Title: Even a tiny positive cosmological constant casts a long shadow

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Abstract:

Even a tiny positive Λ casts a long shadow

Abhay Ashtekar

Institute for Gravitation & the Cosmos, Penn State

First three parts: Joint work with Beatrice Bonga & Aruna Kesavan;
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Garyfest: Santa Barbara, May 1st, 2015

Isolated Systems, Gravitational Waves & the S-matrix

- Confusion re gravitational waves in full GR till 1960s (Exs: Einstein 1916 vs 1936; Eddington)

- The Bondi-Penrose et al Framework

  Notion of null infinity $\mathcal{I}^{\pm}$; (1960s to 1980s)

  Bondi, Metzner, Sachs (BMS) group $\mathcal{B} = \mathcal{S} \ltimes \mathcal{L}$

- Gravitational radiation:

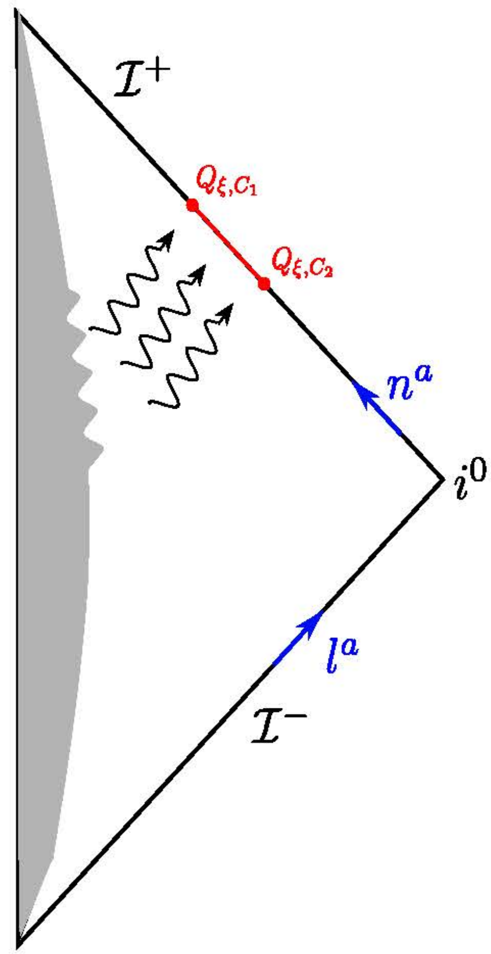
  Gauge invariant Bondi News N_{ab} at $\mathcal{I}^{\pm}$;

  No incoming radiation at $\mathcal{I}^{-}$: $N_{ab} = 0$.

  Balance law for Bondi-energy:

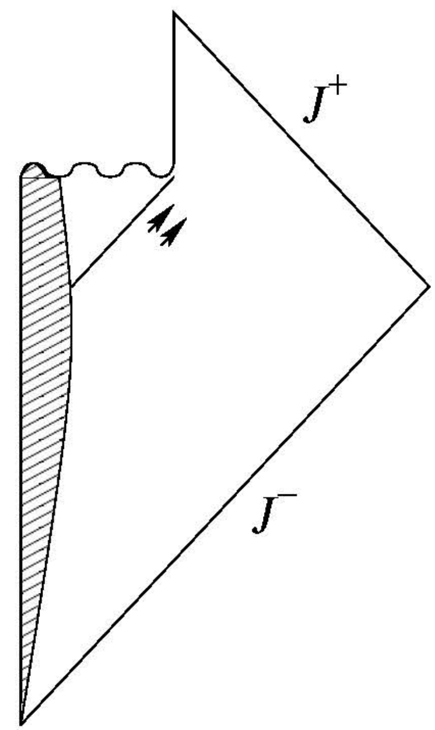
$$Q_{\xi}[C_2] - Q_{\xi}[C_1] = \int_{\Delta\mathcal{I}} \xi \, |N_{ab}|^2$$

  Positive $Q_{\xi}[C]$ and Flux positive (`Gravitational waves are real; you can boil water with them' ...Bondi)



- The BMS group $\mathcal{B}$ admits a unique 4-d Abelian normal subgroup of translations $\mathcal{T}$. Also, if $N_{ab} = 0$, then $\mathcal{B}$ reduces to the Poincare group.

- In quantum theory, one routinely uses $\mathcal{T}$ and $\mathcal{B}$ to define spin and mass of zero rest mass fields, and introduce asymptotic Hilbert spaces for the S-matrix theory, in particular to analyze the issue if information loss during black hole evaporation.



- None of this rich structure discussed so far goes over to the positive Λ case. We do not have even basic notions: Bondi news; Balance law; positive energy or flux; the `no incoming radiation' condition. Don't know what gravitational waves mean in full, non-linear GR for positive Λ , however small !
- Don't have the positive and negative frequency decomposition needed for asymptotic Hilbert spaces in quantum theory.

Organization of the Rest of the Talk

1. Asymptotically de Sitter space-times & difficulties
2. Linear fields on de Sitter Λ
3. New Strategy for positive : Outline

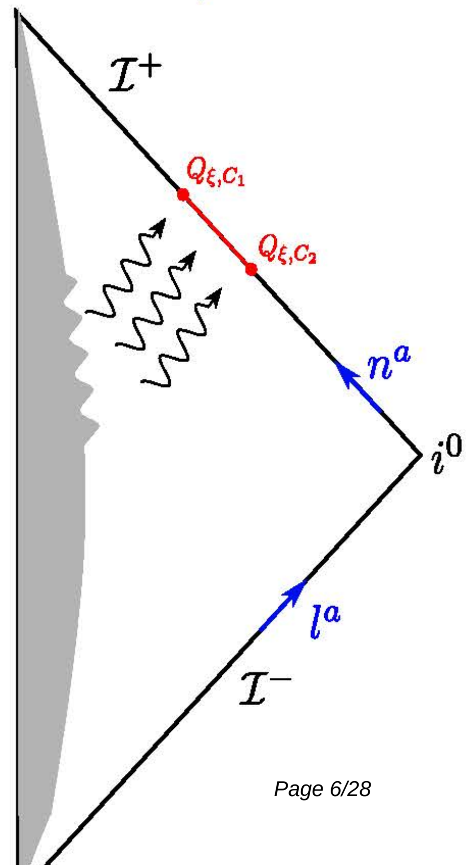
1. Asymptotically de Sitter space-times

- Recall the notion of asymptotic flatness: A physical space-time $(\tilde{M}, \tilde{g})$ is said to be asymptotically Minkowski if it admits a conformal completion (M,g), where M = $\tilde{M} \cup \mathcal{I}$ is a manifold with boundary $\mathcal{I}$, & g = $\Omega^2 \tilde{g}$ on M , s.t.

  i) At the boundary $\mathcal{I}$, we have
  $\Omega = 0$ and $\nabla\Omega \neq 0$;

  ii) $\tilde{g}$ satisfies Einstein Eqs $\tilde{G}_{ab} = 8\pi\, G_N\, \tilde{T}_{ab}$ with $\tilde{T}_{ab}$ falling off sufficiently fast as $\Omega \to 0$;

  iii) and $\mathcal{I}$ is topologically S^2 X R and complete in an appropriate sense.



