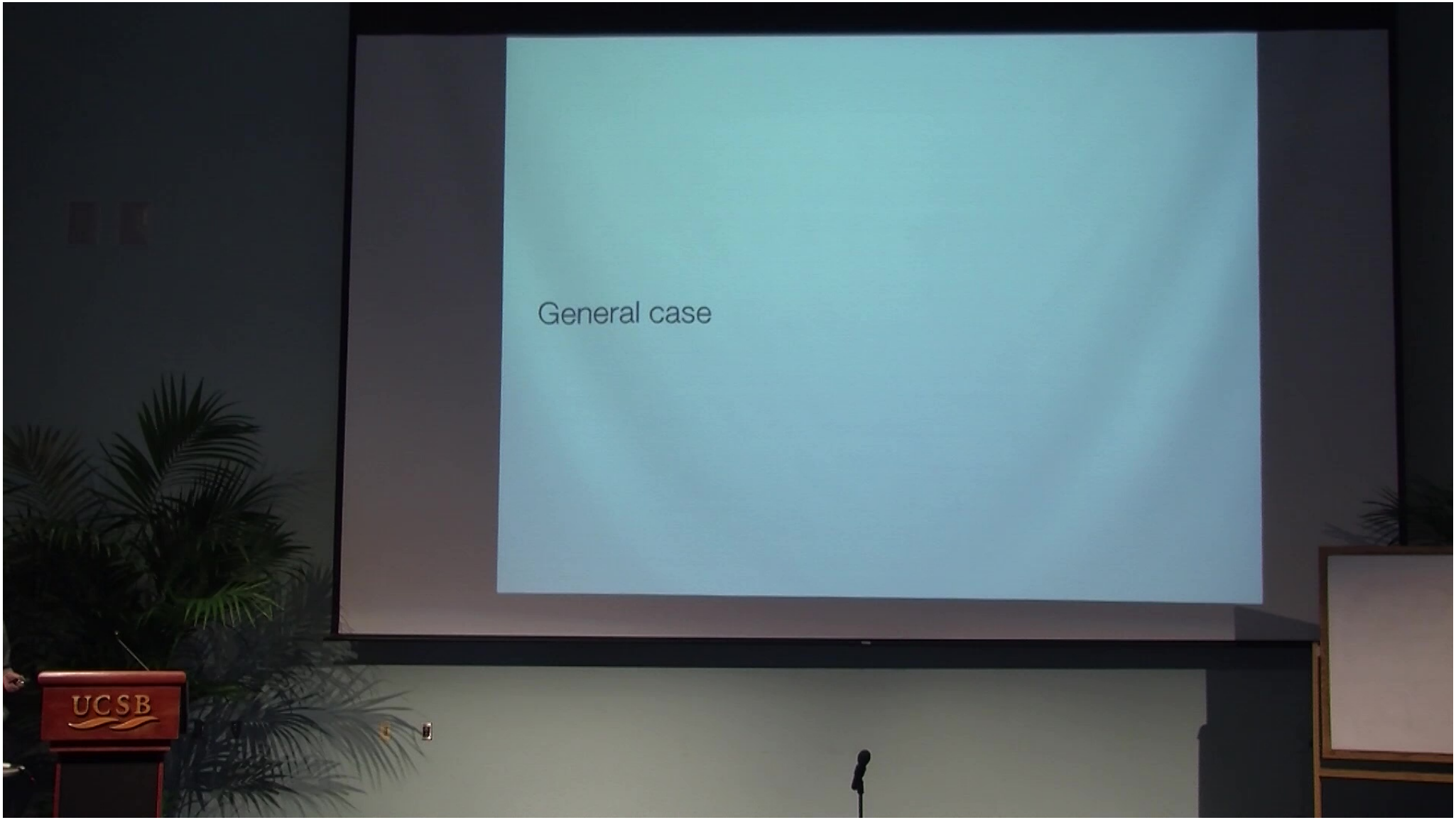
General case
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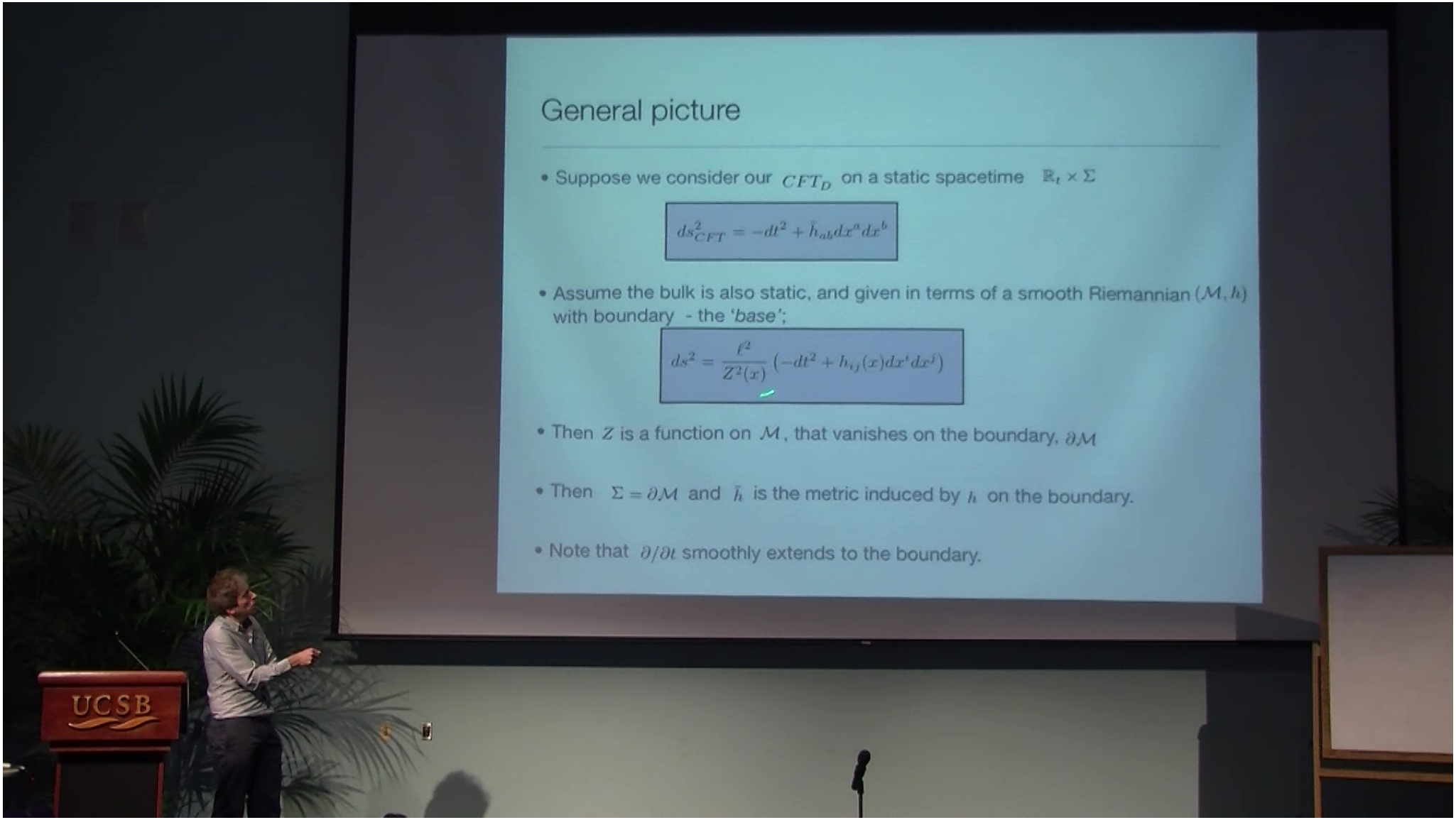
General picture
• Suppose we consider our CFT_D on a static spacetime $\mathbb{R}_t \times \Sigma$
$ds^2_{CFT} = -dt^2 + \bar{h}_{ab}dx^a dx^b$
• Assume the bulk is also static, and given in terms of a smooth Riemannian $(\mathcal{M}, h)$ with boundary - the 'base';
$ds^2 = \frac{\ell^2}{Z^2(x)}\left(-dt^2 + h_{ij}(x)dx^i dx^j\right)$
• Then Z is a function on $\mathcal{M}$, that vanishes on the boundary, $\partial\mathcal{M}$
• Then $\Sigma = \partial\mathcal{M}$ and $\bar{h}$ is the metric induced by h on the boundary.
