Title: TBA

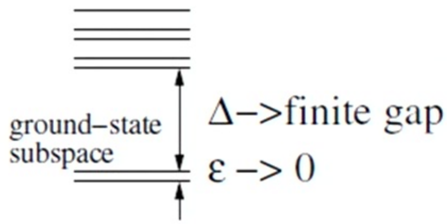
Date: May 28, 2015 03:45 PM

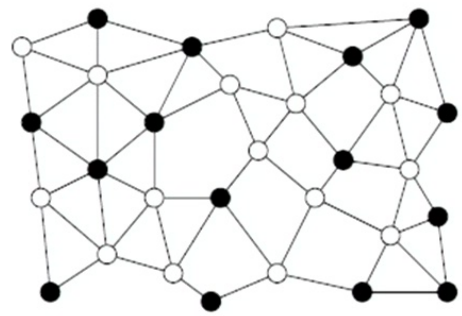
URL: http://pirsa.org/15050130

Abstract:

Local quantum systems and gapped quantum systems

- A **local quantum system** is described by $(\mathcal{V}_N, H_N)$
  $\mathcal{V}_N$: a Hilbert space with a tensor structure $\mathcal{V}_N = \otimes_{i=1}^N \mathcal{V}_i$
  H_N: a local Hamiltonian acting on $\mathcal{V}_N$:
  $H_N = \sum \hat{O}_{ij}$





- *A ground state is not a single state in $\mathcal{V}_N$, but a subspace $\mathcal{V}_{grnd\ space} \subset \mathcal{V}_N$.*

- A **gapped quantum system** (a concept for $N \to \infty$ limit):
  $\{(\mathcal{V}_{N_1}, H_{N_1}); (\mathcal{V}_{N_2}, H_{N_2}); (\mathcal{V}_{N_3}, H_{N_3}); \cdots\}$ with gapped spectrum.

- *A gapped quantum system is not a single Hamiltonian, but a sequency of Hamiltonian with larger and larger sizes.*

Local unitary trans. defines gapped quantum phases

- Two gapped states, $|\Psi(0)\rangle$ and $|\Psi(1)\rangle$ (or more precisely, two gapped ground state subspaces), are in the same phase iff they are related through a local unitary (LU) evolution

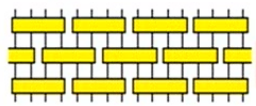
$$|\Psi(1)\rangle = P\left(e^{-\mathrm{i}\int_0^1 dg'\, H(g')}\right)|\Psi(0)\rangle$$

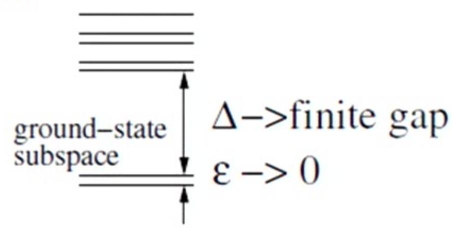
where $H(g) = \sum_{\boldsymbol{i}} O_{\boldsymbol{i}}(g)$ and $O_{\boldsymbol{i}}(g)$ are local hermitian operators.

- $|\Psi(0)\rangle$ and $|\Psi(1)\rangle$ can be smoothly connect without closing the gap. Hastings-Wen cond-mat/0503554;Bravyi-Hastings-Michalakis arXiv:1001.0344

- LU evolution = *local unitary transformation*:

$$|\Psi(1)\rangle = P\left(e^{-\mathrm{i}\,T\int_0^1 dg\, H(g)}\right)|\Psi(0)\rangle$$

$=$  $|\Psi(0)\rangle$



- The local unitary transformations define an equivalence relation: *Two gapped states related by a LU trans. are in the same phase.* **A gapped quantum phase is an equivalence class of local unitary transformations** – a conjecture.

A gapped quantum liquid phase: [Zeng-Wen, arXiv:1406.5090]

- **A gapped quantum phase:**
  $H_{N_1}, H_{N_2}, H_{N_3}, H_{N_4}, \cdots$
  $H'_{N_1}, H'_{N_2}, H'_{N_3}, H'_{N_4}, \cdots$

$$\begin{array}{cccc} \Psi_{N_1} & \Psi_{N_2} & \Psi_{N_3} & \Psi_{N_4} \\ \updownarrow LU & \updownarrow LU & \updownarrow LU & \updownarrow LU \\ \Psi'_{N_1} & \Psi'_{N_2} & \Psi'_{N_3} & \Psi'_{N_4} \end{array}$$

  OK definition only for translation invariant systems.

