Title: Transport in Strange Metals

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Abstract:

General relativity and the cuprates

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ABSTRACT: We add a periodic potential to the simplest gravitational model of a superconductor and compute the optical conductivity. In addition to a superfluid component, we find a normal component that has Drude behavior at low frequency followed by a power law fall-off. Both the exponent and coefficient of the power law are temperature independent and agree with earlier results computed above T_c. These results are in striking agreement with measurements on some cuprates. We also find a gap $\Delta = 4.0\ T_c$, a rapidly decreasing scattering rate, and "missing spectral weight" at low frequency, all of which also agree with experiments.

KEYWORDS: Holography and condensed matter physics (AdS/CMT), Gauge-gravity correspondence, AdS-CFT Correspondence

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Transport in strange metals

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May 1, 2015

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Cologne

Outline

1. Quasiparticle transport in ordinary metals

    Bloch vs. Peierls

2. Transport without quasiparticles in strange metals

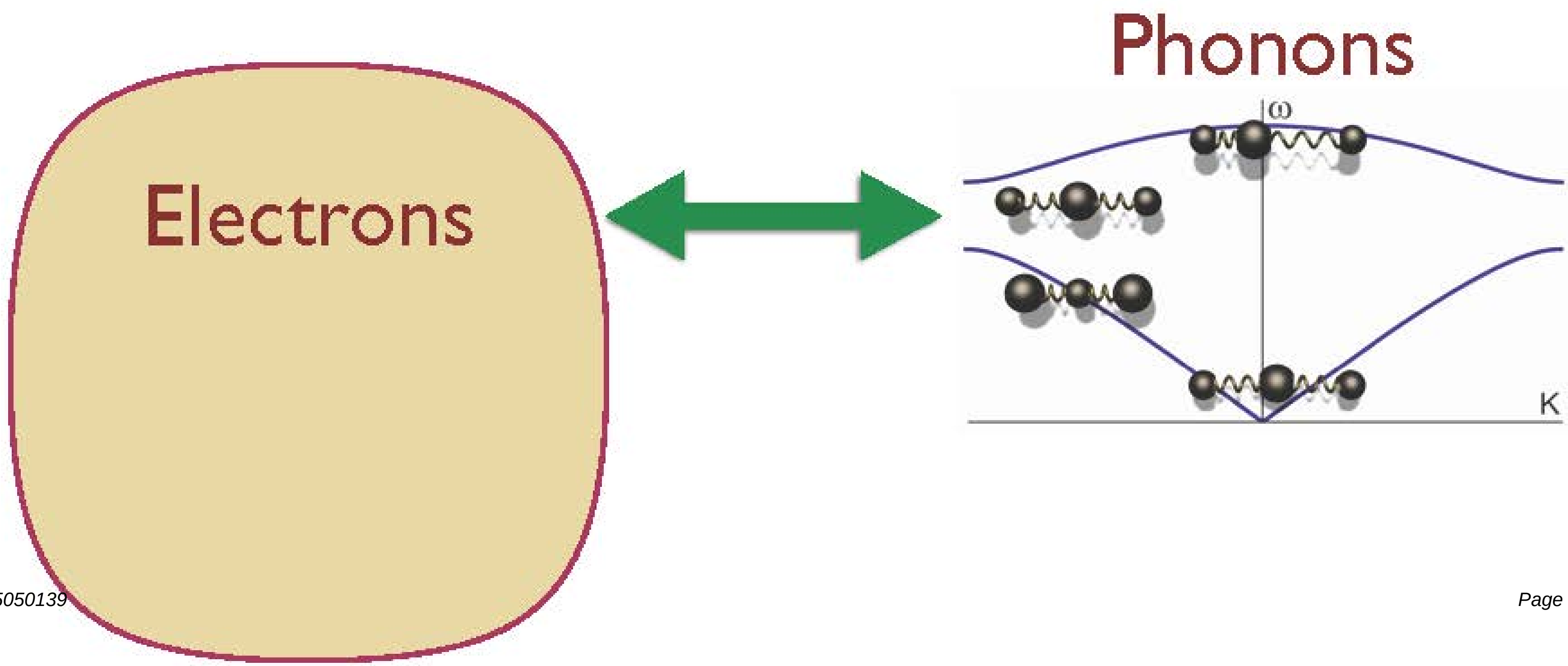
    Memory functions, holography, and hydrodynamics

3. The spin density wave quantum critical point

    Transport with hyperscaling violation

Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch's law (1930) : a resistivity $\rho(T) \sim T^5$.



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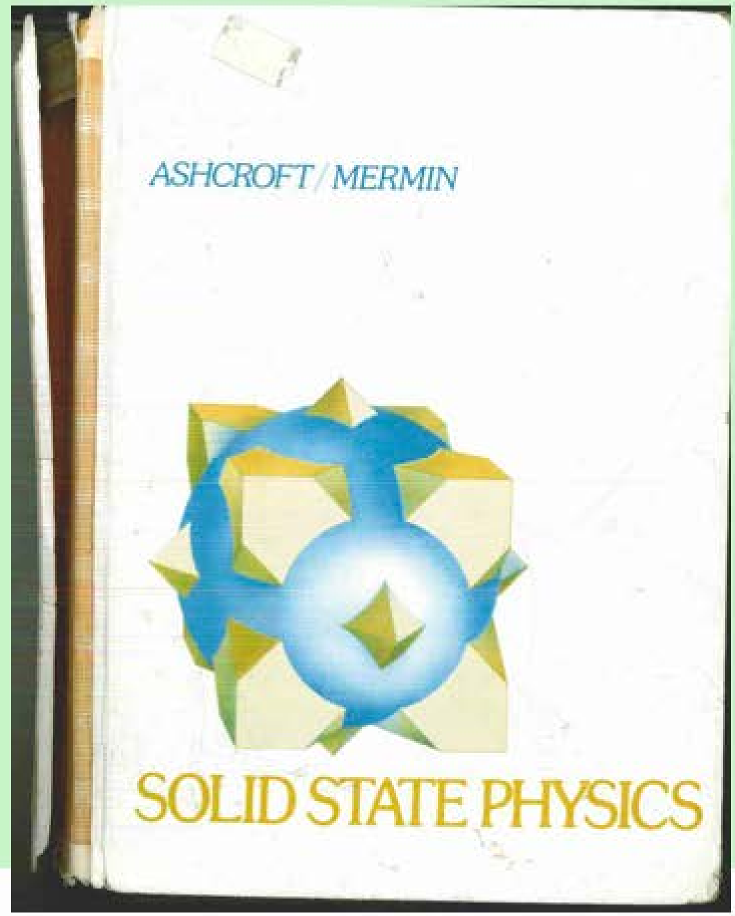

However, this ignores "phonon drag"

PHONON DRAG

Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5.

28 R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932)

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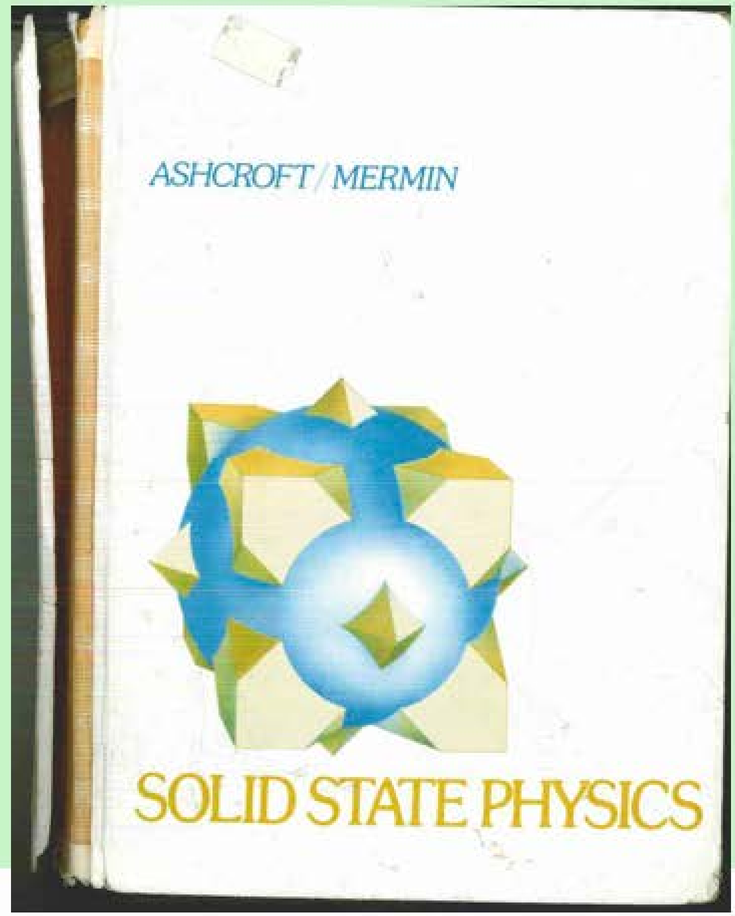

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Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5. This behavior has yet to be observed,

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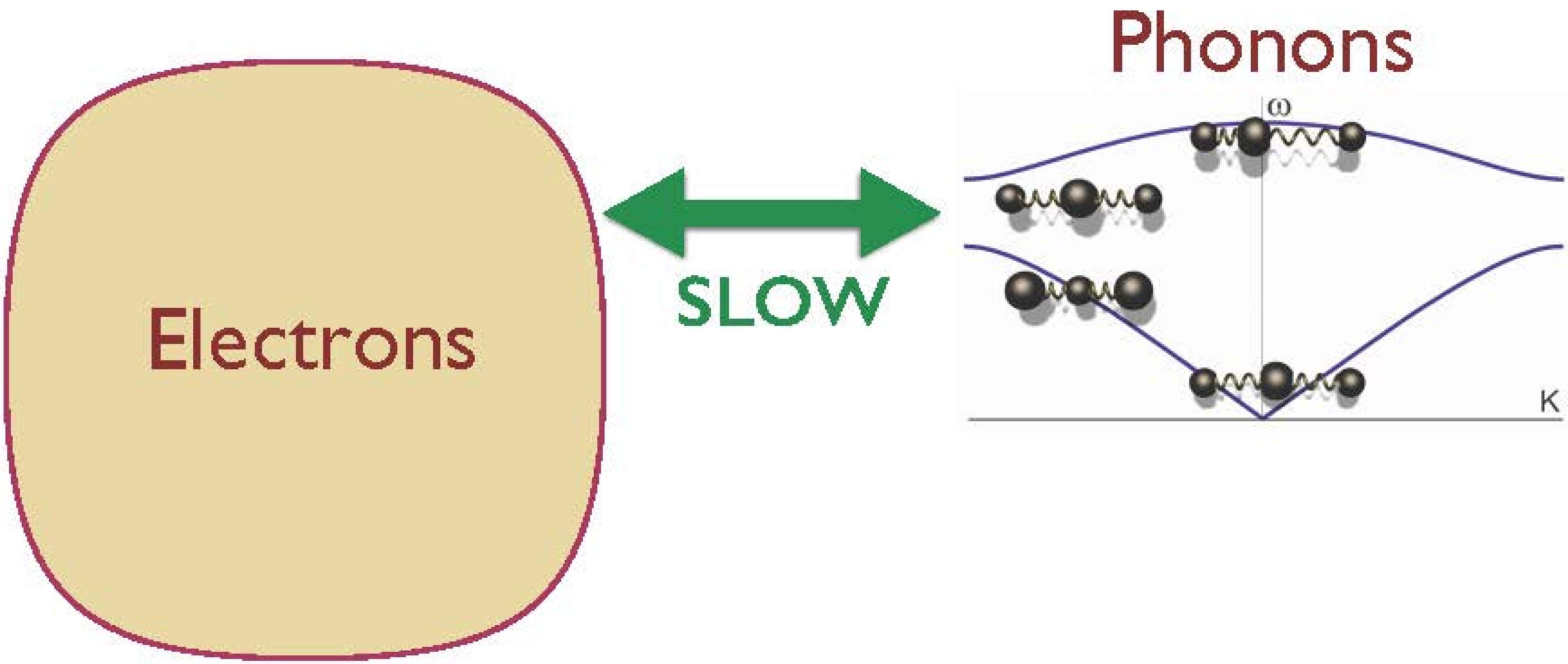
Rates of Momentum Flow



