Title: Geometries of any dimension without twist and shear

Date: May 29, 2015 02:00 PM

URL: http://pirsa.org/15050116

Abstract: <p>We study a general class of D-dimensional spacetimes that admit a non-twisting and shear-free null vector field. This includes the famous non-expanding Kundt family and the expanding Robinson-Trautman family of spacetimes. In particular, we show that the algebraic structure of the Weyl tensor is I(b) or more special, and derive surprisingly simple conditions under which the optically privileged null direction is a multiple WAND. All possible algebraically special types, including the refinement to subtypes, are thus identified. No field equations are applied, so that the results are valid not only in Einstein's theory but also in its generalizations. Differences between the D=4 and D>4 cases are summarized, and we give a short discussion of some interesting particular subcases (exact gravitational waves, gyratons, non-rotating p-form black holes etc.).</p>

Geometries of any dimension without twist and shear

J. Podolský

in collaboration with

R. Švarc, M. Ortaggio, M. Žofka

Institute of Theoretical Physics
Charles University in Prague

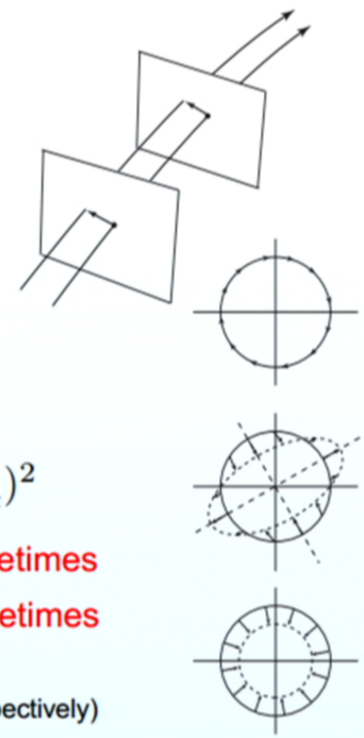
Perimeter Institute June 2015

definition of the geometries

more than 50 years ago

- Wolfgang Kundt (1961, 1962)
- Ivor Robinson, Andrzej Trautman (1960, 1962)

introduced and studied four-dimensional geometries that admit a geodesic null congruence generated by $\boldsymbol{k}$ which is



- twist-free: $0 = \mathrm{Tr}\, A^2 \quad \equiv -k_{[\alpha;\beta]}\, k^{\alpha;\beta}$
- shear-free: $0 = \mathrm{Tr}\, \sigma^2 \quad \equiv \quad k_{(\alpha;\beta)}\, k^{\alpha;\beta} - \frac{1}{D-2}(k^{\alpha}{}_{;\alpha})^2$
- non-expanding: $0 = \Theta \quad \equiv \quad \frac{1}{D-2} k^{\alpha}{}_{;\alpha}$ Kundt spacetimes
  expanding: $0 \neq \Theta$ Robinson–Trautman spacetimes

for their recent reviews see the monographs (chapters 31+28 and 18+19, respectively)

- Stephani, Kramer, MacCallum, Hoenselaers and Herlt 2003 *Exact Solutions of Einstein's Field Equations*
- Griffiths and JP 2009 *Exact Space-Times in Einstein's General Relativity*

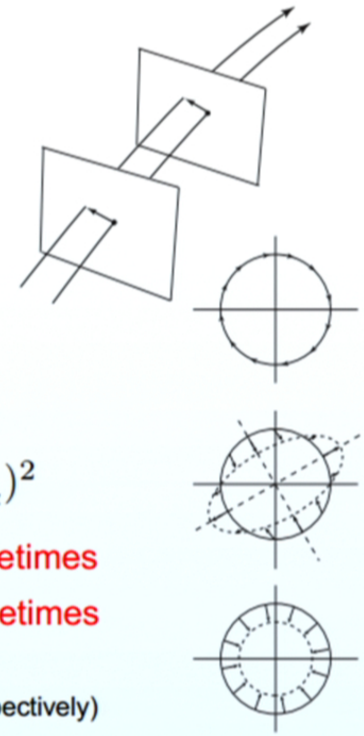
our goal was to investigate their generalization to arbitary dimension D

definition of the geometries

more than 50 years ago

- Wolfgang Kundt (1961, 1962)
- Ivor Robinson, Andrzej Trautman (1960, 1962)

introduced and studied four-dimensional geometries that admit a geodesic null congruence generated by $\boldsymbol{k}$ which is



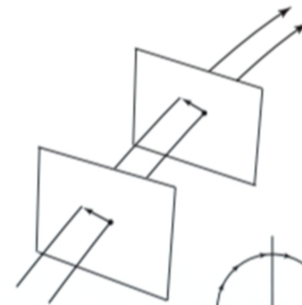
- twist-free: $0 = \mathrm{Tr}\, A^2 \equiv -k_{[\alpha;\beta]}\, k^{\alpha;\beta}$
- shear-free: $0 = \mathrm{Tr}\, \sigma^2 \equiv k_{(\alpha;\beta)}\, k^{\alpha;\beta} - \frac{1}{D-2}(k^{\alpha}{}_{;\alpha})^2$
- non-expanding: $0 = \Theta \equiv \frac{1}{D-2} k^{\alpha}{}_{;\alpha}$ Kundt spacetimes
  expanding: $0 \neq \Theta$ Robinson–Trautman spacetimes

for their recent reviews see the monographs (chapters 31+28 and 18+19, respectively)

- Stephani, Kramer, MacCallum, Hoenselaers and Herlt 2003 *Exact Solutions of Einstein's Field Equations*
- Griffiths and JP 2009 *Exact Space-Times in Einstein's General Relativity*

our goal was to investigate their generalization to arbitary dimension D

definition of the geometries



more than 50 years ago

- Wolfgang Kundt (1961, 1962)
- Ivor Robinson, Andrzej Trautman (1960, 1962)

introduced and studied four-dimensional geometries that admit a geodesic null congruence generated by $\boldsymbol{k}$ which is

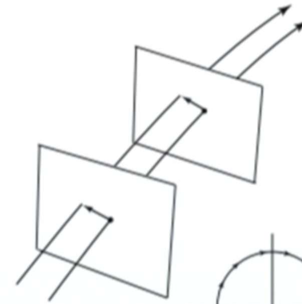
- twist-free: $0 = \text{Tr}\, A^2 \quad \equiv -k_{[\alpha;\beta]}\, k^{\alpha;\beta}$
- shear-free: $0 = \text{Tr}\, \sigma^2 \quad \equiv \quad k_{(\alpha;\beta)}\, k^{\alpha;\beta} - \frac{1}{D-2}(k^{\alpha}_{\ ;\alpha})^2$
- non-expanding: $0 = \Theta \quad \equiv \quad \frac{1}{D-2} k^{\alpha}_{\ ;\alpha}$ Kundt spacetimes
  expanding: $0 \neq \Theta$ Robinson–Trautman spacetimes

for their recent reviews see the monographs (chapters 31+28 and 18+19, respectively)

- Stephani, Kramer, MacCallum, Hoenselaers and Herlt 2003 *Exact Solutions of Einstein's Field Equations*
- Griffiths and JP 2009 *Exact Space-Times in Einstein's General Relativity*

our goal was to investigate their generalization to arbitary dimension D

definition of the geometries



more than 50 years ago

- Wolfgang Kundt (1961, 1962)
- Ivor Robinson, Andrzej Trautman (1960, 1962)

introduced and studied four-dimensional geometries that
admit a geodesic null congruence generated by $\boldsymbol{k}$ which is

- twist-free: $0 = \mathrm{Tr}\, A^2 \quad \equiv -k_{[\alpha;\beta]}\, k^{\alpha;\beta}$
- shear-free: $0 = \mathrm{Tr}\, \sigma^2 \quad \equiv \quad k_{(\alpha;\beta)}\, k^{\alpha;\beta} - \frac{1}{D-2}(k^{\alpha}{}_{;\alpha})^2$
- non-expanding: $0 = \Theta \quad \equiv \quad \frac{1}{D-2} k^{\alpha}{}_{;\alpha}$ Kundt spacetimes
  expanding: $0 \neq \Theta$ Robinson–Trautman spacetimes

for their recent reviews see the monographs (chapters 31+28 and 18+19, respectively)

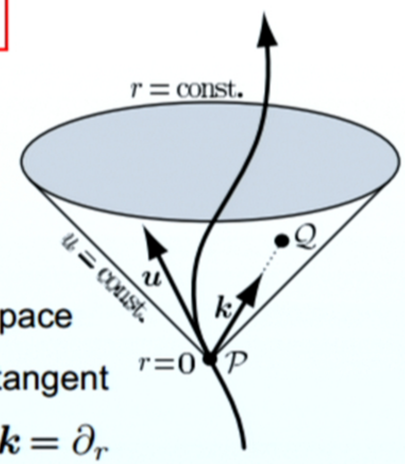
- Stephani, Kramer, MacCallum, Hoenselaers and Herlt 2003 *Exact Solutions of Einstein's Field Equations*
- Griffiths and JP 2009 *Exact Space-Times in Einstein's General Relativity*

our goal was to investigate their generalization to arbitary dimension D

general metric

in suitable coordinates all such metrics can always be written as

$$\mathrm{d}s^2 = g_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q + 2\,g_{up}\,\mathrm{d}u\,\mathrm{d}x^p - 2\,\mathrm{d}u\,\mathrm{d}r + g_{uu}\,\mathrm{d}u^2$$



where $g_{pq},\ g_{up},\ g_{uu}$ are functions of the coordinates x, u, r:

- $x \equiv (x^p) \equiv (x^2, x^3, \dots, x^{D-1})$:
  spatial coordinates in the transverse $(D-2)$-dim space
- $u =$ const: foliation by null wave-surfaces to which $\boldsymbol{k}$ is tangent
- r: affine parameter along the geodesics generated by $\boldsymbol{k} = \partial_r$

