Title: Aspects of 6d (1,0) theories and their toroidal reduction to 4d - Michele Del Zotto

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Abstract: <p>I am going to discuss applications of String/M/F theory dualities to argue about the toroidal compactification to four dimensions of 6d (1,0) theories.</p>

NAHM: $\exists$ SCA $D \leq 6$

$\exists$ SCFT? $D \leq 4$ ok

$D = 5, 6$ ~ more complicated

QFT: Seiberg ... $\underline{D=6}$ $(1,0)$

$\exists \equiv$ prediction of ST. $(\phi, \ldots), (A_\mu, \ldots), (\xi, B^-_{\mu\nu}, \ldots)$

<u>logic</u>: start from phase w/ broken conformal $\longrightarrow$ argue for $\exists$ of conformal point

<u>examples</u>: M M5 brane

<u>6d (2,0)</u> <u>A_{M-1}</u>

$(1,0)$: M M5s probing $\mathbb{C}^2/\Gamma$ in M theory

: M M5s – M9

heterotic E_8 instantons.

$(\xi, B^-_{\mu\nu}, \dots)$

examples: M M5 branes

6d $(2,0)$ A_{M-1}

IIB $\mathbb{C}^2/\Gamma$

$\Gamma_G \subset SU(2)$

ADE

6d $(2,0)$

G

(1,0) : M M5s probing $\mathbb{C}^2/\Gamma$ in M theory

: M M5s – M9

heterotic E_8 instantons.

$\exists$ has deep implications

$\exists$ (2,0) ⇝ Montonen-Olive duality

/ T^2 ⇝ $N=4$ SYM

$SL(2,\mathbb{Z})$

/ Σ ⇝ 4D $N=2$ class S

5 brane

(2,0) A_{M-1}

IIB $\mathbb{C}^2/\Gamma$

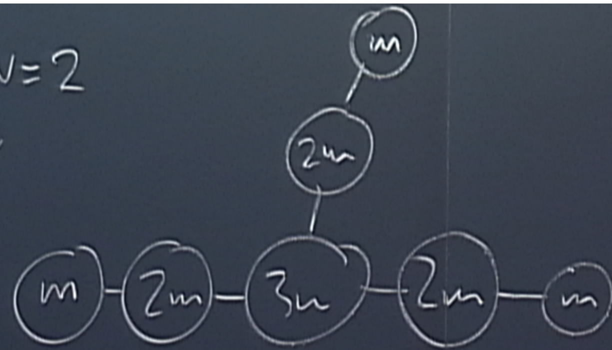
$\Gamma_G \subset SU(2)$

ADE

6d (2,0)

