Title: A Monotonicity Theorem for Two-dimensional Boundaries and Defects

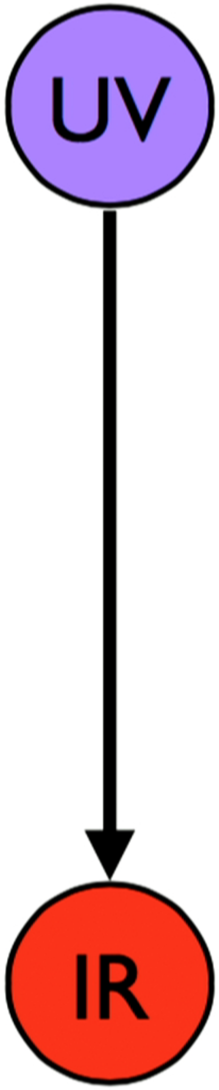
Date: May 28, 2015 11:00 AM

URL: http://pirsa.org/15050124

Abstract: <p>I will propose a proof for a monotonicity theorem, or c-theorem, for a three-dimensional Conformal Field Theory (CFT) on a space with a boundary, and for a two-dimensional defect coupled to a higher-dimensional CFT. The proof is applicable only to renormalization group flows that are localized at the boundary or defect, such that the bulk theory remains conformal along the flow, and that preserve locality, reflection positivity, and Euclidean invariance along the defect. The method of proof is a generalization of Komargodskiâ€™s proof of Zamolodchikovâ€™s c-theorem. The key ingredient is an external â€œdilatonâ€• field introduced to match Weyl anomalies between the ultra-violet (UV) and infra-red (IR) fixed points. Reflection positivity in the dilatonâ€™s effective action guarantees that a certain coefficient in the boundary/defect Weyl anomaly must take a value in the UV that is larger than (or equal to) the value in the IR. This boundary/defect c-theorem may have important implications for many theoretical and experimental systems, ranging from graphene to branes in string theory and M-theory.</p>

Outline:

- Review: Monotonicity Theorems
- The Systems
- The Weyl Anomaly
- The Proof
- Summary and Outlook



Quantum Field Theory (QFT)

Wilsonian Renormalization Group (RG)

"integrate out" DOF

Below a mass threshold,
integrate out massive DOF

massive DOF
"decouples"

Monotonicity Theorems

Make our intuition precise, for RG flows in QFT

Provide a precise way to count number of DOF

Provide rigorous proof that the number of DOF DECREASES along RG flow

for any coupling strength!

Place stringent theoretical constraints on what is possible in RG flows

c-theorem
2-dimensional QFT
Zamolodchikov JETP 43, 12, 565, 1986

F-theorem
3-dimensional QFT
Jafferis, Klebanov, Pufu, and Safdi 1103.1181
Casini and Huerta 1202.5650

a-theorem
4-dimensional QFT
Cardy PLB 215 (1988) 749
Komargodski and Schwimmer 1107.3987

g-theorem
2-dimensional CFT
with a boundary
Affleck and Ludwig PRL 67 (1991) 161
Friedan and Konechny hep-th/0312197

The c-theorem

A.B. Zamolodchikov
JETP Vol. 43 No. 12 p. 565, 1986

for RG flows in

RENORMALIZABLE EUCLIDEAN

QFTs in $d = 2$

Assumptions

① Euclidean Symmetry
(Poincaré symmetry)

② Locality

③ Reflection Positivity
(Unitarity)

1. Euclidean Symmetry

Non-dynamical background metric $g_{\mu\nu}(x)$

Action functional $S(g_{\mu\nu}, \lambda)$

Coupling constants $\lambda = (\lambda_1, \lambda_2, \ldots)$

Generating functional $Z[g_{\mu\nu}, \lambda]$

1. Euclidean Symmetry

Stress-Energy Tensor

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \ln Z[g_{\mu\nu}, \lambda]$$

$$T_{\mu\nu} = T_{\nu\mu}$$

$$g_{\mu\nu} \to \delta_{\mu\nu}$$

Translational and Rotational Symmetry

$$\partial_\mu T_{\mu\nu} = 0$$

1. Euclidean Symmetry

Stress-Energy Tensor

complex coordinates $z, \bar{z}$

$$[T_{\mu\nu}] = \begin{pmatrix} T_{zz} & T_{z\bar{z}} \\ T_{\bar{z}z} & T_{\bar{z}\bar{z}} \end{pmatrix}$$

$$T_{z\bar{z}} = T_{\bar{z}z}$$

$$T_{\mu}{}^{\mu} = T_{z\bar{z}} = T_{\bar{z}z}$$

2. Locality

$$S(\lambda) = \int d^2 z \, L(\lambda, z, \bar{z})$$

RG flow triggered by relevant local scalar operator

$$L(\lambda, z, \bar{z}) \to L(\lambda, z, \bar{z}) + \lambda_{\mathcal{O}} \mathcal{O}(z, \bar{z})$$

$$\Delta_{\mathcal{O}} < 2$$

3. Reflection Positivity

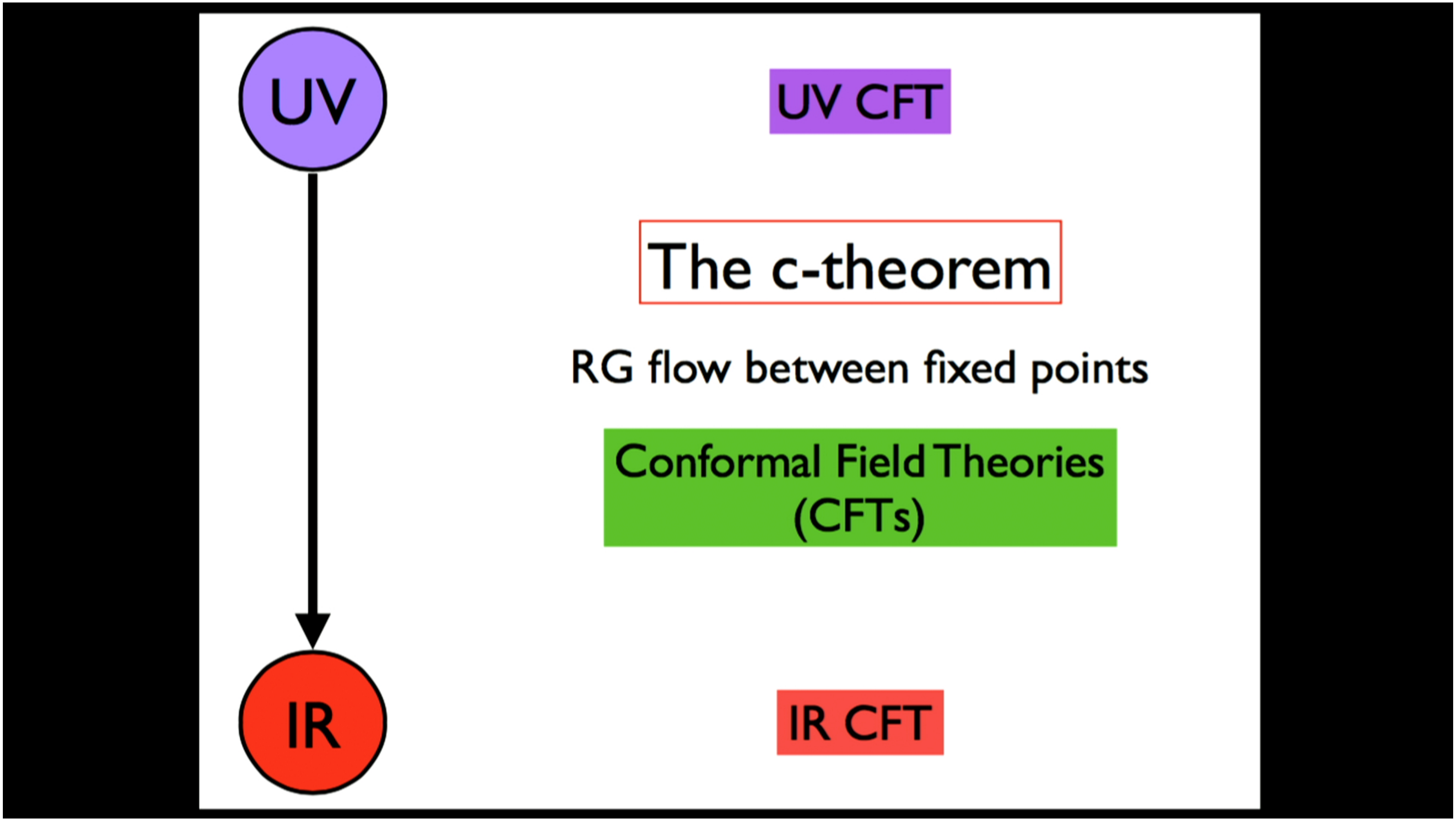
$$||\psi\rangle|^2 = \langle\psi|\psi\rangle \geq 0$$

Reflection Positivity

Two point function of local scalar operator must be non-negative

$$\langle\mathcal{O}^\dagger(x)\mathcal{O}(0)\rangle \geq 0$$

Euclidean "time evolution" preserves norm ≥ 0


UV
UV CFT
The c-theorem
RG flow between fixed points
Conformal Field Theories
(CFTs)
IR
IR CFT