- Kundt non-expanding class $\Theta = 0 \;\Leftrightarrow\; g_{pq} = h_{pq}(u, x)$
- Robinson–Trautman expanding class $\Theta \neq 0 \;\Leftrightarrow\; g_{pq} = R^2(r, u, x)\, h_{pq}(u, x)$
  with $R = \exp\left(\int \Theta\,\mathrm{d}r\right)$

important members of the Kundt family

- pp-waves with gyratons (CCNV spacetimes)
  defined geometrically as admitting a covariantly constant null vector field $\boldsymbol{k}$
  necessarily independent of r: (Brinkmann, 1925)

$$\mathrm{d}s^2 = g_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q + 2\,e_p\,\mathrm{d}u\,\mathrm{d}x^p - 2\,\mathrm{d}u\,\mathrm{d}r + c\;\mathrm{d}u^2$$

- VSI spacetimes
  scalar curvature invariants of all orders vanish
  transverse space is flat, $g_{pq} = \delta_{pq}$: (Coley et al., 2006 etc)

$$\mathrm{d}s^2 = \delta_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q + 2\,(e_p + f_p\,r)\,\mathrm{d}u\,\mathrm{d}x^p - 2\,\mathrm{d}u\,\mathrm{d}r + (a\,r^2 + b\,r + c)\,\mathrm{d}u^2$$

- Bertotti–Robinson, Nariai, Plebański–Hacyan, and Minkowski spacetimes
  direct-product spaces, electrovacuum with Λ, algebraic type D and O
  CSI backgrounds on which the Kundt waves and gyratons propagate:

$$\mathrm{d}s^2 = g_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q - 2\,\mathrm{d}u\,\mathrm{d}r + a\,r^2\,\mathrm{d}u^2$$

important members of the Robinson–Trautman family

- de Sitter, anti-de Sitter, and Minkowski spacetimes
  conformally flat vacuum spaces with Λ, algebraic type O

$$\mathrm{d}s^2 = r^2 \frac{\delta_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q}{(1+\frac{1}{4}\,\delta_{mn}\,x^m x^n)^2} - 2\,\mathrm{d}u\,\mathrm{d}r - \left[1 - \frac{2\Lambda}{(D-2)(D-1)}\,r^2\right]\mathrm{d}u^2$$

- static black holes of type D with various horizon geometries and topologies
  since h_{pq} is any Riemannian Einstein space: $\mathcal{R}_{pq} = K(D-3)\,h_{pq},\ K = 0, \pm 1$
  Schwarzschild–Kottler–Tangherlini for a constant curvature $h_{pq}(x) = (1+\frac{1}{4}K\,\delta_{mn}\,x^m x^n)^{-2}\,\delta_{pq}$

$$\mathrm{d}s^2 = r^2\,h_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q - 2\,\mathrm{d}u\,\mathrm{d}r - \left[K - \frac{2\Lambda}{(D-2)(D-1)}\,r^2 - \frac{\mu}{r^{D-3}}\right]\mathrm{d}u^2$$

- expanding gravitational waves of type N do not exist in higher dimensions D

$$\mathrm{d}s^2 = r^2 \frac{\delta_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q}{P^2(x,u)} - 2\,\mathrm{d}u\,\mathrm{d}r - \left[K - 2r\,(\log P)_{,u} - \frac{\Lambda}{3}\,r^2\right]\mathrm{d}u^2$$

where $P(x,u)$ is any solution of $\Delta \log P = K$

explicit form of curvature tensors for the general metric

$$\mathrm{d}s^2 = g_{pq}(r,u,x)\,\mathrm{d}x^p\,\mathrm{d}x^q + 2\,g_{up}(r,u,x)\,\mathrm{d}u\,\mathrm{d}x^p - 2\,\mathrm{d}u\,\mathrm{d}r + g_{uu}(r,u,x)\,\mathrm{d}u^2$$

we calculated all coordinate components of $\Gamma^a_{bc}\,, R_{abcd}\,, R_{ab}\,, R\,, C_{abcd}$ *lengthy...*

using a useful notation for tensors in the $(D-2)$-dimensional transverse space with the Riemannian metric g_{pq}:

- $e_{pq} \equiv g_{u(p||q)} - \frac{1}{2} g_{pq,u}$
- $E_{pq} \equiv g_{u[p,q]} + \frac{1}{2} g_{pq,u}$
- $f_{pq} \equiv g_{u(p,r||q)} + \frac{1}{2} g_{up,r} g_{uq,r}$
- symbol $_{||}$ denotes covariant derivative with respect to g_{pq}
- ${}^S R_{mpnq}$, ${}^S R_{pq}$, ${}^S C_{mpnq}$ are the Riemann, Ricci, Weyl tensors of g_{pq}

the curvature tensors read:

see JP, Švarc: CQG (2015)

explicit form of curvature tensors for the general metric

$$ds^2 = g_{pq}(r,u,x)\,dx^p\,dx^q + 2\,g_{up}(r,u,x)\,du\,dx^p - 2\,du\,dr + g_{uu}(r,u,x)\,du^2$$

we calculated all coordinate components of $\Gamma^a_{bc}\,, R_{abcd}\,, R_{ab}\,, R\,, C_{abcd}$ *lengthy...*

using a useful notation for tensors in the $(D-2)$-dimensional transverse space with the Riemannian metric g_{pq}:

- $e_{pq} \equiv g_{u(p||q)} - \frac{1}{2} g_{pq,u}$
- $E_{pq} \equiv g_{u[p,q]} + \frac{1}{2} g_{pq,u}$
- $f_{pq} \equiv g_{u(p,r||q)} + \frac{1}{2} g_{up,r} g_{uq,r}$
- symbol $_{||}$ denotes covariant derivative with respect to g_{pq}
- ${}^S R_{mpnq}$, ${}^S R_{pq}$, ${}^S C_{mpnq}$ are the Riemann, Ricci, Weyl tensors of g_{pq}

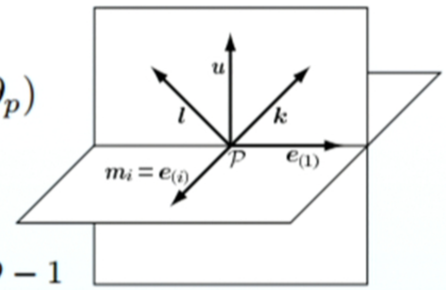
the curvature tensors read:

see JP, Švarc: CQG (2015)

algebraic classification: the Weyl tensor null alignment

to determine the algebraic type of any spacetime we must project the Weyl tensor coordinate components C_{abcd} onto a normalized null frame $\{\boldsymbol{k},\, \boldsymbol{l},\, \boldsymbol{m}_2, \boldsymbol{m}_3, ..., \boldsymbol{m}_{D-1}\}$

$$\boldsymbol{k} = \partial_r\,, \qquad \boldsymbol{l} = \tfrac{1}{2} g_{uu}\,\partial_r + \partial_u\,, \qquad \boldsymbol{m}_i = m_i^p\,(\,g_{up}\,\partial_r + \partial_p)$$



- $\boldsymbol{k}$ and $\boldsymbol{l}$ are future-oriented null vectors: $\quad \boldsymbol{k} \cdot \boldsymbol{l} = -1$
- $\boldsymbol{m}_i$ are $(D-2)$ transverse spatial vectors: $\boldsymbol{m}_i \cdot \boldsymbol{m}_j = \delta_{pq}$

$i, j = 2, 3, ..., D-1$

components of the Weyl tensor in this null frame are the Newman–Penrose scalars $\Psi_{A^{ij..}}$

$$\begin{aligned}
\Psi_{0^{ij}} &\equiv C_{abcd}\,k^a\,m_i^b\,k^c\,m_j^d \\
\Psi_{1^{ijk}} &\equiv C_{abcd}\,k^a\,m_i^b\,m_j^c\,m_k^d \qquad & \Psi_{1T^i} &\equiv C_{abcd}\,k^a\,l^b\,k^c\,m_i^d \\
\Psi_{2^{ijkl}} &\equiv C_{abcd}\,m_i^a\,m_j^b\,m_k^c\,m_l^d \qquad & \Psi_{2S} &\equiv C_{abcd}\,k^a\,l^b\,l^c\,k^d \\
\Psi_{2^{ij}} &\equiv C_{abcd}\,k^a\,l^b\,m_i^c\,m_j^d \qquad & \Psi_{2T^{ij}} &\equiv C_{abcd}\,k^a\,m_i^b\,l^c\,m_j^d \\
\Psi_{3^{ijk}} &\equiv C_{abcd}\,l^a\,m_i^b\,m_j^c\,m_k^d \qquad & \Psi_{3T^i} &\equiv C_{abcd}\,l^a\,k^b\,l^c\,m_i^d \\
\Psi_{4^{ij}} &\equiv C_{abcd}\,l^a\,m_i^b\,l^c\,m_j^d
\end{aligned}$$

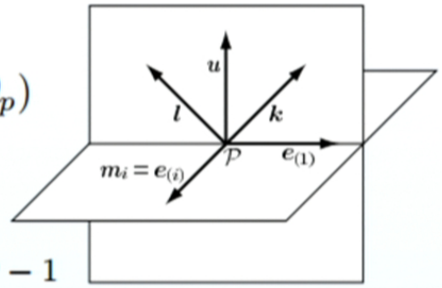
grouped by their boost weight:

if we perform a boost $\boldsymbol{k}' = \lambda \boldsymbol{k}$, $\boldsymbol{l}' = \lambda^{-1} \boldsymbol{l}$ then they scale as $\Psi'_{0^{ij}} = \lambda^2 \Psi_{0^{ij}}$, $\Psi'_{1^{ijk}} = \lambda \Psi_{1^{ijk}}$, etc.