• Note that $\partial/\partial t$ smoothly extends to the boundary.
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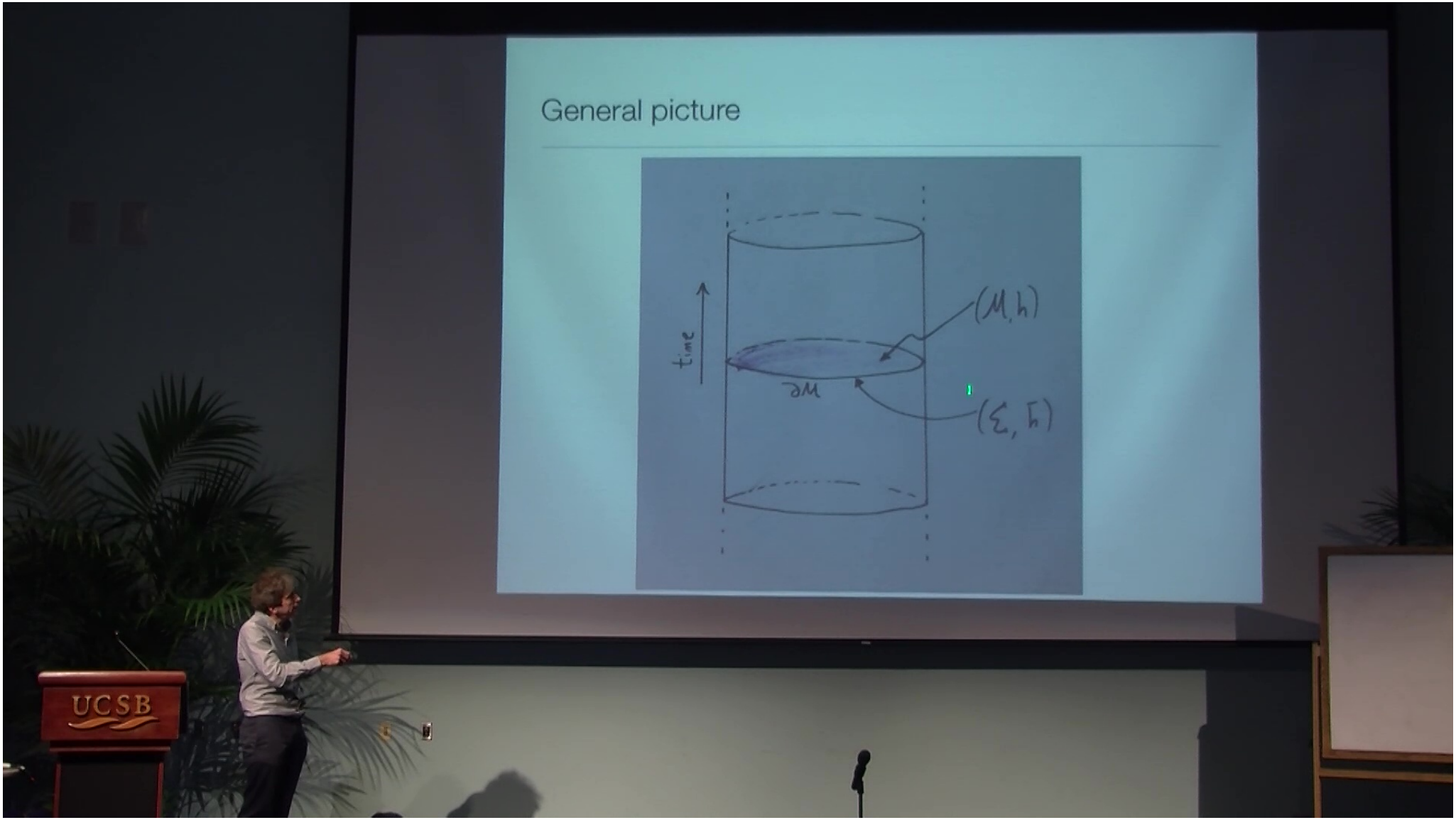
General picture
time
$(\mathcal{M}, h)$
$\partial\mathcal{M}$
$(\Sigma, \bar{h})$
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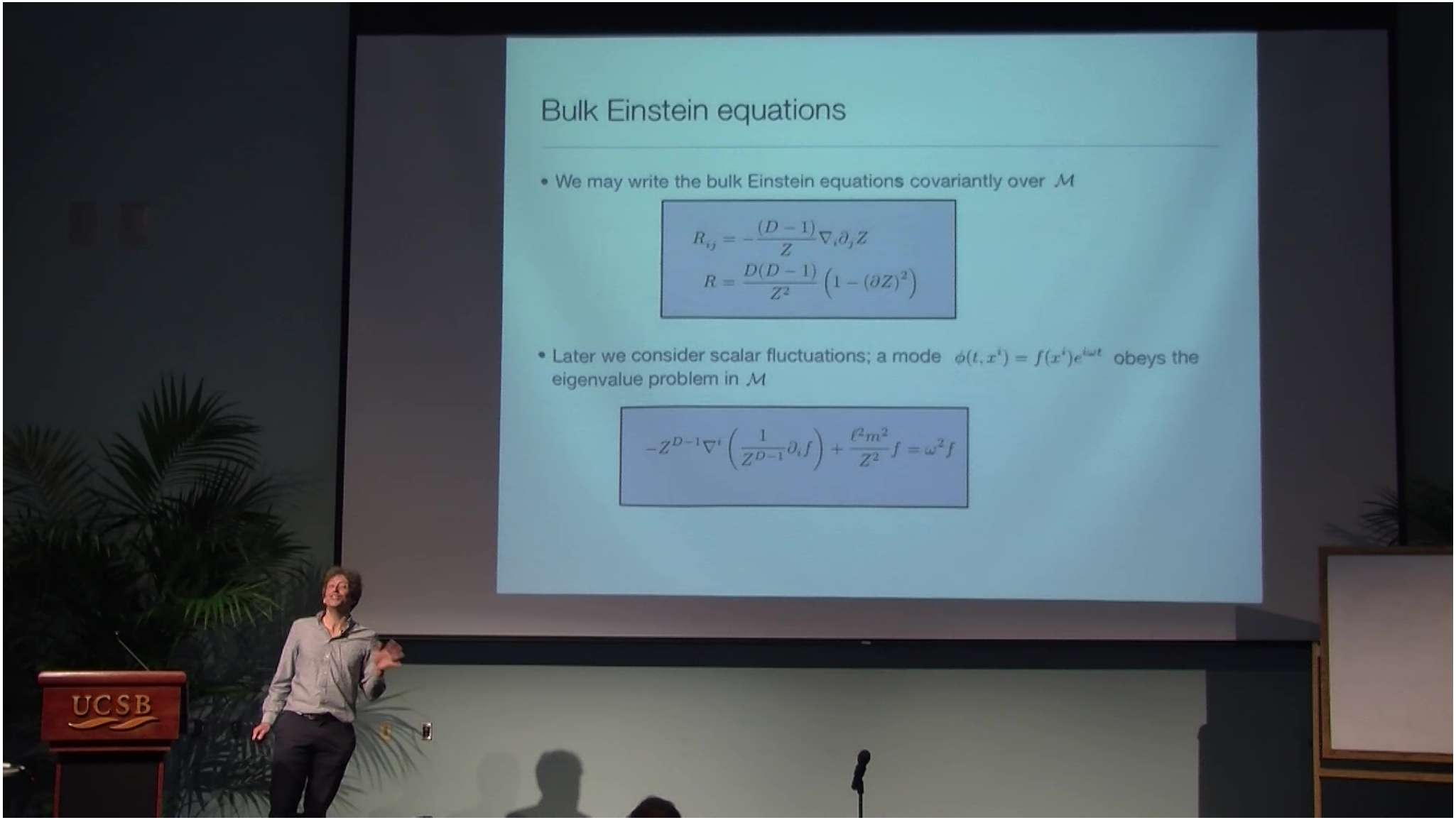
Bulk Einstein equations
• We may write the bulk Einstein equations covariantly over $\mathcal{M}$
$R_{ij} = -\frac{(D-1)}{Z}\nabla_i\partial_j Z$
$R = \frac{D(D-1)}{Z^2}\left(1-(\partial Z)^2\right)$