Conformal Field Theory

Non-dynamical background metric $g_{\mu\nu}(x)$

Conformal Transformation

Diffeomorphism

$$x^\mu \to x'^\mu(x)$$

such that

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$T_\mu{}^\mu = \frac{1}{\sqrt{g}} \frac{\delta}{\delta\Omega} \ln Z$$

Conformal invariance

$$T_\mu{}^\mu = 0$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to \delta_{\mu\nu}$$

$$d > 2$$

Rotations	$x^\mu \to \Lambda^\mu{}_\nu\, x^\nu$
Translations	$x^\mu \to x^\mu + c^\mu$
Dilatations	$x^\mu \to \lambda\, x^\mu$
Special Conformal	$x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

$$SO(d+1,1)$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to \delta_{\mu\nu}$$

$$d = 2$$

Conformal Transformation

$$z \to w(z)$$

$$\bar{z} \to \bar{w}(\bar{z})$$

Conformal Field Theory

$$g_{\mu\nu}(x) \to \delta_{\mu\nu}$$

$$d = 2$$

Holomorphic and anti-holomorphic DECOUPLE

$$\partial_\mu T_{\mu\nu} = 0$$

$$\partial_z T_{\bar{z}\bar{z}} + \partial_{\bar{z}} T_{z\bar{z}} = 0$$

$$T_\mu{}^\mu = T_{z\bar{z}} = T_{\bar{z}z} = 0$$

$$\partial_z T_{\bar{z}\bar{z}} = 0 \qquad \partial_{\bar{z}} T_{zz} = 0$$

Conformal Field Theory

$$T_{zz}(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}} \qquad T_{\bar{z}\bar{z}}(\bar{z}) = \sum_{n=-\infty}^{\infty} \frac{\bar{L}_n}{\bar{z}^{n+2}}$$

Virasoro algebra

$$[L_m, L_n] = (m-n)L_{n+m} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$[L_m, \bar{L}_n] = 0$$

$SO(d+1,1) = SO(3,1)$ **subgroup** $L_{\pm 1}$ and L_0, $\bar{L}_{\pm 1}$ and $\bar{L}_0$

Conformal Field Theory

Central Charge

Thermodynamic entropy

Cardy NPB 270 (186) 1986

System size L Temperature T

$$T \gg 1/L$$

$$S_{\text{thermo}} = \frac{\pi}{3} c L T + \dots$$

Conformal Field Theory

Central Charge

Entanglement Entropy (EE)

Holzhey, Larsen, Wilczek hep-th/9403108 Calabrese + Cardy hep-th/0405152

Short-distance cutoff a Interval of length ℓ

$$S_{\mathrm{EE}} = \frac{c}{3} \ln \frac{2\ell}{a} + \dots$$

Conformal Field Theory

Central Charge

$$||\psi\rangle|^2 = \langle\psi|\psi\rangle \geq 0$$

Vacuum $|0\rangle$

$$L_m|0\rangle = 0 \qquad \forall\ m \geq 0$$

$$|L_{-m}|0\rangle|^2 = \langle 0|[L_m, L_{-m}]|0\rangle = \frac{c}{12}m(m^2-1) \geq 0$$

$$c \geq 0$$

The c-theorem

$$\langle T_{zz}(z,\bar{z})T_{zz}(0,0)\rangle = \frac{F(z\bar{z})}{z^4}$$

$$\langle T_\mu{}^\mu(z,\bar{z})T_{zz}(0,0)\rangle = \frac{G(z\bar{z})}{z^3\bar{z}}$$

$$\langle T_\mu{}^\mu(z,\bar{z})T_\mu{}^\mu(0,0)\rangle = \frac{H(z\bar{z})}{z^2\bar{z}^2}$$

Fixed point $\Rightarrow$ $F = c/2 \quad G = 0 \quad H = 0$

The c-theorem

$$\langle T_{zz}(z,\bar{z})T_{zz}(0,0)\rangle = \frac{F(z\bar{z})}{z^4}$$

$$\langle T_\mu{}^\mu(z,\bar{z})T_{zz}(0,0)\rangle = \frac{G(z\bar{z})}{z^3\,\bar{z}}$$

$$\langle T_\mu{}^\mu(z,\bar{z})T_\mu{}^\mu(0,0)\rangle = \frac{H(z\bar{z})}{z^2\bar{z}^2}$$

Reflection Positivity $\Rightarrow \quad H \geq 0$

The c-theorem

c-function

$$C \equiv 2F - G - \frac{3}{8}H$$

Fixed point $\quad\Rightarrow\quad C = c$

The c-theorem

c-function

$$C \equiv 2F - G - \frac{3}{8}H$$

$$\partial_\mu T_{\mu\nu} = 0$$

$$\partial_{\bar{z}} T_{zz} + \partial_z T_{\bar{z}z} = 0$$

Multiply by $T_{zz}(0,0)$ and $T_{\bar{z}z}(0,0)$

and take $\langle \ldots \rangle$

The c-theorem

c-function

$$C \equiv 2F - G - \frac{3}{8}H$$

$$r \equiv \sqrt{z\bar{z}}$$

$$r\frac{\partial F}{\partial r} + \frac{1}{4}\left(r\frac{\partial G}{\partial r} - 3G\right) = 0$$

$$r\frac{\partial G}{\partial r} - G + \frac{1}{4}\left(r\frac{\partial H}{\partial r} - 2H\right) = 0$$

eliminate G

The c-theorem

c-function

$$C \equiv 2F - G - \frac{3}{8}H$$

$$r \equiv \sqrt{z\bar{z}}$$

$$r\frac{\partial C}{\partial r} = -\frac{3}{2}H \leq 0$$

The c-theorem

$$\frac{\partial C}{\partial r} \leq 0 \qquad \Rightarrow \qquad c_{\mathrm{UV}} \geq c_{\mathrm{IR}}$$

Strong form

Weak form

Other Proofs

Holography

Freedman, Gubser, Pilch, Warner hep-th/9904017
Myers and Sinha 1006.1263, 1011.5819

Null energy Condition — Weak form

Entanglement Entropy

Casini and Huerta hep-th/0405111

Strong Sub-Additivity — Strong Form

Weyl Anomaly Matching

Komargodski and Schwimmer 1107.3987 Komargodski 1112.4538

Reflection Positivity — Weak form

Generalizations?

non-local and/or non-unitary QFTs?

QFTs with less symmetry?

higher dimensions?

QFTs without Euclidean symmetry?

What if $d_{\mathrm{UV}} \neq d_{\mathrm{IR}}$?

What if the relevant operator is not a scalar?

What if the fixed points have Lifshitz scaling?

$\vec{x} \to \lambda \vec{x} \qquad t \to \lambda^z t \qquad z =$ dynamical exponent

What if $z_{\mathrm{UV}} \neq z_{\mathrm{IR}}$?

The F-theorem

Jafferis, Klebanov, Pufu, and Safdi 1103.1181

Casini and Huerta 1202.5650

for RG flows in

RENORMALIZABLE

EUCLIDEAN SYMMETRIC

LOCAL

QFTs in $d = 3$

The F-theorem

At fixed point: conformally map to S^3

$$F \equiv -\ln Z_{S^3}^{\text{ren.}}$$

$$F_{\text{UV}} \geq F_{\text{IR}}$$

Entanglement Entropy

Casini and Huerta 1202.5650

Strong Sub-Additivity

Weak form

The F-theorem

At fixed point: conformally map to S^3

$$F \equiv -\ln Z_{S^3}^{\text{ren.}}$$

$$F_{\text{UV}} \geq F_{\text{IR}}$$

Holography

Freedman, Gubser, Pilch, Warner hep-th/9904017
Myers and Sinha 1006.1263, 1011.5819

Null energy Condition

Weak form

The a-theorem

Cardy PLB 215 (1988) 749

Komargodski and Schwimmer 1107.3987

for RG flows in

RENORMALIZABLE

EUCLIDEAN SYMMETRIC

LOCAL

QFTs in $d = 4$

Weyl Anomaly

CFT in any d

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Conformal invariance

$$T_{\mu}{}^{\mu} = 0$$

Weyl Anomaly

CFT in any d

Non-trivial $g_{\mu\nu}$

Quantum Effects
Break Conformal Invariance

$$T_{\mu}{}^{\mu} \neq 0$$

Weyl Anomaly

$$d = 4$$

$$T_\mu{}^\mu = a\,E - c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

Euler density

$$E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Weyl tensor

$$W_{\mu\nu\rho\sigma}$$

central charges a and c

The a-theorem

$$a_{\mathrm{UV}} \geq a_{\mathrm{IR}}$$

Weyl Anomaly Matching

Komargodski and Schwimmer 1107.3987 Komargodski 1112.4538

Reflection Positivity **Weak form**

Holography

Freedman, Gubser, Pilch, Warner hep-th/9904017
Myers and Sinha 1006.1263, 1011.5819