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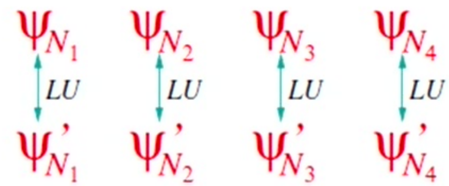

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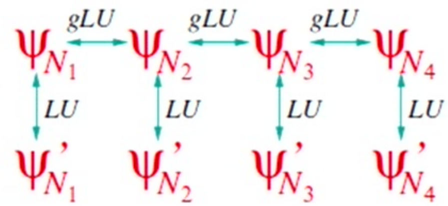

- $\Psi_{N_{i+1}} \overset{LA}{\sim} \Psi_{N_i} \otimes \Psi^{dp}_{N_{i+1}-N_i}$. Generalized local unitary (gLU) trans.
  where
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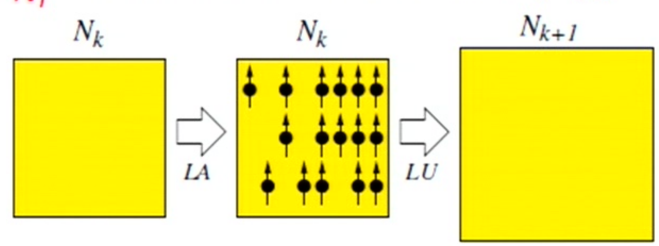

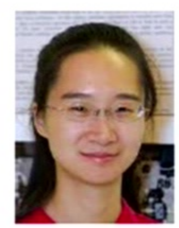

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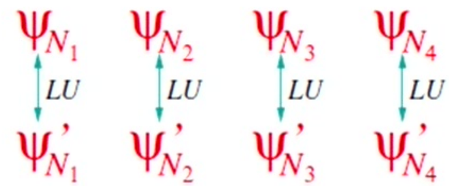

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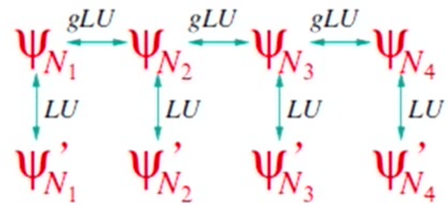

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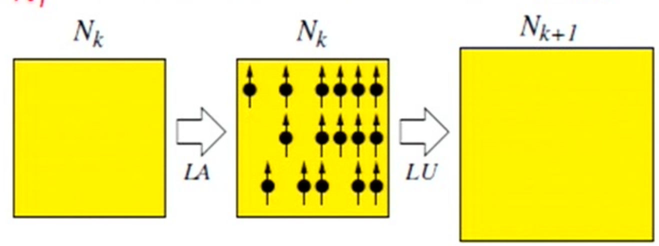
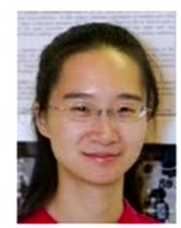




- gLU transformations allow us to take the thermal dynamical limit ($N_k \to \infty$ limit) without translation symmetry.

Examples

Different gapped quantum systems are described by different sequences of Hamiltonians $\{H_{N_k}\}$:

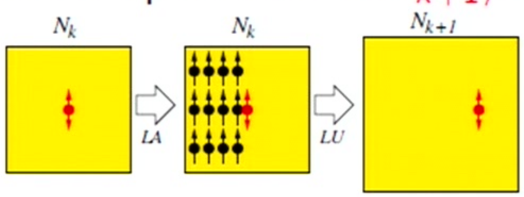
- Trivial gapped quantum liquid system: $H_{N_k}^{\text{trivial-liquid}} = \sum_{i=1}^{N_k} \sigma_i^z$

- Gapped quantum system (non-liquid): $H_{N_k}^{\text{non-liquid}} = \sum_{i=1, i\neq N_k/2}^{N_k} \sigma_i^z$

  (The ground states $\Psi_{N_k}^{\text{non-liquid}}$ have a free spin at $x = N_k/2$)