• Later we consider scalar fluctuations; a mode $\phi(t,x^i) = f(x^i)e^{i\omega t}$ obeys the eigenvalue problem in $\mathcal{M}$
$-Z^{D-1}\nabla^i\left(\frac{1}{Z^{D-1}}\partial_i f\right) + \frac{\ell^2 m^2}{Z^2} f = \omega^2 f$
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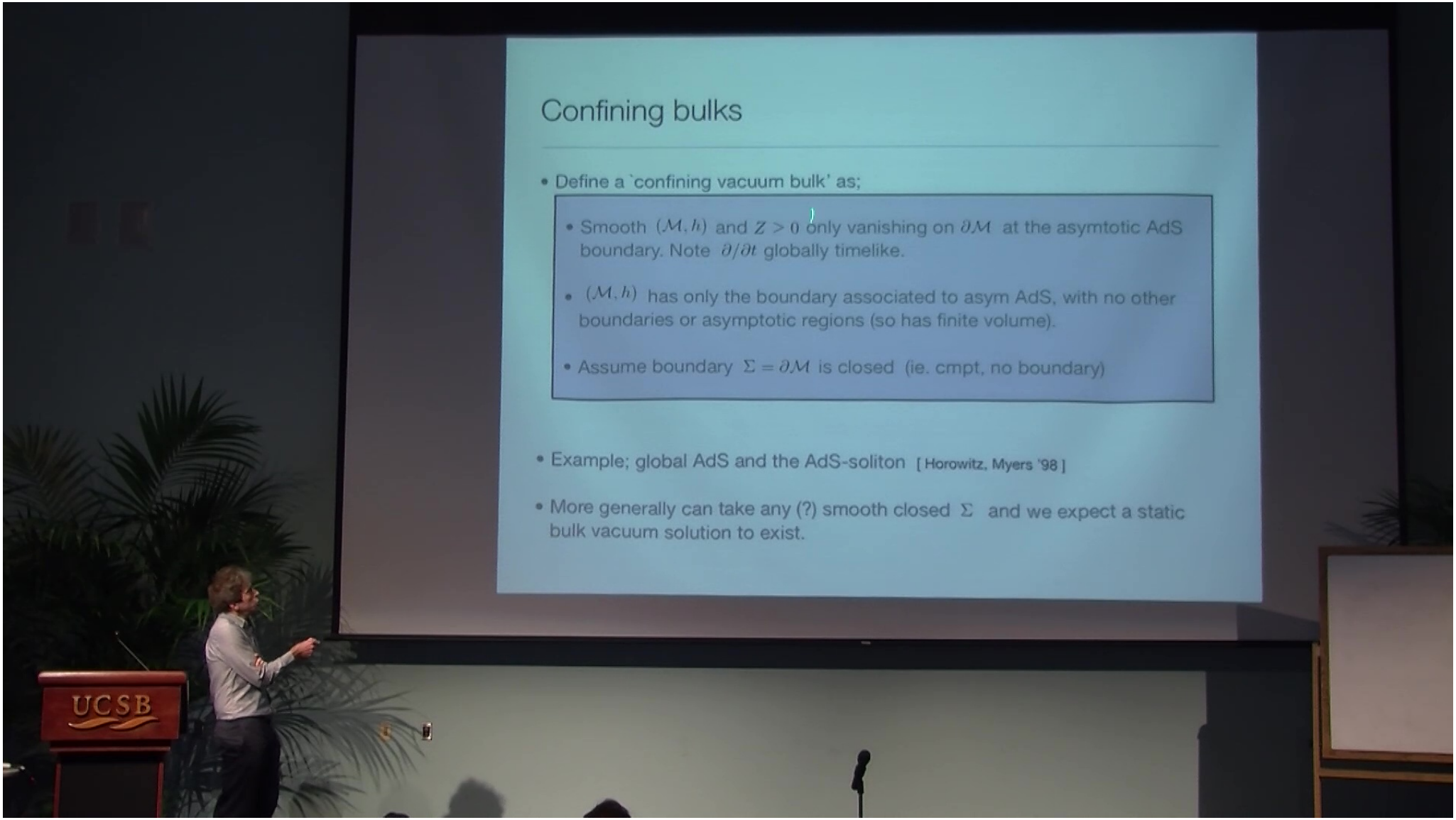
Confining bulks
• Define a `confining vacuum bulk' as;
• Smooth $(\mathcal{M}, h)$ and $Z > 0$ only vanishing on $\partial\mathcal{M}$ at the asymtotic AdS boundary. Note $\partial/\partial t$ globally timelike.