G

in between 4D $N=2$
known systems
$SL(2,\mathbb{Z})$
not in class S



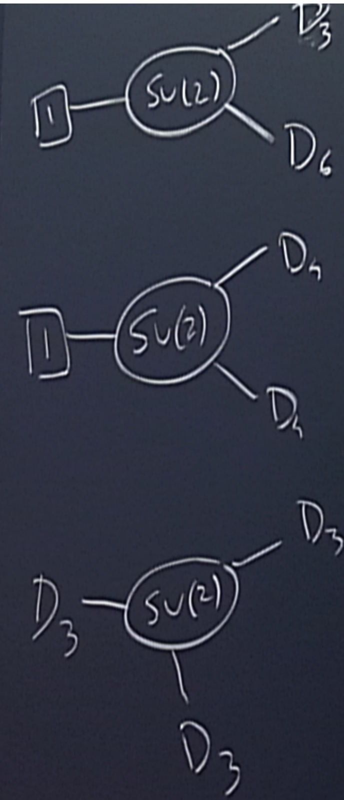
- $\hat{E}_{6,7,8}$ quiver theories

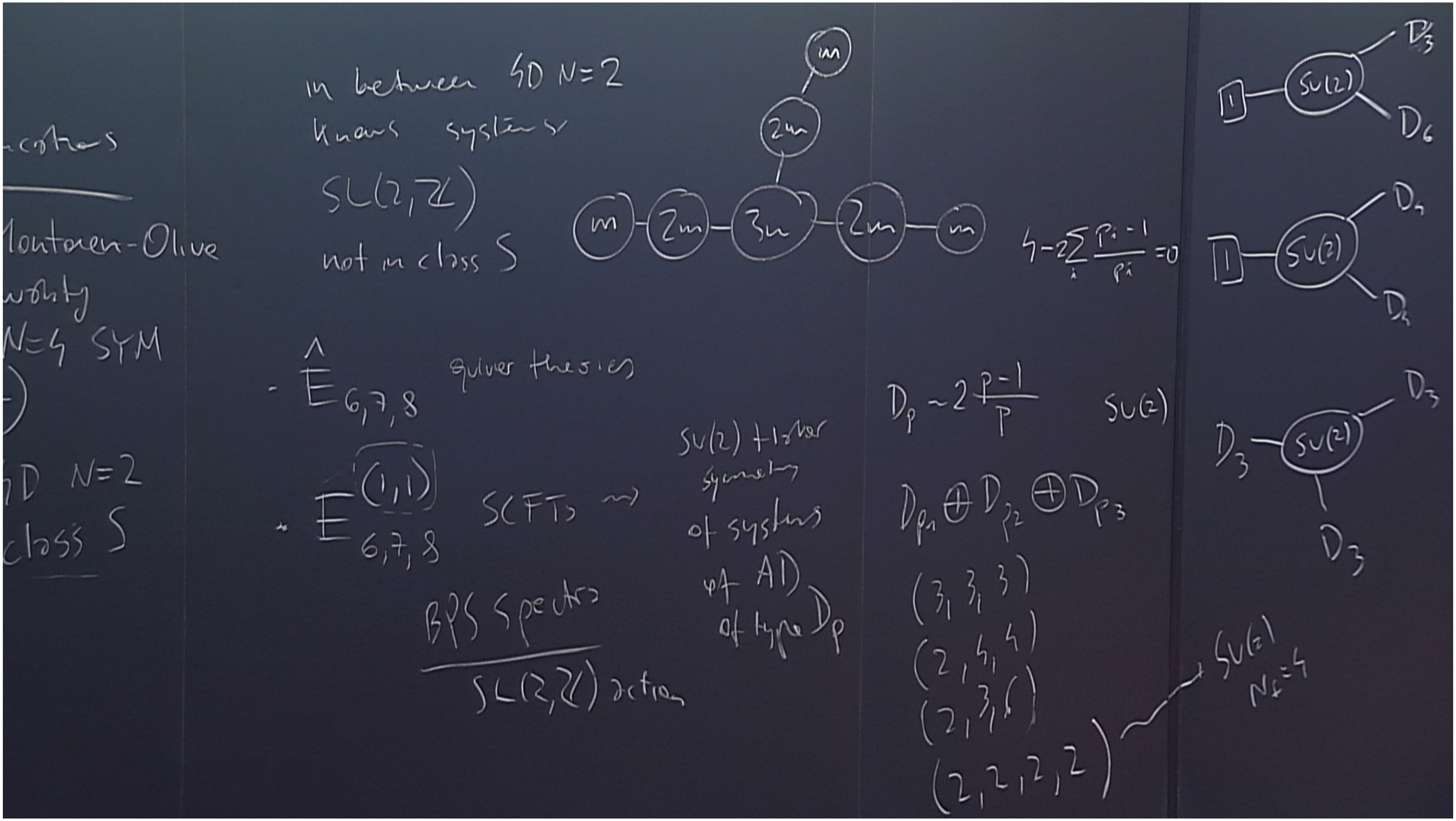
- $E^{(1,1)}_{6,7,8}$ SCFTs $\rightsquigarrow$ SU(2) + flavor symmetry of systems of AD of type D_p