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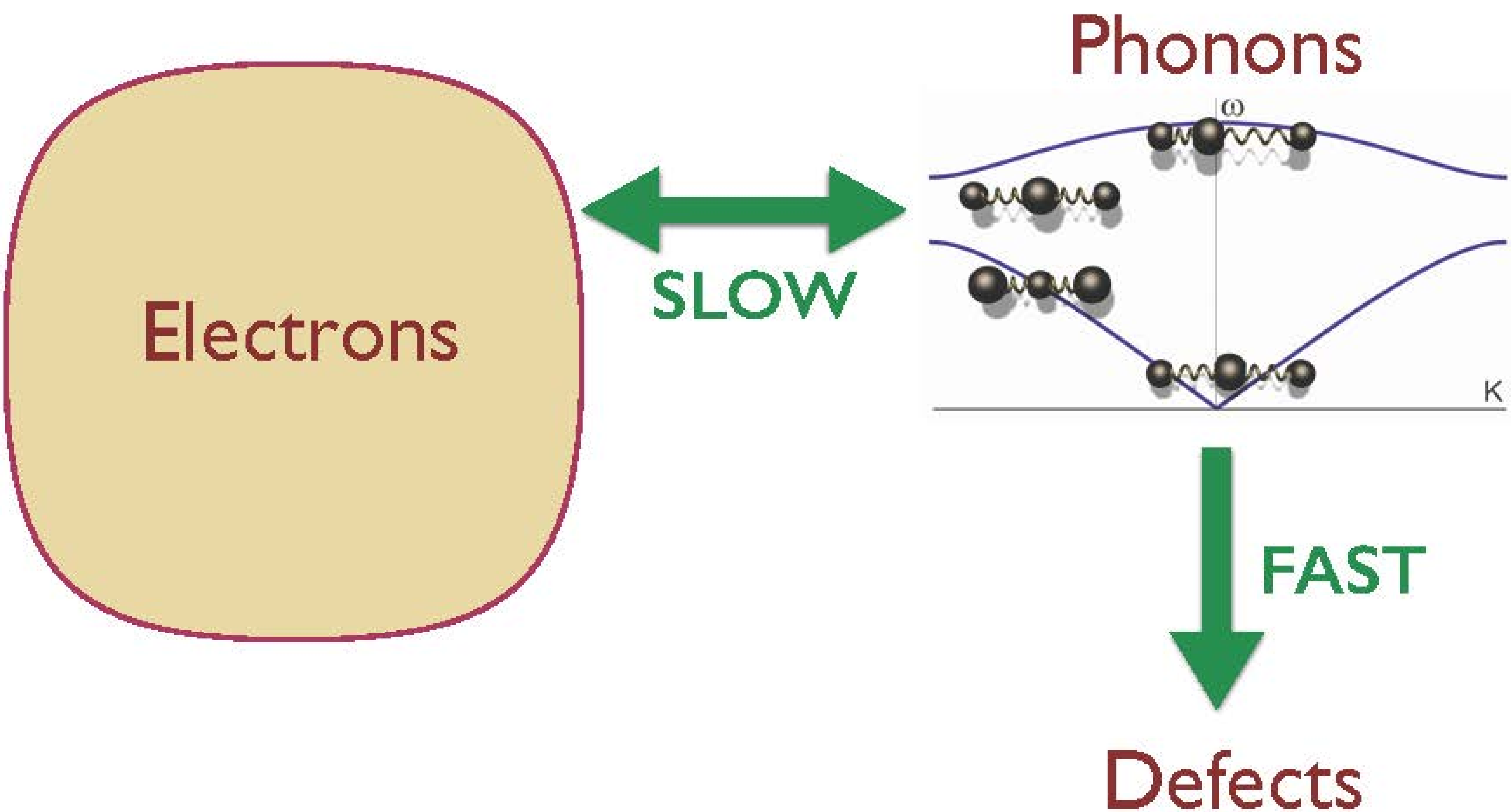

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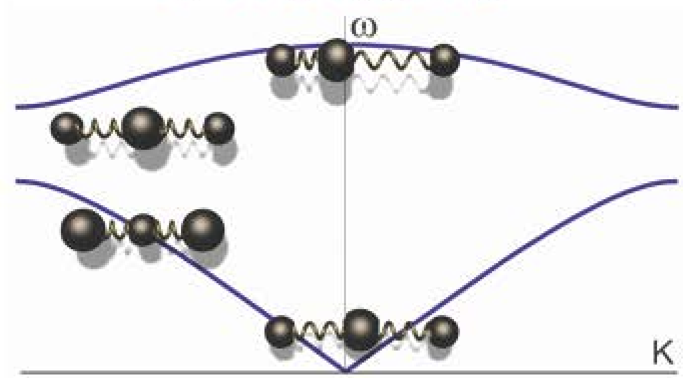
Phonons



Electrons

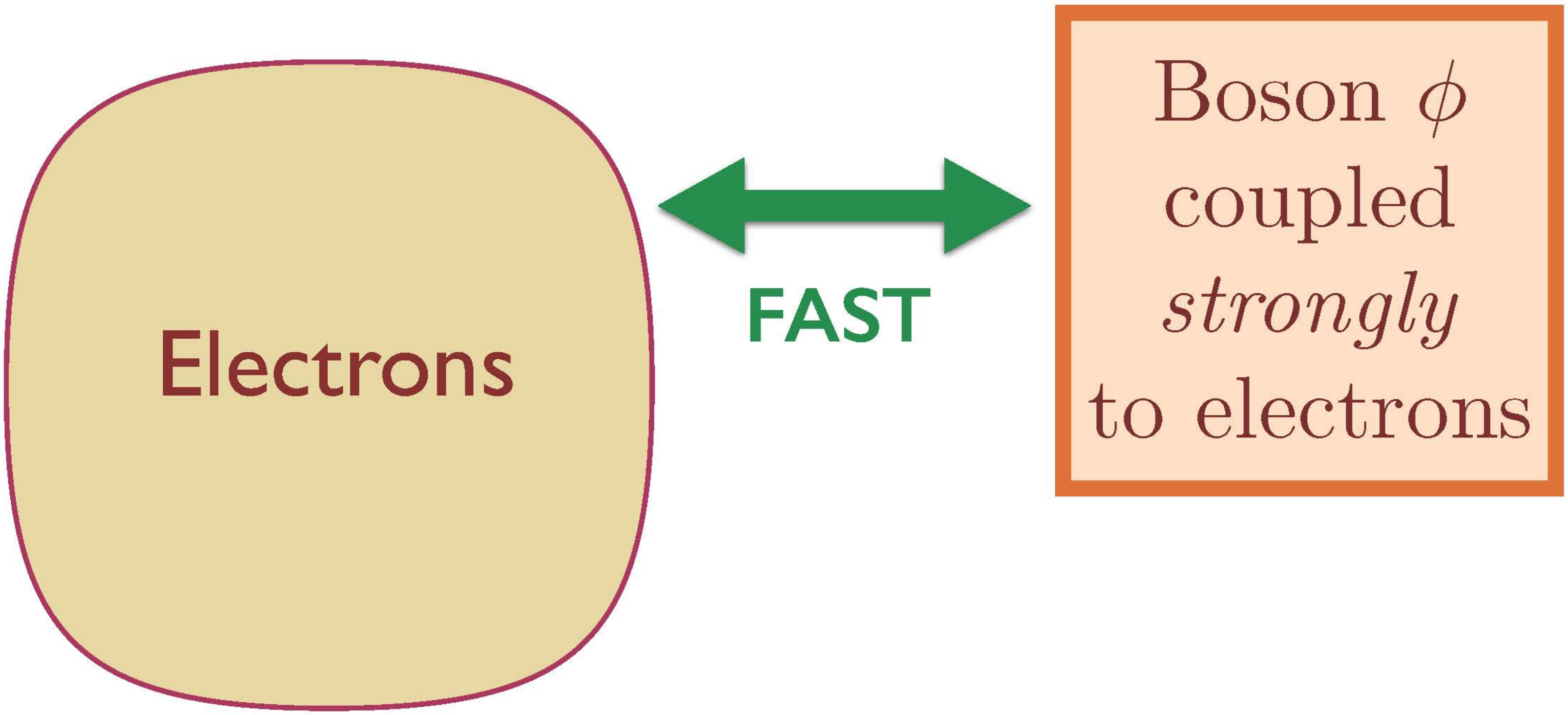
SLOW

Process controlling resistivity (Bloch)

FAST

Defects

Rates of Momentum Flow



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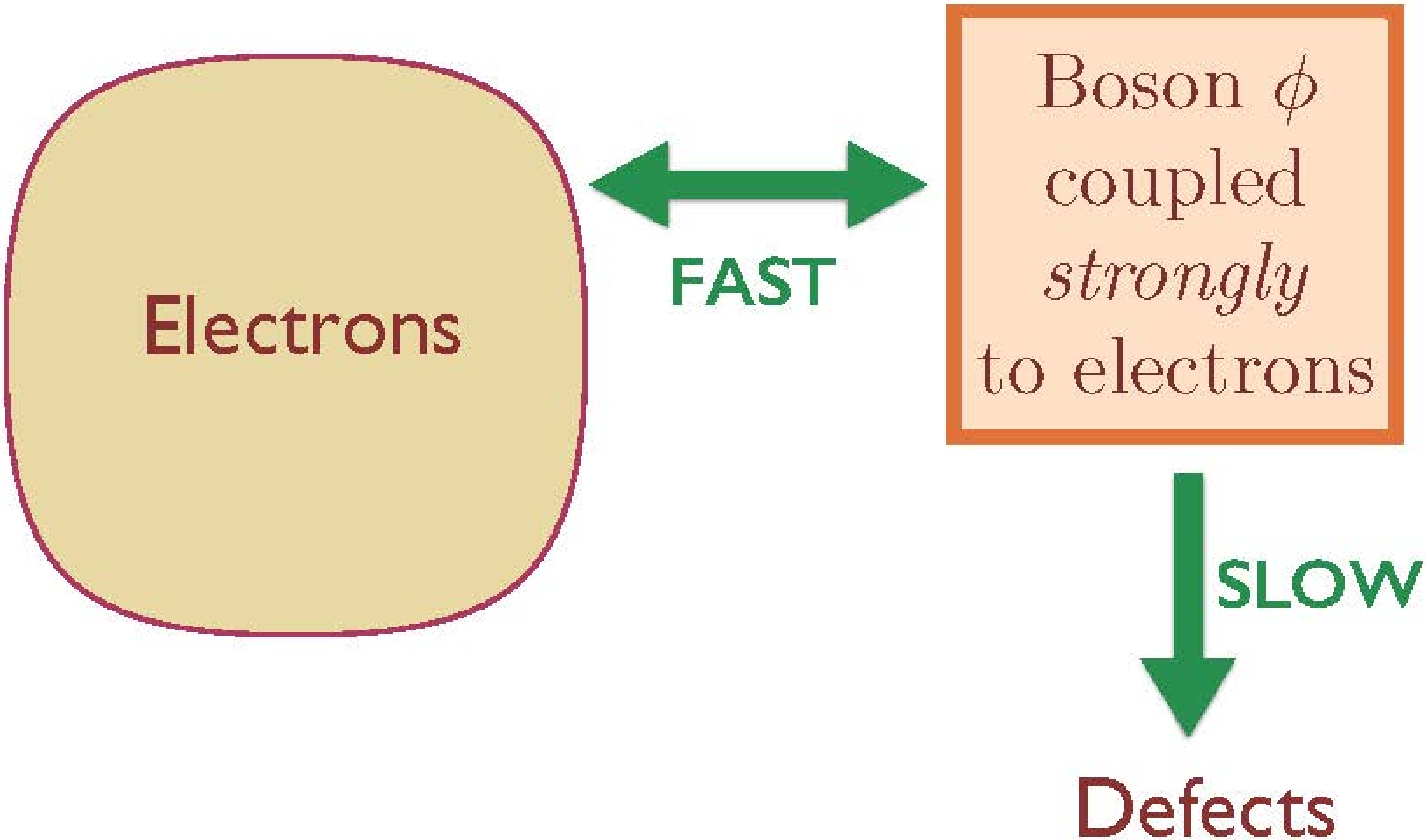

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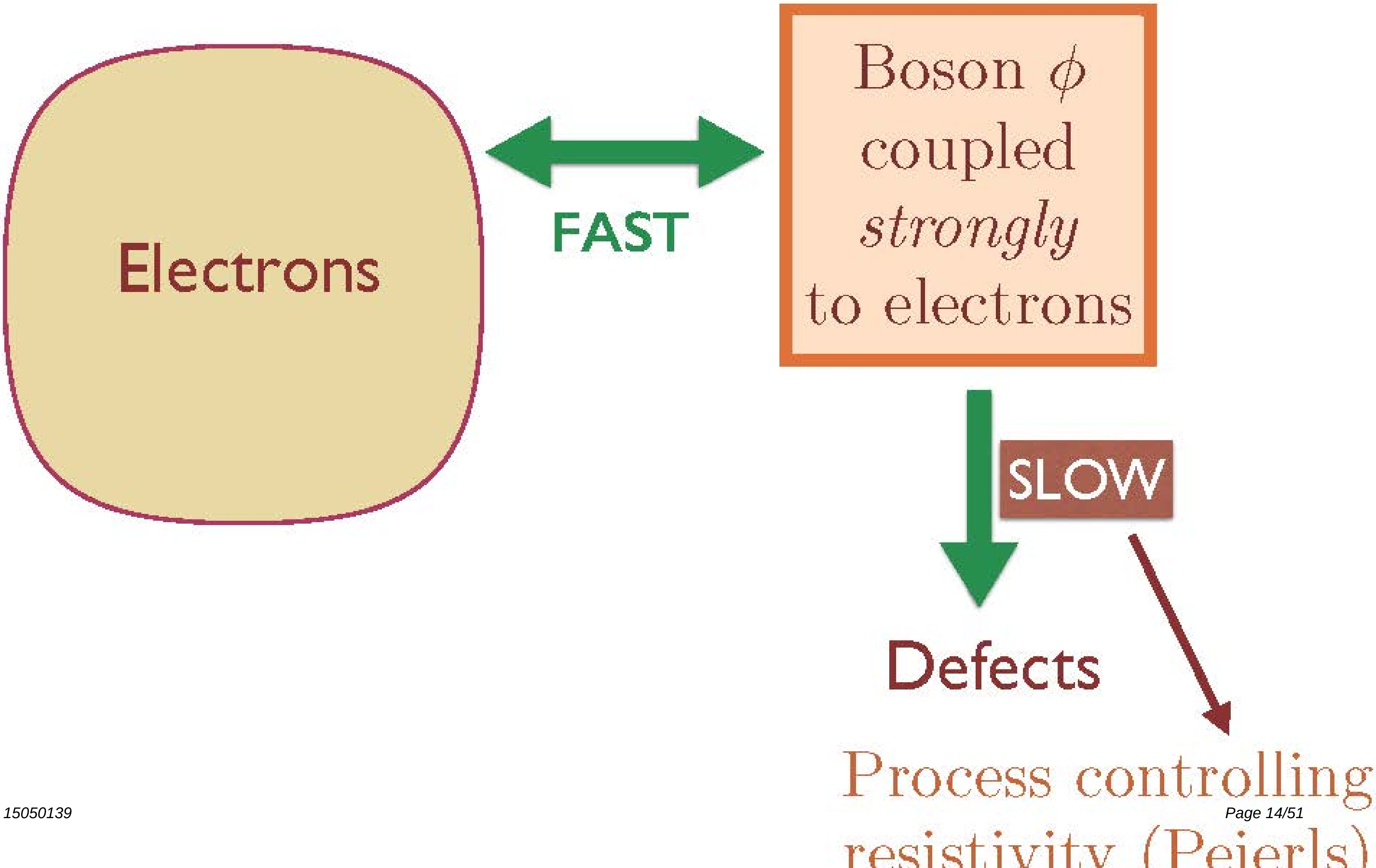