Null energy Condition **Weak form**

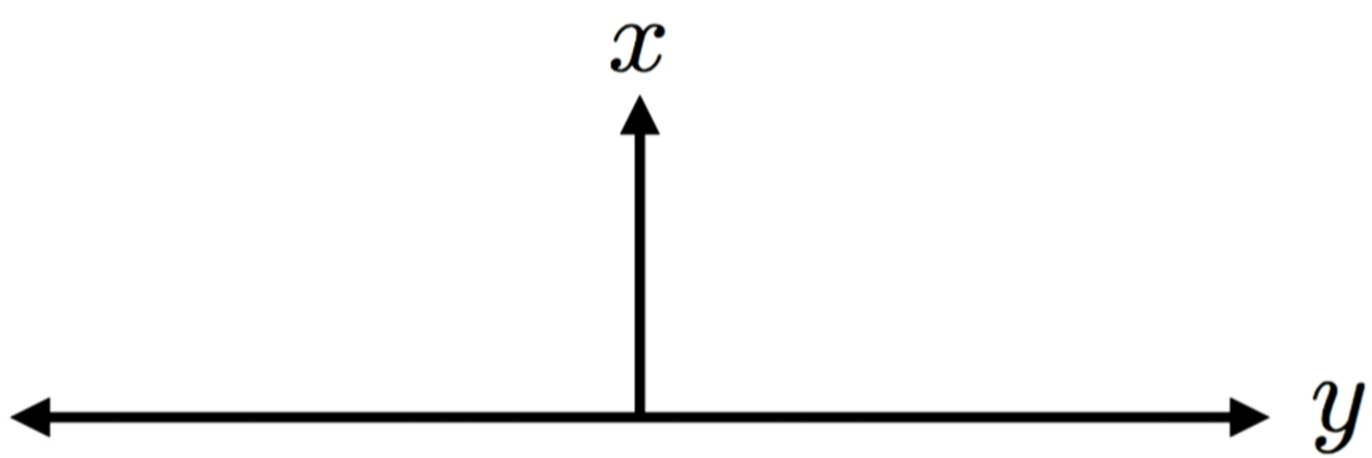
The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

Local, reflection-positive CFT in $d = 2$

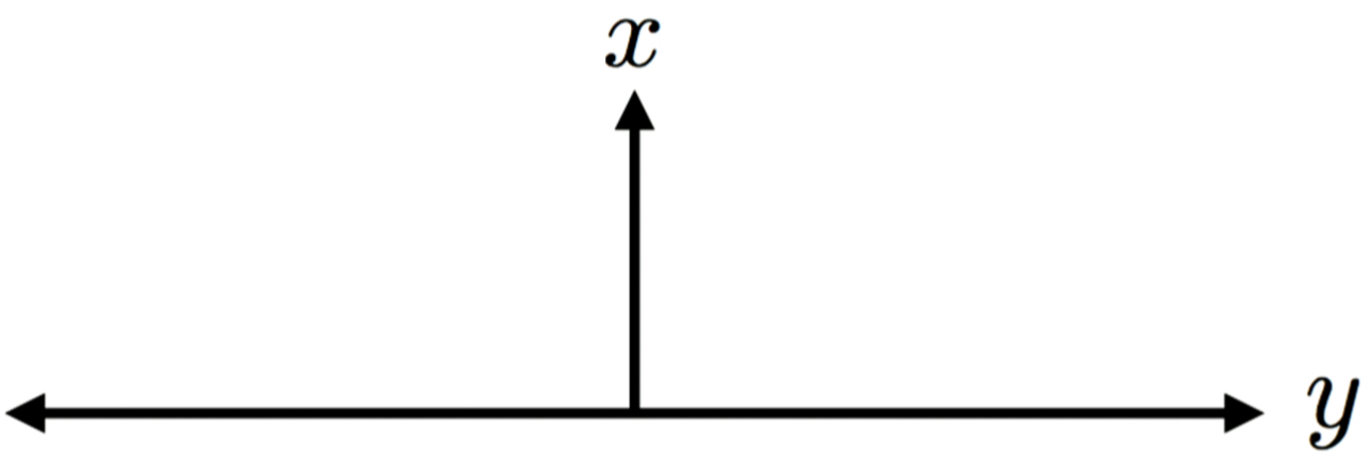
on a space with a boundary

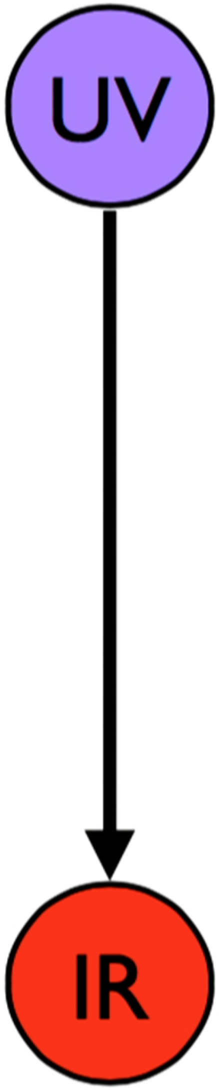


The g-theorem

Conformal boundary conditions

Boundary CFT
(BCFT)





UV BCFT

CFT with boundary condition "α"

Boundary RG flow

$$S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} \to S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} + \int dx\, dy\, \delta(x) \lambda_{\mathcal{O}} \mathcal{O}(y)$$

$$\Delta_{\mathcal{O}} < 1$$

IR BCFT

CFT with boundary condition "β"

Boundary RG flow

$$S(\lambda)^{\rm UV}_{\rm BCFT} \to S(\lambda)^{\rm UV}_{\rm BCFT} + \int dx\, dy\, \delta(x) \lambda_{\mathcal{O}} \mathcal{O}(y)$$

Bulk theory remains conformal

$$T_{\mu\nu} = [T_{\mu\nu}]_{\rm bulk} + \delta(x)\,[T_{\mu\nu}]_{\partial}$$

Invariance under diffeomorphisms along the boundary

$$[T_{\mu\perp}]_{\partial} = [T_{\perp\mu}]_{\partial} = 0 \qquad [T_{\perp\perp}]_{\partial} = 0$$

$$\partial_\mu\,[T_{\mu\nu}]_{\partial} \propto [T_{\perp\perp}]_{\rm bulk}$$

Boundary RG flow

$$S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} \to S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} + \int dx\, dy\, \delta(x) \lambda_{\mathcal{O}} \mathcal{O}(y)$$

Bulk theory remains conformal

$$T_{\mu\nu} = [T_{\mu\nu}]_{\mathrm{bulk}} + \delta(x)\, [T_{\mu\nu}]_{\partial}$$

$$\left[T_\mu{}^\mu\right]_{\mathrm{bulk}} = 0$$

$$T_\mu{}^\mu = \delta(x)\, \left[T_\mu{}^\mu\right]_\partial \neq 0$$

The g-theorem

At fixed point: conformally map to disk

$$g_\alpha \equiv Z_\alpha^{\mathrm{ren.}}$$

"Boundary entropy"

$$\ln g_\alpha$$

Counts DOF localized at boundary

The g-theorem

Entanglement Entropy (EE)

Calabrese + Cardy hep-th/0405152

Interval including the boundary

$$S_{\text{EE}} = \frac{c}{6} \ln \frac{2\ell}{a} + \ln g_\alpha + \ldots$$

$$\ln g_\alpha = S_{\text{EE}}^{\text{BCFT}} - \frac{1}{2} S_{\text{EE}}^{\text{CFT}}$$

The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

Define a g-function G

Euclidean symmetry, locality, reflection positivity

$$\frac{\partial G}{\partial r} \leq 0 \qquad \Rightarrow \qquad g_{\mathrm{UV}} \geq g_{\mathrm{IR}}$$

Strong form　　　　Weak form

The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

Define a g-function G

Euclidean symmetry, locality, reflection positivity

$$\frac{\partial G}{\partial r} \leq 0 \quad \Rightarrow \quad g_{\mathrm{UV}} \geq g_{\mathrm{IR}}$$

They can't prove that $\ln g$ is bounded from below!

The g-theorem

Thermodynamic entropy

Affleck and Ludwig PRL 67 (1991) 161

System size L Temperature T

$$T \gg 1/L$$

$$S_{\text{thermo}} = \frac{\pi}{3} cLT + \ln g_\alpha + \ldots$$

The g-theorem

Affleck and Ludwig PRL 67 (1991) 161

Friedan and Konechny hep-th/0312197

Define a g-function G

Euclidean symmetry, locality, reflection positivity

$$\frac{\partial G}{\partial r} \leq 0 \quad \Rightarrow \quad g_{\mathrm{UV}} \geq g_{\mathrm{IR}}$$

They can't prove that $\ln g$ is bounded from below!

Generalizations?

Higher-dimensional g-theorems?

Proposals

Yamaguchi	hep-th/0207171
Takayanagi et al.	1105.5165, 1108.5152, 1205.1573
Estes, Jensen, O'B., Tsatis, Wrase	1403.6475
Gaiotto	1403.8052

Many tests in particular examples

No proofs yet!