algebraic classification: the Weyl tensor null alignment

to determine the algebraic type of any spacetime we must project the Weyl tensor coordinate components C_{abcd} onto a normalized null frame $\{\boldsymbol{k}, \boldsymbol{l}, \boldsymbol{m}_2, \boldsymbol{m}_3, ..., \boldsymbol{m}_{D-1}\}$

$$\boldsymbol{k} = \partial_r \,, \qquad \boldsymbol{l} = \tfrac{1}{2} g_{uu}\, \partial_r + \partial_u \,, \qquad \boldsymbol{m}_i = m_i^p \,(g_{up}\, \partial_r + \partial_p)$$



- $\boldsymbol{k}$ and $\boldsymbol{l}$ are future-oriented null vectors: $\boldsymbol{k} \cdot \boldsymbol{l} = -1$
- $\boldsymbol{m}_i$ are $(D-2)$ transverse spatial vectors: $\boldsymbol{m}_i \cdot \boldsymbol{m}_j = \delta_{pq}$

$i, j = 2, 3, ..., D-1$

components of the Weyl tensor in this null frame are the Newman–Penrose scalars $\Psi_{A^{ij..}}$

$$\Psi_{0^{ij}} \equiv C_{abcd}\, k^a\, m_i^b\, k^c\, m_j^d$$

$$\Psi_{1^{ijk}} \equiv C_{abcd}\, k^a\, m_i^b\, m_j^c\, m_k^d \qquad \Psi_{1T^i} \equiv C_{abcd}\, k^a\, l^b\, k^c\, m_i^d$$

$$\Psi_{2^{ijkl}} \equiv C_{abcd}\, m_i^a\, m_j^b\, m_k^c\, m_l^d \qquad \Psi_{2S} \equiv C_{abcd}\, k^a\, l^b\, l^c\, k^d$$

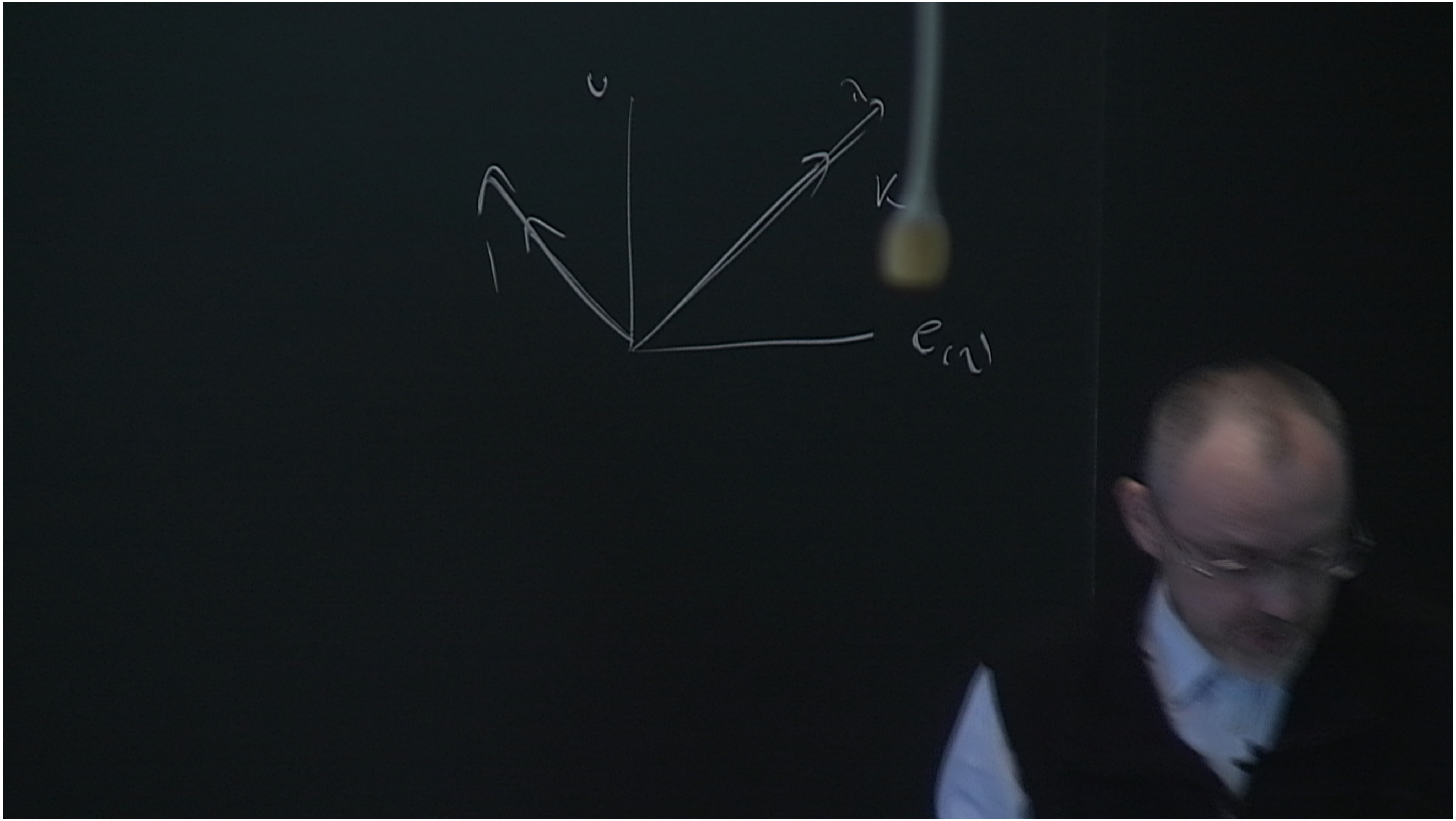
$$\Psi_{2^{ij}} \equiv C_{abcd}\, k^a\, l^b\, m_i^c\, m_j^d \qquad \Psi_{2T^{ij}} \equiv C_{abcd}\, k^a\, m_i^b\, l^c\, m_j^d$$

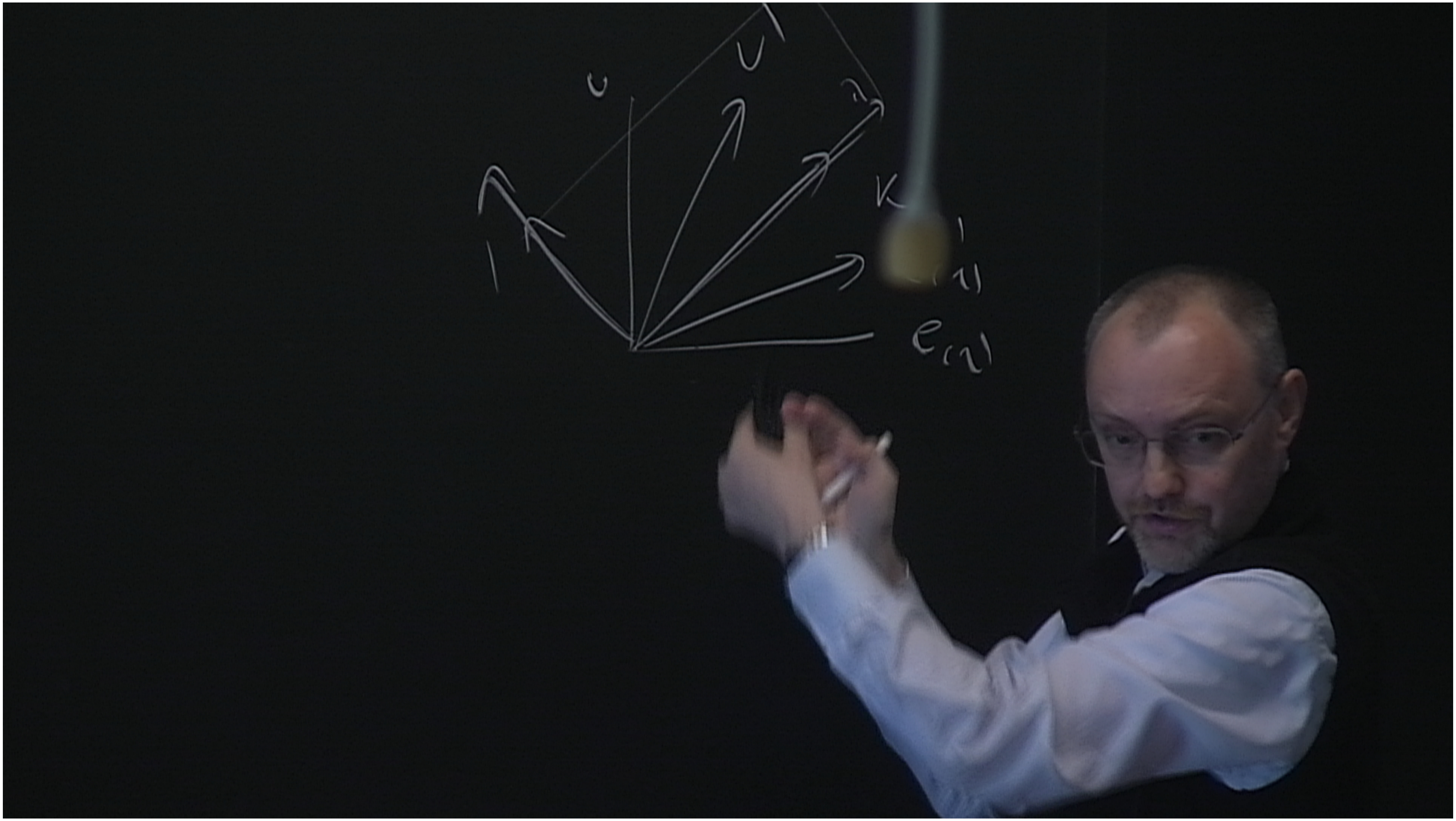
$$\Psi_{3^{ijk}} \equiv C_{abcd}\, l^a\, m_i^b\, m_j^c\, m_k^d \qquad \Psi_{3T^i} \equiv C_{abcd}\, l^a\, k^b\, l^c\, m_i^d$$

$$\Psi_{4^{ij}} \equiv C_{abcd}\, l^a\, m_i^b\, l^c\, m_j^d$$

grouped by their boost weight:

if we perform a boost $\boldsymbol{k}' = \lambda \boldsymbol{k}$, $\boldsymbol{l}' = \lambda^{-1} \boldsymbol{l}$ then they scale as $\Psi'_{0^{ij}} = \lambda^2 \Psi_{0^{ij}}$, $\Psi'_{1^{ijk}} = \lambda \Psi_{1^{ijk}}$, etc.