Asymptotically flat case: summary (contd)

- Field equations imply that $\mathcal{I}$ is null, hence ruled by the integral curves of its null normal, $n^a = \nabla^a \Omega$. Hence, the asymptotic symmetry group is reduced from Diff ($\mathcal{I}$) to the BMS group $\mathcal{B}$, which admits a preferred 4-d (Abelian, normal) sub-group of BMS-translations $\mathcal{T}$, that is then used to define energy-momentum, positive and negative frequency decomposition, etc.

Asymptotically de Sitter space-times

- A physical space-time $(\tilde{M}, \tilde{g})$ is said to be asymptotically de Sitter if it admits a conformal completion (M,g), where M = $\tilde{M} \cup \mathcal{I}$ is a manifold with boundary $\mathcal{I}$, & g = $\Omega^2 \tilde{g}$ on M , such that :

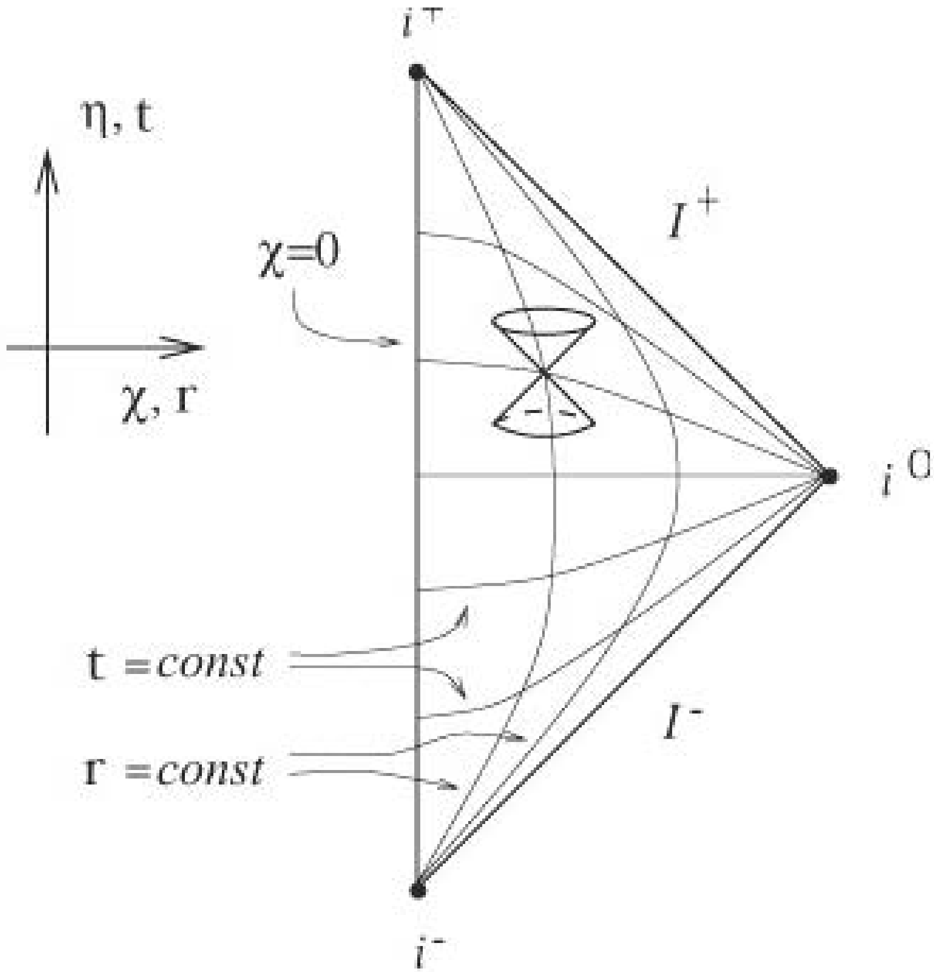
  i) At the boundary $\mathcal{I}$, we have $\Omega = 0 \text{ and } \nabla\Omega \neq 0$;

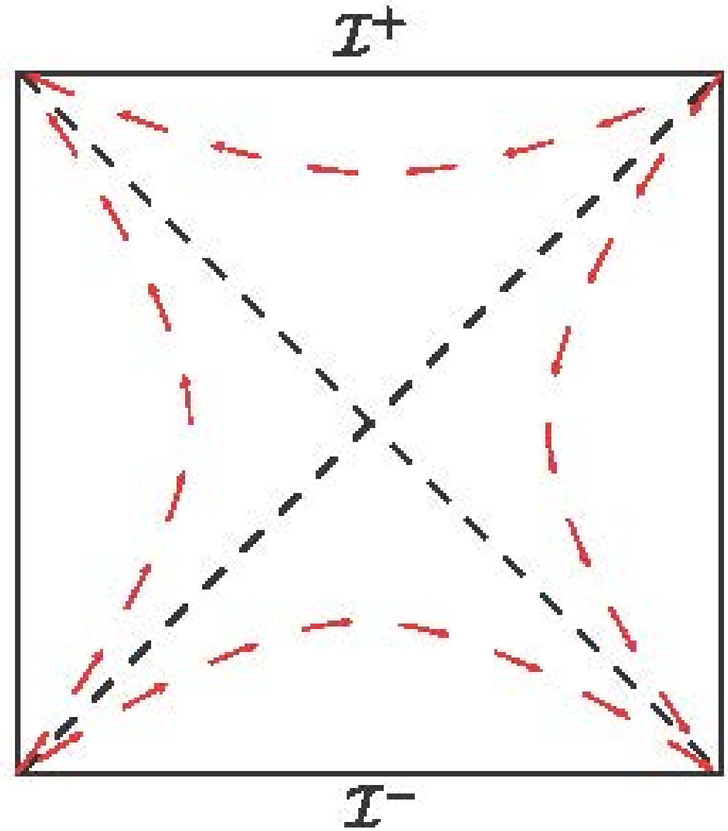
  ii) $\tilde{g}$ satisfies Einstein Eqs $\tilde{G}_{ab} = 8\pi G_N \tilde{T}_{ab} - \Lambda \tilde{g}_{ab}$ with $\tilde{T}_{ab}$ falling off sufficiently fast as $\Omega \to 0$; and,

  iii) $\mathcal{I}$ is topologically S^3 (minus punctures, e.g. S^2 X R) and complete in an appropriate sense.

Field equations now imply that $\mathcal{I}$ is space-like rather than null. Hence, no extra structure like a preferred ruling. Hence the asymptotic symmetry group is just Diff ($\mathcal{I}$) ! Not clear how to define Energy & linear (or, angular) momentum.