GOAL

Prove a g-theorem for Local, reflection-positive BCFT in $d = 3$

Proposals

Nozaki, Takayanagi, Ugajin 1205.1573

Estes, Jensen, O'B., Tsatis, Wrase 1403.6475

Weyl Anomaly Matching

Komargodski and Schwimmer 1107.3987

Komargodski 1112.4538

Examples

Graphene with a boundary

Critical Ising model in $d = 3$ with a boundary

M-theory: M2-branes with a boundary

String theory: various brane intersections

Holographic BCFTs

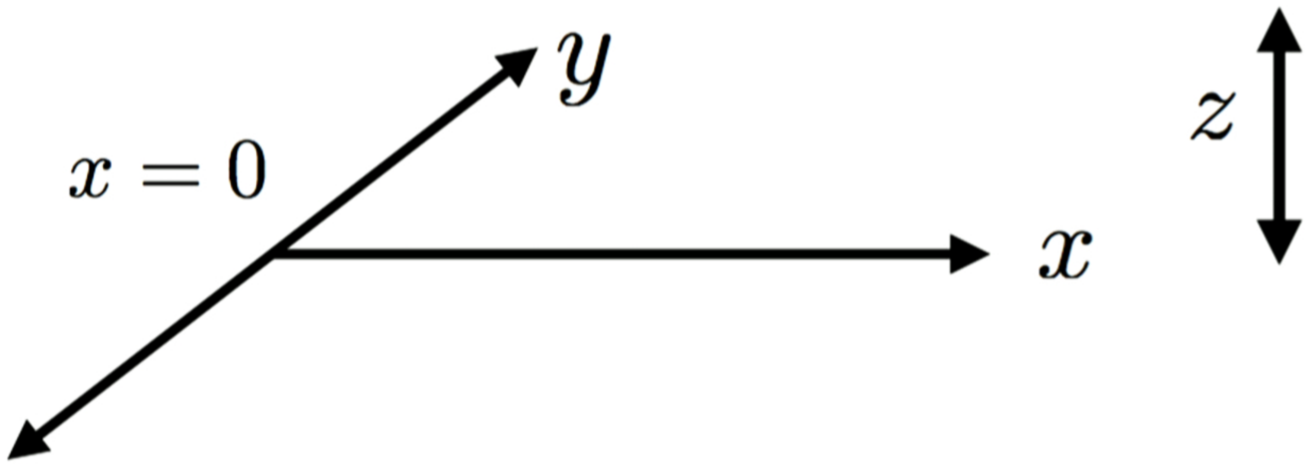
Outline:

- Review: Monotonicity Theorems
- The Systems
- The Weyl Anomaly
- The Proof
- Summary and Outlook

The Systems

BCFT in $d = 3$

With a planar boundary



The Systems

$$SO(d+1,1) = SO(4,1)$$

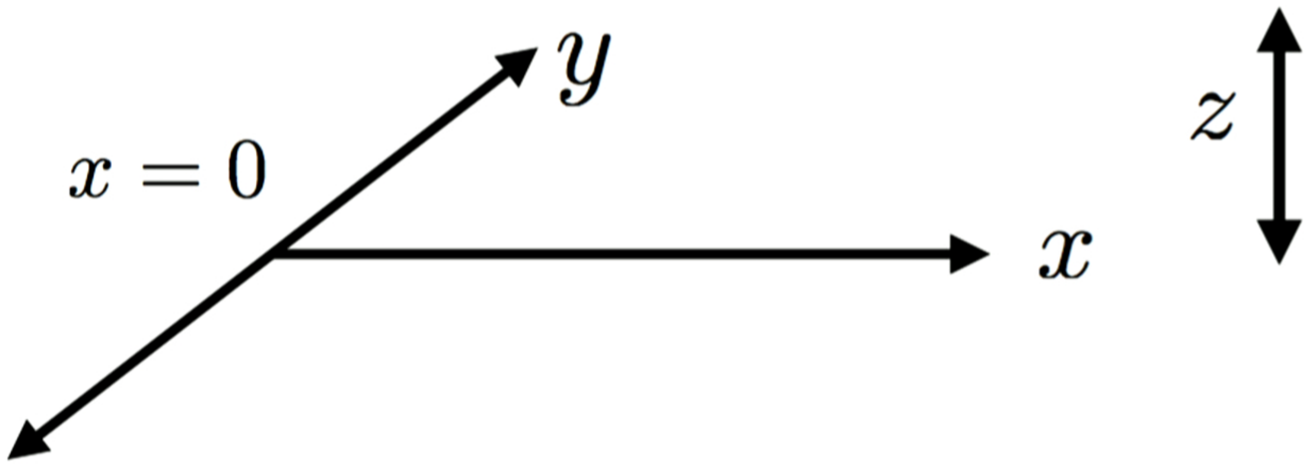
Rotations	$x^\mu \to \Lambda^\mu{}_\nu \, x^\nu$
Translations	$x^\mu \to x^\mu + c^\mu$
Dilatations	$x^\mu \to \lambda \, x^\mu$
Special Conformal	$x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

Broken to subgroup that preserves $x = 0$

The Systems

Translations $x^\mu \to x^\mu + c^\mu$

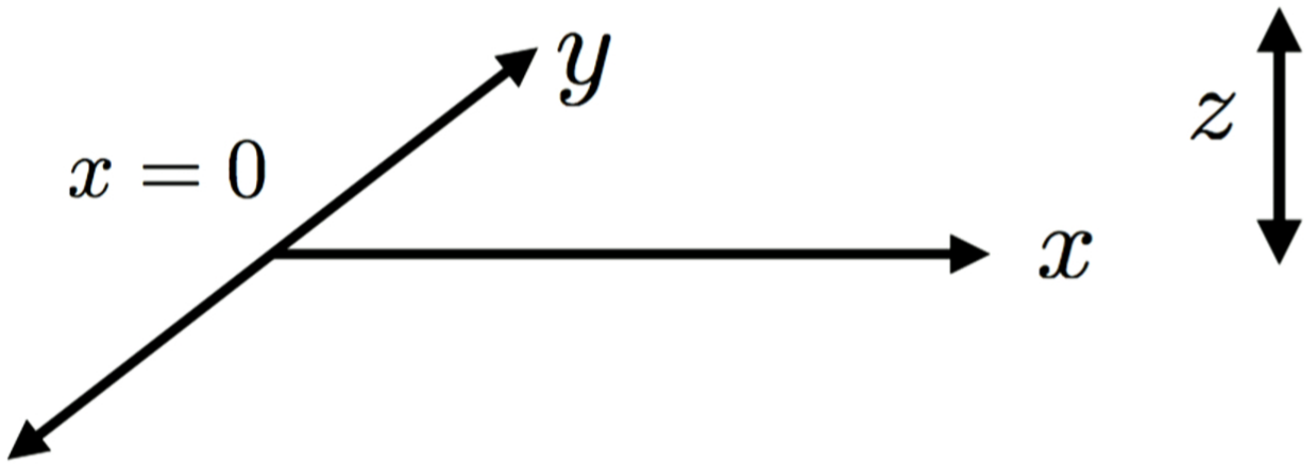
Broken to translations along (y, z)



The Systems

Dilatations $x^\mu \to \lambda\, x^\mu$

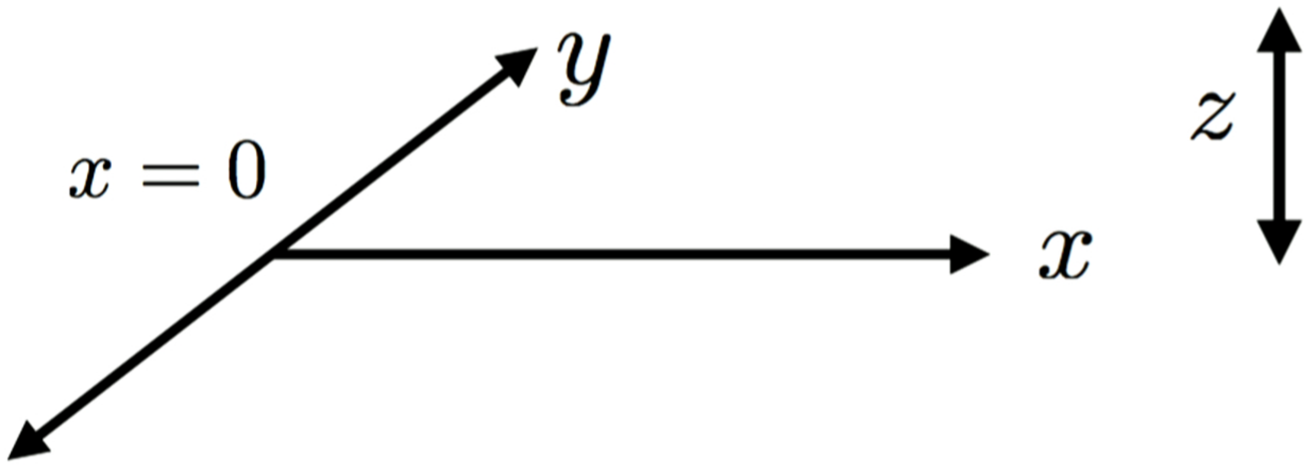
Unbroken



The Systems

Special Conformal $x^\mu \to \dfrac{x^\mu + b^\mu x^2}{1 + 2x^\nu b_\nu + b^2 x^2}$