• $(\mathcal{M}, h)$ has only the boundary associated to asym AdS, with no other boundaries or asymptotic regions (so has finite volume).
• Assume boundary $\Sigma = \partial\mathcal{M}$ is closed (ie. cmpt, no boundary)
• Example; global AdS and the AdS-soliton [Horowitz, Myers '98]
• More generally can take any (?) smooth closed Σ and we expect a static bulk vacuum solution to exist.
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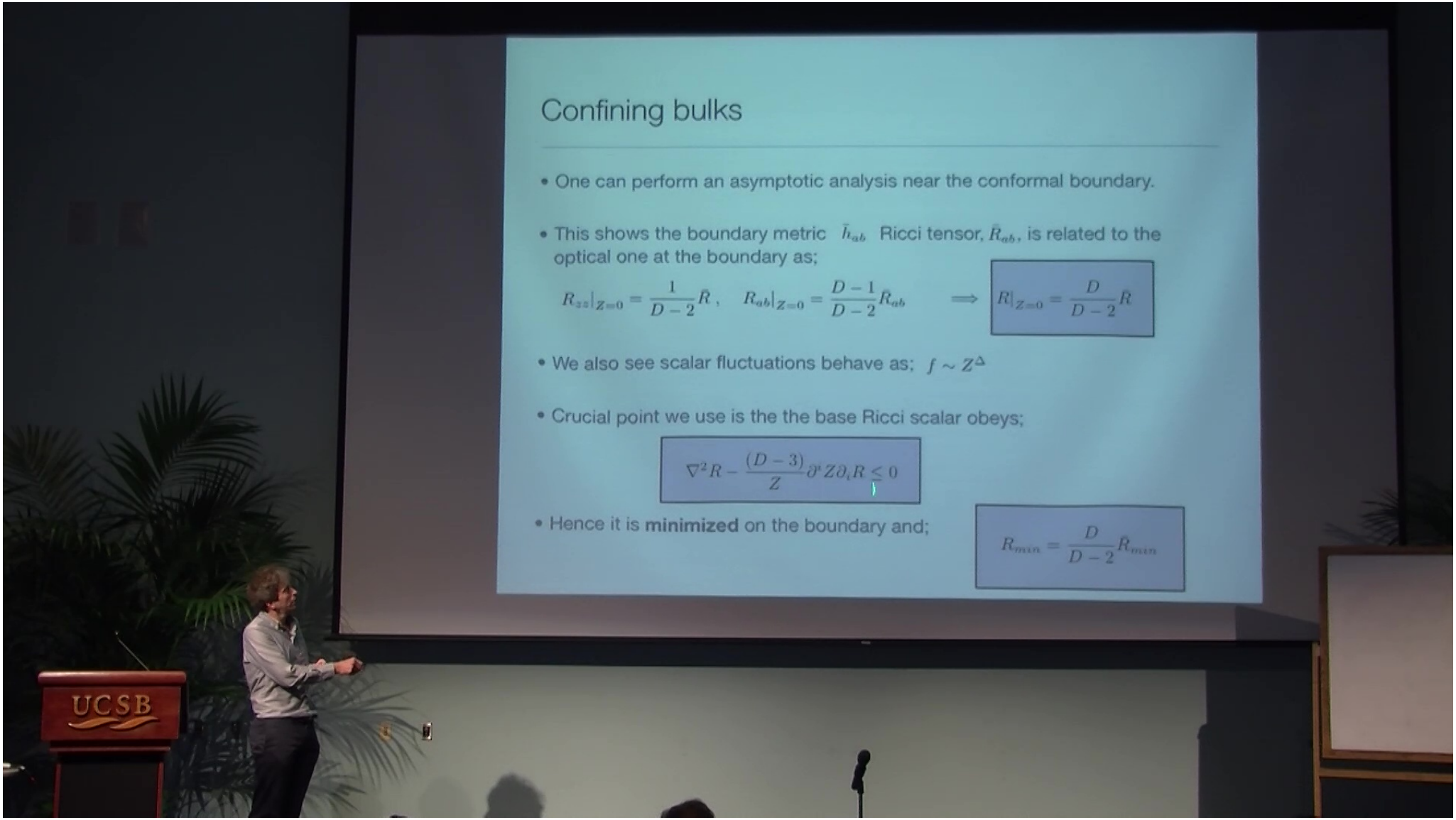
Confining bulks
• One can perform an asymptotic analysis near the conformal boundary.