Contrasting Minkowski and deSitter

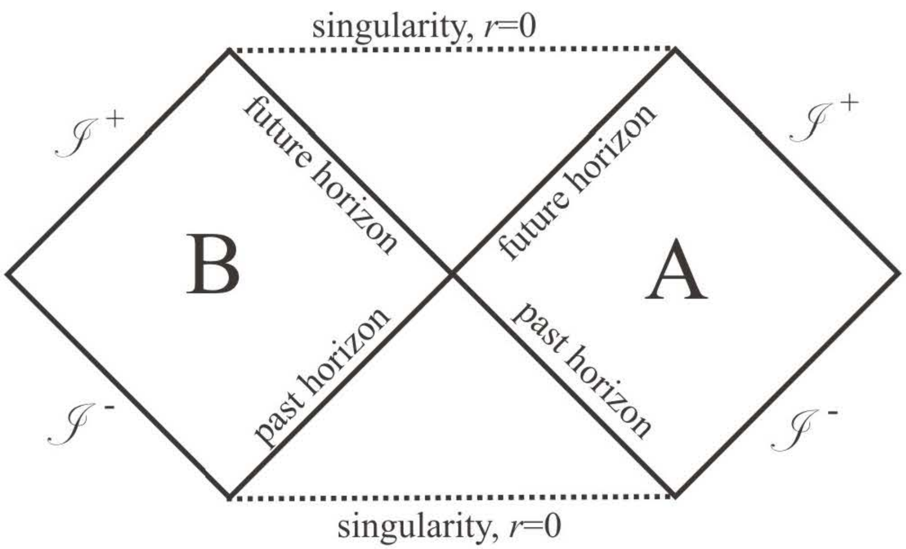




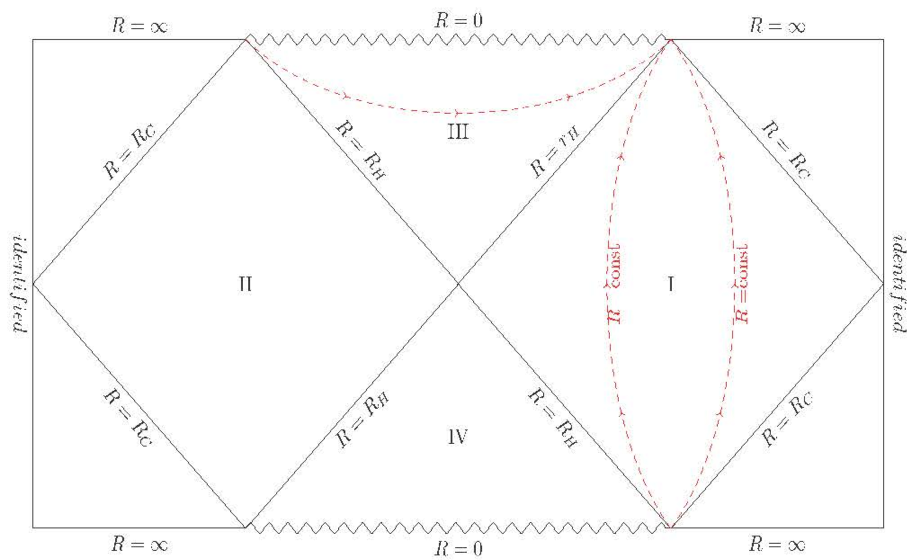
Time translation Killing fields are time-like near Minkowskian $\mathcal{I}$, whence energy fluxes of test fields across $\mathcal{I}$ are positive. In de Sitter, **all** Killing fields are space-like near $\mathcal{I}$. So fluxes associated with them, including the `energy flux' can be arbitrarily negative in de Sitter space-time irrespective of how small Λ is.

Asymptotic flatness vs Asymptotically deSitter

Eternal BH with $\Lambda = 0$



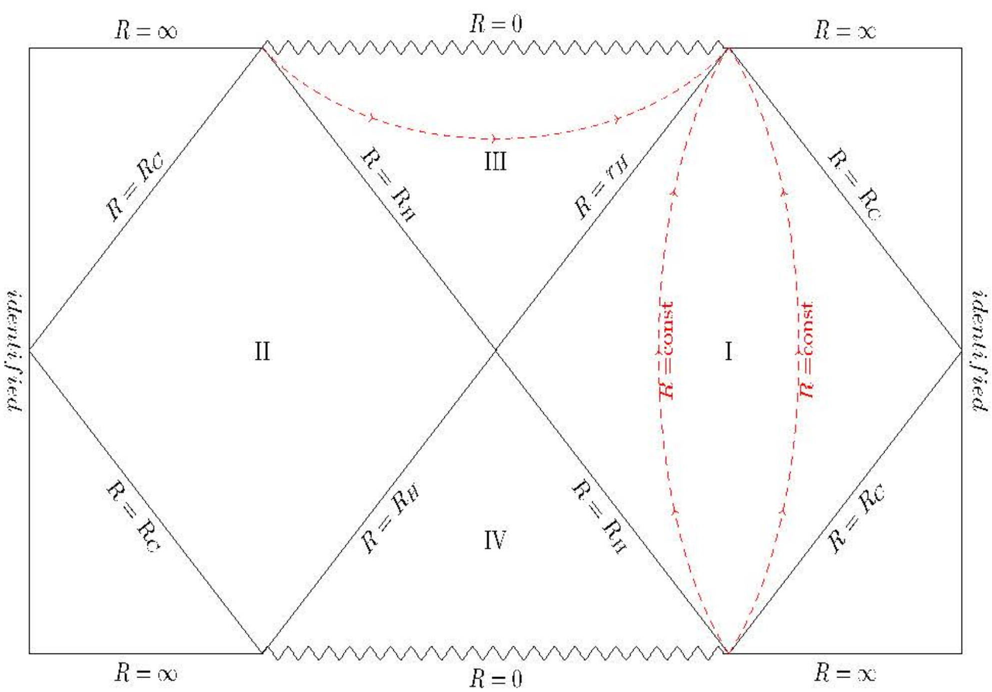
Eternal BH with positive Λ



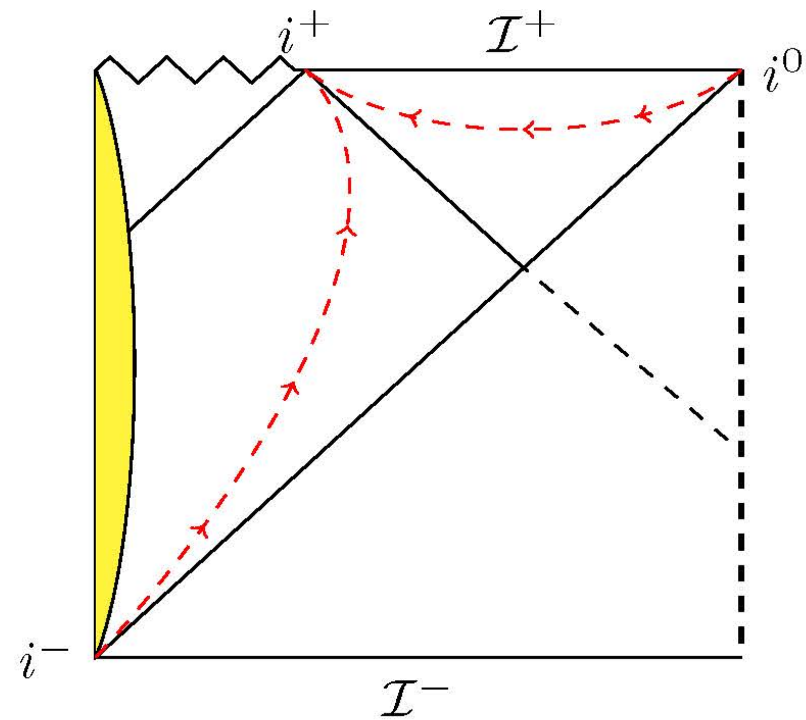
With positive Λ , the maximal extension has an infinite number of Asymptotic regions. One generally terminates the sequence via identification. Spatial topology is then $S^2 \times S^1$ rather than $S^2 \times R$.