- Fail to have gLU relation $\Psi_{N_k}^{\text{non-liquid}} \to \Psi_{N_{k+1}}^{\text{non-liquid}}$:

A free spin at $x = 3N_{k+1}/4$



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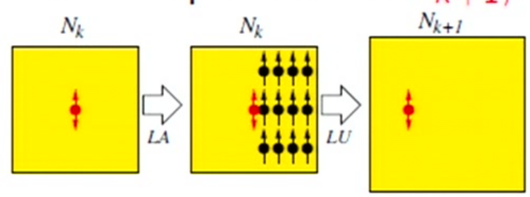

The above two states of system size N_{k+1} are not LU related to the ground state $\Psi_{N_{k+1}}^{\text{non-liquid}}$ with a free spin at $x = N_{k+1}/2$

A gapped quantum liquid phase: [Zeng-Wen, arXiv:1406.5090]

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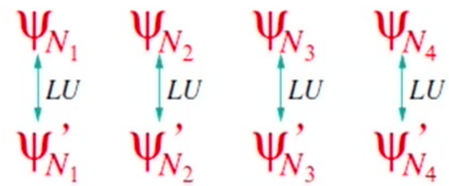

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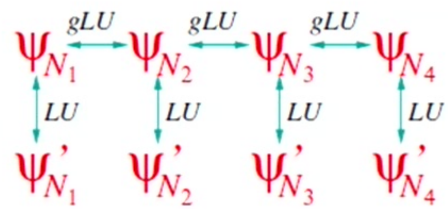

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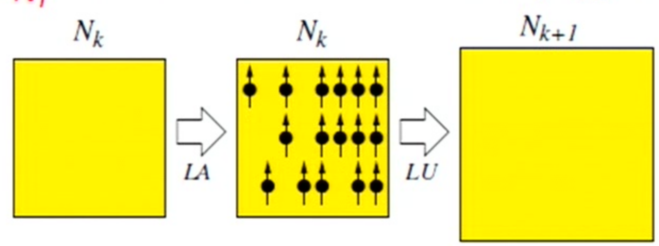

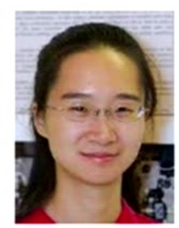

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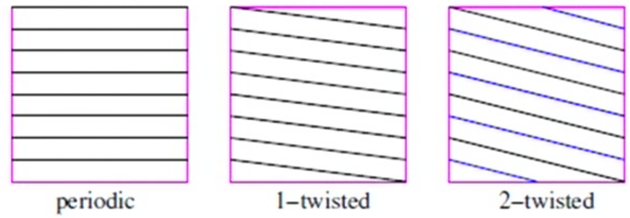
Examples

- Transverse Ising model in symmetry breaking phase
  $\to$ a gaped quantum liquid. Ground state degeneracy $GSD = 2$

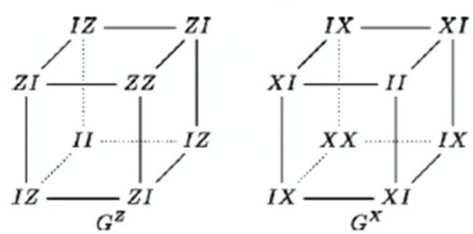
- Stacking 2+1D FQH states $\to$ gapped quantum state,
  but not liquids.

- Layered $\nu = 1/m$ FQH state:
  Ground state degeneracy can be
  $GSD = m^{L_z}, m, m^2$



- Haah's cubic code on 3D cubic lattice:

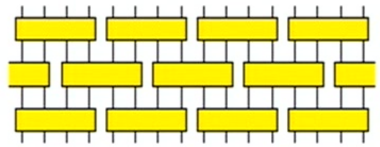
$$H = -\sum_{cubes} (G^Z + G^X),$$



Jeongwan Haah, Phys. Rev. A 83, 042330 (2011) arXiv:1101.1962

Bosonic/fermionic gapped quantum liquid phases

Both local bosonic and fermionic systems have the following local property: $\mathcal{V}_{\text{tot}} = \otimes_i \mathcal{V}_i$ Gu-Wang-Wen arXiv:1010.1517