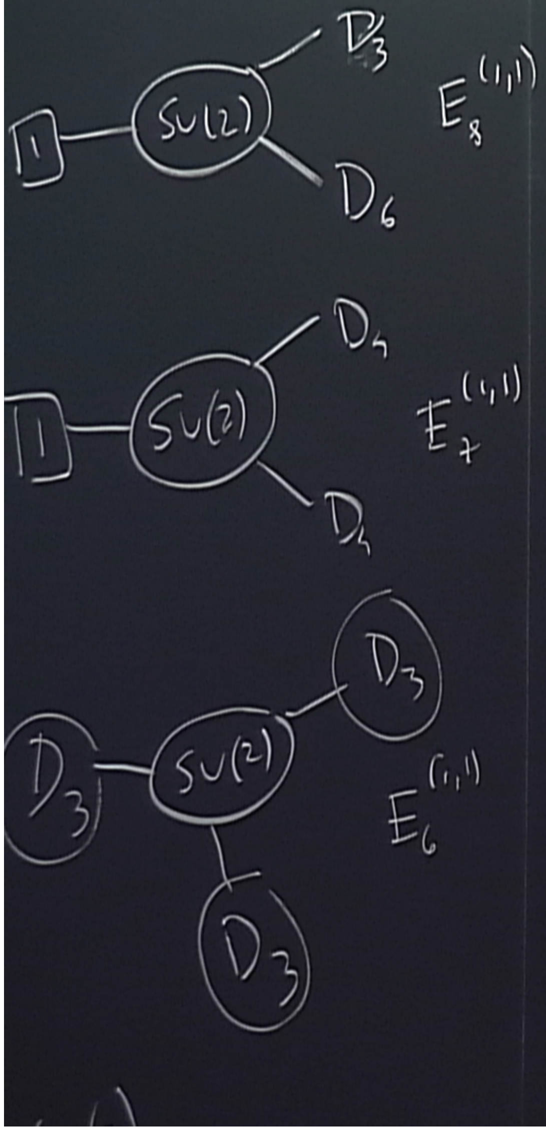
$D_p \sim 2\frac{p-1}{p}$ SU(2)

$D_{p_1} \oplus D_{p_2} \oplus D_{p_3}$

$(3,3,3)$
$(2,4,4)$
$(2,3,6)$
$(2,2,2,2)$




in between 4D N=2
known systems
SL(2,Z)
not in class S
Montonen-Olive
N=4 SYM
4D N=2
class S
quiver theories
Ê 6,7,8
E 6,7,8
SCFTs
SU(2) + flavor symmetry
of systems of AD of type Dp
BPS spectra
SL(2,Z) action
m
2m
3m
Dp ~ 2 (p-1)/p
SU(2)
Dp1 ⊕ Dp2 ⊕ Dp3
(3, 3, 3)
(2, 4, 4)
(2, 3, 6)
(2, 2, 2, 2)
SU(2)
Nf=4
D3
D6
D4
SU(2)
1



$\exists$ (1,0).

- generalization

- rich family of 4D $N=2$
  $\longleftrightarrow$ 6d (1,0) / T^2
  (Ganor, Morrison, Seiberg mechanism)
  1 Het E_8 instanton
  $E_{8,7,6}$, D_4, $A_{3,2,1}$

- proof conjecture OSTT

Strategy: exploit string dualities

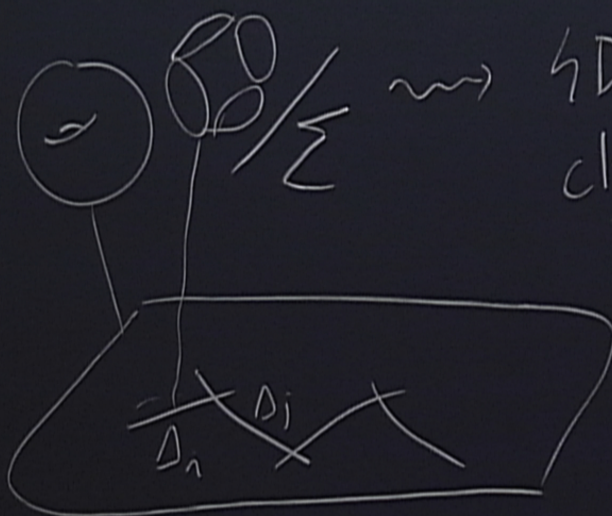
heterotic
E_8 instantons.