Asymptotic flatness vs Asymptotically deSitter

Eternal Asym. dS BH



Asym. ds BH resulting from a Spherical collapse



In the collapsing case, one cannot identify. So space-time has a time-like boundary on the right. Cannot specify incoming states just on $\mathcal{I}^-$!

Restricted symmetries and Bondi-like Charges

- Can we strengthen the boundary conditions at $\mathcal{I}$ to reduce the Diff ($\mathcal{I}$) to a manageable size? A natural strategy; commonly used in the literature: Demand that, q_{ab}, the intrinsic +,+,+ metric at $\mathcal{I}$ be conformally flat, as in de Sitter.

- Not only is the group reduced; but it is reduced to the de Sitter group! Following Bondi, One can an now define charges $Q_\xi[C]$ at $\mathcal{I}$ in full GR as in asymptotically flat space-times: $Q_\xi[C] = \oint_C E_{ab}\xi^a dS^b$. Expected answers in Kerr-deSitter .

Gravitational radiation considerations: Problem

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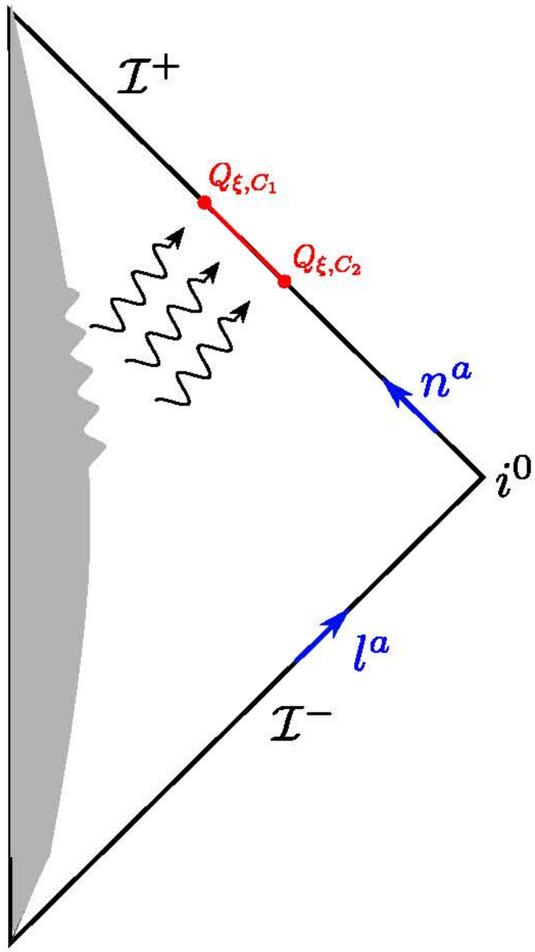
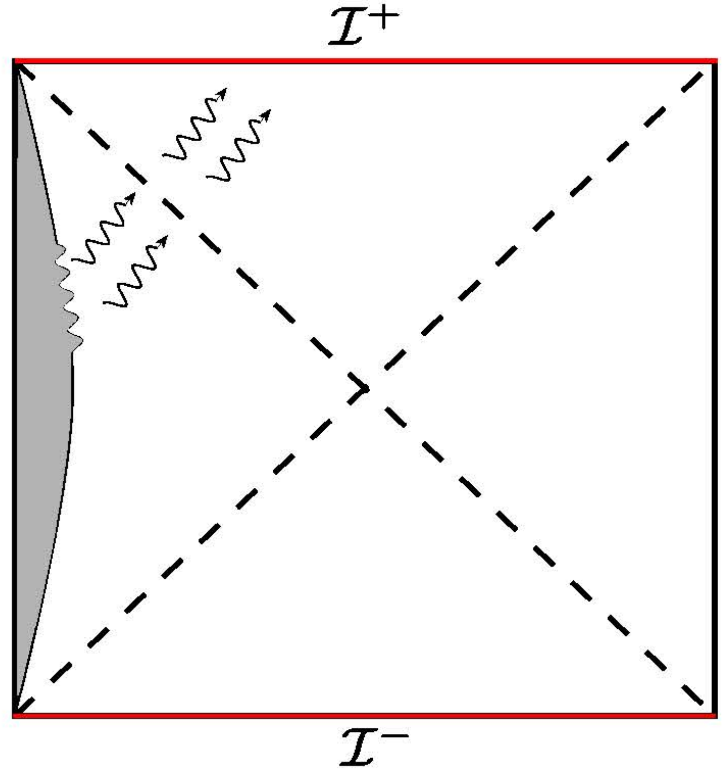
- However, there are two serous problems:
  i) Conformal flatness of $\mathcal{I}$ $\Leftrightarrow B_{ab} = 0$ at $\mathcal{I}$.
  Since $\mathcal{I}$ is space-like, half the solutions simply thrown out!
  ii) Secondly, $Q_\xi[C]$ well-defined, but absolutely conserved no flux of energy, momentum, etc through $\mathcal{I}$!

Contrasting zero and positive Λ cases





In asymptotically flat space-times, non-trivial flux of Bondi-energy through $\mathcal{I}$. If Λ is positive and $B_{ab} = 0$, fluxes across $\mathcal{I}$ vanish identically irrespective of how tiny Λ is!

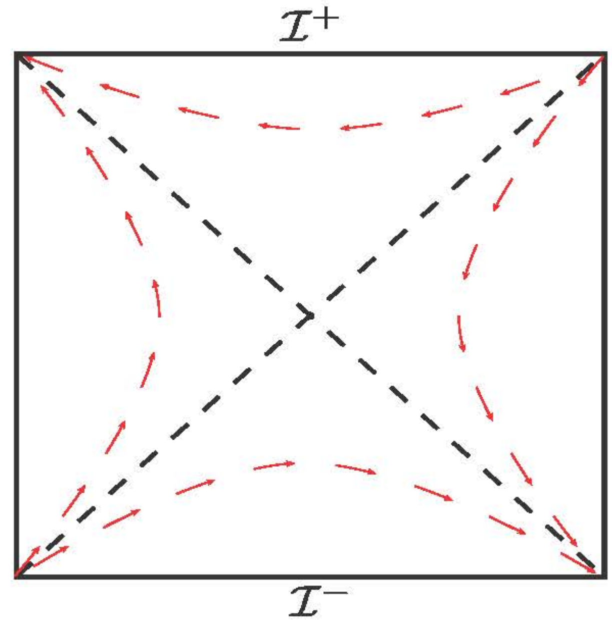
2. Linear Gravitational Waves on de Sitter

- Can seek some guidance from linearized gravitational waves as in the $\Lambda = 0$ case (where non-linear effects fall-off rapidly as one approaches $\mathcal{I}^+$.