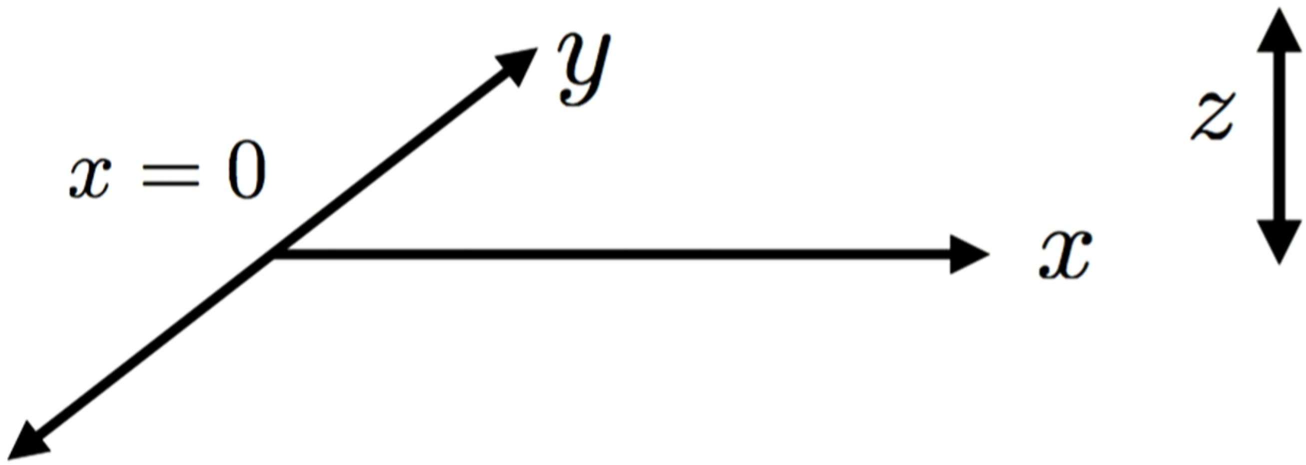
Broken to $b^x = 0$



The Systems

$$SO(d+1,1) \to SO(d,1)$$

$$SO(4,1) \to SO(3,1)$$



The Systems

Boundary RG Flows

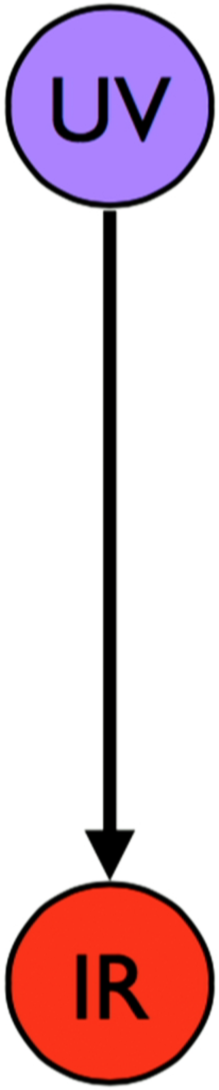
$$S(\lambda)^{\rm UV}_{\rm BCFT} \to S(\lambda)^{\rm UV}_{\rm BCFT} + \int dx\, dy\, dz\, \delta(x)\, \lambda_{\mathcal{O}} \mathcal{O}(y,z)$$

scalar $\mathcal{O}$ with $\Delta_{\mathcal{O}} < 2$

$$T_{\mu\nu} = [T_{\mu\nu}]_{\rm bulk} + \delta(x)\, [T_{\mu\nu}]_{\partial}$$

Bulk theory remains conformal

$$\left[T_\mu{}^\mu\right]_{\rm bulk} = 0 \qquad T_\mu{}^\mu = \delta(x)\, \left[T_\mu{}^\mu\right]_\partial \neq 0$$



UV BCFT

Single real, free, massless, scalar
Neumann B.C.

Boundary RG Flow

$$S(\lambda)^{\rm UV}_{\rm BCFT} \to S(\lambda)^{\rm UV}_{\rm BCFT} + \int d^3x\, \delta(x)\, m^2 \Phi^2(\vec{x})$$

$$\Delta_{\Phi^2} = 1$$

IR BCFT

Single real, free, massless, scalar
Dirichlet B.C.

The Systems

Boundary RG Flows

$$S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} \to S(\lambda)^{\mathrm{UV}}_{\mathrm{BCFT}} + \int dx\, dy\, dz\, \delta(x)\, \lambda_{\mathcal{O}} \mathcal{O}(y,z)$$

Weyl Anomaly Matching

Komargodski and Schwimmer 1107.3987   Komargodski 1112.4538

Reflection Positivity   **Weak form**

Weyl Anomaly

CFT in any d

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Conformal invariance

$$T_{\mu}{}^{\mu} = 0$$

Weyl Anomaly

CFT in any d

Non-trivial $g_{\mu\nu}$

Quantum Effects
Break Conformal Invariance

$$T_\mu{}^\mu \neq 0$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #1

Write down all curvature invariants built from $g_{\mu\nu}$ with the correct dimension

$$d = 4$$

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + \dots$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #2

Wess-Zumino consistency

$$g_{\mu\nu} \to e^{2\Omega_1} e^{2\Omega_2} g_{\mu\nu} \quad = \quad g_{\mu\nu} \to e^{2\Omega_2} e^{2\Omega_1} g_{\mu\nu}$$

Fixes some coefficients

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + \dots$$

Weyl Anomaly

What is the general form of $T_\mu{}^\mu$?

Step #3

Add local counterterms to $S(g_{\mu\nu}, \lambda)$
Determine how they enter $T_\mu{}^\mu$

Fixes more coefficients

$$T_\mu{}^\mu = c_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R^2 + \ldots$$

Weyl Anomaly

CFT in any d

d odd $\qquad T_\mu{}^\mu = 0$

d even $\qquad T_\mu{}^\mu \neq 0$

$d = 2 \qquad T_\mu{}^\mu = \frac{c}{24\pi} R$

Weyl Anomaly

$$d = 4$$

$$T_\mu{}^\mu = a\,E - c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

Euler density

$$E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Weyl tensor

$$W_{\mu\nu\rho\sigma}$$

“central charges” a and c

Weyl Anomaly

$$g_{\mu\nu}(x) \rightarrow e^{2\Omega(x)} g_{\mu\nu}(x)$$

$\int d^d x \sqrt{g}\, T_\mu{}^\mu$ is invariant

$$T_\mu{}^\mu = a\,E - c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

Type A

$\sqrt{g}\,E$

Changes by a total derivative

Type B

$\sqrt{g}\,W^2$

Invariant

Weyl Anomaly

BCFT in $d = 3$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\text{bulk}} + \delta(x)\left[T_\mu{}^\mu\right]_\partial$$

$$\left[T_\mu{}^\mu\right]_{\text{bulk}} = 0$$

What is the general form of $\left[T_\mu{}^\mu\right]_\partial$?

Geometry of Submanifolds

"worldsheet" σ^1, σ^2

"target space" x^μ

Embedding $x^\mu(\sigma^a)$

Induced metric

$$\hat{g}_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b}$$

$\hat{R}_{abcd} \qquad \hat{R}_{ab} \qquad \hat{R}$

Geometry of Submanifolds

Extrinsic Curvature
"Second Fundamental Form"

Gaussian Normal Coordinates

$$K_{ab} = \frac{1}{2}\partial_x \hat{g}_{ab}(x, \sigma)$$

Mean curvature

$$K \equiv \hat{g}^{ab} K_{ab}$$

Weyl Anomaly

What is the general form of $[T_\mu{}^\mu]_\partial$?

Schwimmer + Theisen 0802.1017

See also:

Berenstein, Corrado, Fischler, Maldacena hep-th/9809188

Graham + Witten hep-th/9901021

Henningson + Skenderis hep-th/9905163

Gustavsson hep-th/0310037, 0404150

Asnin 0801.1469

Weyl Anomaly

$$[T_\mu{}^\mu]_\partial = c_1\,\hat{R} + c_2(K_{ab}K^{ab} - \frac{1}{2}K^2)$$

Boundary "central charges"

c_1 and c_2

Weyl Anomaly

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu}$$

$$\left[T_{\mu}{}^{\mu} \right]_{\partial} = c_1 \, \hat{R} + c_2 (K_{ab} K^{ab} - \frac{1}{2} K^2)$$

Type A

$$\sqrt{\hat{g}} \hat{R} \to \sqrt{\hat{g}} \left[\hat{R} - 2 \nabla^2 \Omega \right]$$

Changes by a total derivative

Type B

$$\sqrt{\hat{g}} (K_{ab} K^{ab} - \frac{1}{2} K^2)$$

Invariant

Weyl Anomaly

GOAL

$$[T_\mu{}^\mu]_\partial = c_1\,\hat{R} + c_2(K_{ab}K^{ab} - \frac{1}{2}K^2)$$

Boundary RG Flow

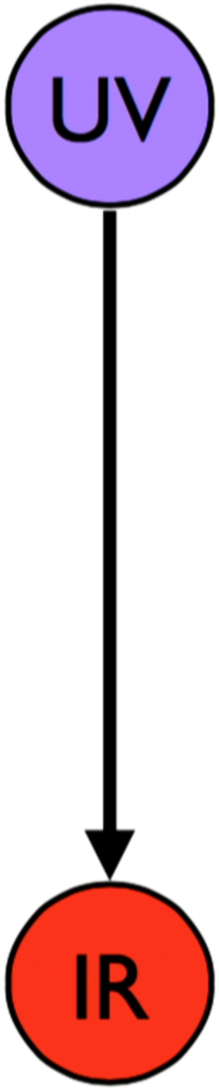
$$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$$