$\exists\,(2,0)$ $\rightsquigarrow$ Montonen-Olive duality
$/T^2$ $\rightsquigarrow$ $N=4$ SYM
$SL(2,\mathbb{Z})$

not in c

$\hat{E}_{6,7,8}$

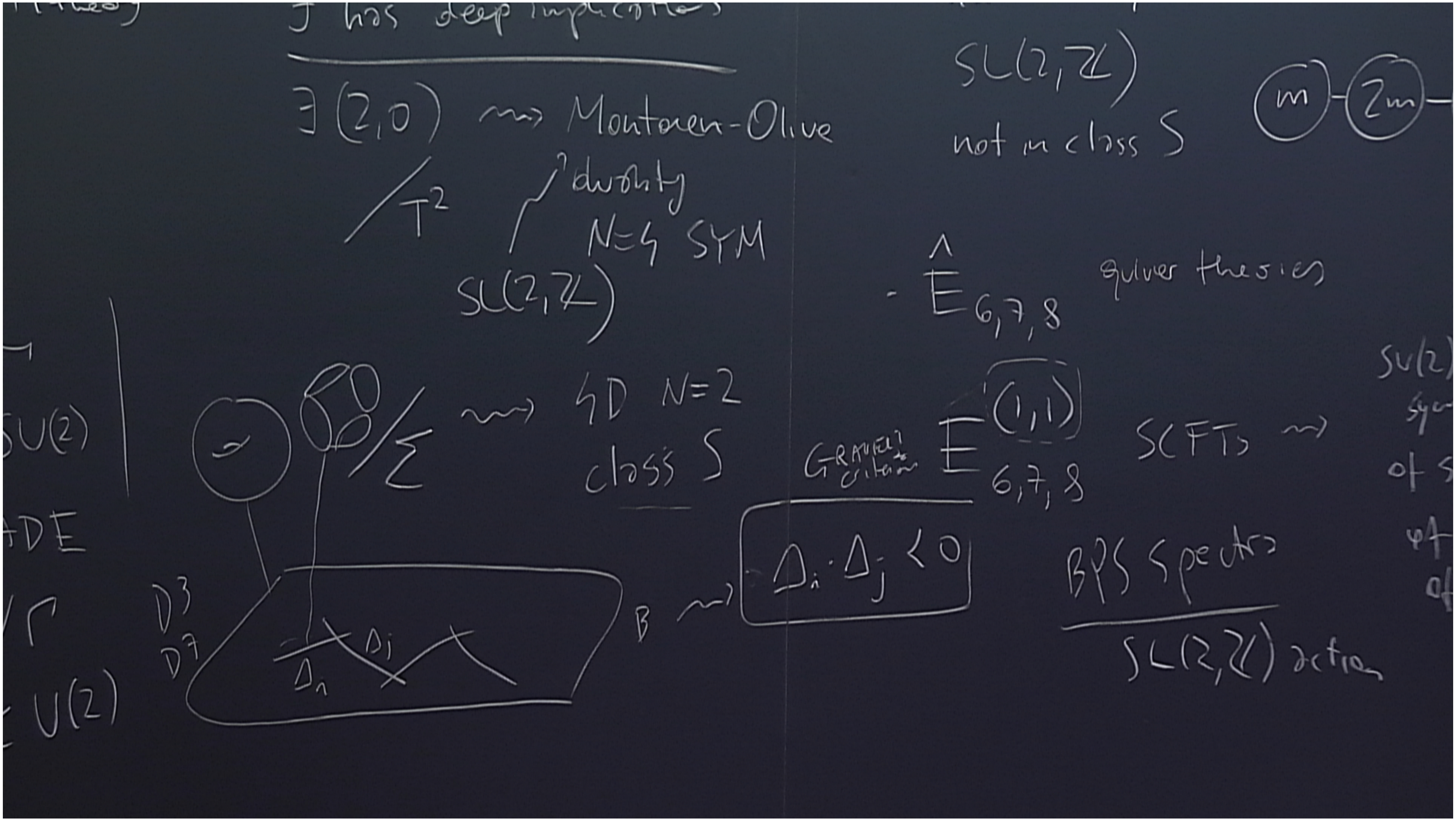
IIB	$\mathbb{C}^2/\Gamma$
6d (2,0)	$\Gamma_G \subset SU(2)$
G	ADE
6d (1,0)	$\mathbb{C}^2/\Gamma$, $\Gamma \subset U(2)$