- In the $\Lambda > 0$ case, we now have:
  * Explicit consequences of the $B_{ab} = 0$ condition.
  * Expressions of energy, momentum and angular-momentum fluxes carried by gravitational waves;
  * Positive energy-flux in physically relevant situations;

(test-)Fields in de Sitter Space-time

- Symmetries: subgroup $G = T \ltimes \mathrm{SO}(3)$ of isometries that leaves H^+ (or H^-) invariant is 7 dimensional; but T is not Abelian: $[T, S_i] = H\, S_i$ where $H = \sqrt{(\Lambda/3)}$
The time translation KVF t^a vanishes at the bifurcation surface C



- Fluxes across H^+, of deSitter energy, momentum and angular momentum associated with the 7 Killing fields K^a, generating G : $f_K = T_{ab} K^a n^b$; positive for $K^a = T^a$!
Energy flux can be negative on $\mathcal{I}$ because T^a is not future directed time-like in other quadrants.

The $B_{ab} = 0$ Condition in linear theory

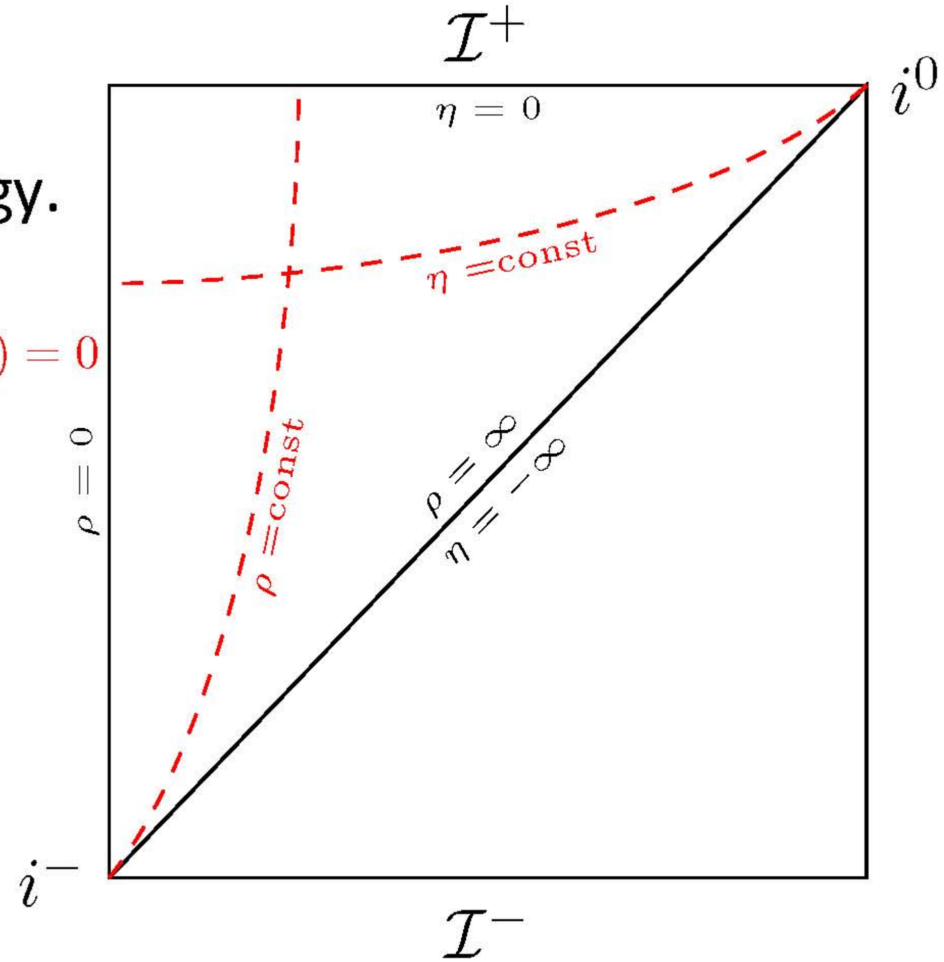
Linearized gravitational waves:

Perturbation theory used in cosmology.

Explicit calculations show that the condition $B_{ab} = 0$ at $\mathcal{I}$ requires $B(k,H) = 0$ leaving only the `decaying' modes (for which h_1 vanishes at $\mathcal{I}$), where

$$h_1(\vec{x},\eta) = E(k,H)(\sin k\eta - k\eta \cos k\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

$$h_2(\vec{x},\eta) = B(k,H)(k\eta \sin k\eta - \cos k\eta)\, e^{i\vec{k}\cdot\vec{x}}$$



de Sitter metric:

$$ds^2 = (1/H\eta)^2\,(-d\eta^2 + d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta\, d\phi^2))$$

Note: Limit $\Lambda \to 0$ corresponds to $H \to \infty$

de Sitter-momentum fluxes

- Energy-momentum and angular momentum carried by gravitational waves: Start with the covariant phase space of linear gravitational perturbations h_{ab}. Symplectic structure (derived from the action)

$$\omega(h,h') = (\ell/8\pi G)\int_M (h_{ab}E'^{\,ab} - h'_{ab}E^{ab})\, dV.$$

- We can compute Hamiltonians corresponding to de Sitter symmetries. Energy defined by a de Sitter time translation T^a:

$$H_T \equiv \tfrac{1}{2}\,\omega(h,\,\mathcal{L}_T h) = \tfrac{\ell}{8\pi G}\int_M E^{ab}(\mathcal{L}_T h_{ab} - \tfrac{2}{3}(D_c T^c)h_{ab})\, dV$$

Now, $h_{ab} = 0 \text{ at } \mathcal{I}$ if $B_{ab} = 0$ there, and the flux vanishes. Same is true for fluxes of linear and angular momentum. (Correct/ standard answer the $\Lambda \to 0$ limit, but the limit is subtle!)

- Thus, if $B_{ab} = 0$, not only we rule out by fiat ½ the DOF but the remaining DOF do not carry any de Sitter fluxes!

Why do all fluxes vanish?