• This shows the boundary metric $\bar{h}_{ab}$ Ricci tensor, $\bar{R}_{ab}$, is related to the optical one at the boundary as;
$R_{zz}|_{Z=0} = \frac{1}{D-2}\bar{R}\,, \quad R_{ab}|_{Z=0} = \frac{D-1}{D-2}\bar{R}_{ab} \quad \Longrightarrow \quad R|_{Z=0} = \frac{D}{D-2}\bar{R}$
• We also see scalar fluctuations behave as; $f \sim Z^{\Delta}$
• Crucial point we use is the the base Ricci scalar obeys;
$\nabla^2 R - \frac{(D-3)}{Z}\partial^i Z \partial_i R \leq 0$
• Hence it is **minimized** on the boundary and;
$R_{min} = \frac{D}{D-2}\bar{R}_{min}$
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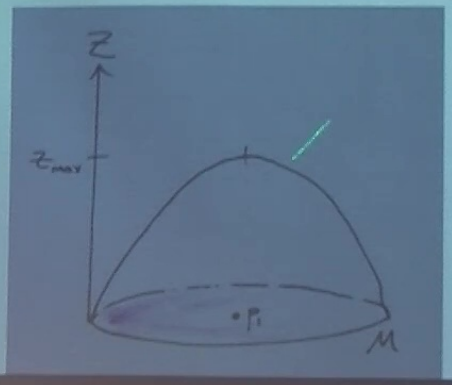
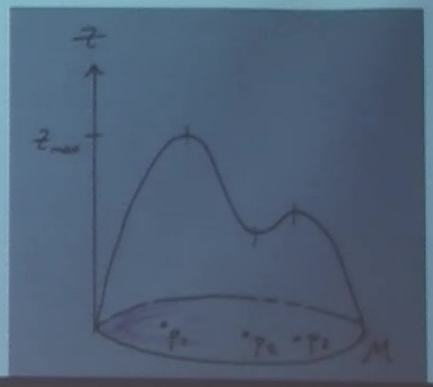
Bulk redshift

- The function Z determines the local redshift in the bulk. $Z(x)/\ell$ relates the proper time at x^i to the bulk coordinate time t, and thus the CFT time.
- Take extrema of Z at points $p_{(n)}$. Let the maximum value be Z_{max}
- Then; $D(D-1) = Z^2_{(n)} R_{(n)}$ and these points cannot be minima of Z



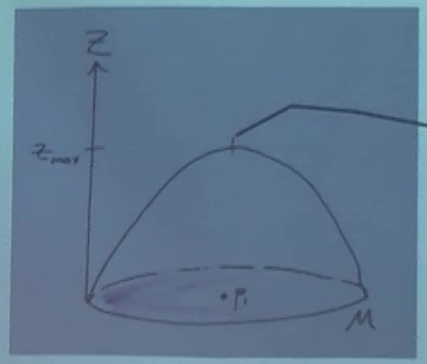


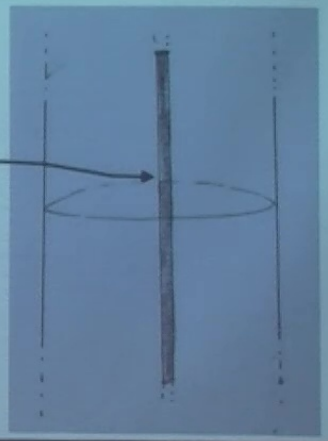
Small black holes

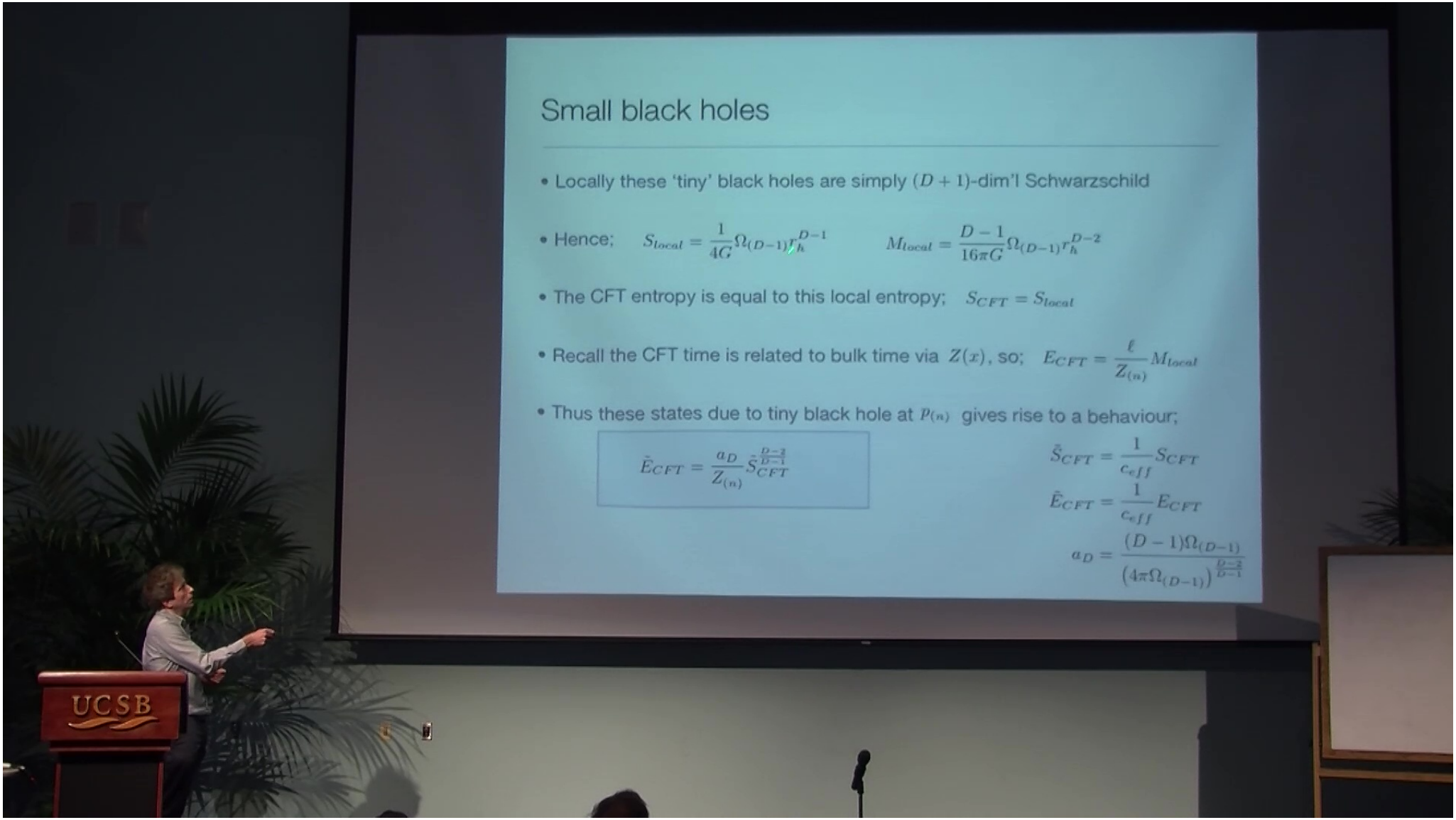
- A timelike geodesic, proper time τ, executes a motion on $\mathcal{M}$

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i{}_{jk}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau} = \partial^i\left(Z^2\right) + \frac{4}{Z}\partial_j Z\frac{dx^j}{d\tau}\frac{dx^i}{d\tau}$$