$/\Sigma$ $\rightsquigarrow$ 4D $N=2$
class S

GRAUERT criterion $E_{6,7,8}$ $(1,1)$

$B \rightsquigarrow \boxed{\Delta_i \cdot \Delta_j < 0}$


has deep implications
∃(2,0) ⇝ Montonen-Olive duality
/T²
N=4 SYM
SL(2,Z)
SU(2)
ADE
U(2)
/Σ ⇝ 4D N=2 class S
D3
D7
Δi
Δ1
B ⇝ Δi · Δj < 0
SL(2,Z)
not in class S
m
2m
Ê6,7,8
quiver theories
(1,1)
E6,7,8
SCFTs ⇝
BPS Spectra / SL(2,Z) action
SU(2)

4D $\mathcal{N}=2$

$D_p \rightsquigarrow D_p(G)$ SCFTs

G flavor symm.

$b = \frac{p-1}{p}\, h$

$D_p(SU(mp))$

is lagrangian

$(m) - (2m) - \cdots - ((p-1)m) - \boxed{pm}$

$p = (3,3,3) \sim SU(3m)$

$(1,0)$: M

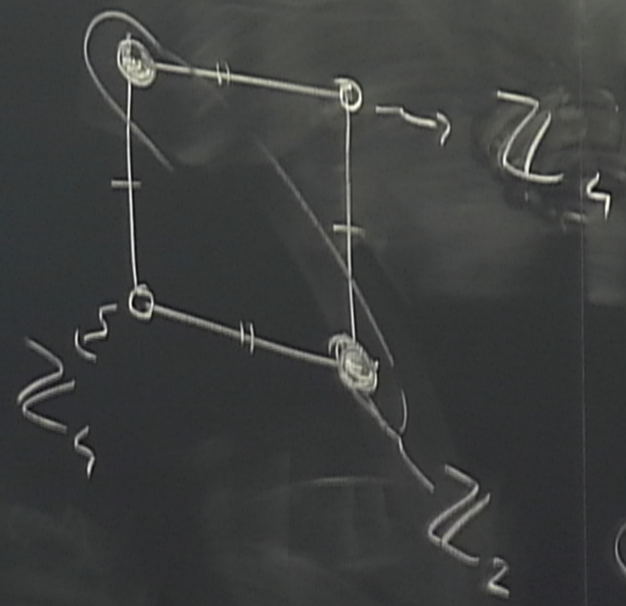
: M

examples: M M5 brane

6d $(2,0)$ A_{M-1}

orbifolded tori $T^2/\mathbb{Z}_{2,3,4,6}$

any $\quad \tau = e^{2\pi i/3} \quad \tau = i \quad \tau = e^{2\pi i/3}$



$\mathbb{C}^2/\Gamma$

4D $\mathcal{N}=2$

$D_p \rightsquigarrow D_p(G)$ SCFTs

G flavor symm.