$H' \sim UHU^\dagger, \quad U =$ 


- Bosonic liquid phases are defined by gLU trans. $U = \prod U_{ijk}$:
  (1) $[U_{ijk}, U_{i'j'k'}] = 0$
  (2) U_{ijk} acts within $V_i \otimes V_j \otimes V_k$. e.g. $U_{ijk} = e^{i(b_i b_j b_k^\dagger + h.c.)}$
- Fermionic liquid phases are defined by gLU trans. $U^f = \prod U^f_{ijk}$:
  (1) $[U^f_{ijk}, U^f_{i'j'k'}] = 0$, *but U^f_{ijk} may not act within* $V_i \otimes V_j \otimes V_k$.
  e.g. $U^f_{ijk} = e^{i(t_{ij} c_i c_j + h.c.)}$, where $c_i = \sigma^x_i \prod_{j<i} \sigma^z_j$

Gapped quantum liquids for bosons and fermions have very different mathematical structures

Topological orders = Gapped liquid phases (no symmetry)

For gapped systems with no symmetry:

- According to Landau theory, no symm. to break
  $\to$ all systems belong to one trivial phase

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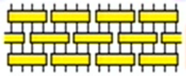
Chen-Gu-Wen arXiv:1004.3835

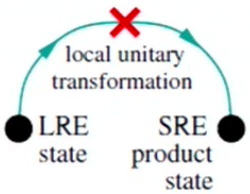
For gapped systems with no symmetry:

- According to Landau theory, no symm. to break
  $\rightarrow$ all systems belong to one trivial phase
- Thinking about entanglement: there are

- **long range entangled (LRE) states**

- **short range entangled (SRE) states**

$|\text{LRE}\rangle \neq$  $|\text{product state}\rangle = |\text{SRE}\rangle$



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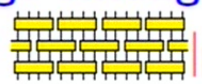
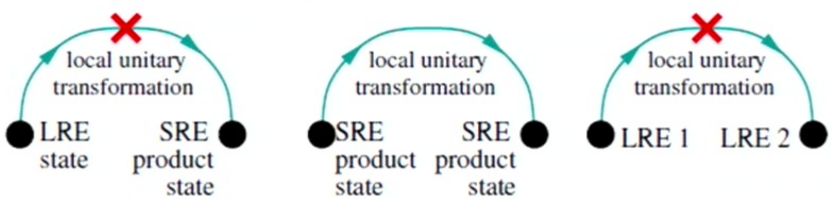
For gapped systems with no symmetry:

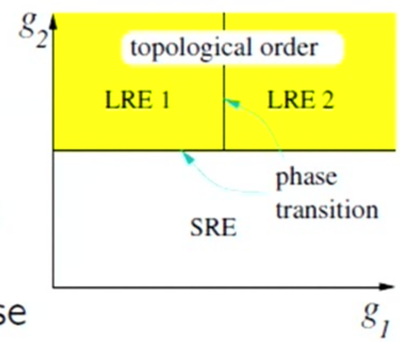
- According to Landau theory, no symm. to break $\to$ all systems belong to one trivial phase
- Thinking about entanglement: there are

- **long range entangled (LRE) states** $\to$ many phases

- **short range entangled (SRE) states** $\to$ one phase

$|\text{LRE}\rangle \neq$  $|\text{product state}\rangle = |\text{SRE}\rangle$





- All SRE states belong to the same trivial phase
- LRE states can belong to many different phases: different patterns of long-range entanglements defined by LU trans. = different **topological orders** Wen, Phys. Rev. B40, 7387 (1989)

Example of Topologically ordered states

To make topological order, we need to sum over many different product states, but we should not sum over everything.

$\sum_{\text{all spin config.}} |\uparrow\downarrow ..\rangle = |\rightarrow\rightarrow ..\rangle$

Xiao-Gang Wen | Lectures on topological order: Long range entanglement

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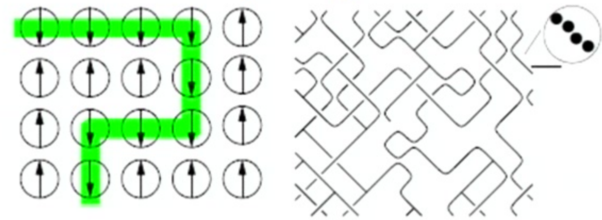
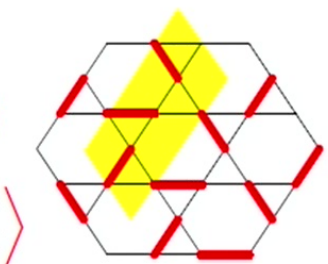