- Hence we may patch in a very small black hole at the extrema pts. $p_{(n)}$ of Z






Small black holes
• Locally these 'tiny' black holes are simply $(D+1)$-dim'l Schwarzschild
• Hence: $S_{local} = \frac{1}{4G}\Omega_{(D-1)} r_h^{D-1}$ $M_{local} = \frac{D-1}{16\pi G}\Omega_{(D-1)} r_h^{D-2}$
• The CFT entropy is equal to this local entropy; $S_{CFT} = S_{local}$
• Recall the CFT time is related to bulk time via $Z(x)$, so; $E_{CFT} = \frac{\ell}{Z_{(n)}} M_{local}$
• Thus these states due to tiny black hole at $P_{(n)}$ gives rise to a behaviour;
$\tilde{E}_{CFT} = \frac{a_D}{Z_{(n)}} \tilde{S}_{CFT}^{\frac{D-2}{D-1}}$
$\tilde{S}_{CFT} = \frac{1}{c_{eff}} S_{CFT}$
$\tilde{E}_{CFT} = \frac{1}{c_{eff}} E_{CFT}$
$a_D = \frac{(D-1)\Omega_{(D-1)}}{\left(4\pi\Omega_{(D-1)}\right)^{\frac{D-2}{D-1}}}$
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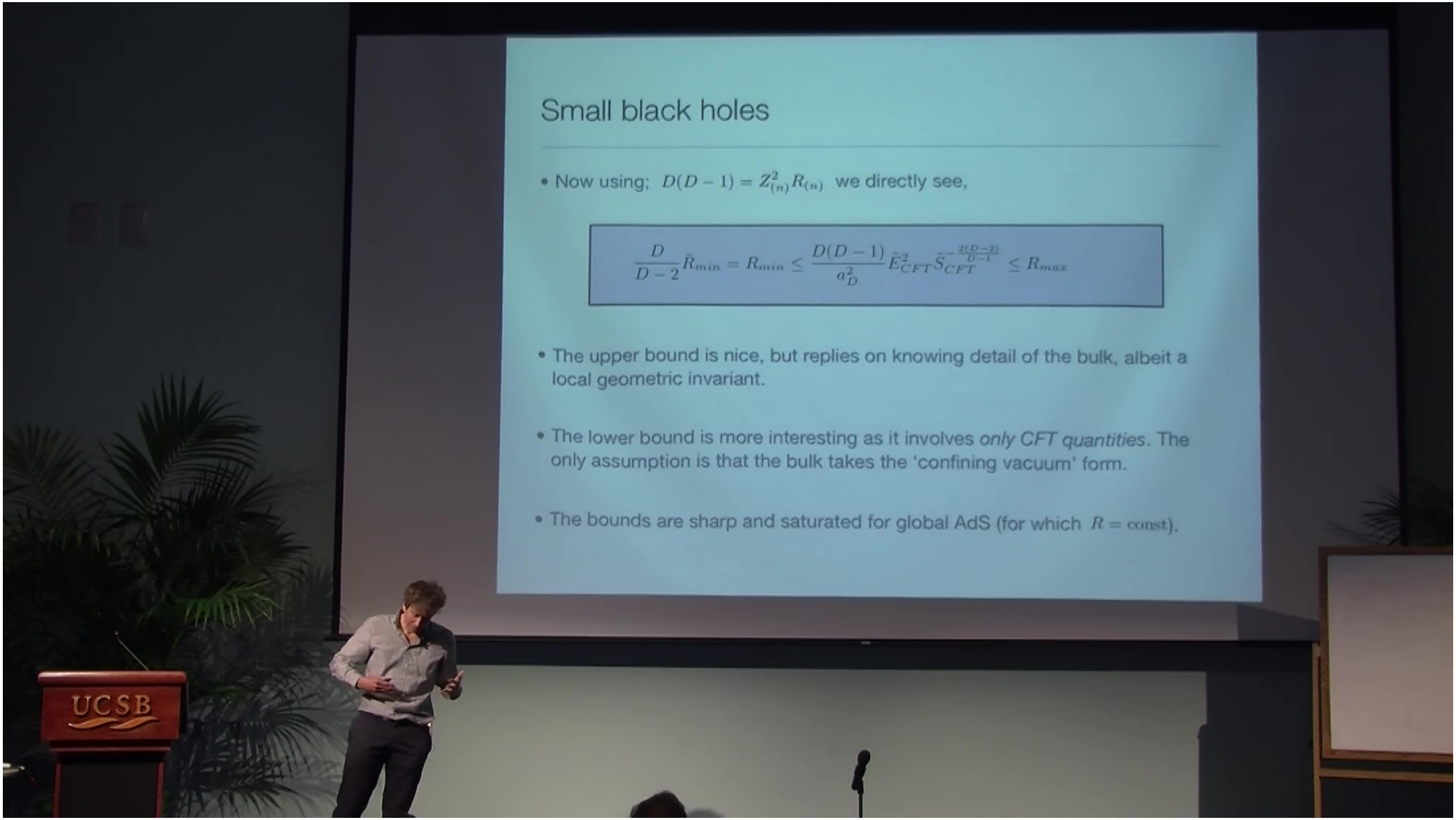
Small black holes
• Now using; $D(D-1) = Z_{(n)}^2 R_{(n)}$ we directly see,
$$\frac{D}{D-2}\bar{R}_{min} = R_{min} \leq \frac{D(D-1)}{a_D^2}\tilde{E}_{CFT}^2 \tilde{S}_{CFT}^{-\frac{2(D-2)}{D-1}} \leq R_{max}$$
• The upper bound is nice, but replies on knowing detail of the bulk, albeit a local geometric invariant.
• The lower bound is more interesting as it involves *only CFT quantities*. The only assumption is that the bulk takes the 'confining vacuum' form.
• The bounds are sharp and saturated for global AdS (for which $R = \text{const}$).