$b = \frac{p-1}{p} h(G)$

$\underline{D_p(SU(mp))}$

is lagrangian

$(m) - (2m) - \cdots - ((p-1)m) - \boxed{pm}$

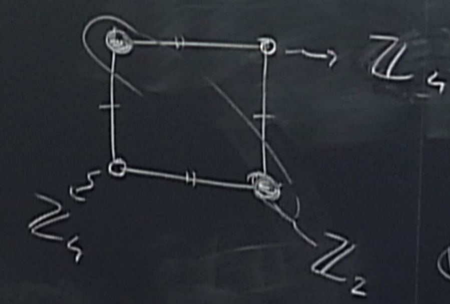
$p = (3,3,3) \rightsquigarrow SU(3m)$

$(2,4,4) \rightsquigarrow E_7$

$(2,3,6) \rightsquigarrow E_8$

orbifolded tori $T^2/\mathbb{Z}_{2,3,4,6}$

any $\tau = e^{2\pi i/3}$ $\tau = i$ $\tau = e^{2\pi i/6}$



$\mathbb{C}^2$

Γ_G

$\mathbb{C}^2$

Γ

s

os symm.

orbifolded tori $T^2/\mathbb{Z}_{2,3,4,6}$

any, $\tau = e^{2\pi i/3}$, $\tau = i$, $\tau = e^{2\pi i/3}$

minimal 6d (1,0) models

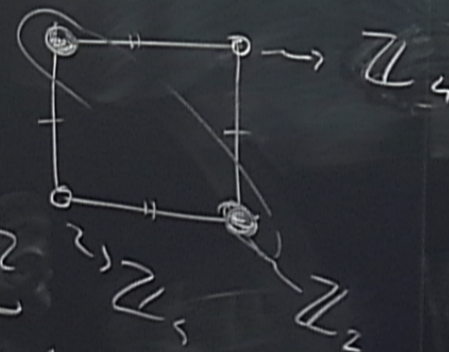
$O(-n) \to \mathbb{P}^1$

$n = 1, 2, 3, 4, 5, 6, 7, 8, 12$

1 het E_8

1 (2,0)