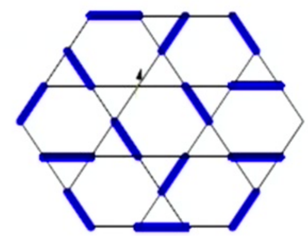
- *sum* over a subset of spin config.:

$|\Phi^{Z_2}_{\text{loops}}\rangle = \sum \left| \text{loops} \right\rangle$

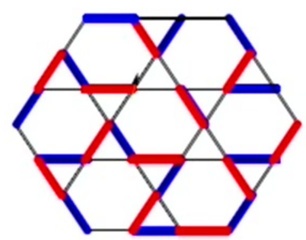
$|\Phi^{DS}_{\text{loops}}\rangle = \sum (-)^{\#\text{ of loops}} \left| \text{loops} \right\rangle$

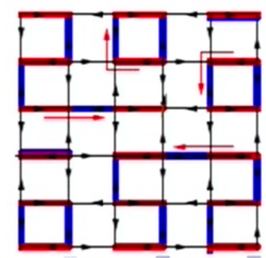
$|\Phi^{\theta}_{\text{loops}}\rangle = \sum (e^{i\theta})^{\#\text{ of loops}} \left| \text{loops} \right\rangle$



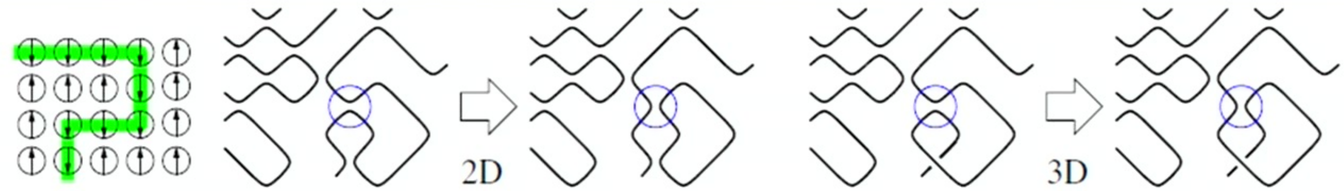


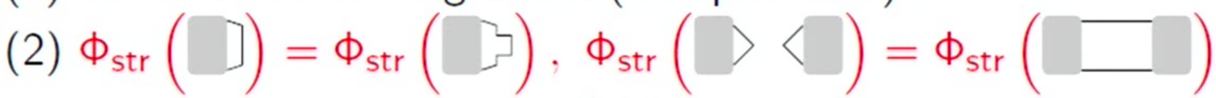
- Can the above wavefunction be the ground states of local Hamiltonians?





Sum over a subset: local rule → global wave function



- Local rules of a string liquid:
  (1) Dance while holding hands (no open ends)
  

  → Global wave function Φ_{str}  $= 1$
- Local rules of another string liquid:
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  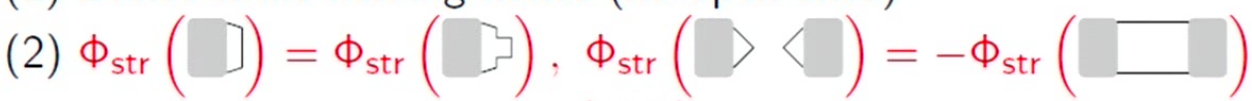

  → Global wave function Φ_{str}  $= (-)^{\#\text{ of loops}}$
- Two topo. orders: Z_2 topo. order Read-Sachdev PRL 66, 1773 (91), Wen PRB 44, 2664 (91), Moessner-Sondhi PRL 86 1881 (01) and double-semion topo. order. Freedman etal cond-mat/0307511, Levin-Wen cond-mat/0404617