u
e(2)



principal alignment Weyl types and subtypes

- the scalars $\Psi_{A^{ij..}}$ of various boost weights vanish or not in a suitable null frame
- equivalent to the existence of the multiple WAND: Weyl aligned null direction $\boldsymbol{k}$

(sub)type	vanishing components					
I	$\Psi_{0^{ij}}$					
I(a)	$\Psi_{0^{ij}}$	Ψ_{1T^i}				
I(b)	$\Psi_{0^{ij}}$	$\tilde{\Psi}_{1ijk}$				
II	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$				
II(a)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2S}			
II(b)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	$\tilde{\Psi}_{2T(ij)}$			
II(c)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	$\tilde{\Psi}_{2ijkl}$			
II(d)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2ij}			
III	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2S} $\tilde{\Psi}_{2T(ij)}$ $\tilde{\Psi}_{2ijkl}$ Ψ_{2ij}			
III(a)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2S} $\tilde{\Psi}_{2T(ij)}$ $\tilde{\Psi}_{2ijkl}$ Ψ_{2ij}	Ψ_{3T^i}		
III(b)	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2S} $\tilde{\Psi}_{2T(ij)}$ $\tilde{\Psi}_{2ijkl}$ Ψ_{2ij}	$\tilde{\Psi}_{3ijk}$		
N	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$	Ψ_{2S} $\tilde{\Psi}_{2T(ij)}$ $\tilde{\Psi}_{2ijkl}$ Ψ_{2ij}	Ψ_{3T^i} $\tilde{\Psi}_{3ijk}$		
D	$\Psi_{0^{ij}}$	Ψ_{1T^i} $\tilde{\Psi}_{1ijk}$		Ψ_{3T^i} $\tilde{\Psi}_{3ijk}$	$\Psi_{4^{ij}}$	another WAND $\boldsymbol{l}$

for definitions see Coley, Milson, Pravda, Pravdová: CQG (2004), Ortaggio, Pravda, Pravdová: CQG (2013)

immediate conclusions on the algebraic structure

for all twist-free shear-free geometries in any dimension D

- the Weyl scalars of the highest boost weight $+2$ always vanish:

$$\Psi_{0^{ij}} = 0 \qquad \Rightarrow$$

for any such spacetime the optically privileged null vector $\boldsymbol{k} = \partial_r$ is a WAND

- moreover, the boost weight $+1$ Weyl scalars $\tilde{\Psi}_{1^{ijk}}$ always vanish:

$$\Psi_{0^{ij}} = 0, \qquad \tilde{\Psi}_{1^{ijk}} = 0 \qquad \Rightarrow$$

all Kundt or Robinson–Trautman geometries are of type I(b), or more special

- the remaining boost weight $+1$ Weyl scalars Ψ_{1T^i} vanish if, an only if,

$$(g_{up,r} - 2\Theta g_{up})_{,r} = 2\Theta_{,p} \qquad \Leftrightarrow$$

spacetimes are of algebraic type II, or more special, and $\boldsymbol{k} = \partial_r$ is a *double* WAND
in particular, Kundt geometry ($\Theta = 0$) is of type II $\Leftrightarrow$ $g_{up} = e_p(u,x) + f_p(u,x)\, r$

the key functions for type II spacetimes simplify to

$$P = (\tfrac{1}{2}g_{uu,r} - \Theta g_{uu})_{,r} + \frac{1}{(D-2)(D-3)}\,{}^{S}R + \frac{1}{D-2}g^{mn}f_{m||n} - \frac{1}{4}\frac{D-4}{D-2}g^{mn}f_m f_n - 2\Theta_{,u}$$

$$Q_{pq} = {}^{S}R_{pq} + \tfrac{1}{2}(D-4)(f_{(p||q)} + \tfrac{1}{2}f_p f_q)$$

$$F_{pq} = f_{[p,q]}$$

$$V_p = \tfrac{1}{2}(f_{p,u} - g_{uu,rp} - g_{up}g_{uu,rr} + g^{mn}f_m E_{np}) + \frac{1}{2}\frac{D-4}{D-3}g^{mn}g_{u[p}f_{m]}f_n$$
$$-\frac{1}{D-3}\,g^{mn}[g_{m[p,u||n]} + g_{u[m,p]||n} + e_{m[p}f_{n]} + \tfrac{1}{2}g_{up}f_{m||n} - \tfrac{1}{2}g_{um}(f_{(n||p)} - 3f_{[n,p]})]$$
$$+g_{up}g_{uu}\Theta_{,r} + 2g_{up}\Theta_{,u} + g_{uu}\Theta_{,p} + \Theta(g_{up}g_{uu,r} + g_{uu,p})$$

$$X_{pmq} = g_{p[m,u||q]} + g_{u[q,m]||p} + e_{p[m}f_{q]} - g_{u[q}f_{m]||p} - g_{up}f_{[m,q]} - \tfrac{1}{2}g_{u[q}f_{m]}f_p$$

$$W_{pq} = -\tfrac{1}{2}g_{uu||p||q} - \tfrac{1}{2}g_{pq,uu} + g_{u(p,u||q)} - \tfrac{1}{2}g_{uu,r}e_{pq} - g_{uu,r(p}g_{q)u} - \tfrac{1}{2}g_{up}g_{uq}g_{uu,rr}$$
$$+\tfrac{1}{2}g_{uu,(p}f_{q)} + \tfrac{1}{2}g_{uu}f_{(p||q)} + g_{u(p}f_{q),u} + \tfrac{1}{4}g^{mn}(g_{um}g_{un}f_p f_q + f_m f_n g_{up}g_{uq})$$
$$-\tfrac{1}{2}g^{mn}g_{um}f_n g_{u(p}f_{q)} + g^{mn}(E_{mp}E_{nq} + f_m E_{n(p}g_{q)u} - g_{um}E_{n(p}f_{q)}) + g_{uu}g_{up}g_{uq}\Theta_{,r}$$
$$+2g_{up}g_{uq}\Theta_{,u} + 2g_{uu}g_{u(p}\Theta_{,q)} + \Theta(2g_{uu,(p}g_{q)u} + g_{uu}g_{u(p||q)} + g_{up}g_{uq}g_{uu,r} - \tfrac{1}{2}g_{uu}g_{pq,u})$$

versus 73 lines of C_{abcd} ...

algebraically special Kundt geometries

Kundt spacetimes for which the optically privileged field $\boldsymbol{k} = \partial_r$ is a multiple WAND are

$$\mathrm{d}s^2 = g_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q + 2\,(e_p + f_p\,r)\,\mathrm{d}u\,\mathrm{d}x^p - 2\,\mathrm{d}u\,\mathrm{d}r + g_{uu}\,\mathrm{d}u^2$$

g_{pq}, e_p, f_p are functions of x and u only $\qquad g_{uu}(r, u, x)$

this metric contains all vacuum spacetimes, with Λ, aligned elmag field, pure radiation

see JP, Žofka: CQG (2009)

Kundt: summary of all algebraic types in $D = 4$

type	definition	NP relation	necessary and sufficient conditions
II(a)	$\Psi_{2S} = 0$	Re$\Psi_2 = 0$	$g_{uu} = a\,r^2 + b\,r + c, \quad a = \frac{1}{4} f^p f_p - \frac{1}{2}({}^S R + f)$
II(b)			always
II(c)			always
II(d)	$\Psi_{2ij} = 0$	Im$\Psi_2 = 0$	$F_{pq} = 0$
III	II(ad)		
III(a)	$\Psi_{3T^i} = 0$	Re$\Psi_3 = 0$ Im$\Psi_3 = 0$	$a_{,p} + f_p\, a = 0, \quad a = \frac{1}{4} f^p f_p - \frac{1}{2}({}^S R + f)$ $b_{,p} - f_{p,u} = e_p({}^S R + f) - \frac{1}{2} f^q e_q f_p + f^q E_{qp} + 2X_p$
III(b)			always
N	III(a)	$\Psi_2 = 0 = \Psi_3$	
O	all vanish	all vanish	N with $W_{pq} = \frac{1}{2}\, g_{pq}\, W$
D	$\Psi_{3T^i} = 0$ $\Psi_{4ij} = 0$	$\Psi_3 = 0$ $\Psi_4 = 0$	$\frac{1}{2}(r f_p\, g_{uu,rr} + g_{uu,rp} - f_{p,u}) + e_p(\frac{1}{2}\, g_{uu,rr} - \frac{1}{4} f^q f_q)$ $= r\,(\frac{1}{2} f^q F_{qp} + Y_p) - \frac{1}{4} f^q e_q f_p + \frac{1}{2} f^q E_{qp} + X_p$ $W_{pq} = \frac{1}{2}\, g_{pq}\, W$