Nozaki, Takayanagi, Ugajin 1205.1573

Estes, Jensen, O'B., Tsatis, Wrase 1403.6475

Outline:

- Review: Monotonicity Theorems
- The Systems
- The Weyl Anomaly
- The Proof
- Summary and Outlook



The Proof

Weyl Anomaly Matching

Komargodski and Schwimmer 1107.3987

Komargodski 1112.4538

local, reflection-positive QFT in any d

RG flow between fixed point CFTs

Dilaton

Non-dynamical background metric $g_{\mu\nu}(x)$

Non-dynamical background scalar $\tau(x)$

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

Dilaton

Non-dynamical background metric $g_{\mu\nu}(x)$

Non-dynamical background scalar $\tau(x)$

$$\lambda_{\mathcal{O}}\mathcal{O}(x) \to e^{(\Delta_{\mathcal{O}}-d)\tau(x)}\lambda_{\mathcal{O}}\mathcal{O}(x)$$

Dilaton

Non-dynamical background metric $g_{\mu\nu}(x)$

Non-dynamical background scalar $\tau(x)$

$$S(\lambda)_\tau = \int d^d x \sqrt{g}\, L(\lambda, \vec{x})_\tau$$

$$= \int d^d x \sqrt{g} \left[L(\lambda, \vec{x})_{\tau=0} + \tau \left[T_\mu{}^\mu \right]_{\tau=0} + \mathcal{O}(\tau^2) \right]$$

Dilaton

Non-dynamical background metric $g_{\mu\nu}(x)$

Non-dynamical background scalar $\tau(x)$

Weyl Anomaly Matching

d even

Dilaton

Integrate out massive DOF

Obtain effective action

$$S_{\text{eff}} \equiv -\ln Z$$

Regular and local in τ

Expand in τ

$$S_{\text{eff}} = S_{\text{eff}}^{\tau=0} + S_{\text{eff}}^{\tau}$$

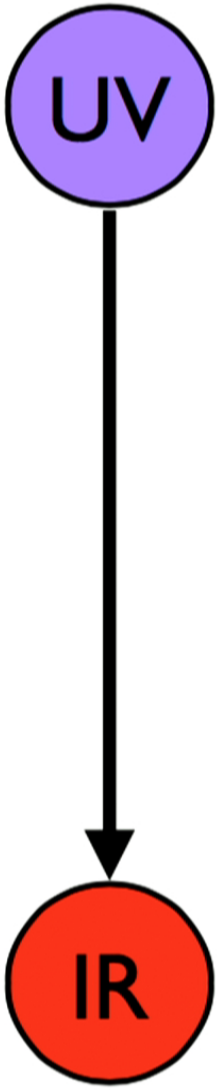
Dilaton

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\frac{\delta}{\delta \Omega} \ln Z$$

Expand in τ

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\,\left[T_\mu{}^\mu\right]_{\tau=0} + \frac{\delta S^\tau_{\text{eff}}}{\delta \Omega}$$



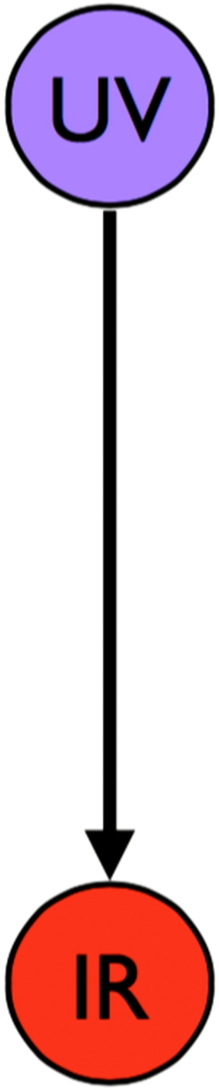
$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_\mu{}^\mu\right]^{\text{UV}}_{\tau=0} = 0$$

$$g_{\mu\nu} = \delta_{\mu\nu}$$

$$\tau = 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_\mu{}^\mu\right]_{\tau=0} \neq 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_\mu{}^\mu\right]^{\text{IR}}_{\tau=0} = 0$$



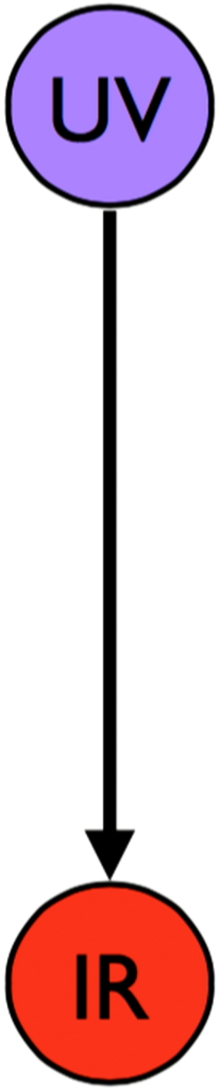
$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_{\mu}{}^{\mu}\right]^{\text{UV}}_{\tau=0} = 0$$

$$g_{\mu\nu} = \delta_{\mu\nu}$$

$$\tau \neq 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_{\mu}{}^{\mu}\right]_{\tau=0} + \frac{\delta S^{\tau}_{\text{eff}}}{\delta \Omega} = 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\left[T_{\mu}{}^{\mu}\right]^{\text{IR}}_{\tau=0} = 0$$



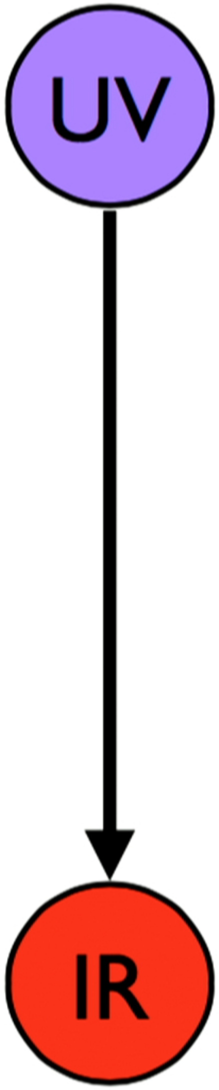
$$\frac{\delta S_{\mathrm{eff}}}{\delta \Omega} = -\sqrt{g}\,\left[T_{\mu}{}^{\mu}\right]^{\mathrm{UV}}_{\tau=0} \neq 0$$

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

$$\tau = 0$$

$$\left[T_{\mu}{}^{\mu}\right]^{\mathrm{UV}}_{\tau=0} \neq \left[T_{\mu}{}^{\mu}\right]^{\mathrm{IR}}_{\tau=0}$$

$$\frac{\delta S_{\mathrm{eff}}}{\delta \Omega} = -\sqrt{g}\,\left[T_{\mu}{}^{\mu}\right]^{\mathrm{IR}}_{\tau=0} \neq 0$$



$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\left[T_\mu{}^\mu\right]^{\text{UV}}_{\tau=0} \neq 0$$

$$g_{\mu\nu} \neq \delta_{\mu\nu}$$

$$\tau \neq 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\left[T_\mu{}^\mu\right]_{\tau=0} + \frac{\delta S^\tau_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\left[T_\mu{}^\mu\right]^{\text{UV}}_{\tau=0} \neq 0$$

$$\frac{\delta S_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\left[T_\mu{}^\mu\right]^{\text{IR}}_{\tau=0} + \frac{\delta S^\tau_{\text{eff}}}{\delta \Omega} = -\sqrt{g}\left[T_\mu{}^\mu\right]^{\text{UV}}_{\tau=0} \neq 0$$

Dilaton

Weyl Anomaly Matching

Schwimmer and Theisen 1011.0696

At the IR fixed point, $S^{\tau}_{\rm eff}$ must produce

$$\frac{\delta S^{\tau}_{\rm eff}}{\delta \Omega} = -\sqrt{g}\left[\left[T_{\mu}{}^{\mu}\right]^{\rm UV}_{\tau=0} - \left[T_{\mu}{}^{\mu}\right]^{\rm IR}_{\tau=0}\right]$$

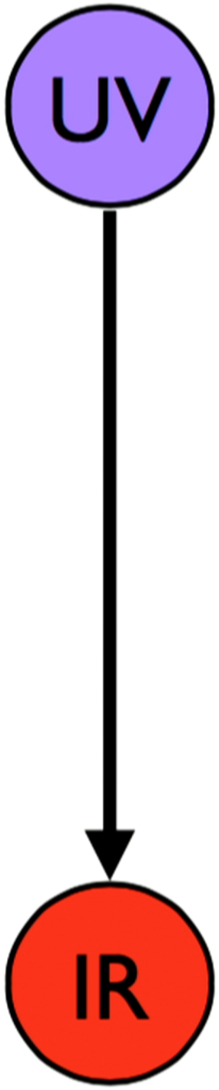
Dilaton

Boundary RG Flow

$$S^{\mathrm{UV}}_{\mathrm{BCFT}} \to S^{\mathrm{UV}}_{\mathrm{BCFT}} + \int dx\, dy\, dz\, \delta(x)\, \lambda_{\mathcal{O}} \mathcal{O}(y,z)$$

$$\lambda_{\mathcal{O}}\, \mathcal{O}(y,z) \to e^{(\Delta_{\mathcal{O}}-2)\tau(y,z)} \lambda_{\mathcal{O}}\, \mathcal{O}(y,z)$$

τ is localized at the boundary,
and depends only on boundary coordinates!



$$\left[T_\mu{}^\mu\right]_\partial^{\rm UV} = c_1^{\rm UV}\,\hat{R} + c_2^{\rm UV}(K_{ab}K^{ab} - \frac{1}{2}K^2)$$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\rm bulk} + \delta(x)\,\left[T_\mu{}^\mu\right]_\partial$$

Boundary RG Flow

$$S_{\rm BCFT}^{\rm UV} \to S_{\rm BCFT}^{\rm UV} + \int dx\,dy\,dz\,\delta(x)\,\lambda_{\mathcal{O}}\mathcal{O}(y,z)$$

$$\left[T_\mu{}^\mu\right]_{\rm bulk} = 0$$

$$\left[T_\mu{}^\mu\right]_\partial^{\rm IR} = c_1^{\rm IR}\,\hat{R} + c_2^{\rm IR}(K_{ab}K^{ab} - \frac{1}{2}K^2)$$

The Proof

Strategy

Komargodski and Schwimmer 1107.3987
Komargodski 1112.4538

In S^{τ}_{eff} write the $\tau\nabla^2\tau$ term in TWO WAYS

① Weyl anomaly matching

coefficient $\propto c_1^{\text{UV}} - c_1^{\text{IR}}$

② Reflection Positivity $\Longrightarrow$ coefficient ≥ 0

$$c_1^{\text{UV}} \geq c_1^{\text{IR}}$$

The Proof

$$S^{\tau}_{\text{eff}} = -\int d^3x\, \delta(x)\, \sqrt{g}\ \tau\left[(c_1^{\text{UV}} - c_1^{\text{IR}})\,\hat{R} + (c_2^{\text{UV}} - c_2^{\text{IR}})(K_{ab}K^{ab} - \frac{1}{2}K^2)\right]$$

$$+\mathcal{O}(\tau^2)$$

$$g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu} \qquad \tau \to \tau + \Omega$$

$$\sqrt{\hat{g}}\hat{R} \to \sqrt{\hat{g}}\left[\hat{R} - 2\nabla^2\Omega\right]$$

$$\frac{\delta S^{\tau}_{\text{eff}}}{\delta\Omega} = -\sqrt{g}\left[\left[T_{\mu}{}^{\mu}\right]^{\text{UV}}_{\tau=0} - \left[T_{\mu}{}^{\mu}\right]^{\text{IR}}_{\tau=0}\right] + \sqrt{g}\,(c_1^{\text{UV}} - c_1^{\text{IR}})\, 2\nabla^2\tau$$

The Proof

$$S^{\tau}_{\rm eff} = -\!\int\! d^3x\,\delta(x)\,\sqrt{g}\;\tau\,\left[(c_1^{\rm UV} - c_1^{\rm IR})\,\hat{R} + (c_2^{\rm UV} - c_2^{\rm IR})(K_{ab}K^{ab} - \frac{1}{2}K^2)\right]$$

$$-\int\! d^3x\,\delta(x)\,\sqrt{g}\;(c_1^{\rm UV} - c_1^{\rm IR})\,\tau\nabla^2\tau \;\; + \ldots$$

$$\frac{\delta S^{\tau}_{\rm eff}}{\delta\Omega} = -\sqrt{g}\left[\left[T_{\mu}{}^{\mu}\right]^{\rm UV}_{\tau=0} - \left[T_{\mu}{}^{\mu}\right]^{\rm IR}_{\tau=0}\right]$$

The Proof

$$S^{\tau}_{\rm eff} = -\int d^3x\, \delta(x)\, \sqrt{g}\ \tau \left[(c_1^{\rm UV} - c_1^{\rm IR})\, \hat{R} + (c_2^{\rm UV} - c_2^{\rm IR})(K_{ab}K^{ab} - \frac{1}{2}K^2)\right]$$

$$- \int d^3x\, \delta(x)\, \sqrt{g}\ (c_1^{\rm UV} - c_1^{\rm IR})\, \tau \nabla^2 \tau \ + \ldots$$

Survives the flat-space limit
$g_{\mu\nu} \rightarrow \delta_{\mu\nu}$

$$\frac{\delta S^{\tau}_{\rm eff}}{\delta \Omega} = -\sqrt{g}\left[\left[T_{\mu}{}^{\mu}\right]^{\rm UV}_{\tau=0} - \left[T_{\mu}{}^{\mu}\right]^{\rm IR}_{\tau=0}\right]$$

The Proof

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Another form for the two-derivative term

$$Z = \int \mathcal{D}[\text{fields}]\, e^{-S(\lambda)_\tau}$$

$$S_\tau = \int d^3x \sqrt{g} \left[L(\lambda, \vec{x})_{\tau=0} + \tau \left[T_\mu{}^\mu \right]_{\tau=0} + \mathcal{O}(\tau^2) \right]$$

$$S_{\text{eff}} = -\ln Z = -\langle e^{-\int d^3x\, \tau(x) T_\mu{}^\mu(x) + \dots} \rangle_{\tau=0}$$

The Proof

$$g_{\mu\nu} = \delta_{\mu\nu}$$

Another form for the two-derivative term

$$\langle e^{-\int d^3x\,\tau(x) T_\mu{}^\mu(x) + \dots} \rangle = 1 - \int d^3x\,\tau(x)\,\langle T_\mu{}^\mu(x)\rangle$$
$$+\frac{1}{2}\int d^3x\,d^3y\,\tau(x)\tau(y)\,\langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle + \dots$$

↑

Taylor expand about x

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^\mu_\mu(x) T^\mu_\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\, (y-x)^\rho (y-x)^\sigma \langle T_\mu{}^\mu(x) T_\mu{}^\mu(y)\rangle\right]$$

$$T_\mu{}^\mu = \left[T_\mu{}^\mu\right]_{\text{bulk}} + \delta(x)\left[T_\mu{}^\mu\right]_\partial$$

$$\left[T_\mu{}^\mu\right]_{\text{bulk}} = 0$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma\,\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle\right]$$

$$\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle = \delta(x)\delta(y)\langle\left[T_\mu{}^\mu(x)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle\right]$$

Reflection Positivity

$$\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle = \delta(x)\delta(y)\langle\left[T_\mu{}^\mu(x)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle \geq 0$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^{\mu}_{\mu}(x) T^{\mu}_{\mu}(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle\right]$$

Translation invariance along the boundary

$$\delta(x)\int d^3y\,\delta(y)\,(y-x)^\rho(y-x)^\sigma\langle\left[T_\mu{}^\mu(x)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

$$=\delta(x)\frac{1}{2}\delta^{\rho\sigma}\int d^3y\,\delta(y)\,y^2\langle\left[T_\mu{}^\mu(0)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

The Proof

Another form for the two-derivative term

$$\frac{1}{2}\int d^3x\, d^3y\, \tau(x)\tau(y)\, \langle T^\mu_\mu(x) T^\mu_\mu(y)\rangle$$

$$\supset \frac{1}{4}\int d^3x\, \tau(x)\partial_\rho\partial_\sigma\tau(x)\left[\int d^3y\,(y-x)^\rho(y-x)^\sigma\langle T_\mu{}^\mu(x)T_\mu{}^\mu(y)\rangle\right]$$

Reflection Positivity

$$\delta(x)\int d^3y\,\delta(y)\,(y-x)^\rho(y-x)^\sigma\langle\left[T_\mu{}^\mu(x)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle$$

$$=\delta(x)\frac{1}{2}\delta^{\rho\sigma}\int d^3y\,\delta(y)\,y^2\langle\left[T_\mu{}^\mu(0)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle \geq 0$$

The Proof

$$\int d^3x\,\delta(x)\ \tau\nabla^2\tau\ (c_1^{\rm UV}-c_1^{\rm IR})$$

$$=\int d^3x\,\delta(x)\ \tau\nabla^2\tau\left[\frac{1}{8}\int d^3y\,\delta(y)\,y^2\langle\left[T_\mu{}^\mu(0)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle\right]$$

$$c_1^{\rm UV}-c_1^{\rm IR}=\left[\frac{1}{8}\int d^3y\,\delta(y)\,y^2\langle\left[T_\mu{}^\mu(0)\right]_\partial\left[T_\mu{}^\mu(y)\right]_\partial\rangle\right]\geq 0$$

$$c_1^{\rm UV}\geq c_1^{\rm IR}$$

$$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$$

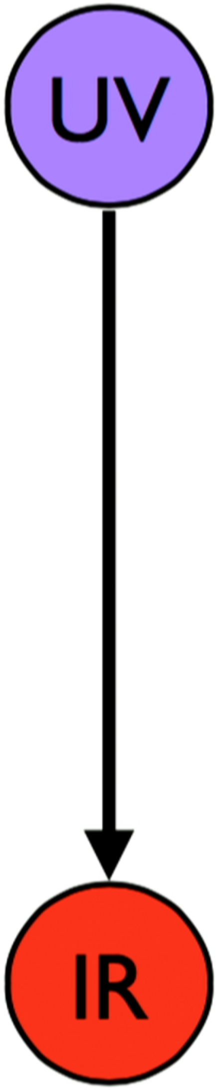
Does c_1 count DOF?