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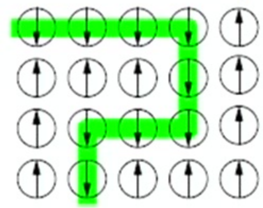
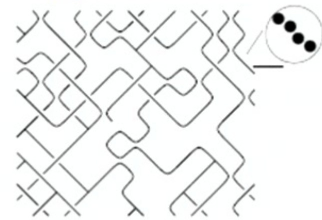
$\sum_{\text{all spin config.}} |\uparrow\downarrow ..\rangle = |\rightarrow\rightarrow ..\rangle$





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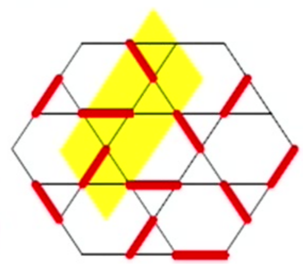

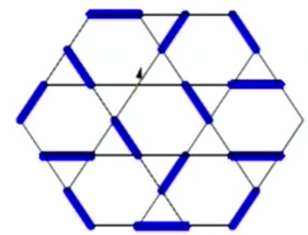

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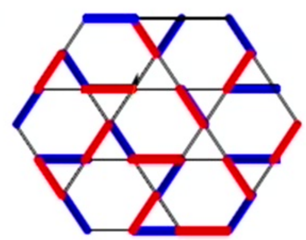
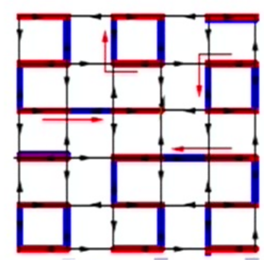
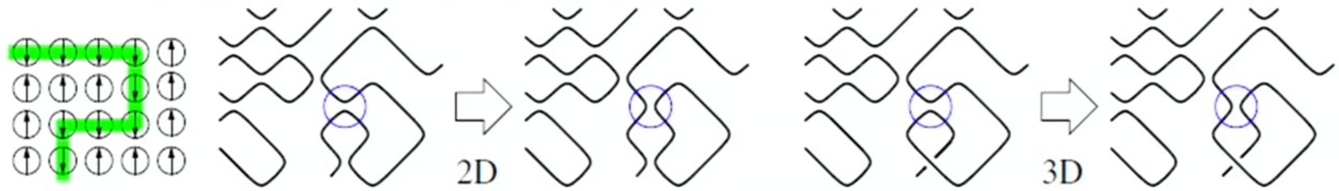




Xiao-Gang Wen | Lectures on topological order: Long range entanglement

Sum over a subset: local rule → global wave function



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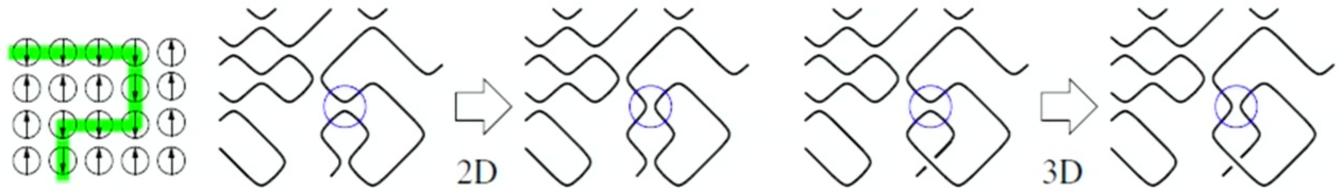
Sum over a subset: local rule → global wave function



- Local rules of a string liquid:
  (1) Dance while holding hands (no open ends)
  (2) $\Phi_{str}\left(\right) = \Phi_{str}\left(\right)$, $\Phi_{str}\left(\right) = \Phi_{str}\left(\right)$
  → Global wave function $\Phi_{str}\left(\right) = 1$
- Local rules of another string liquid:
  (1) Dance while holding hands (no open ends)
  (2) $\Phi_{str}\left(\right) = \Phi_{str}\left(\right)$, $\Phi_{str}\left(\right) = -\Phi_{str}\left(\right)$
  → Global wave function $\Phi_{str}\left(\right) = (-)^{\#\text{ of loops}}$
- Two topo. orders: Z_2 topo. order Read-Sachdev PRL 66, 1773 (91), Wen PRB 44, 2664 (91), Moessner-Sondhi PRL 86 1881 (01) and double-semion topo. order. Freedman etal cond-mat/0307511, Levin-Wen cond-mat/0404617