$i, j = 2, 3$ $\quad p, q = 2, 3$ $\quad \Psi_{2ij}$ antisymmetric $\quad \Psi_{4ij}$ symmetric and traceless

geodesic deviation in vacuum Kundt spacetimes

canonical components of the gravitational field:

$$\ddot{Z}^{(1)} = \frac{2\,\Lambda}{(D-1)(D-2)}\,Z^{(1)} + \Psi^{\rm int}_{2S}\,Z^{(1)} - \frac{1}{\sqrt{2}}\,\Psi^{\rm int}_{3T^j}\,Z^{(j)}$$

longitudinal spatial direction

$$\ddot{Z}^{(i)} = \frac{2\,\Lambda}{(D-1)(D-2)}\,Z^{(i)} - \Psi^{\rm int}_{2T(ij)}\,Z^{(j)} - \frac{1}{\sqrt{2}}\,\Psi^{\rm int}_{3T^i}\,Z^{(1)} - \frac{1}{2}\,\Psi^{\rm int}_{4^{ij}}\,Z^{(j)}$$

transverse spatial directions $(i)=(2),(3),\ldots$

conformally flat	type D	type III	type N
isotropic motion	Newton-like	longitudinal	transverse

$$\Psi^{\rm int}_{2S} = \Psi_{2S} \qquad \Psi^{\rm int}_{2T(ij)} = \Psi_{2T(ij)}$$

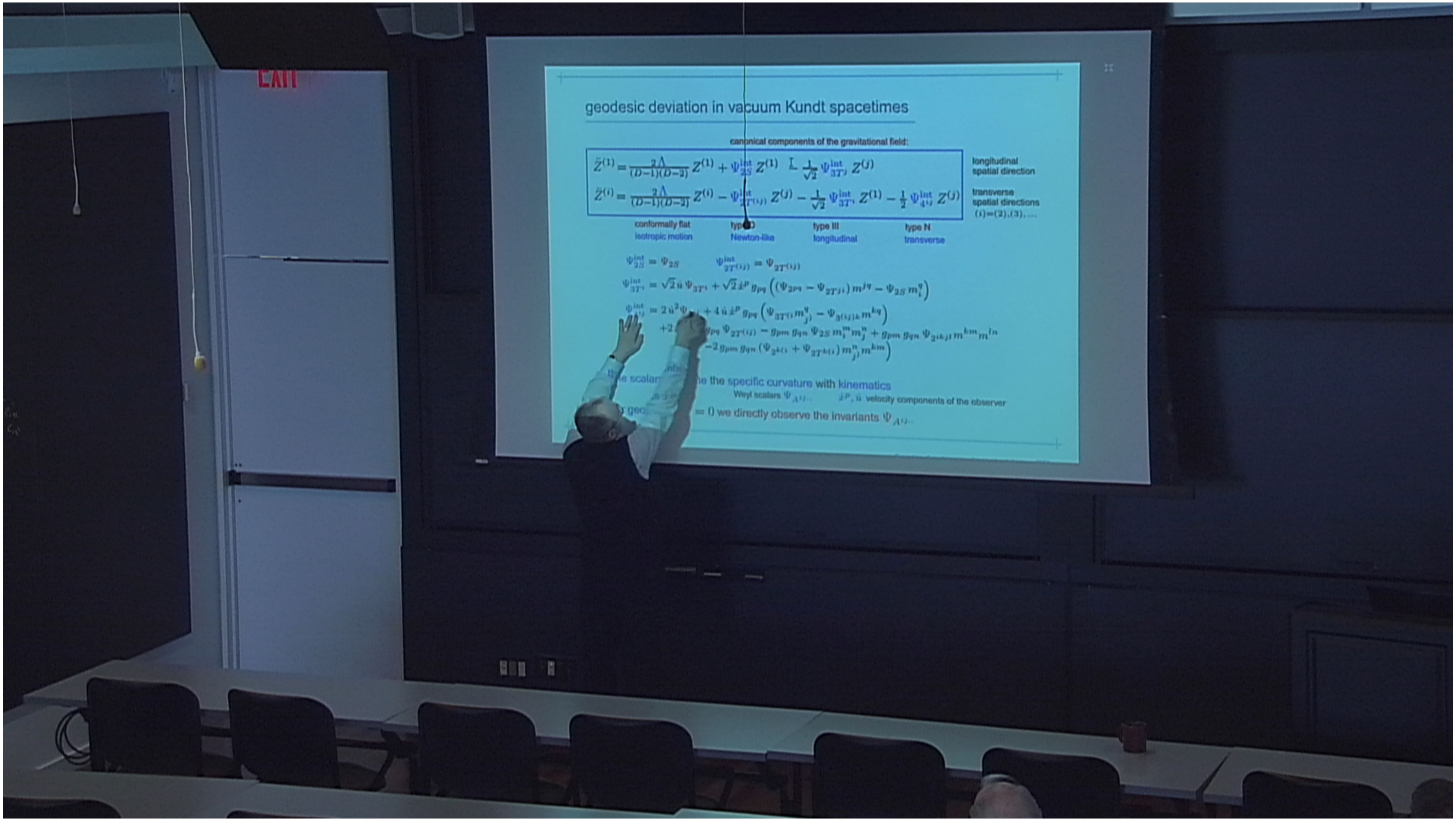
$$\Psi^{\rm int}_{3T^i} = \sqrt{2}\,\dot{u}\,\Psi_{3T^i} + \sqrt{2}\,\dot{x}^p\,g_{pq}\left((\Psi_{2^{pq}} - \Psi_{2T^{ji}})\,m^{jq} - \Psi_{2S}\,m^q_i\right)$$

$$\begin{aligned}\Psi^{\rm int}_{4^{ij}} = {} & 2\,\dot{u}^2\Psi_{4^{ij}} + 4\,\dot{u}\,\dot{x}^p\,g_{pq}\left(\Psi_{3T^{(i}}m^q_{j)} - \Psi_{3^{(ij)k}}\,m^{kq}\right)\\ & +2\,\dot{x}^p\dot{x}^q\Big(g_{pq}\,\Psi_{2T^{(ij)}} - g_{pm}\,g_{qn}\,\Psi_{2S}\,m^m_i m^n_j + g_{pm}\,g_{qn}\,\Psi_{2^{ikjl}}\,m^{km}m^{ln}\\ & \qquad -2\,g_{pm}\,g_{qn}\,(\Psi_{2^{k(i}} + \Psi_{2T^{k(i}})\,m^n_{j)}\,m^{km}\Big)\end{aligned}$$

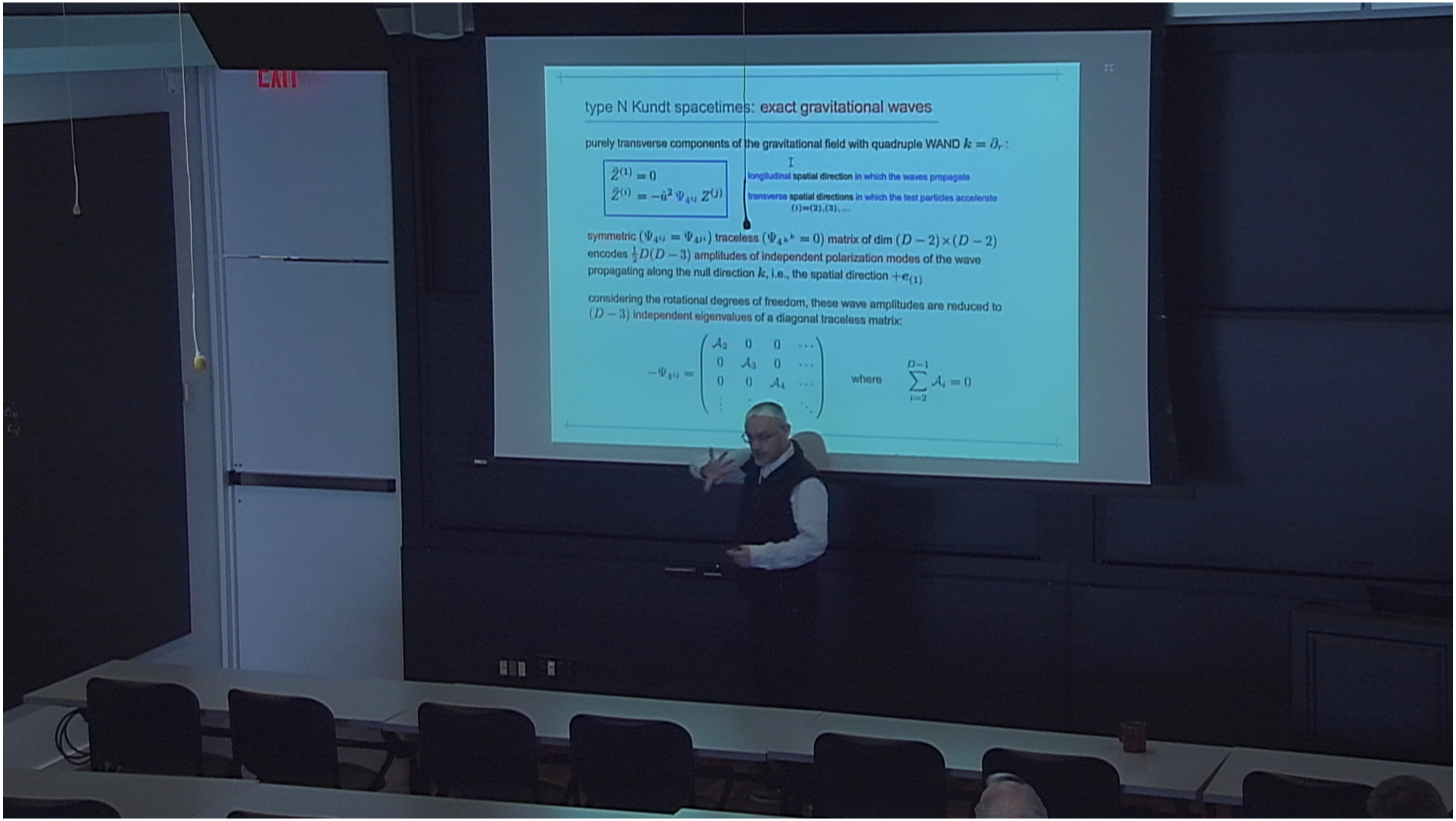
- the scalars combine the specific curvature with kinematics