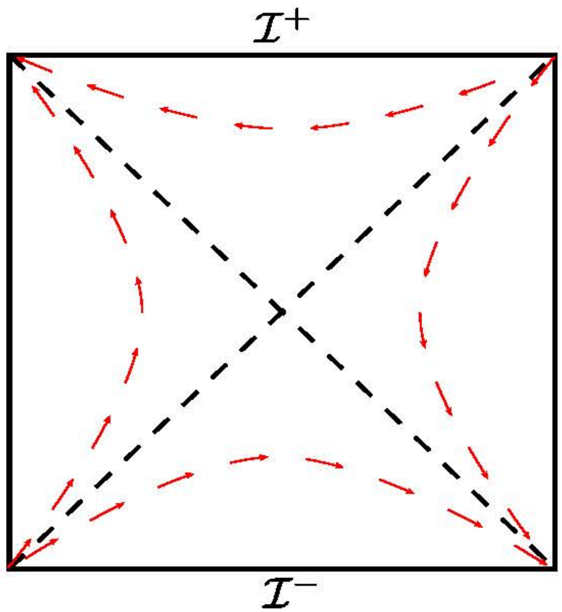
- In retrospect, however, this is not surprising. In the asymptotically flat case, if impose the condition: $B^{ab} = {}^{\star}C^{abcd} n_a n_b = 0$ at $\mathcal{I}$, we find (AA, 1980s) :

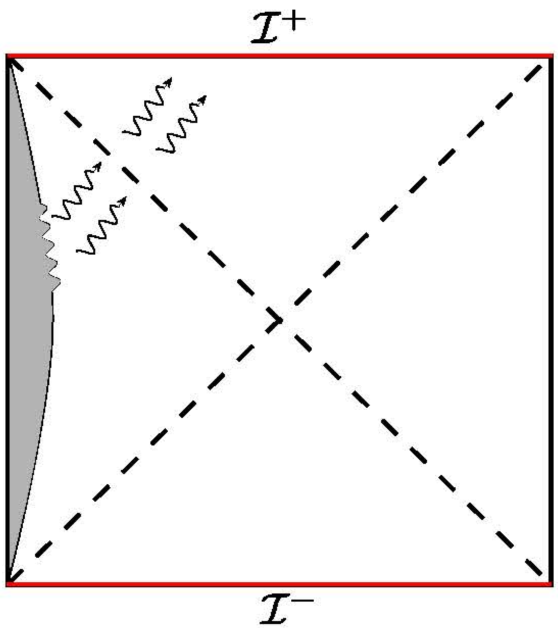
i) The BMS group reduces to the Poincare, just as Diff ($\mathcal{I}$) reduced to the de Sitter group here; and,

ii) The Bondi-news N_{ab} vanishes identically on $\mathcal{I}$; there is no flux of gravitational radiation across $\mathcal{I}$!

- Thus, the condition is too restrictive. But, if we remove it, we lose the entire machinery we routinely use in the asymptotically flat case: No `charges' representing de Sitter energy, momentum etc, let alone the positive energy theorem; no analog of the gauge invariant Bondi news $N_{ab}$; no access to the structure needed in simulations of BH mergers to calculate the `kicks' via emission of 3-momentum; no obvious Hilbert spaces of asymptotic states to for quantum theory!

Properties of the energy-flux



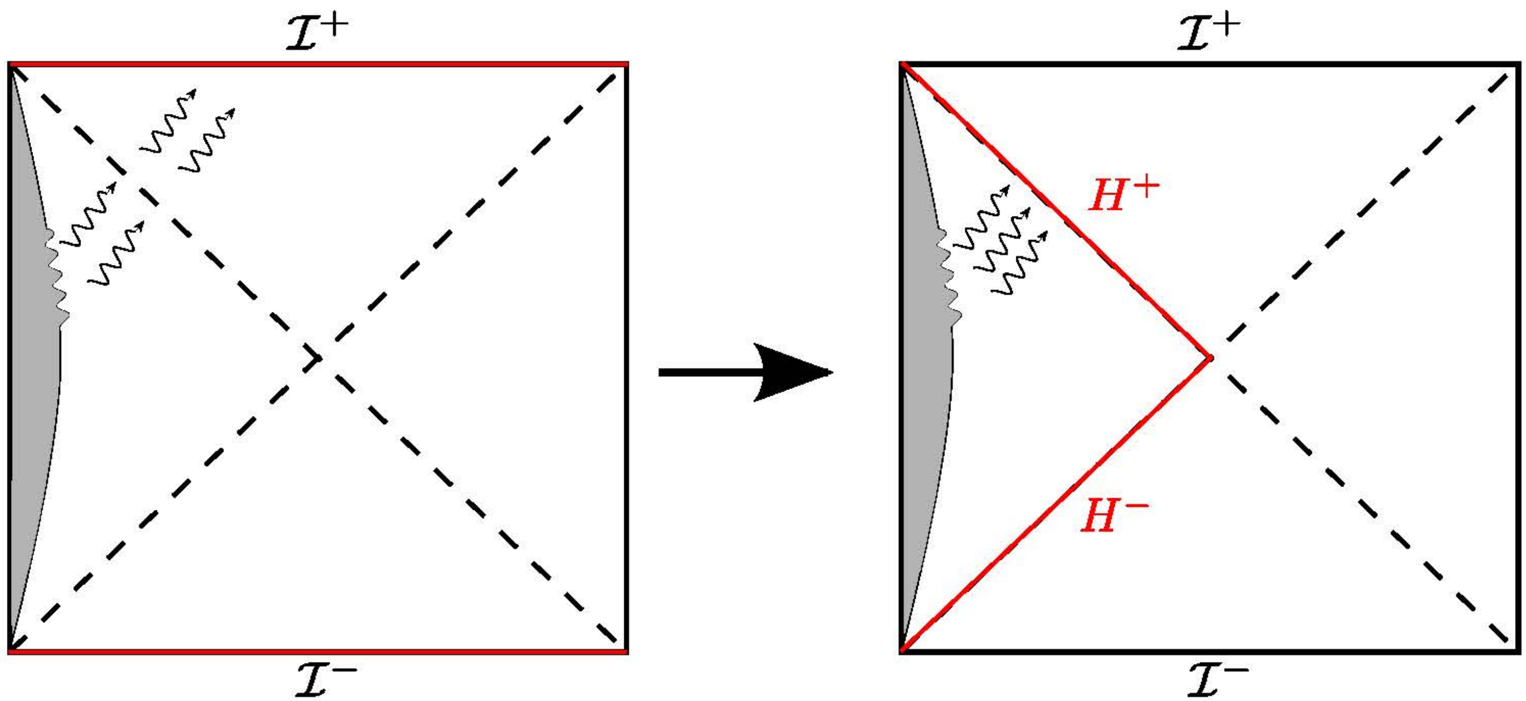


- Positivity: The Killing field T^a is future pointing and time-like in the left quadrant. This implies that H_T is positive in all physically interesting situations shown in the two figures

- The explicit expression of H_T : Agrees with the (2nd order) linearization of the flux one would get in the exact theory if we used $Q_\xi[C] = \oint E_{ab}\xi^a dS^b$ for charge integrals in the exact theory! Powerful hint for the Exact theory.