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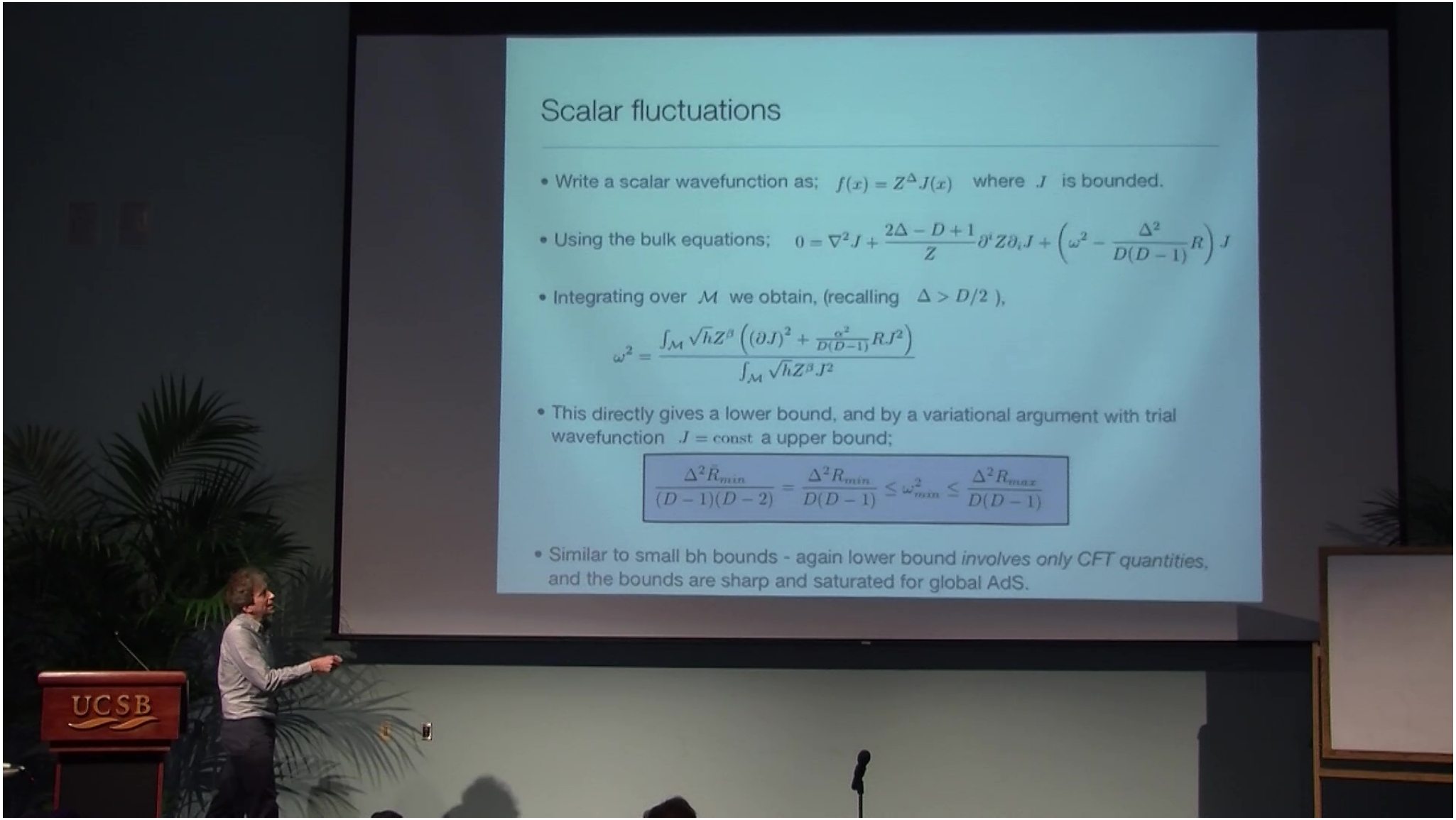
Scalar fluctuations
• Write a scalar wavefunction as; $f(x) = Z^{\Delta} J(x)$ where J is bounded.
• Using the bulk equations; $0 = \nabla^2 J + \frac{2\Delta - D + 1}{Z} \partial^i Z \partial_i J + \left(\omega^2 - \frac{\Delta^2}{D(D-1)} R \right) J$
• Integrating over $\mathcal{M}$ we obtain, (recalling $\Delta > D/2$),
$\omega^2 = \frac{\int_{\mathcal{M}} \sqrt{h} Z^{\beta} \left((\partial J)^2 + \frac{\alpha^2}{D(D-1)} R J^2 \right)}{\int_{\mathcal{M}} \sqrt{h} Z^{\beta} J^2}$
• This directly gives a lower bound, and by a variational argument with trial wavefunction $J = \text{const}$ a upper bound;
$\frac{\Delta^2 \bar{R}_{min}}{(D-1)(D-2)} = \frac{\Delta^2 R_{min}}{D(D-1)} \leq \omega^2_{min} \leq \frac{\Delta^2 R_{max}}{D(D-1)}$
• Similar to small bh bounds - again lower bound *involves only CFT quantities*, and the bounds are sharp and saturated for global AdS.
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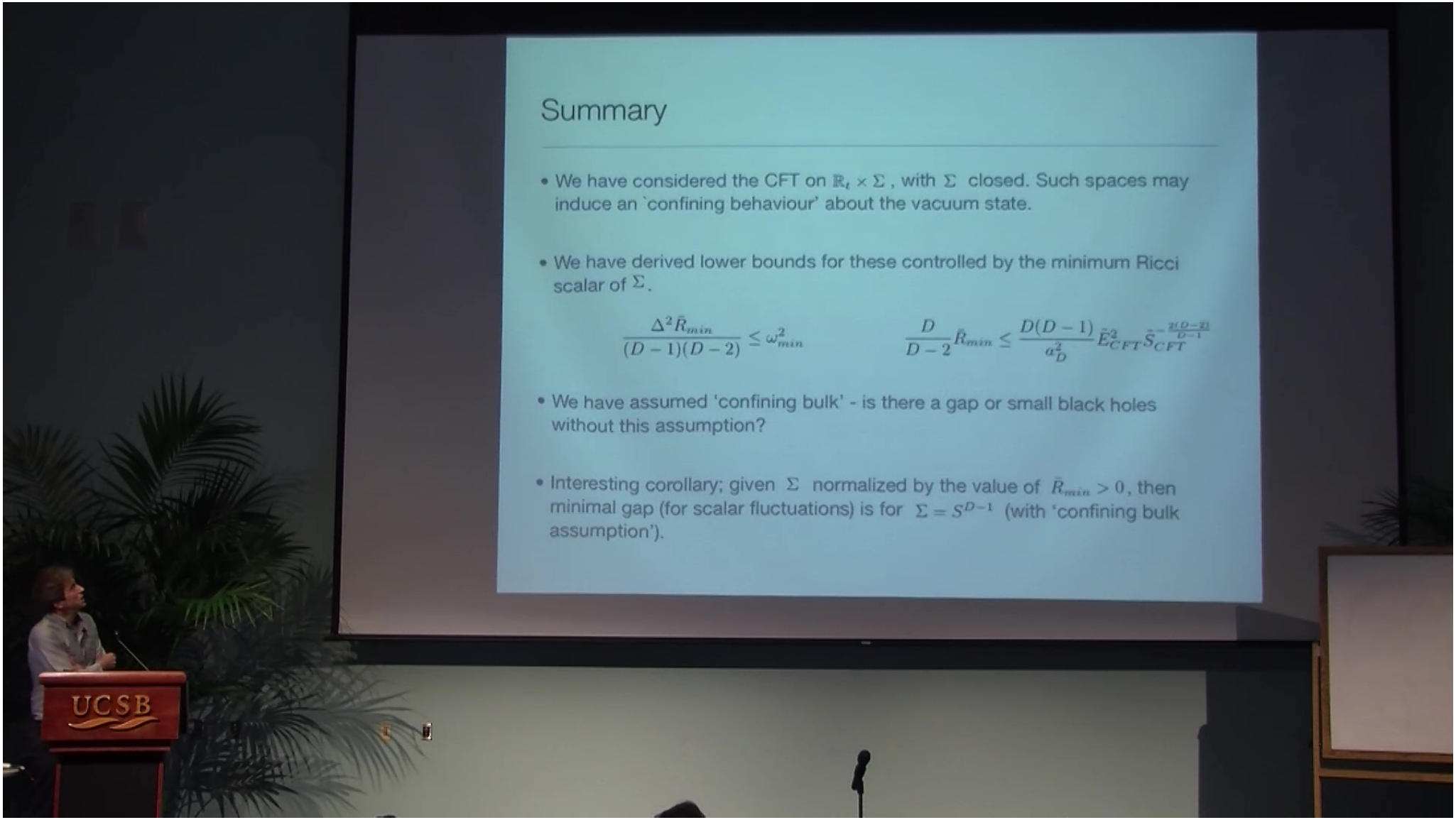
Summary
• We have considered the CFT on $\mathbb{R}_t \times \Sigma$, with Σ closed. Such spaces may induce an `confining behaviour' about the vacuum state.