  Weyl scalars $\Psi_{A^{ij..}}$ $\quad \dot{x}^p, \dot{u}$ velocity components of the observer

- for geodesics $\dot{x}^p = 0$ we directly observe the invariants $\Psi_{A^{ij..}}$



geodesic deviation in vacuum Kundt spacetimes
canonical components of the gravitational field:
longitudinal spatial direction
transverse spatial directions
conformally flat
isotropic motion
Newton-like
type III
longitudinal
type N
transverse
specific curvature with kinematics
Weyl scalars
velocity components of the observer
we directly observe the invariants



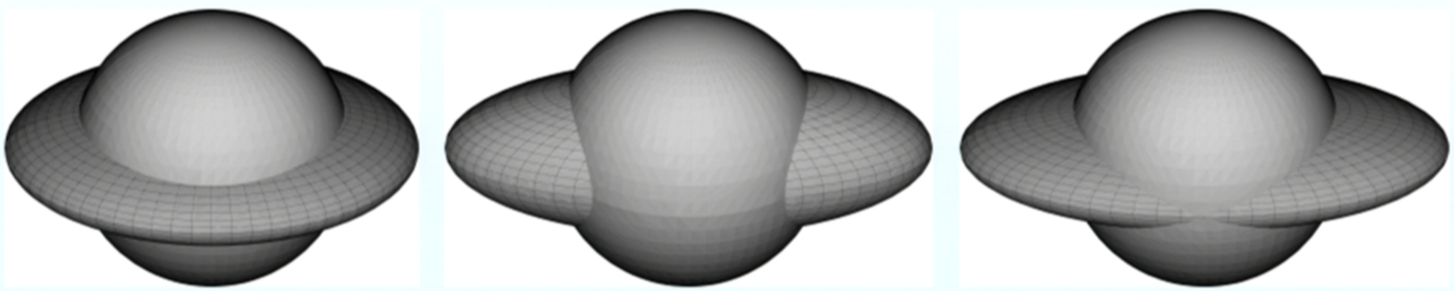
type N Kundt spacetimes: exact gravitational waves
purely transverse components of the gravitational field with quadruple WAND $k = \partial_r$:
$\ddot{Z}^{(1)} = 0$
$\ddot{Z}^{(i)} = -\dot{u}^2\, \Psi_{4^{ij}}\, Z^{(j)}$
longitudinal spatial direction in which the waves propagate
transverse spatial directions in which the test particles accelerate
$(i)=(2),(3),\ldots$
symmetric ($\Psi_{4^{ij}} = \Psi_{4^{ji}}$) traceless ($\Psi_{4^k}{}^k = 0$) matrix of dim $(D-2)\times(D-2)$
encodes $\frac{1}{2}D(D-3)$ amplitudes of independent polarization modes of the wave
propagating along the null direction k, i.e., the spatial direction $+e_{(1)}$
considering the rotational degrees of freedom, these wave amplitudes are reduced to
$(D-3)$ independent eigenvalues of a diagonal traceless matrix:
$-\Psi_{4^{ij}} = \begin{pmatrix} \mathcal{A}_2 & 0 & 0 & \cdots \\ 0 & \mathcal{A}_3 & 0 & \cdots \\ 0 & 0 & \mathcal{A}_4 & \cdots \\ \vdots & & & \ddots \end{pmatrix}$ where $\sum_{i=2}^{D-1} \mathcal{A}_i = 0$

gravitational waves in any dimension D

relative motion of test particles (initially at rest) can be explicitly integrated to

$$Z^{(i)}(\tau) = \begin{cases} Z_0^{(i)} \cosh\left(\sqrt{\mathcal{A}_i}\,|\dot{u}|\,\tau\right) & \text{for} \quad \mathcal{A}_i > 0 \quad \text{particles recede} \\ Z_0^{(i)} \cos\left(\sqrt{-\mathcal{A}_i}\,|\dot{u}|\,\tau\right) & \text{for} \quad \mathcal{A}_i < 0 \quad \text{particles approach} \\ Z_0^{(i)} & \text{for} \quad \mathcal{A}_i = 0 \quad \text{no influence} \end{cases}$$

three distinct possibilities in $D = 5$: tidal deformation of a unit sphere of test particles



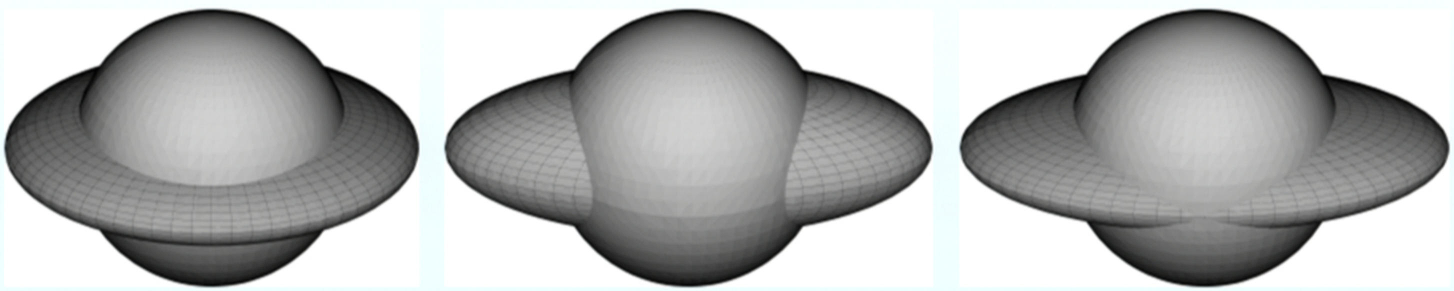
2 eigenvalues positive, 1 negative | 1 eigenvalue positive, 2 negative | 1 positive, 1 negative, 1 zero

gravitational waves in any dimension D

relative motion of test particles (initially at rest) can be explicitly integrated to

$$Z^{(i)}(\tau) = \begin{cases} Z_0^{(i)} \cosh\left(\sqrt{\mathcal{A}_i}\,|\dot{u}|\,\tau\right) & \text{for } \mathcal{A}_i > 0 \quad \text{particles recede} \\ Z_0^{(i)} \cos\left(\sqrt{-\mathcal{A}_i}\,|\dot{u}|\,\tau\right) & \text{for } \mathcal{A}_i < 0 \quad \text{particles approach} \\ Z_0^{(i)} & \text{for } \mathcal{A}_i = 0 \quad \text{no influence} \end{cases}$$

three distinct possibilities in $D = 5$: tidal deformation of a unit sphere of test particles



2 eigenvalues positive, 1 negative — 1 eigenvalue positive, 2 negative — 1 positive, 1 negative, 1 zero

new observable effects due to higher dimensions

suppose a gravitational wave propagating in the direction $e_{(1)}$ of a D-dim spacetime
in the transverse $(D-2)$-dim subspace we observe:

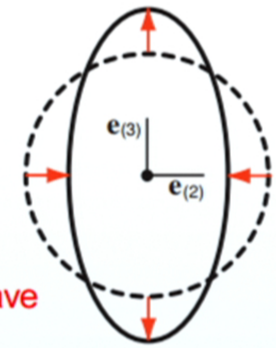
$\boxed{D=4}$ classical general relativity

eigenvalues of the matrix are $\mathcal{A}_2, \mathcal{A}_3$

$\boxed{\mathcal{A}_3 = -\mathcal{A}_2}$ traceless property

$$-\Psi_{4^{ij}} = \begin{pmatrix} \mathcal{A}_2 & 0 \\ 0 & -\mathcal{A}_2 \end{pmatrix}$$

simultaneously (non)trivial $\begin{cases} \mathcal{A}_2 = 0 & \text{NO wave} \\ \mathcal{A}_2 \neq 0 & \text{wave} \end{cases}$

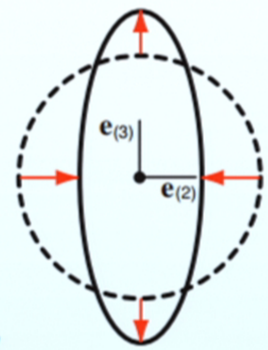


$\boxed{D=5}$ higher dimensional gravity

eigenvalues are $\mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4$

$\boxed{\mathcal{A}_4 = -(\mathcal{A}_2 + \mathcal{A}_3)}$

$$-\Psi_{4^{ij}} = \left(\begin{array}{cc|c} \mathcal{A}_2 & 0 & 0 \\ 0 & \mathcal{A}_3 & 0 \\ \hline 0 & 0 & -(\mathcal{A}_2+\mathcal{A}_3) \end{array}\right)$$



any $\mathcal{A}_2$ and $\mathcal{A}_3$:

↑ observable by detectors as a VIOLATION of the TT-property in our $(1+3)$-dim universe

↑ not directly observable by our detectors

field equations, of course, put specific restrictions

in our studies of twist-free shear-free spacetimes we considered

Einstein's field equations with a cosmological constant Λ:

$$R_{\alpha\beta} - \tfrac{1}{2} R\, g_{\alpha\beta} + \Lambda\, g_{\alpha\beta} = 8\pi\, T_{\alpha\beta}$$

for

- vacuum: $T_{\alpha\beta} = 0$
- aligned pure radiation: $T_{\alpha\beta} = \Phi^2 k_\alpha k_\beta$
- aligned Maxwell field: $T_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\mu} F_\beta{}^\mu - \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}\right)$