3. Strategy for the Full, non-linear theory: Outline

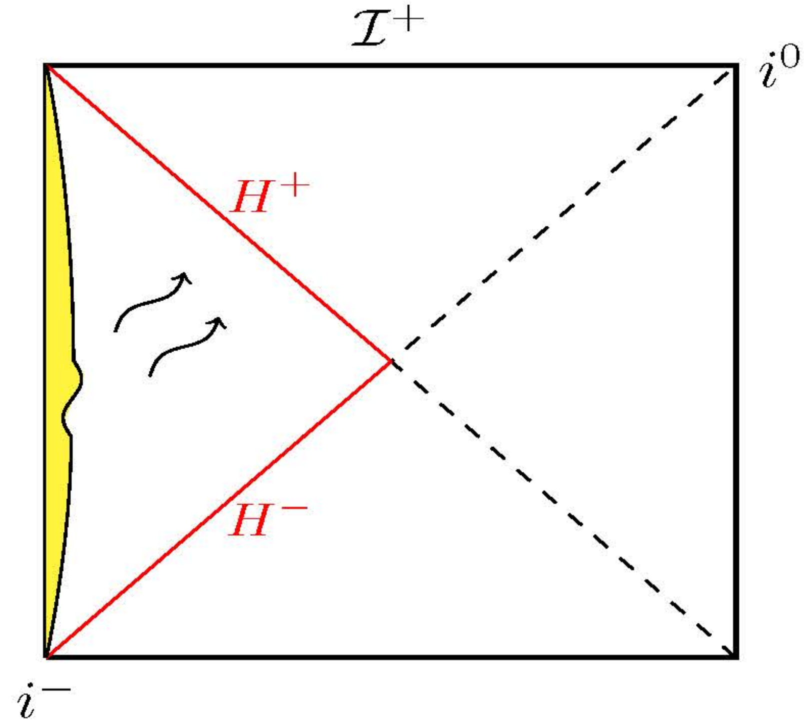
- The strategy is to construct the theory using cosmological horizons in place of $\mathcal{I}^-$. Two possibilities being pursued:



An oscillating star emitting gravitational waves. These are registered at the future horizon. Requiring that the past horizon be a Weakly Isolated Horizon (WIH) naturally incorporates the no incoming radiation boundary condition.

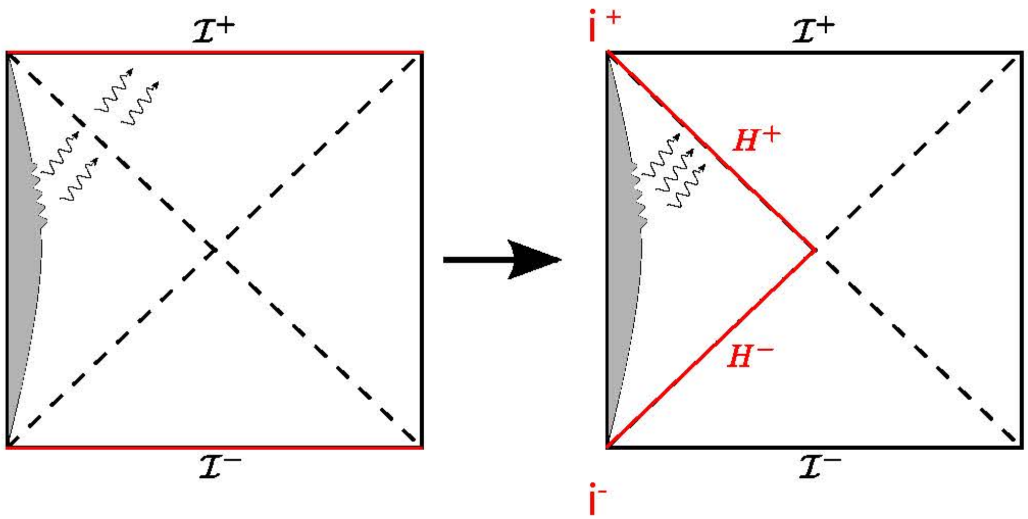
Meaning of `Radiation'

- In Minowski space, we ask for the 1/r part of the field based on peeling theorems. They don't carry over (Bicak et al, Penrose). Cannot simply ask for retarded fields because of the `Coulombic' parts of the field.



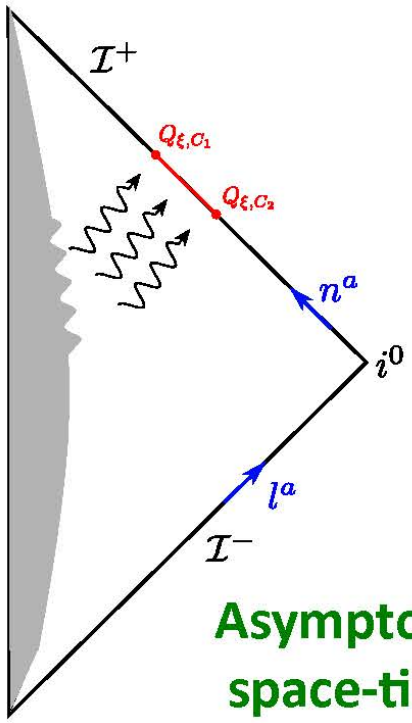
- Mimic the procedure used at Minkowski $\mathcal{I}$: e.g. at H^- the radiative modes of the Maxwell field encoded in $\underleftarrow{F_{ab} l^b}$. Vanishing of these two functions implies all fluxes at H^- i.e. $f_K = T_{ab} K^a l^b$, are zero: `No incoming radiation' condition can be imposed satisfactorily at H^-. Radiative modes $\underleftarrow{F_{ab} n^b}$ at H^+ determine fluxes created by the source.

Full, non-linear GR: The Setup



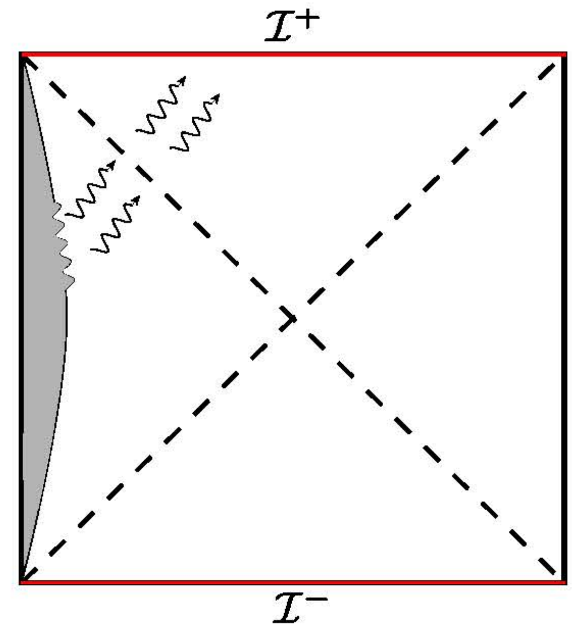
Focus on gravitating systems that remain in a spatially bounded region. Then we obtain a point i$^-$ on $\mathcal{I}^-$ and a point i$^+$ on $\mathcal{I}^+$.

Assume that, H$^-$, the future event horizon of i$^-$ is a weakly isolated horizon (WIH): A null non-expanding submanifold, S^2 X R, equipped with a null normal l^a which is a symmetry of the intrinsic metric and \`exitrinsic curvature' of H$^-$. Implements the \`no incoming radiation' condition. (AA, Beetle, Fairhurst,Lewandowski,...) . Area constant. H$^-$ is the local $\mathcal{I}^-$. H$^+$, the past event horizon of i$^+$, serves as the local $\mathcal{I}^+$
This will be our notion of an isolated system in presence of positive Λ .

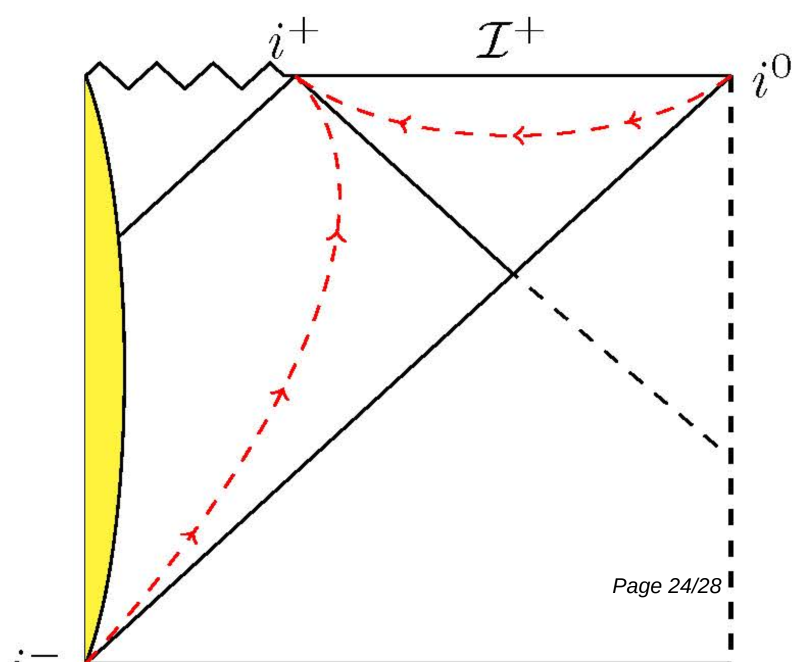


Asymptotically flat space-time

Exploit the properties of the Killing field



Asymptotically de Sitter Space-time



Collapse To a BH

For the region bounded by the past horizon H⁻, information coming from the right time-like boundary (world-tube of i° is irrelevant!

Symmetries & Charge integrals at H^+

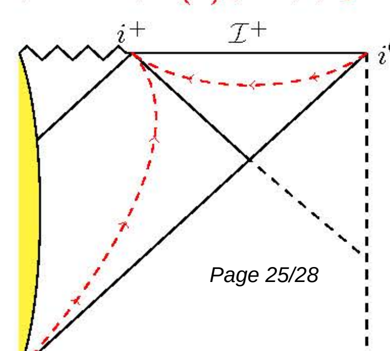
- Using the structure at the bifurcate cross-section C_0, one can introduce a 4 dimensional symmetry group *G* also on the future horizon H^+. Using the time translation symmetry field T^a one can do a +/- frequency decomposition and construct asymptotic states on the two horizons for S matrix theory.

- Charge integrals more subtle because of the presence of gravitational radiation on H^+. We have a proposal with several desired properties but it may have to be refined as we analyze further properties : For energy, (T^a is called n^a for easy comparison with asymptotically flat case):

$$Q_n[C] = (1/8\pi G)\oint_C d^2V\, r\,[\mathrm{Re}(\Psi_2 + \bar\sigma_{(l)}\sigma_{(n)}) + \theta_n((1/r) - (\theta_{(l)}/2))]$$

$$\sim (1/8\pi G)\oint_C d^2V\, r\,[\mathrm{Re}(\Psi_2 + \bar\sigma_{(l)}\dot\sigma_{(l)})$$

at $\mathcal{I}^+$ in the asymptotically flat case.



Charge integrals and balance laws at H^+

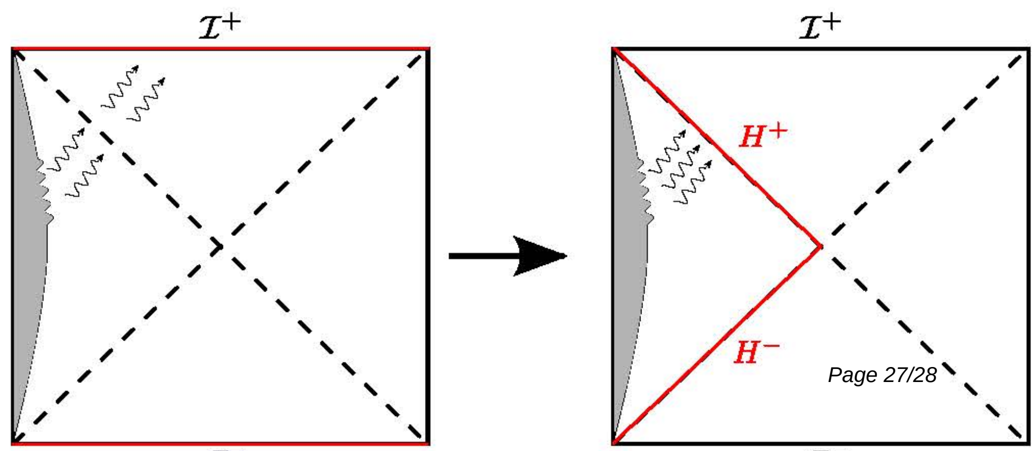
- The energy `charge' $Q_n[C]$ on H^+ is closely related to area as one would expect from the study of quasi-local horizons in the dynamical context (AA, & Krishnan, Booth & Fairhurst, …) . This provides a dual picture not available at $\mathcal{I}^+$ of asymptotically flat space-times: $$Q_n[C] = (r/2G)\,[1 - (r^2/l^2) + 2\dot{r}]$$

Note the close similarity with Schwarzschild de Sitter. $Q_n[C]$ is guaranteed to be positive if the horizon radius r can be shown to be always less than the cosmological radius ℓ .

- There is a balance law , very similar to Bondi's. Note that as the energy is radiated away across the cosmological horizon, its energy decreases and area increases.

4. Summary

- For positive Λ, literature has focused primarily on $\mathcal{I}$, assuming conformally flat intrinsic geometry. But this is too restrictive because it halves the number of modes and, furthermore, ill suited to study gravitational radiation in full GR and for quantum considerations.
- Inclusion of Λ, however small, introduces qualitatively new, conceptual issues. Exs: $\mathcal{I}$ and hence all symmetry vector fields there are space-like; energy can be arbitrarily negative; an extra time-like boundary in gravitational collapse changing the S-matrix theory paradigm; no `peeling' at $\mathcal{I}$, making it difficult to impose no incoming radiation condition …
- New framework: Focus instead on H^- (and H^+ ?) adapted to the isolated system of interest.



Symmetries and Charge Integrals at H^-

- If H^- is axi- symmetric, the symmetry group on H^- is 7 dimensional. Furthermore, we can define `conserved charges' Q_K [C] associated with any cross-section C of H^- and a symmetry vector field K^a on H^-. These are absolutely conserved because there is no radiation across H^-. Expected results for Kerr-de Sitter. Even without axi-symmetry, group is 1 dimensional and energy (or mass) is well-defined.

- Expectation (Work in Progress): If T_{ab} satisfies energy conditions, then the energy, Q_T[C] is positive (ADM type energy associated with H^-). Idea is to use the Witten-type spinorial equation (and appropriate boundary conditions for the spinor at a cross section C of H^- (possibly C_0)).

- Balance between `rich structure' to do physics and mathematics and `rich set of examples'. Kerr-de Sitter has this structure and so do the few numerical relativity simulations of gravitational collapse that have been worked out (Shibata group, Shapiro group)