• We have derived lower bounds for these controlled by the minimum Ricci scalar of Σ.
$\frac{\Delta^2 \bar{R}_{min}}{(D-1)(D-2)} \leq \omega^2_{min}$
$\frac{D}{D-2}\bar{R}_{min} \leq \frac{D(D-1)}{a_D^2}\tilde{E}^2_{CFT}\tilde{S}_{CFT}^{-\frac{2(D-2)}{D-1}}$
• We have assumed 'confining bulk' - is there a gap or small black holes without this assumption?
• Interesting corollary; given Σ normalized by the value of $\bar{R}_{min} > 0$, then minimal gap (for scalar fluctuations) is for $\Sigma = S^{D-1}$ (with 'confining bulk assumption').
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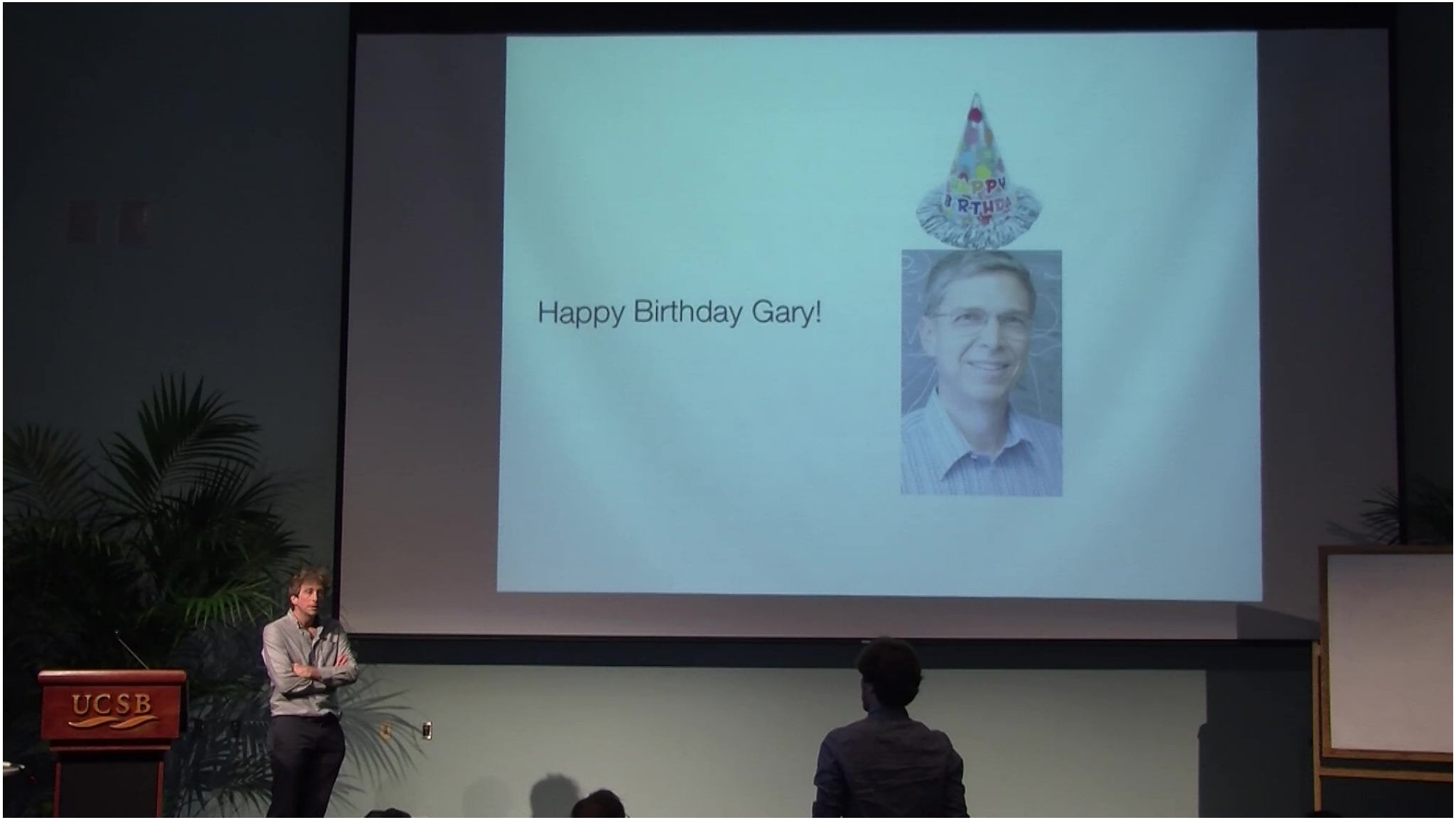
Happy Birthday Gary!
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