  $F_{[\alpha\beta;\gamma]} = 0$ and $F^{\mu\nu}{}_{;\nu} = 0$

  such that $F_{\alpha\beta} k^\beta = E\, k_\alpha$

(electro) vacuum Kundt spacetimes

the most general such metric in D dimensions is of type II and takes the form

$$ds^2 = g_{pq}\,dx^p\,dx^q + 2\,(e_p + f_p\,r)\,dx^p\,du - 2\,du\,dr + (a\,r^2 + b\,r + c)\,du^2$$

where g_{pq}, e_p, f_p, a, b, c are functions of x and u only,
which are constrained by the remaining 6 explicit Einstein equations

- $^{\rm s}R_{pq} = \frac{2\Lambda}{D-2}\,g_{pq} - \frac{B^2 - 2E^2}{D-2}\,g_{pq} + 2B_{pm}B_{qn}\,g^{mn} + \frac{1}{2}(f_p f_q + f_{p,q} + f_{q,p}) - {}^{\rm s}\Gamma^m_{pq}\,f_m$
- $\frac{1}{2}g^{pq}a_{||p||q} + \frac{3}{2}f^p a_{,p} + a(\varphi + \frac{1}{2}f^p f_p) - \frac{1}{2}g^{pq}g^{mn}f_{q,n}(f_{p,m} - f_{m,p})$
  $= -2\,g^{pq}(E_{,p} + E f_p)(E_{,q} + E f_q)$
- etc: for more details see JP, Žofka: CQG (2009)

the corresponding electromagnetic field is

$$\boldsymbol{F} = E\,dr \wedge du + (r\,E_{,p} + \xi_p)\,dx^p \wedge du + \tfrac{1}{2}B_{pq}\,dx^p \wedge dx^q$$

where E, ξ_p, B_{pq} are functions of x and u only,
which are constrained by the remaining two Maxwell equations
gravitational waves, possibly with gyratons see Krtouš, JP, Zelnikov, Kadlecová: PRD (2012)

(electro) vacuum Kundt spacetimes

the most general such metric in D dimensions is of type II and takes the form

$$ds^2 = g_{pq}\,dx^p\,dx^q + 2\,(e_p + f_p\,r)\,dx^p\,du - 2\,du\,dr + (a\,r^2 + b\,r + c)\,du^2$$

where g_{pq}, e_p, f_p, a, b, c are functions of x and u only,
which are constrained by the remaining 6 explicit Einstein equations

- $^{S}R_{pq} = \frac{2\Lambda}{D-2}\,g_{pq} - \frac{B^2 - 2E^2}{D-2}\,g_{pq} + 2B_{pm}B_{qn}\,g^{mn} + \frac{1}{2}(f_p f_q + f_{p,q} + f_{q,p}) - {^{S}\Gamma^m_{pq}}\,f_m$
- $\frac{1}{2}g^{pq}a_{||p||q} + \frac{3}{2}f^p a_{,p} + a(\varphi + \frac{1}{2}f^p f_p) - \frac{1}{2}g^{pq}g^{mn}f_{q,n}(f_{p,m} - f_{m,p})$
  $= -2\,g^{pq}(E_{,p} + E\,f_p)(E_{,q} + E\,f_q)$
- etc: for more details see JP, Žofka: CQG (2009)

the corresponding electromagnetic field is

$$\boldsymbol{F} = E\,dr \wedge du + (r\,E_{,p} + \xi_p)\,dx^p \wedge du + \frac{1}{2}B_{pq}\,dx^p \wedge dx^q$$

where E, ξ_p, B_{pq} are functions of x and u only,
which are constrained by the remaining two Maxwell equations
gravitational waves, possibly with gyratons see Krtouš, JP, Zelnikov, Kadlecová: PRD (2012)

geodesic deviation in vacuum Kundt spacetimes

canonical components of the gravitational field:

$$\ddot{Z}^{(1)} = \frac{2\,\Lambda}{(D-1)(D-2)}\,Z^{(1)} + \Psi^{\rm int}_{2S}\,Z^{(1)} + \frac{1}{\sqrt{2}}\,\Psi^{\rm int}_{3T^j}\,Z^{(j)}$$

longitudinal spatial direction

$$\ddot{Z}^{(i)} = \frac{2\,\Lambda}{(D-1)(D-2)}\,Z^{(i)} - \Psi^{\rm int}_{2T(ij)}\,Z^{(j)} - \frac{1}{\sqrt{2}}\,\Psi^{\rm int}_{3T^i}\,Z^{(1)} - \frac{1}{2}\,\Psi^{\rm int}_{4^{ij}}\,Z^{(j)}$$

transverse spatial directions $(i)=(2),(3),\ldots$

conformally flat	type D	type III	type N
isotropic motion	Newton-like	longitudinal	transverse

$$\Psi^{\rm int}_{2S} = \Psi_{2S} \qquad \Psi^{\rm int}_{2T(ij)} = \Psi_{2T(ij)}$$

$$\Psi^{\rm int}_{3T^i} = \sqrt{2}\,\dot{u}\,\Psi_{3T^i} + \sqrt{2}\,\dot{x}^p\,g_{pq}\left((\Psi_{2^{pq}} - \Psi_{2T^{ji}})\,m^{jq} - \Psi_{2S}\,m^q_i\right)$$

$$\begin{aligned}\Psi^{\rm int}_{4^{ij}} = {}& 2\,\dot{u}^2\,\Psi_{4^{ij}} + 4\,\dot{u}\,\dot{x}^p\,g_{pq}\left(\Psi_{3T^{(i}}m^q_{j)} - \Psi_{3^{(ij)k}}\,m^{kq}\right)\\ &+2\,\dot{x}^p\dot{x}^q\Big(g_{pq}\,\Psi_{2T(ij)} - g_{pm}\,g_{qn}\,\Psi_{2S}\,m^m_i m^n_j + g_{pm}\,g_{qn}\,\Psi_{2^{ikjl}}\,m^{km}m^{ln}\\ &\qquad -2\,g_{pm}\,g_{qn}\,(\Psi_{2^{k(i}} + \Psi_{2T^{k(i}})\,m^n_{j)}\,m^{km}\Big)\end{aligned}$$

- the scalars combine the specific curvature with kinematics

  Weyl scalars $\Psi_{A^{ij..}}$ $\quad \dot{x}^p, \dot{u}$ velocity components of the observer
- for geodesics $\dot{x}^p = 0$ we directly observe the invariants $\Psi_{A^{ij..}}$

fundamental difference:

- in $D = 4$: $$2H(r,x,u) = \Delta \log P - 2r(\log P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3}r^2$$

  where P is a complicated function of x and u

- in $D > 4$: $$2H(r) = K - \frac{\mu}{r^{D-3}} - \frac{2\Lambda}{(D-2)(D-1)}r^2$$

  where K is just a constant, $K = +1, 0, -1$
  spatial scalar curvature is $\mathcal{R} = K(D-2)(D-3)$

for more details see JP, Ortaggio: CQG (2006)

fundamental difference:

- in $D = 4$: $$2H(r,x,u) = \Delta \log P - 2r(\log P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3}r^2$$

  where P is a complicated function of x and u

- in $D > 4$: $$2H(r) = K - \frac{\mu}{r^{D-3}} - \frac{2\Lambda}{(D-2)(D-1)}r^2$$

  where K is just a constant, $K = +1, 0, -1$
  spatial scalar curvature is $\mathcal{R} = K(D-2)(D-3)$

for more details see JP, Ortaggio: CQG (2006)

vacuum Robinson–Trautman: all algebraic types

type	$D=4$	$D>4$	
II(a)	$\mu = 0$	$\mu = 0$	$\Leftrightarrow$ D(a)
II(b)	always	always	$\Leftrightarrow$ D(b)
II(c)	always	$\mathcal{C}_{mpnq} = 0$	$\Leftrightarrow$ D(c)
II(d)	always	always	$\Leftrightarrow$ D(d)
III	II(abcd)		
III(a)	$\mu = 0 = \mathcal{R}_{,p}$	equivalent to	O
III(b)	always for $\mu = 0$	equivalent to	O
N	III(ab)		
O	$\mu = 0 = \mathcal{R}_{,p}$ and $c_{\|\|p\|\|q} = \frac{1}{D-2} h_{pq} h^{mn} c_{\|\|m\|\|n}$	equivalent to	D(ac)
D	$\mathcal{R}_{,p} = 0$ and $c_{\|\|p\|\|q} = \frac{1}{D-2} h_{pq} h^{mn} c_{\|\|m\|\|n}$	always	D(bd)

$$\mu = -2m \qquad \mathcal{R} = 2\Delta \log P \qquad c = -2(\log P)_{,u}$$

in higher dimensions $D > 4$:

- only type D and O vacuum RT spacetimes
- no RT gyratons

JP, Ortaggio: CQG (2006)
JP, Švarc: CQG (2015)
JP, Švarc: PRD (2014)

RT spacetimes with p-form Maxwell field

Robinson–Trautman metric with a p-form field   see Ortaggio, JP, Žofka: JHEP (2015)

- for $2\,\mathrm{p} \neq \mathrm{D} > 4$: type D static back holes dressed with type D el & mag field:

$$2H = K - \frac{2\Lambda}{(D-2)(D-1)}\,r^2 - \frac{\mu}{r^{D-3}} + \frac{\kappa_0}{D-2}\left[\frac{2E^2}{(D-3)}\frac{1}{r^{2(D-\mathrm{p}-1)}} - \frac{B^2}{(D-1-2\mathrm{p})}\frac{1}{r^{2(\mathrm{p}-1)}}\right]$$

$$K = 0, \pm 1 \qquad \mu, E, B = \text{const.}$$

$$\boldsymbol{F} = \frac{1}{(\mathrm{p}-2)!}\frac{1}{r^{D+2-2\mathrm{p}}}\, e_{q_1\ldots q_{\mathrm{p}-2}}(x)\,\mathrm{d}u \wedge \mathrm{d}r \wedge \mathrm{d}x^{q_1} \wedge \ldots \wedge \mathrm{d}x^{q_{\mathrm{p}-2}} + \tfrac{1}{\mathrm{p}!}\, b_{q_1\ldots q_{\mathrm{p}}}(x)\,\mathrm{d}x^{q_1} \wedge \ldots \wedge \mathrm{d}x^{q_{\mathrm{p}}}$$