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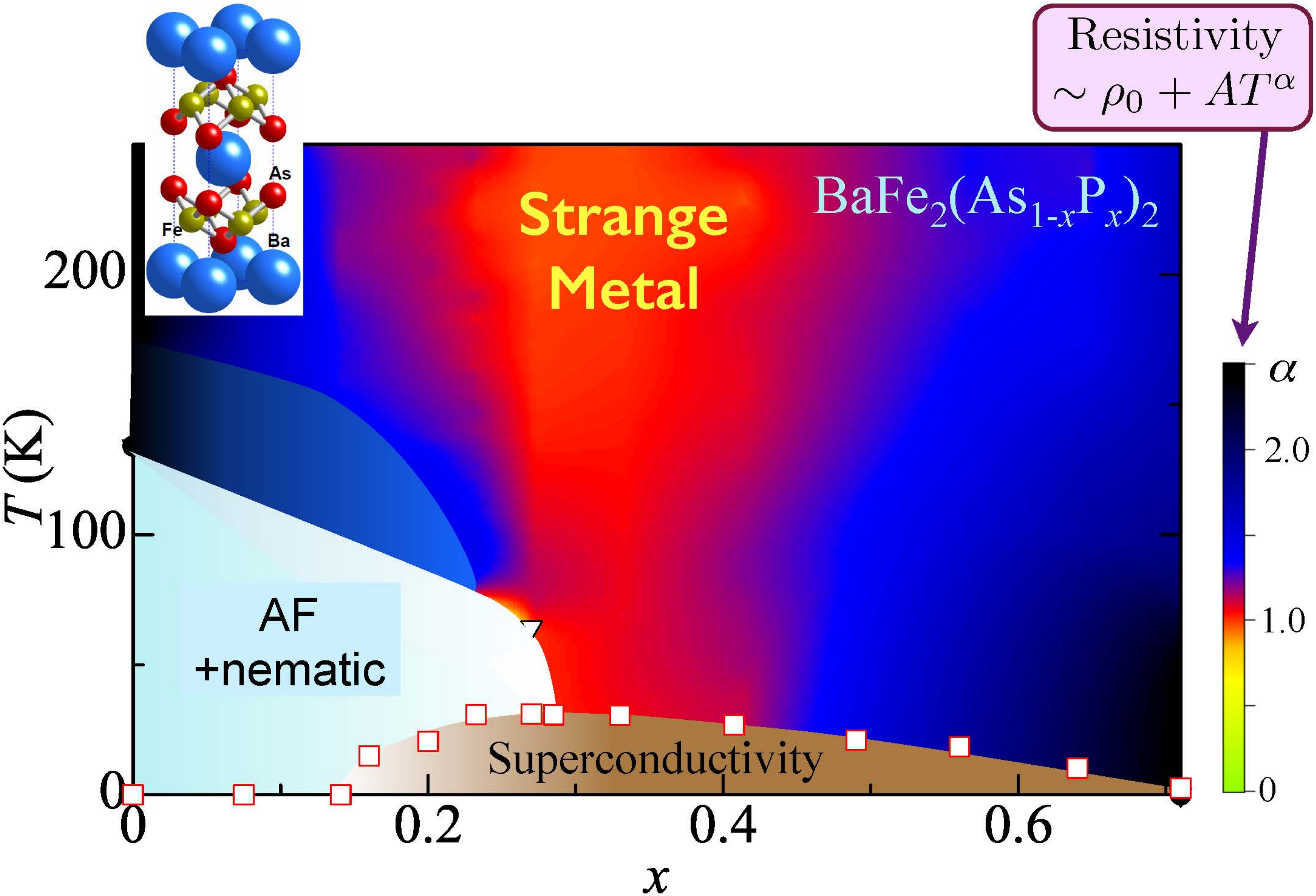
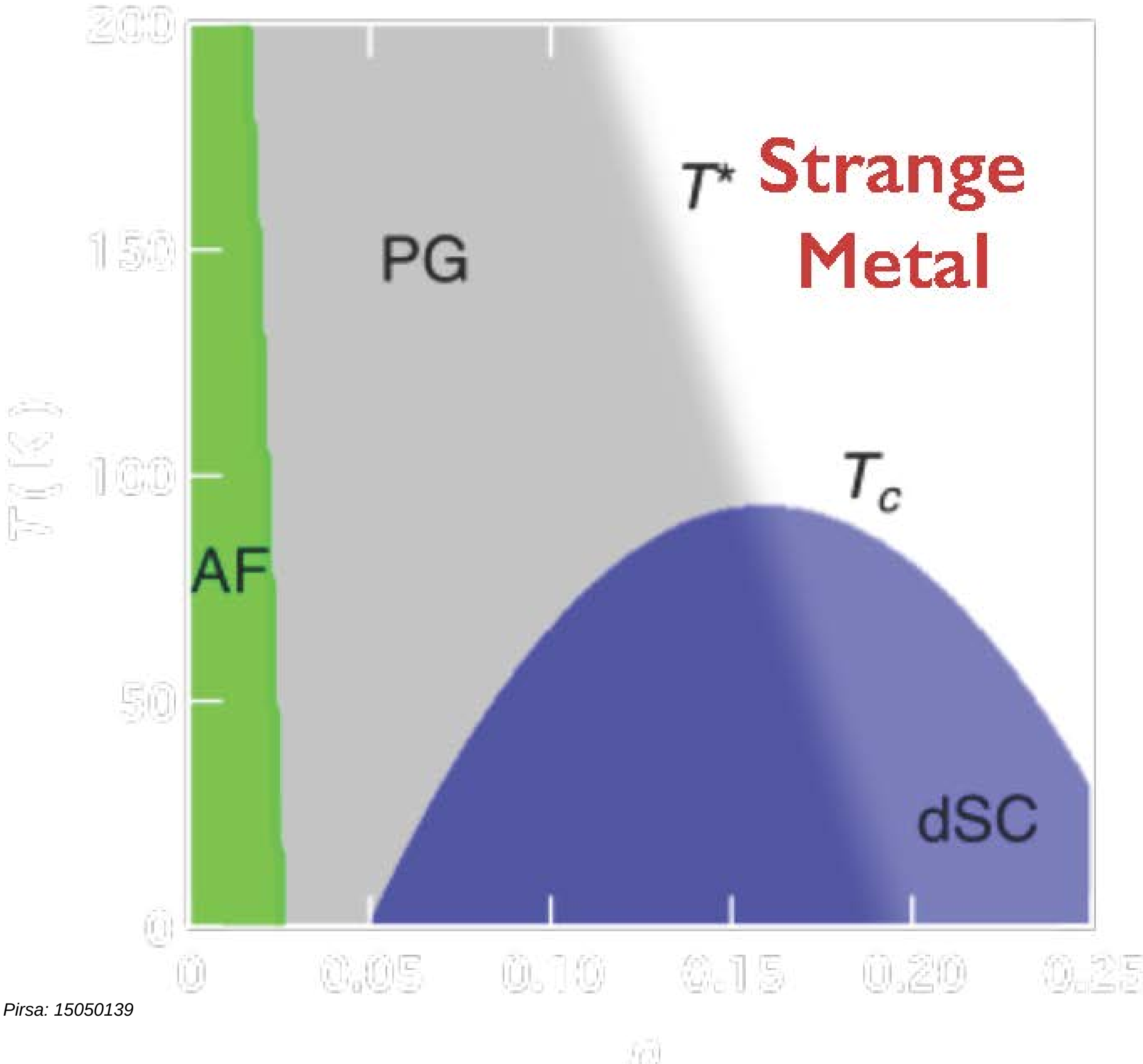
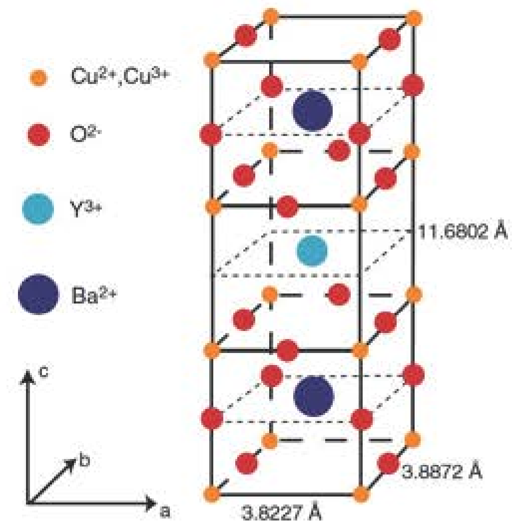
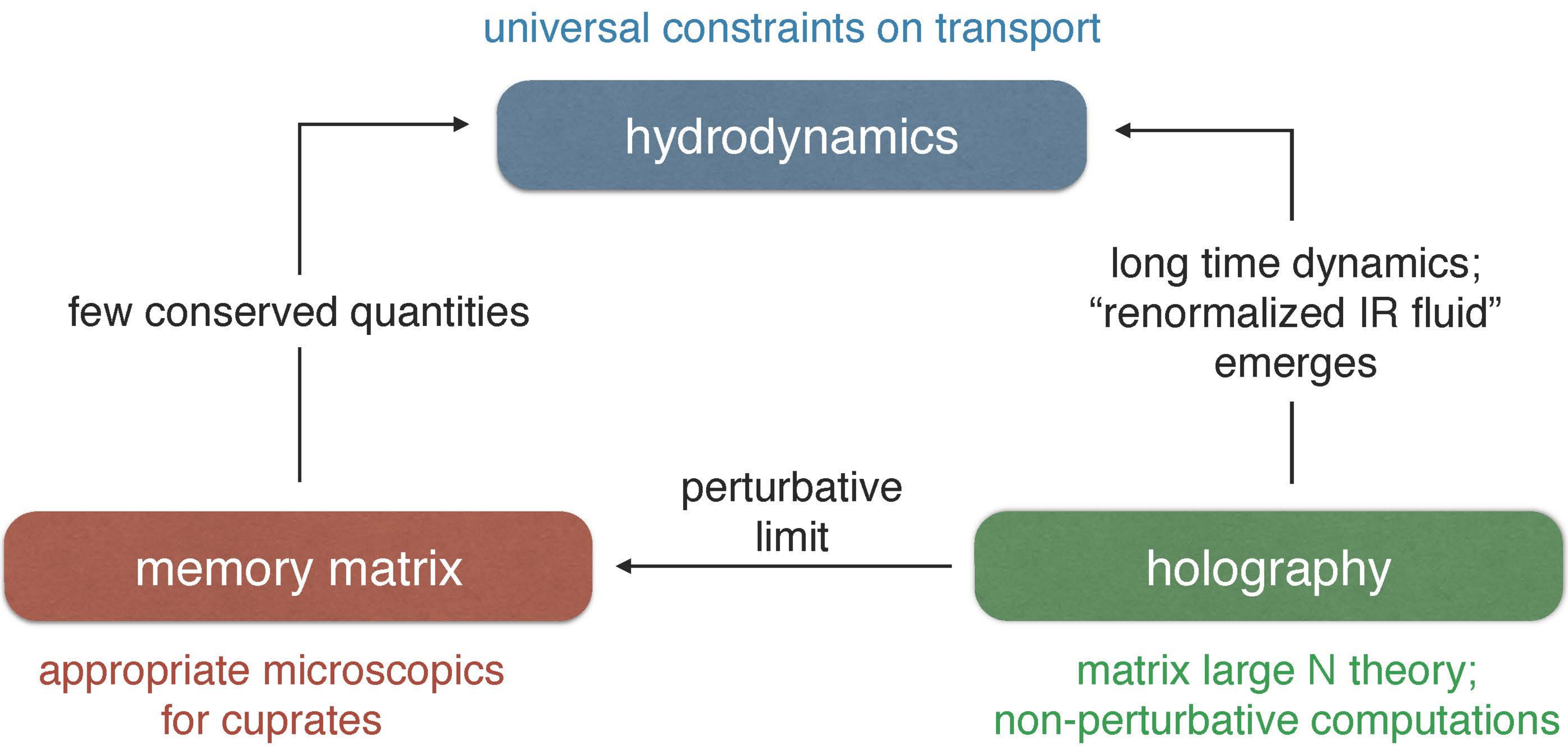


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,





$\mathrm{YBa_2Cu_3O_{6+x}}$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Electrical transport at a strongly-coupled critical theory _with_ particle-hole symmetry, obeying hyperscaling, in d spatial dimensions with dynamic critical exponent z

$$\sigma = \sigma_Q \sim T^{(d-2)/z}$$

Follows from gauge invariance

$(\sigma = 1/\rho = \text{conductivity})$

K. Damle and S. Sachdev, PRB 56, 8714 (1997)

Electrical transport at a strongly-coupled critical theory <u>*without*</u> particle-hole symmetry, <u>*with*</u> a conserved momentum P

$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega)$$

with $\mathcal{Q} \equiv \chi_{J_x, P_x}$ and $\mathcal{M} \equiv \chi_{P_x, P_x}$ thermodynamic response functions

Obtained in hydrodynamics, holography, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, with an *almost* conserved momentum P

$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_L)}$$

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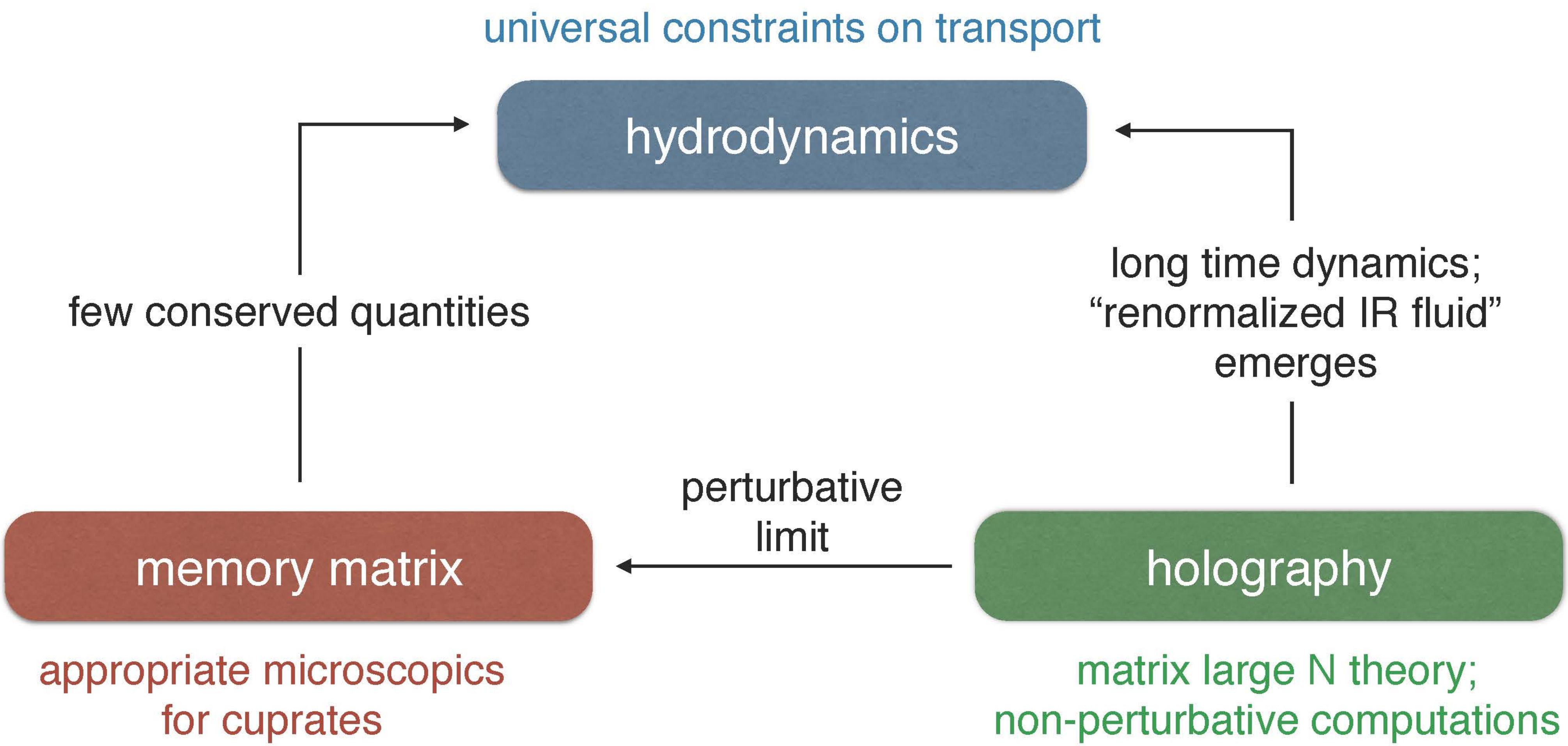
Momentum relaxation by an external source h_L coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h_L(x) \, \mathcal{O}(x).$$

$$\frac{\mathcal{M}}{\tau_L} = \lim_{\omega \to 0} \int d^d q \, |h_L(q)|^2 q_x^2 \frac{\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \text{higher orders in } h_L$$

Obtained by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
S. A. Hartnoll and D. M. Hofman, PRL **108**, 241601 (2012)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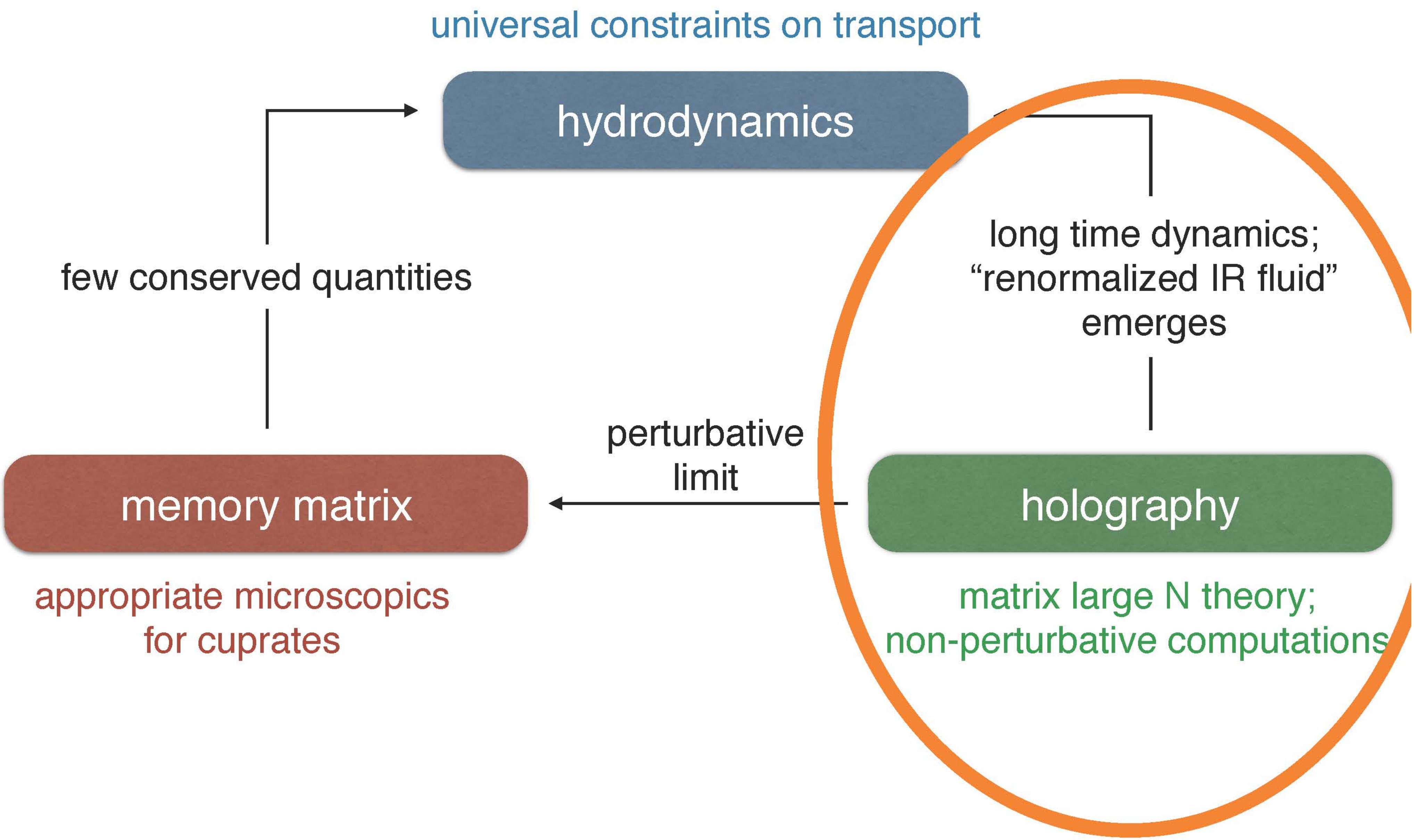
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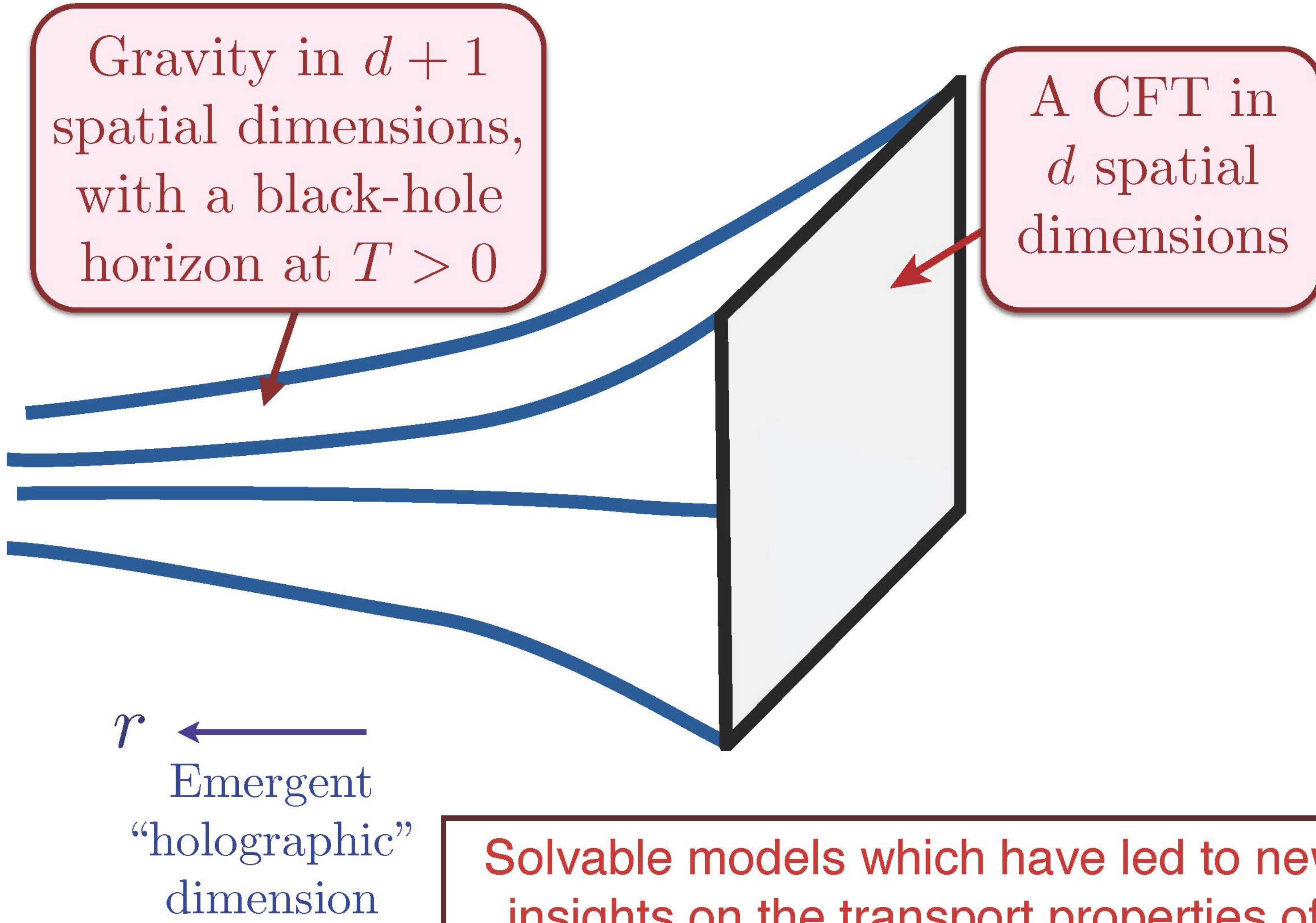
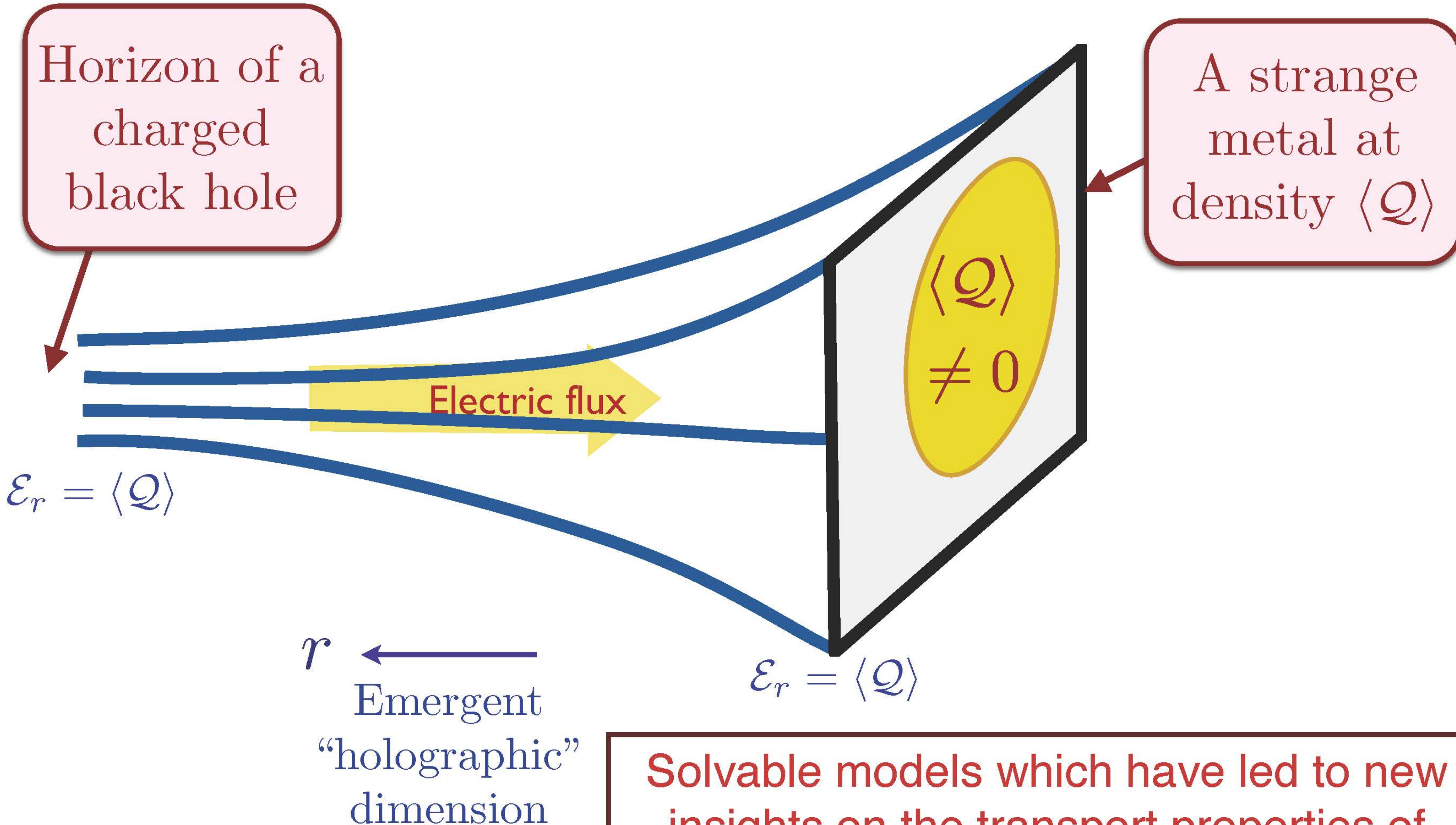
Holography conformal field theory: AdS/CFT



Solvable models which have led to new insights on the transport properties of <u>*quantum matter without quasiparticles*</u>

Holography of a strange metal: a charged black hole



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Optical Conductivity with Holographic Lattices

JHEP 1207 (2012) 168

Gary T. Horowitz[a], Jorge E. Santos[a], David Tong[b]

We add a gravitational background lattice to the simplest holographic model of matter at finite density and calculate the optical conductivity. With the lattice, the zero frequency delta function found in previous calculations (resulting from translation invariance) is broadened and the DC conductivity is finite. The optical conductivity exhibits a Drude peak with a cross-over to power-law behavior at higher frequencies. Surprisingly, these results bear a strong resemblance to the properties of some of the cuprates.

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Computed σ by numerical solution of Einstein equations. Found excellent agreement with memory function expression evaluated holographically for theory H_0.

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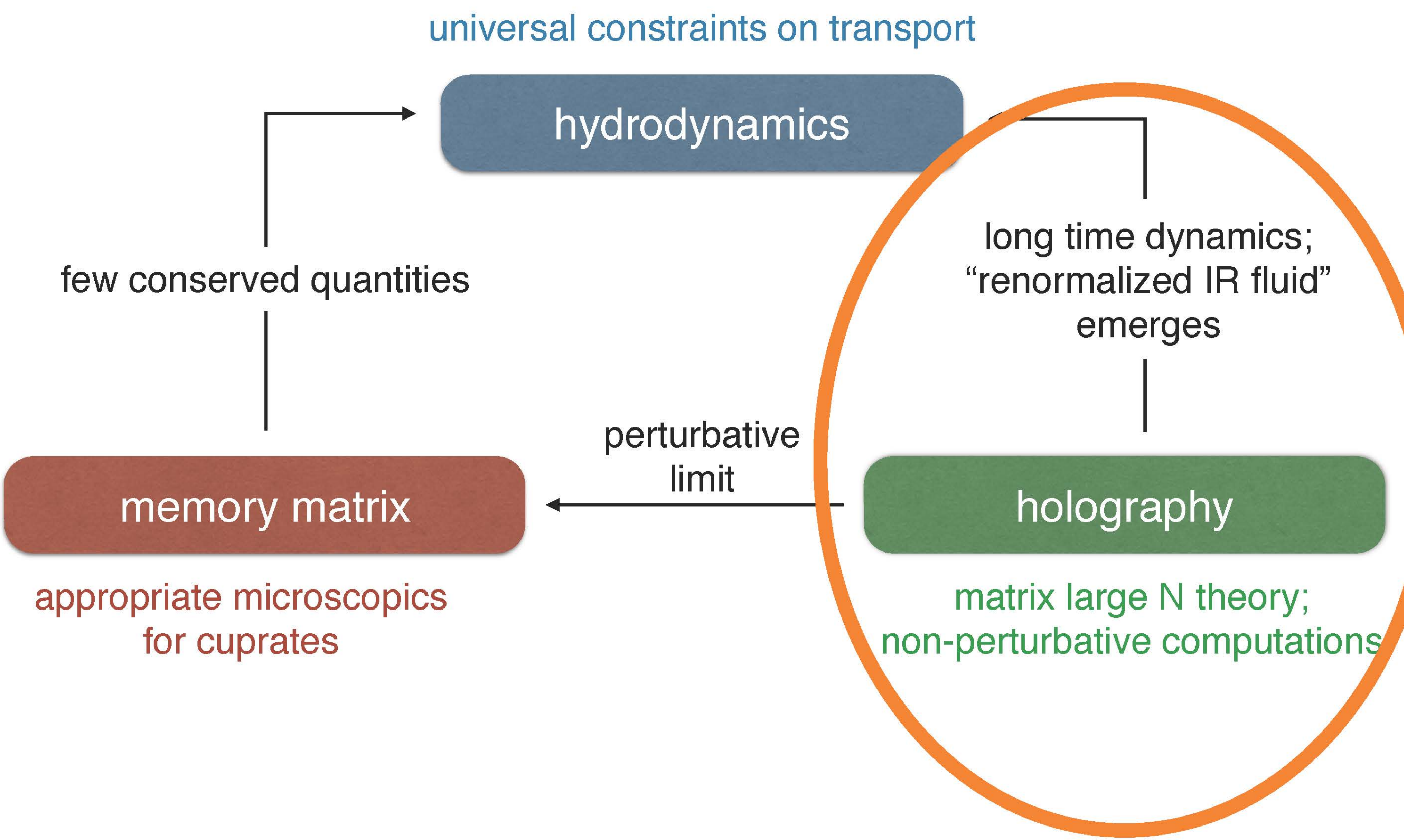
Proof of equivalence between holography (for a large class of background metrics) and memory function formula for τ_L
A. Lucas, JHEP 03, 071 (2015)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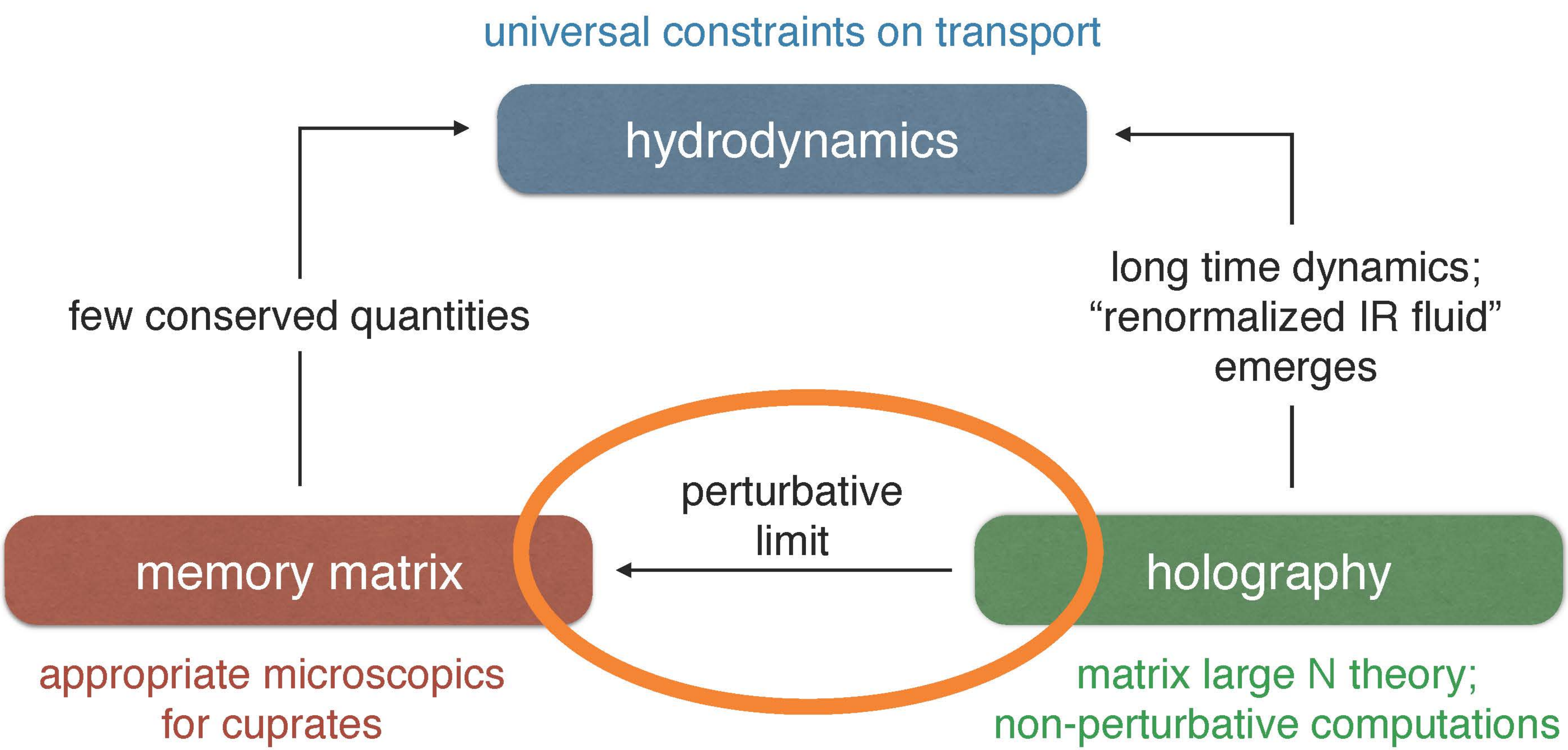
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$$\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau_L} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} B\mathcal{Q}.$$

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M. Blake and A. Donos, PRL **114**, 021601 (2015)

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Blake and Donos: With $\sigma_Q \sim 1/T$ and $\tau_L \sim 1/T^2$, we obtain $\sigma_{xx} \sim 1/T$ and $\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2$, in agreement with strange metal data on cuprates (such data cannot be explained in a quasiparticle model).

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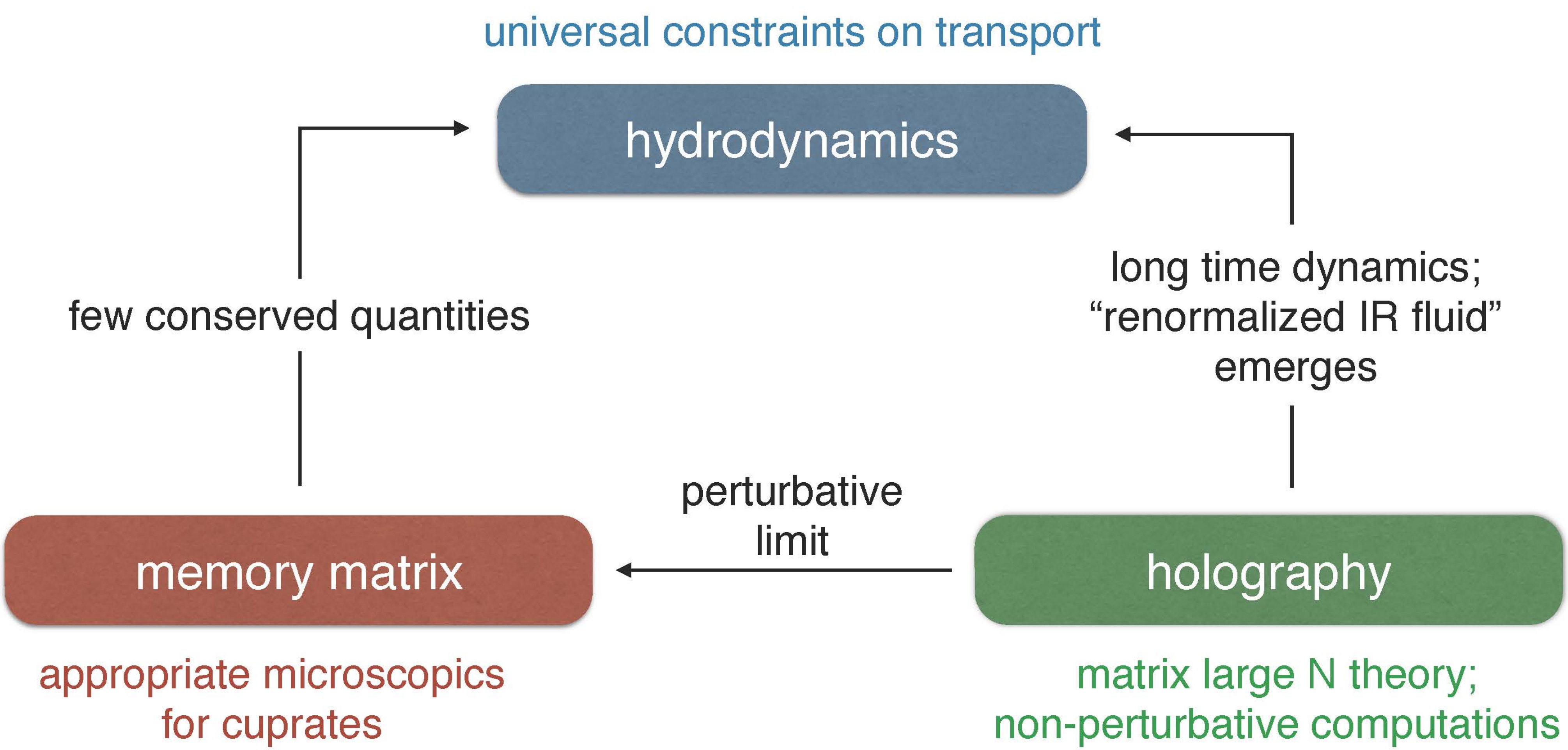
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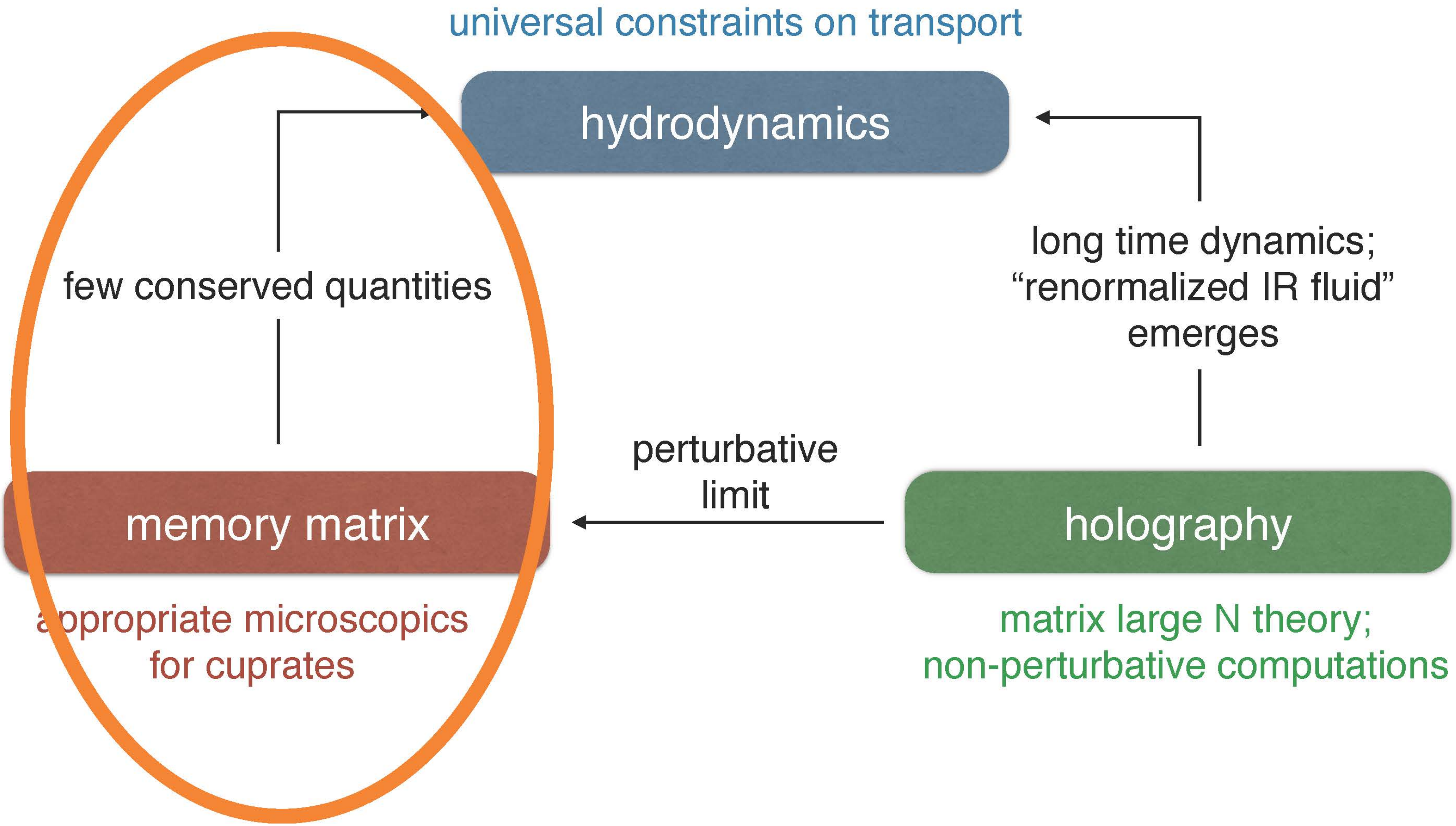
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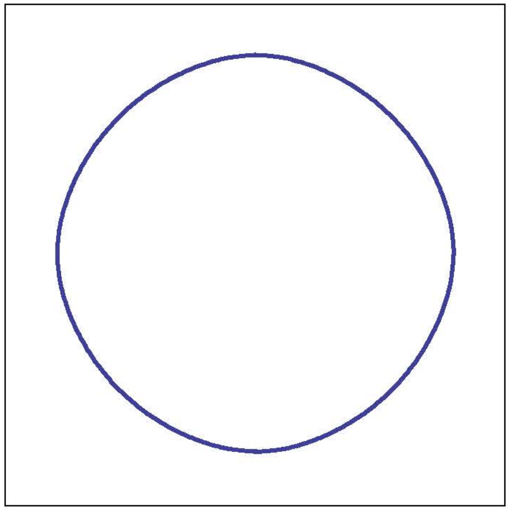
Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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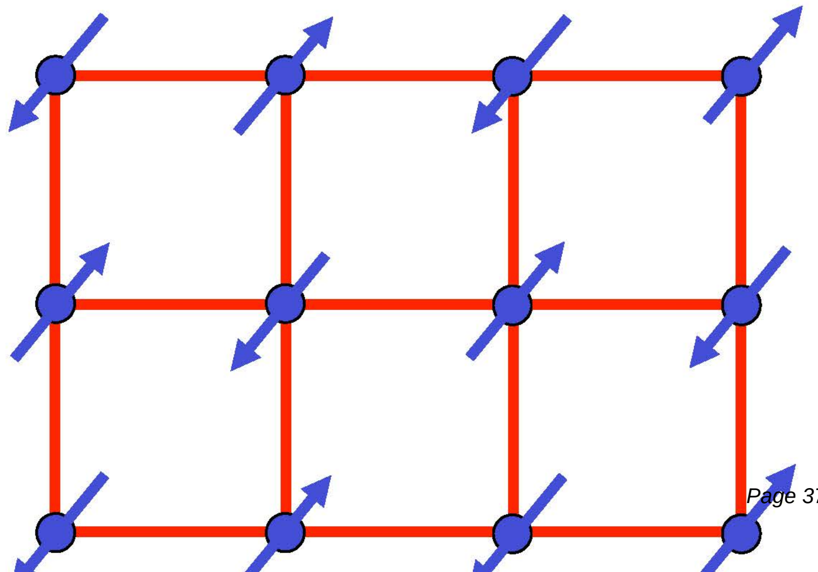
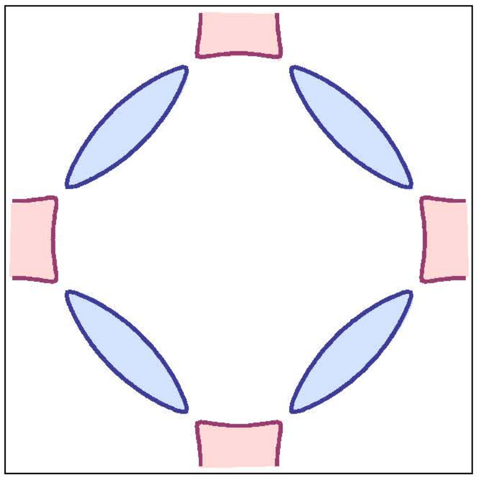


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

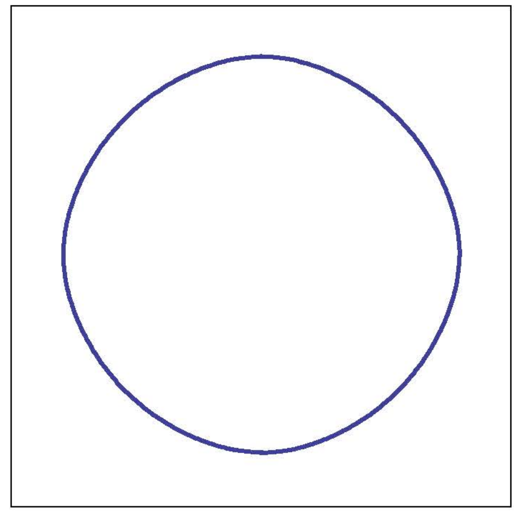
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

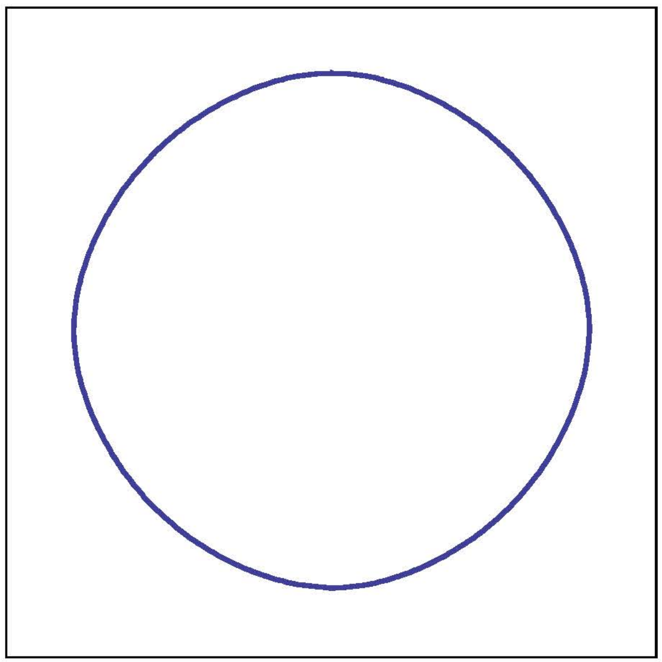
Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Increasing interaction



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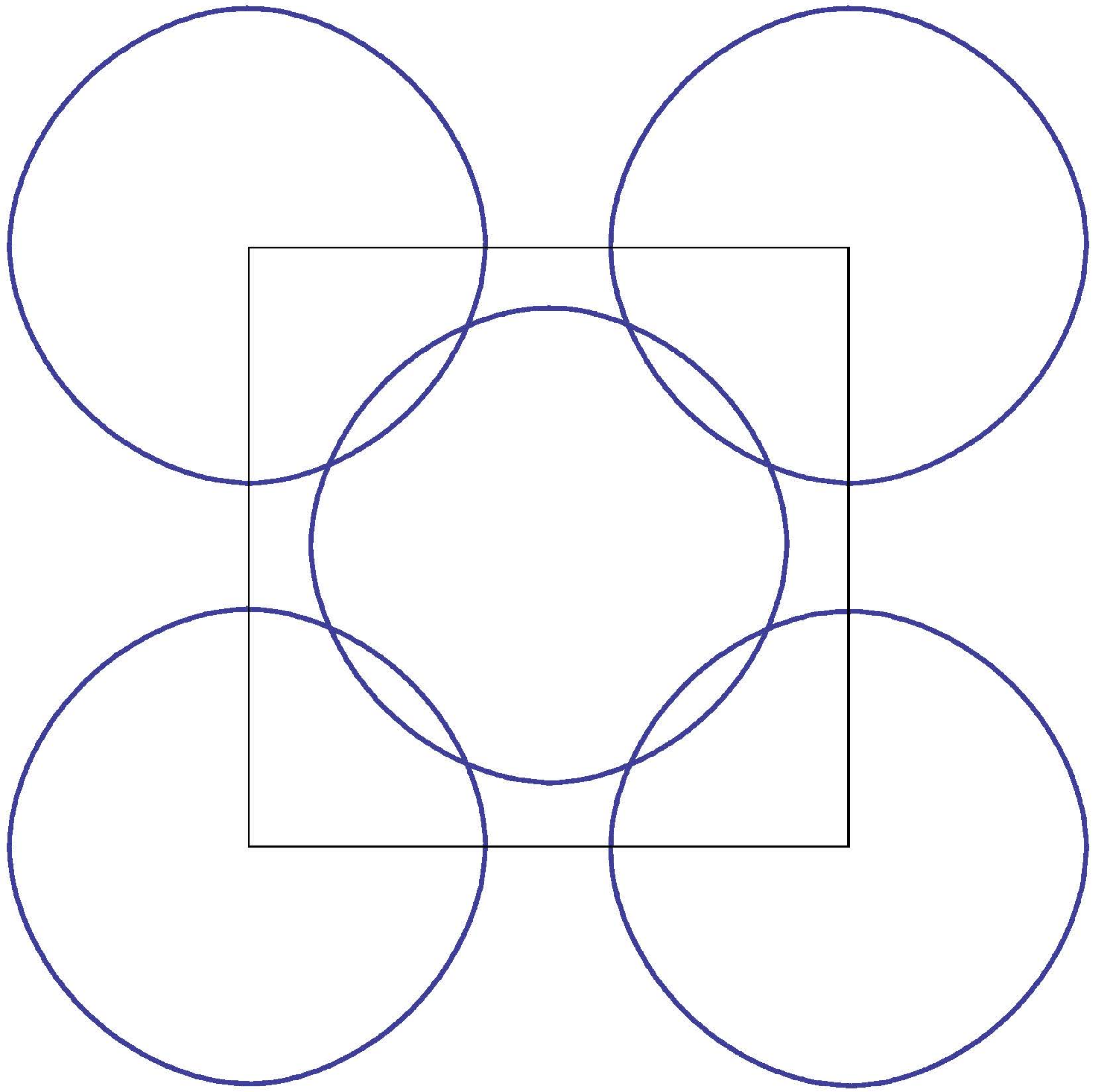
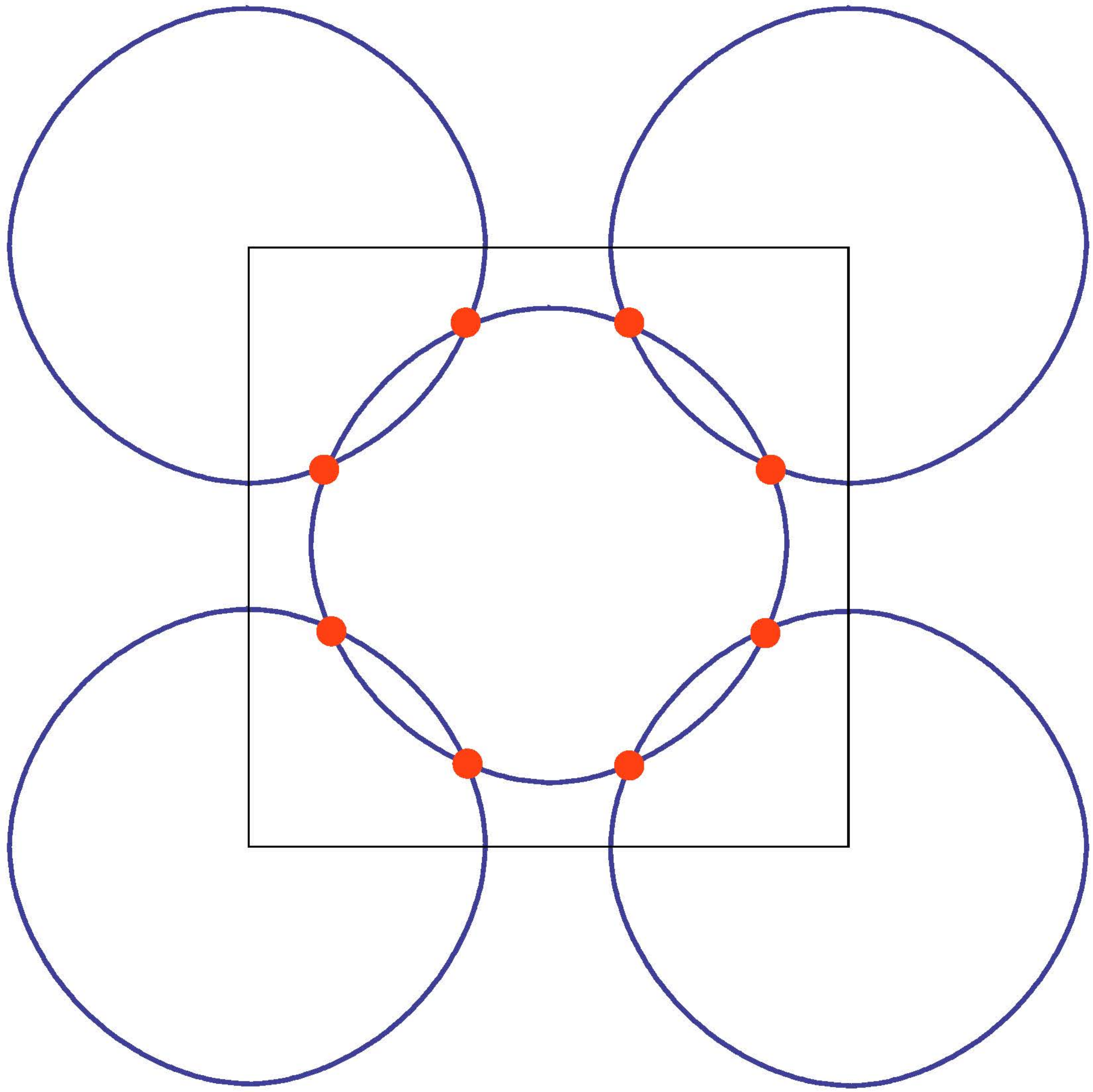
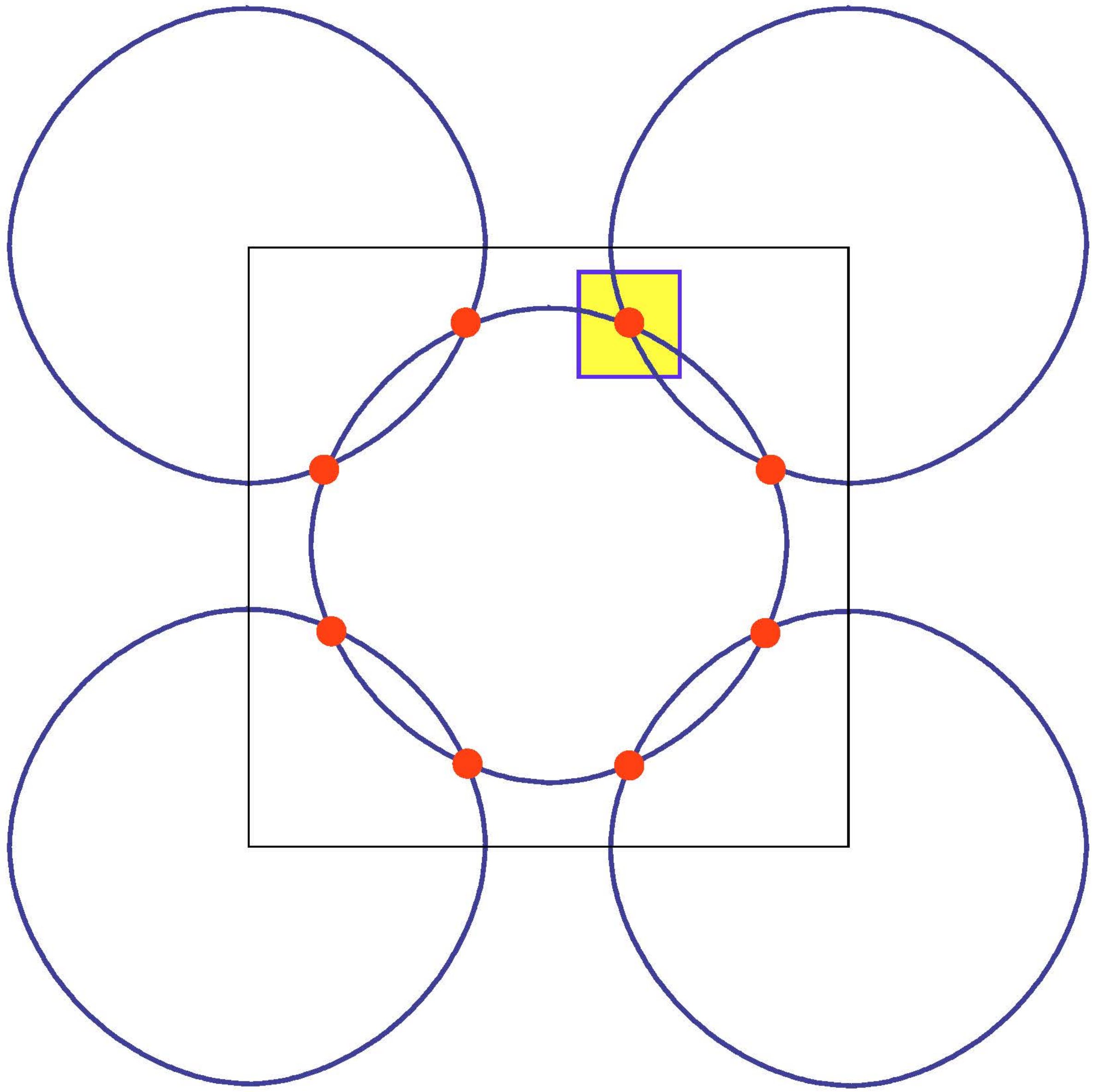


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

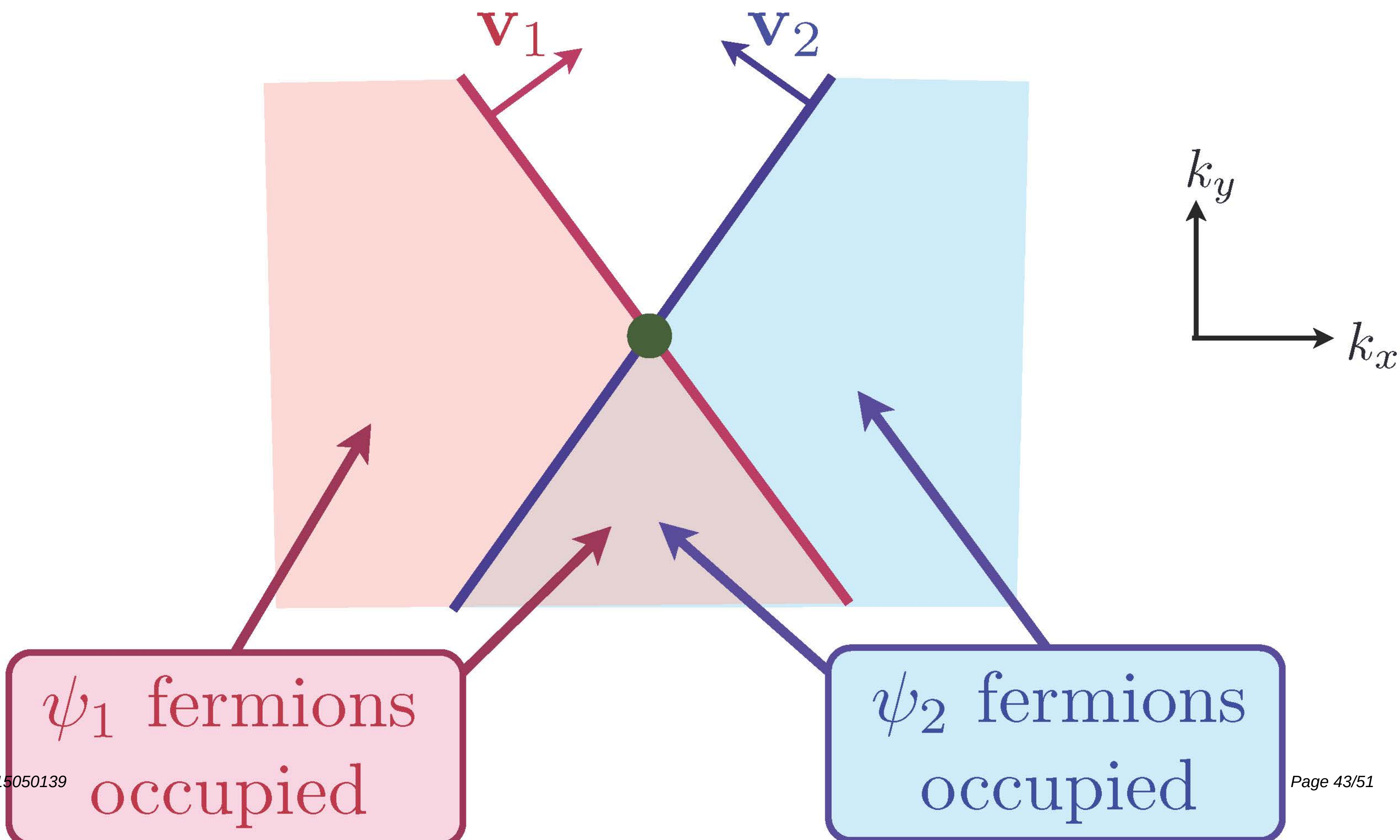


"Hot" spots



Low energy theory for critical point near hot spots

Hot-spot theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, and a "Yukawa" coupling λ. This theory is particle-hole symmetric.



$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega)$$

There is a natural separation into the two contributions to transport:

- Particle-hole symmetric hot-spot theory yields the value of σ_Q.
- Remaining "cold" regions of the Fermi surface yield the contribution of the (nearly) conserved momentum mode.

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- Remaining "cold" regions of the Fermi surface yield the contribution of the (nearly) conserved momentum mode.

But, all known particle-hole symmetric, strongly coupled critical theories obey hyperscaling, and so have $\sigma_Q \sim T^0$ in $d = 2$.

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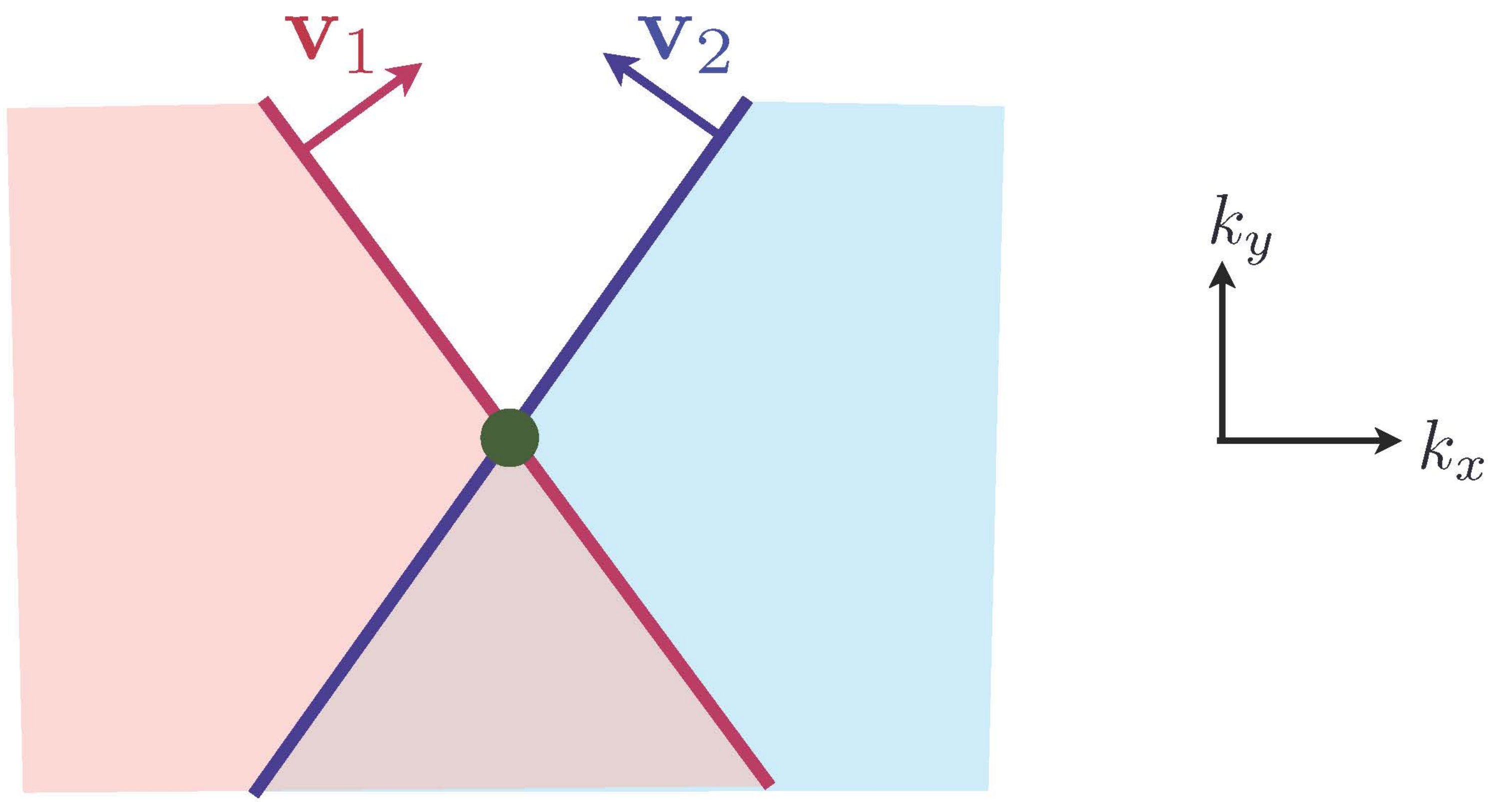

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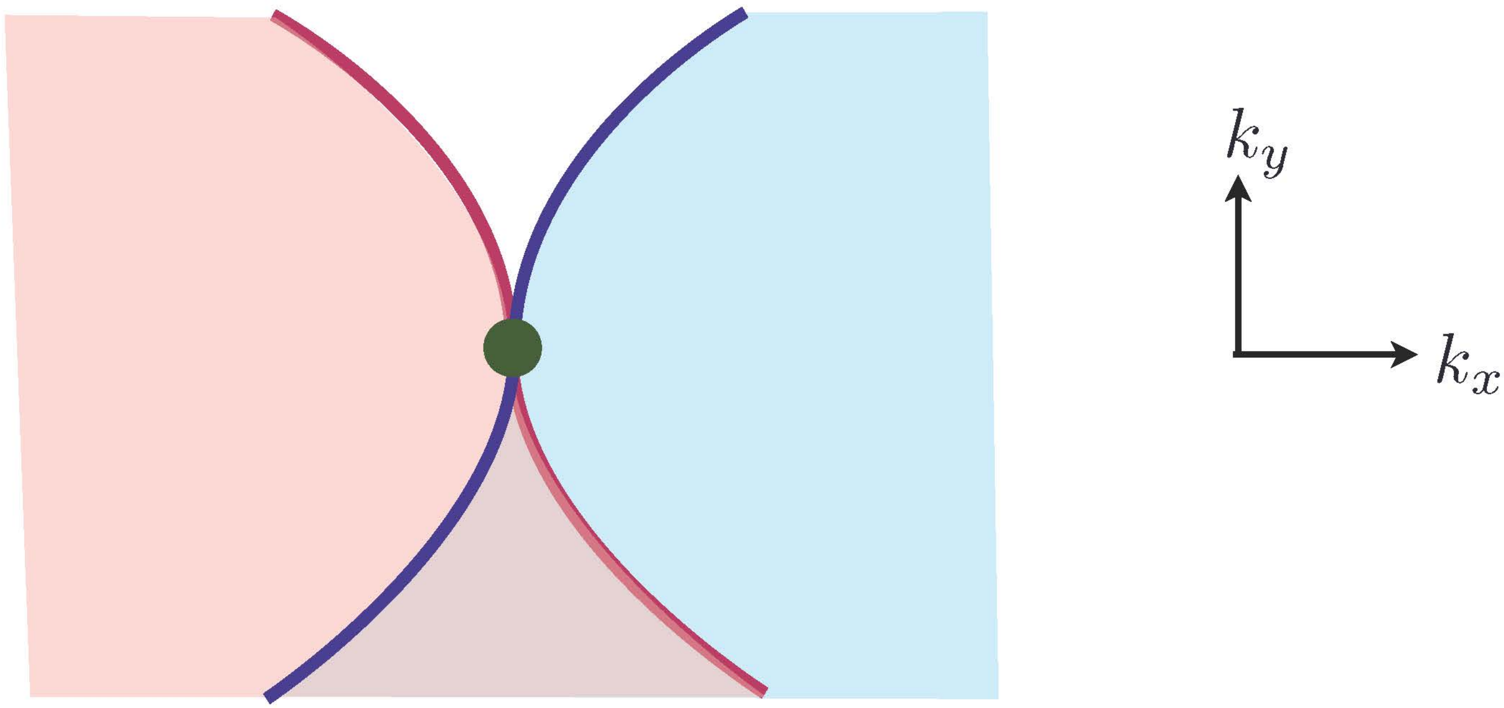
Renormalized Fermi surface has $k_y \sim k_x \ln(1/k_x)$



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- We find that the presence of gapless lines of zero energy excitations at this fixed point leads to *hyperscaling violation*.

- Upto logarithmic corrections, we find the entropy density $S \sim T^{(2-\theta)/z}$, and $\sigma_Q \sim T^{-\theta/z}$ with $\theta = 1$, and $z = 1 + \mathcal{O}(\epsilon)$.

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The observed values $\sigma_Q \sim 1/T$ and $\tau_L \sim 1/T^2$ are not too different!

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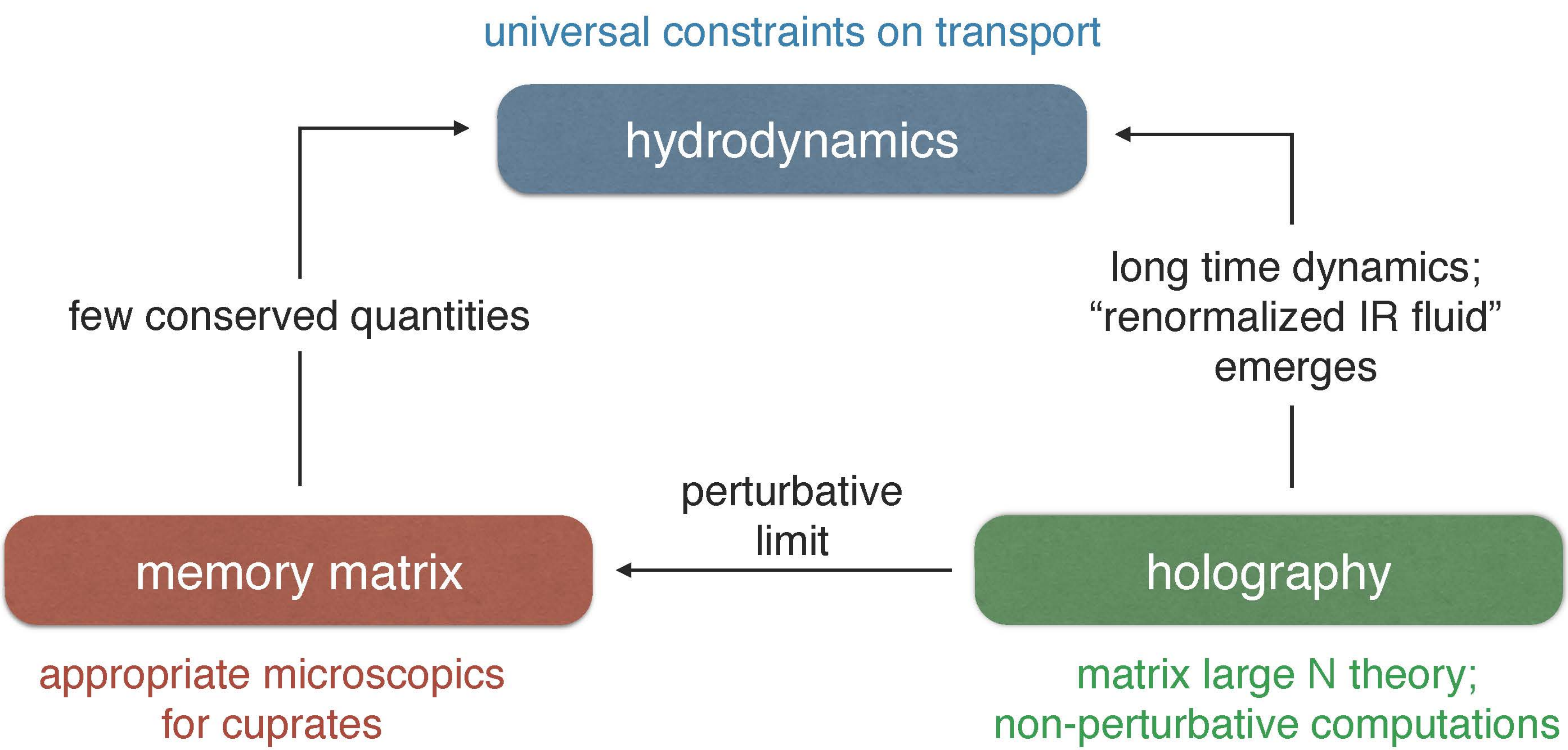
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General relativity and the cuprates

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ABSTRACT: We add a periodic potential to the simplest gravitational model of a superconductor and compute the optical conductivity. In addition to a superfluid component, we find a normal component that has Drude behavior at low frequency followed by a power law fall-off. Both the exponent and coefficient of the power law are temperature independent and agree with earlier results computed above T_c. These results are in striking agreement with measurements on some cuprates. We also find a gap $\Delta = 4.0\ T_c$, a rapidly decreasing scattering rate, and "missing spectral weight" at low frequency, all of which also agree with experiments.

KEYWORDS: Holography and condensed matter physics (AdS/CMT), Gauge-gravity correspondence, AdS-CFT Correspondence

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