Add a single, real, free scalar field or Dirac fermion at the boundary

$$24\pi c_1 \to 24\pi c_1 + 1$$

c_2 unchanged

Both depend on boundary conditions of bulk fields



UV BCFT

Single real, free, massless, scalar
Neumann B.C.

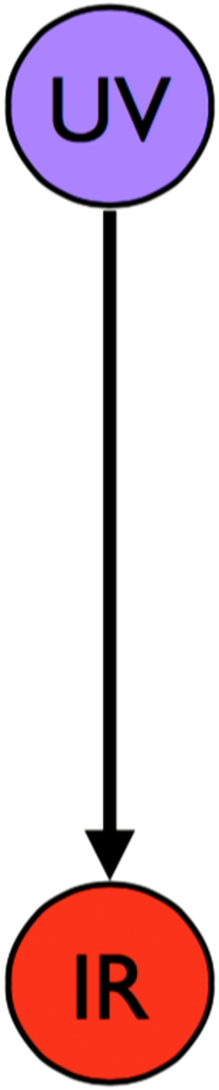
Boundary RG Flow

$$S(\lambda)^{\rm UV}_{\rm BCFT} \to S(\lambda)^{\rm UV}_{\rm BCFT} + \int d^3x\, \delta(x)\, m^2 \Phi^2(\vec{x})$$

$$\Delta_{\Phi^2} = 1$$

IR BCFT

Single real, free, massless, scalar
Dirichlet B.C.



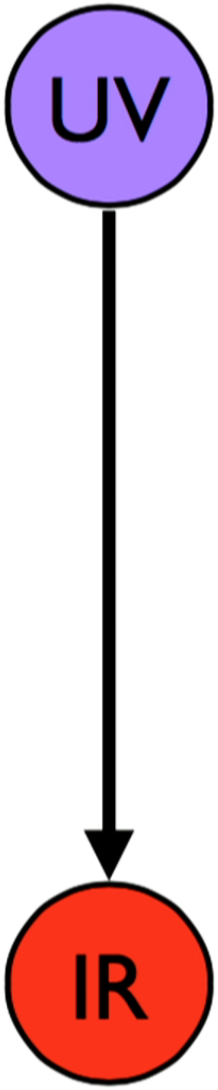
$$c_1^{\mathrm{UV}} = \frac{1}{24\pi}\frac{7}{16}$$

Neumann B.C.

Nozaki, Takayanagi, Ugajin
1205.1573

$$c_1^{\mathrm{IR}} = -\frac{1}{24\pi}\frac{1}{16}$$

Dirichlet B.C.



$$c_1^{\mathrm{UV}} = \frac{1}{24\pi}\frac{7}{16}$$

Neumann B.C.

$$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$$

Is c_1 bounded below?

$$c_1^{\mathrm{IR}} = -\frac{1}{24\pi}\frac{1}{16}$$

Dirichlet B.C.

Defects

Local, reflection-positive CFT in any $d \geq 3$

With a two-dimensional planar defect

Conformal defect

$$SO(d+1,1) \to SO(3,1) \times SO(d-2)$$

$SO(3,1)$: conformal transformations preserving the defect

$SO(d-2)$: rotations about the defect

"Defect CFT" (DCFT)

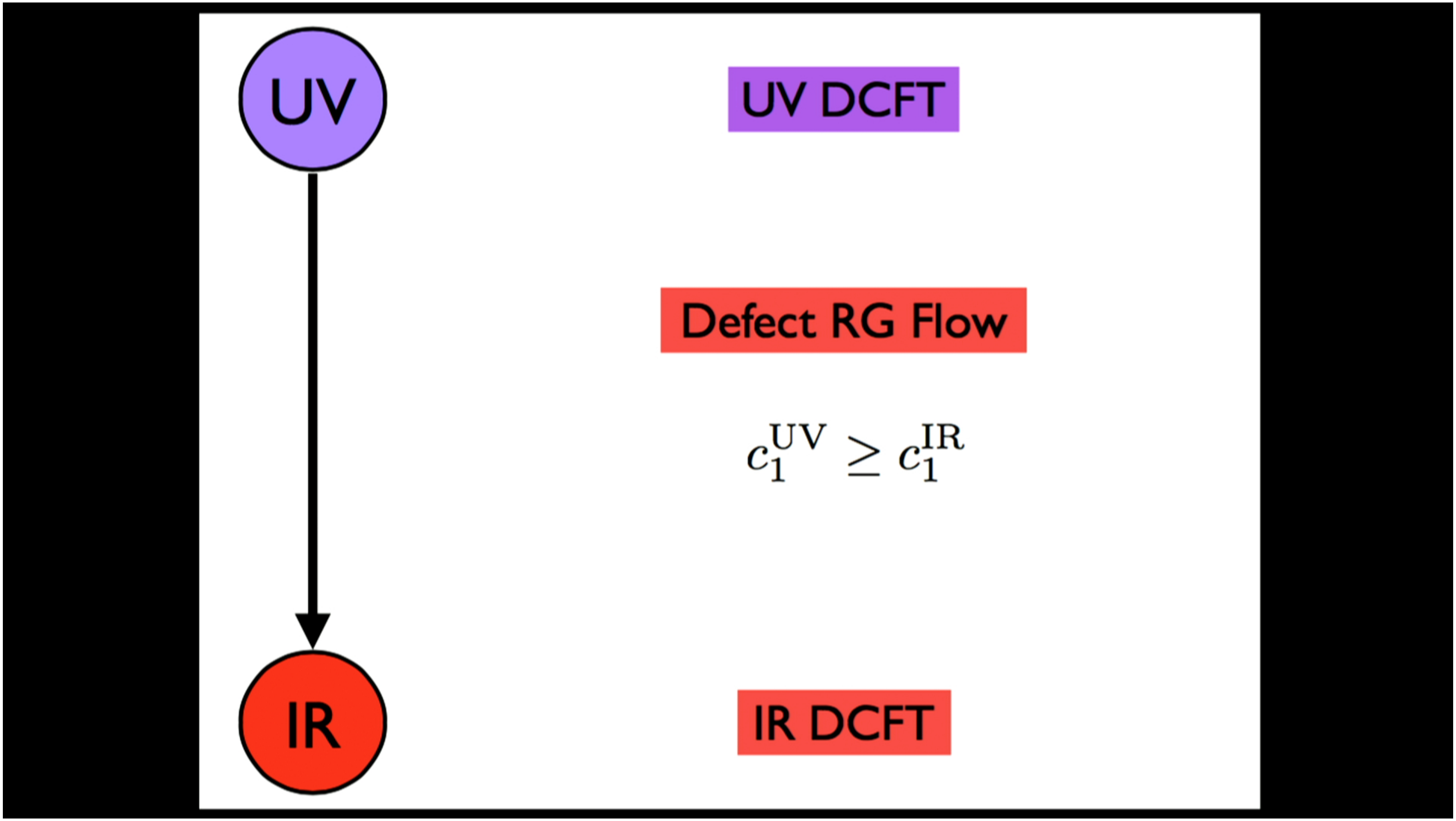
Defects

$$[T_\mu{}^\mu]_{\text{defect}} = c_1\,\hat{R} + c_2(K^\mu_{ab}K^{ab}_\mu - \frac{1}{2}K^\mu K_\mu) + c_3\hat{g}^{ac}\hat{g}^{bd}W_{abcd}$$

Boundary "central charges"

c_1 c_2 c_3

$\hat{g}^{ac}\hat{g}^{bd}W_{abcd}$ **is B-type**


UV
UV DCFT
Defect RG Flow
$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$
IR
IR DCFT

Summary

Local, reflection-positive BCFT in $d = 3$

Local, reflection-positive DCFT in $d \geq 3$
with two-dimensional defect

$$[T_\mu{}^\mu]_{\text{defect}} = c_1 \hat{R} + c_2 (K^\mu_{ab} K^{ab}_\mu - \frac{1}{2} K^\mu K_\mu) + c_3 \hat{g}^{ac} \hat{g}^{bd} W_{abcd}$$

Boundary or Defect RG Flows

$$c_1^{\text{UV}} \geq c_1^{\text{IR}}$$

Summary

$$c_1^{\mathrm{UV}} \geq c_1^{\mathrm{IR}}$$

Higher-dimensional g-theorem

Generalization of the weak form
of Zamolodchikov's c-theorem
to include coupling to higher-dimensional CFT

Proof used only existing ingredients!

Outlook

Immediate questions

Can we define a c_1-function?

Is c_1 bounded below?

Other methods of proof?

What about EE? Or holography?

Examples

Graphene with a boundary

Critical Ising model in $d = 3$ with a boundary

M-theory: M2-branes with a boundary

String theory: various brane intersections

Holographic BCFTs

Outlook

Prove more boundary/defect monotonicity theorems

Yamaguchi	hep-th/0207171
Estes, Jensen, O'B., Tsatis, Wrase	1403.6475
Gaiotto	1403.8052

Find a "universal" proof of monotonicity theorems?

Myers and Sinha	Giombi and Klebanov
1006.1263, 1011.5819	1409.1937

Do monotonicity theorems always survive coupling to a higher-dimensional CFT?

Thank You.