- for $2\,\mathrm{p} = D$ even: non-static type II black holes with elmag radiation of type II or N

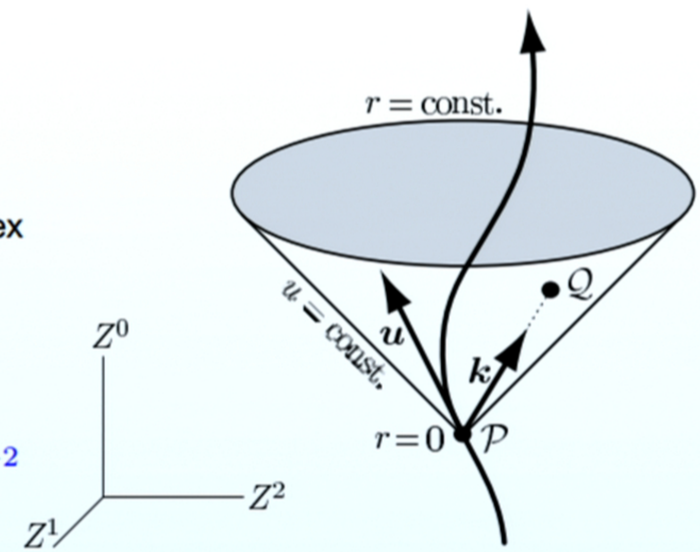
additional term:

$$\left(-\frac{D-2}{r}\, e_{[q_2\ldots q_{\mathrm{p}-1},q_1]} + 2 f_{q_1\ldots q_{\mathrm{p}-1}}(u,x)\right)\mathrm{d}u \wedge \mathrm{d}x^{q_1} \wedge \ldots \wedge \mathrm{d}x^{q_{\mathrm{p}-1}}$$

RT coordinates are adapted to an accelerated source

$$ds^2 = \frac{r^2}{P^2}\,\delta_{pq}\,\mathrm{d}x^p\,\mathrm{d}x^q - 2\,\mathrm{d}u\,\mathrm{d}r - \left[1 - 2\,r(\ln P)_{,u} - \frac{2\Lambda}{(D-2)(D-1)}\,r^2 - \frac{2m(u)}{r^{D-3}}\right]\mathrm{d}u^2$$

- prescribe a general trajectory $z^\alpha(u)$ where u is the proper time of the source
- at any point $\mathcal{P}$ along the trajectory construct the future null cone $u = \mathrm{const.}$
- for such a foliation define $r = 0$ at each vertex (location of the source)
- introduce r as an affine parameter along the null geodesic generated by $\boldsymbol{k}$ which connects $\mathcal{P}$ with any event $\mathcal{Q}$
- choose the spatial coordinates $x^1, \ldots, x^{D-2}$ to identify the points on the subspaces $u = \mathrm{const.}$, $r = \mathrm{const.}$ (typically S^{D-2})



general motion in D-dim background flat space

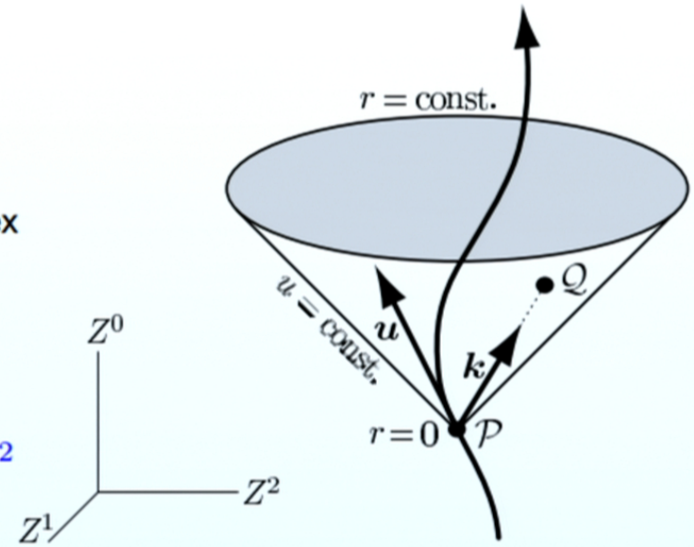
then $P = (\dot z^0 - \dot z^{D-1}) - (\delta_{pq}\,\dot z^q)\,x^p + \frac{1}{4}(\dot z^0 + \dot z^{D-1})\,\delta_{pq}\,x^p x^q$

where the velocity of the source is $\boldsymbol{u} \equiv [\,\dot z^0(u), \dot z^1(u), \ldots, \dot z^{D-1}(u)\,]$

RT coordinates are adapted to an accelerated source

$$ds^2 = \frac{r^2}{P^2}\,\delta_{pq}\,dx^p\,dx^q - 2\,du\,dr - \left[1 - 2\,r(\ln P)_{,u} - \frac{2\Lambda}{(D-2)(D-1)}\,r^2 - \frac{2m(u)}{r^{D-3}}\right] du^2$$

- prescribe a general trajectory $z^\alpha(u)$ where u is the proper time of the source
- at any point $\mathcal{P}$ along the trajectory construct the future null cone $u = \text{const.}$
- for such a foliation define $r = 0$ at each vertex (location of the source)
- introduce r as an affine parameter along the null geodesic generated by $\boldsymbol{k}$ which connects $\mathcal{P}$ with any event $\mathcal{Q}$
- choose the spatial coordinates $x^1, \ldots, x^{D-2}$ to identify the points on the subspaces $u = \text{const.}$, $r = \text{const.}$ (typically S^{D-2})



general motion in D-dim background flat space

then $P = (\dot z^0 - \dot z^{D-1}) - (\delta_{pq}\,\dot z^q)\,x^p + \frac{1}{4}(\dot z^0 + \dot z^{D-1})\,\delta_{pq}\,x^p x^q$

where the velocity of the source is $\boldsymbol{u} \equiv [\,\dot z^0(u), \dot z^1(u), \ldots, \dot z^{D-1}(u)\,]$

photon rocket accelerating in a single spatial direction

$$\mathrm{d}s^2 = -\left(1-\frac{2m(u)}{r^{D-3}}-\frac{2\Lambda}{(D-2)(D-1)}\,r^2-2\,\alpha(u)\,r\,\cos\vartheta-\alpha^2(u)\,r^2\sin^2\vartheta\right)\mathrm{d}u^2$$
$$-2\,\mathrm{d}u\,\mathrm{d}r+2\,\alpha(u)\,r^2\sin\vartheta\,\mathrm{d}u\,\mathrm{d}\vartheta+r^2\left(\mathrm{d}\vartheta^2+\sin^2\vartheta\left(\mathrm{d}\,\theta_2^2+\sum_{i=3}^{D-2}\prod_{j=2}^{i-1}\sin^2\theta_j\,\mathrm{d}\theta_i^2\right)\right)$$

describes straight flight with the acceleration $\alpha(u)$, in suitable parametrization

- the corresponding radiation pattern is

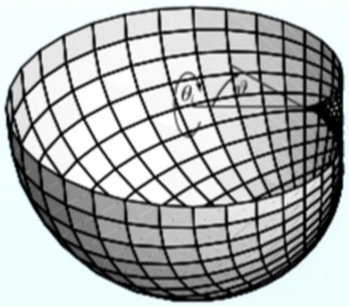
  $n^2(u,\vartheta) = \frac{D-2}{4\pi}\big(-m_{,u}\big)\cos^2\dfrac{\vartheta}{2}$

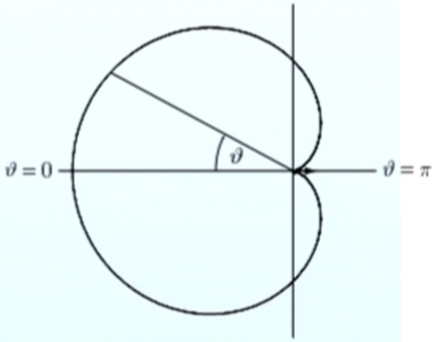
- mass of the rocket decreases exponentially

  $m(u) = m_0\,\exp\left[-(D-1)\int\alpha(u)\,\mathrm{d}u\right]$

- dependence on the final speed v is

  $\dfrac{m(v)}{m_0} = \left(\dfrac{1-v}{1+v}\right)^{(D-1)/2}$ $\alpha = \text{const.}$





radiation emitted along a circular trajectory

- the radiation pattern is

$$n^2(u,\theta_j,\phi) = \tfrac{D-2}{8\pi}\left(-m_{,u}\right)\left[1 + \frac{\left(\prod_{j=1}^{D-3}\sin\theta_j\right)\cos(\phi-\omega\,u)}{\sqrt{1+a^2\omega^2} - a\omega\left(\prod_{j=1}^{D-3}\sin\theta_j\right)\sin(\phi-\omega\,u)}\right]$$

- the mass decreases exponentially

$$m(u) = m_0\,\exp\left(-(D-1)\,a\omega^2\,u\right)$$

- mass of the rocket after the U-turn is

$$\frac{m(v)}{m_0} = \exp\left(-\frac{(D-1)\,\pi\,v}{\sqrt{1-v^2}}\right)$$

less efficient: