

Title: The standard model from non-commutative geometry: what? why? what's new?

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URL: <http://pirsa.org/15050126>

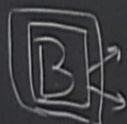
Abstract:

SM from NCG: What? Why? What's New?

"Coming to grips w/ Brouder, Bizi, Besnard"

L_{SM+GR} (+QM) : why?

NCG



Connes Lott
Connes Chamseddine
Barrett
Marcolli

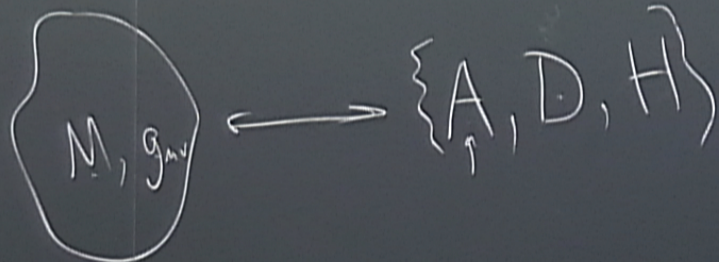
NCG $\begin{cases} \rightarrow \text{Planck scale?} \\ \rightarrow \text{In your face!} \end{cases}$

(Minkowski)

$\begin{pmatrix} x, t \\ x', t' \end{pmatrix}$

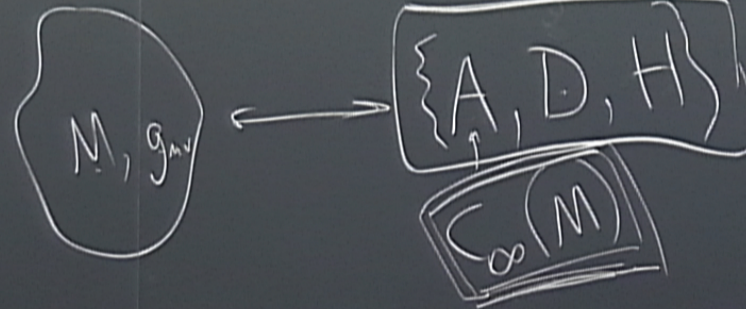
$$\mathbb{D}^2 = g^{\mu\nu} \partial_\mu \partial_\nu \dots$$

NCG: What?



NCG $\begin{cases} \text{Planck scale?} \\ \text{In your face!} \end{cases}$

NCG: What?

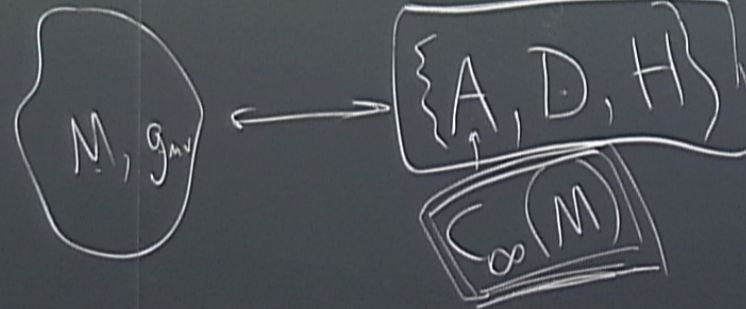


$$\mathbb{D}^2 = g^{\mu\nu} \partial_{\mu} \partial_{\nu} \dots$$

NCG: Why? "Fancy"
"Deep"

NCG $\begin{cases} \text{Planck scale?} \\ \text{In your face!} \end{cases}$

NCG: What?



NCG: Why? "Fancy"
"Deep"

Gauge Theory

1) $SU(3) \times SU(2) \times U(1)$

2) fermions

	$SU(3)$	$SU(2)$	$U(1)$
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$1/6$
u_R	3	1	$2/3$
d_R	3	1	$-1/3$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	2	$-1/2$
ν_R	1	1	0
e_R	1	1	-1
3) h	1	2	$1/2$

NCG

1) $A = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ $\swarrow (\lambda, q, m)$

2) $\left(\begin{array}{c} \vdots \end{array} \right)$ 96×96

3)

x, t
 x', t'

$\{D, H\}$

$\infty(M)$

Gauge Theory

NCG

1) $SU(3) \times SU(2) \times U(1)$

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1) $A = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

(λ, q, m)

2) $\begin{pmatrix} \vdots \end{pmatrix}$

3)

$D+A$

$\uparrow$

$\mathbb{I}$

$D_c \otimes 1 \oplus 1 \otimes D_F$

$\uparrow$

$\delta^m(p_n + A_m)$

96×96

$$A: A \times A \rightarrow A$$

$$B_0 = A \oplus H$$

$$B_0 = \text{assoc}, *$$

$$b_0 b'_0 = \underset{\substack{\uparrow \\ a'h}}{aa'} + \underset{\substack{\uparrow \\ a'h}}{a'h} + \underset{\substack{\uparrow \\ a'h}}{h} a'$$

$$\begin{aligned} (aa')a'' &= a(a'a'') \\ (aa')h &= a(a'h) \\ haa & \\ aha &\leftarrow \end{aligned}$$

$$B = \underbrace{\Sigma A}_{\text{massless photon}} + H \quad D_F$$

$$B \text{ assoc graded } *$$

$$b_0 b'_0 = a a' + \overset{H}{\underbrace{a h}} + \overset{H}{\underbrace{h a'}} \\ \downarrow \quad \downarrow \\ a h \quad a' h'$$

$$(a a') a'' = a (a' a'')$$

$$(a a') h = a (a' h)$$

h a a

a h a ←

$$B = \underline{SLA} + H$$

massless
photon D_F

B assoc graded *

$$\text{Aut}(B) = \text{sym}$$

$\underbrace{b_m}_{B-L}$

$\underbrace{\Phi}_{h_1}$

$$b_0 b'_0 = a a' + \overset{H}{\underbrace{a h}} + \overset{H}{h a'}$$

$\swarrow \quad \searrow$
 $a h \quad a' h'$

$$(a a') a'' = a (a' a'')$$

$$(a a') h = a (a' h)$$

$$h a a$$

$$a h a \leftarrow$$

$$B = \underbrace{S L A} + \overset{H}{\uparrow}$$

massless
photon

D_F

$$S L A \rightarrow M = \bigoplus_n M_n \underbrace{\textcircled{\Phi}}_{h_1}$$

B assoc graded $\neq$

$$\text{Aut}(B) = \text{Sym}$$

$$\textcircled{B_m} \quad B-L$$

B_0

$$B = \underbrace{\Omega A}_{\text{massless photon}} + \overset{\uparrow}{H} D_F$$

B assoc graded $*$

$$\text{Aut}(B) = \text{sym}$$

$$\textcircled{b_m} \text{ B-L}$$

$$\Omega A \rightarrow M = \bigoplus_n M_n \textcircled{\varphi} \underline{h_1}$$

$$\textcircled{B_0}$$

$$\textcircled{BV}^{\text{BRST}} \uparrow$$

1. Motivation

2. S+L NCG

3. Nadir's NCG

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2. S+L NCG

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4. Involutions

$$(M, g) \longleftrightarrow (\underbrace{A, D, H}_{\text{Higgs}}, \underbrace{J, \chi}_{\text{fermions}})$$

$$D = D^\dagger$$

$$DJ = \epsilon' JD$$

$$\{D, \chi\} = 0$$

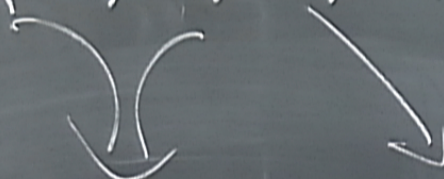
$$[[D, a], JhJ^{-1}] = 0$$

$$J^2 = \epsilon \mathbb{I}$$

$$\chi J = \epsilon'' J \chi$$

$$R_a = J a J^{-1}$$

$$[a, JhJ^{-1}] = 0$$

$$(A, D, H, J, \gamma)$$


$$B = \Omega_0 A \oplus H$$

$$[\omega_n, h, \omega_m] = (\omega_n h) \omega_m - \omega_n (h \omega_m) = 0$$

$$(\omega_n + h)^* = \omega_n^* + h$$

$$(a \psi)^* = \psi^* a^*$$

$$\textcircled{DM} \quad [\omega_n, h, \omega_n'] = (\omega_n h) \quad \omega_n' - \omega_n(h \omega_n') = 0$$

$$(\omega_n + h)^* = \omega_n^* + Jh$$

$$(a \iota)^* = \iota^* a^*$$

$$\pi(d(a))h = [D, a]h$$

$$R_{\pi(d(a))} = J[D, a]^* J^{-1} h$$

$$[a, J \iota^* J^{-1}] = 0$$

$$[[D, a], J \iota^* J^{-1}] = 0$$

$$[\omega_n, h, \omega_n'] = (\omega_n h) \omega_n' - \omega_n (h \omega_n') = 0$$

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$$[[D, a], J \iota^* J^{-1}] = 0$$

$$[[D, a], J[D, \iota^*] J^{-1}] = 0$$

$$-\omega_n(\hbar\omega_n)=0$$

$$a[D, \hbar]$$

$$[a, J^{\dagger}J]=0$$

$$[[D, a], J^{\dagger}J]=0$$

$$[[D, a], J[D, \hbar]J^{\dagger}]=0$$

$$h_n \omega_m = \epsilon^m (-1)^{mn + m(m+1)/2} J \omega_m + J^{-1} h_n$$

$$h \omega_m = J \omega_m + J^{-1}$$

$$J^{-1} J = 0$$

$$h_n \omega_m = \epsilon^m (-1)^{mn + m(m+1)/2} J \omega_m + J^{-1} h_n$$

$$h \omega_m = J \omega_m + J^{-1}$$

$$J^{-1} = 0 \Rightarrow \{[D, a], J[D, b]J^{-1}\} = 0$$

$$[D, a][D, b] = 0$$

$$p_n(h\omega_m) = 0$$

$$(\omega_n + h_m)^{\dagger} := \omega_n^{\dagger} + (-1)^{m(m-1)/2} J h_m J^{-1}$$

$$\omega_m = \epsilon^m (-1)^{mn + m(m+1)/2} J \omega_n^{\dagger} J$$

$$\Rightarrow \{[D, a], J[D, H]J^{-1}\}$$

$$[D, a][D, H] = 0$$