non-Higgsable



$(p-1)m$ — pm

$-n$

$\sim SU(3m)$

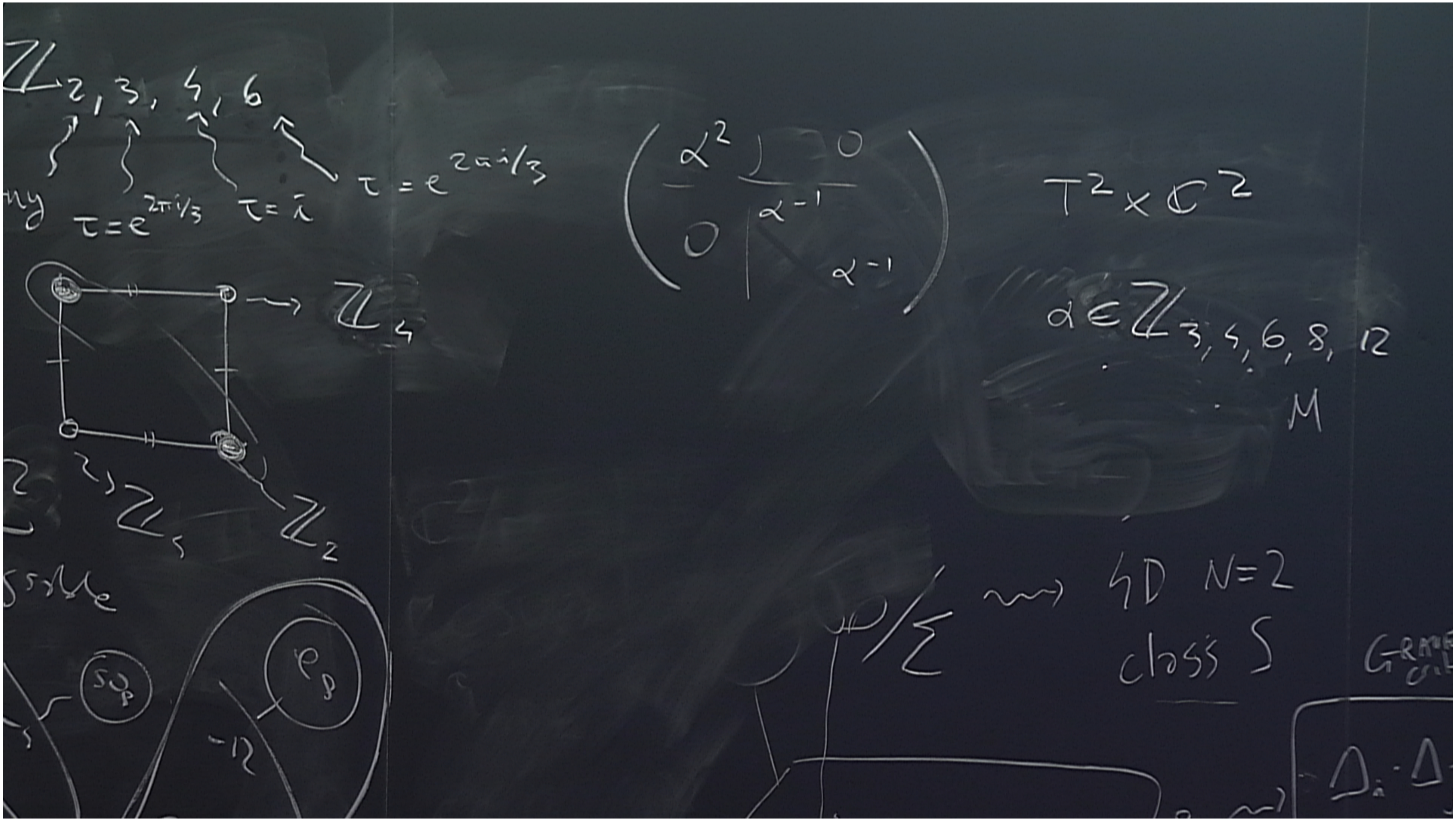
-3 su_3

-4 so_8

-12 e_8

-6 e_6

-8 e_7


ℤ2, 3, 4, 6
τ = e^{2πi/3}
τ = i
τ = e^{2πi/3}
ℤ4
ℤ2
T² × ℂ²
α ∈ ℤ3, 4, 6, 8, 12
M
4D N=2
class S
-12

$$\begin{pmatrix} \alpha^2 & 0 & \\ 0 & \alpha^{-1} & \\ & & \alpha^{-1} \end{pmatrix} \qquad T^2 \times \mathbb{C}^2$$

$$\alpha \in \mathbb{Z}_{3,4,6,8,12}$$

M

$$F / X \times S^1_R \rightsquigarrow M/X$$

$$M/X \times S^1_{R'} \rightsquigarrow \underline{IIA/X}$$

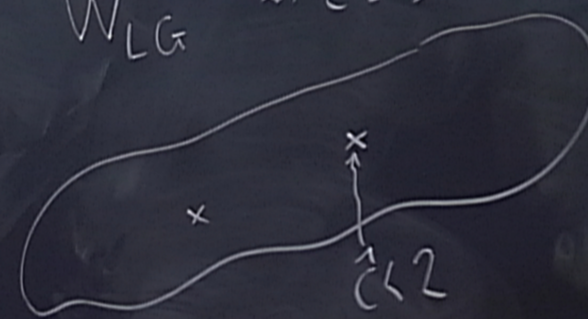
5D $N=2$

M.I.R. $\longleftrightarrow$ $\boxed{IIB/\check{X}}$

Seiberg Witten

$\rightsquigarrow$ Landau Ginzburg $(2,2)$ 2d

$W_{LG} \rightsquigarrow \hat{c} = 3$



$\hat{c} < 2$

$$x_1^3 + x_2^3 + x_3^3 + \alpha x_1 x_2 x_3 + y_1^6 + y_2^6 + t_i y_1 y_2 x_i^2 + \tilde{t}_j (y_1 y_2)^2 x_i$$
$$+ \tilde{\tilde{t}} (y_1 y_2)^3 .$$

$\times \mathbb{C}^2$

$\mathbb{Z}_{3,4,6,8,12}$

M

$$4 - 2 \sum_i \frac{p_i - 1}{p_i} = 0$$

Landau Ginzburg (2,2) 2d

$W_{LG} \rightsquigarrow \hat{c} = 3$

4D N=2

MIR

IIB / $\check{X}$

Seiberg witten

$\hat{c} < 2$

w/ scaling symmetry

$U(1)_R$

$$D_p \sim 2 \frac{p-1}{p} \qquad SU(2)$$

$$D_{p_1} \oplus D_{p_2} \oplus D_{p_3}$$

$(3, 3, 3)$

$(2, 4, 4)$

$(2, 3, 6)$

z_i

$$x_i \in \mathbb{C} \qquad \rightsquigarrow y_i \in \mathbb{C}^*$$

$$W_{LG} = \underbrace{x_1^3 + x_2^3 + x_3^3 + \alpha x_1 x_2 x_3}_{T^2/\mathbb{Z}_3} + \underbrace{y_1^6 + y_2^6}_{\mathbb{C}^2} + \underbrace{t_i(y_1 y_2) x_i^2 + \tilde{t}_j (y_1 y_2)^2 x_j + \tilde{\tilde{t}} (y_1 y_2)^3}.$$

$$+ (y_1^3 + y_2^3)^2$$

$$4 - 2\sum_i \frac{p_i - 1}{p_i} = 0$$

6, 8, 12

M

$y_2 = 1$

Landau Ginzburg (2,2) 2d

$D_p \sim 2\frac{p-1}{p}$ $SU(2)$

$\rightsquigarrow W_{LG} \rightsquigarrow \hat{c} = 3$

$D_{p_1} \oplus D_{p_2} \oplus D_{p_3}$

D_3

$x_i \in \mathbb{C}$ $\rightsquigarrow$ $y_i \in \mathbb{C}^*$

$$W_{LG} = \underbrace{x_1^3 + x_2^3 + x_3^3 + \alpha\, x_1 x_2 x_3}_{T^2/\mathbb{Z}_3} + \underbrace{y_1^6 + y_2^6}_{\mathbb{C}^2} + t(y_1 y_2) x_i^2 + \tilde{t}_j (y_1, y_2)^2 x_i + \tilde{\tilde{t}}(y_1, y_2)^3 .$$

$\hat{c} = 1$

$$+ \left(y_1^3 + y_2^3\right)^2$$

$$4 - 2\sum_i \frac{p_i - 1}{p_i} = 0$$

12

$y_2 = 1$

$$+ \underbrace{\left(y_1^3 + 1\right)^2}_{W^2}$$

$D_p \sim 2\frac{p-1}{p}$ $SU(2)$

$D_{p_1} \oplus D_{p_2} \oplus D_{p_3}$

$(3, 3, 3)$

Landau Ginzburg $(2,2)$ 2d

$\rightsquigarrow$ W_{LG} $\rightsquigarrow \hat{c} = 3$

(IIB / X)

$\boxed{1}$ — $(SU$

$\boxed{1}$ — $(SU$

(D_3) — $(SU$

$\boxed{1}$ — SU(2) — D_3, D_6 $\quad E_8^{(1,1)}$

$\boxed{1}$ — SU(2) — D_4, D_4 $\quad E_7^{(1,1)}$

D_3 — SU(2) — D_3, D_3 $\quad E_6^{(1,1)}$

SU(2)

SU(2) $N_f = 4$

M MS, $\mathbb{C}^2/\Gamma_G$

6d (1,0) $\rightsquigarrow$ $G \times G$ flavor symmetry

$\mathbb{Z}_M$

G type elliptic singularities

$g = (a, a^{-1}, 1) \qquad a \in \mathbb{Z}_{2,3,5,6 \equiv k}$

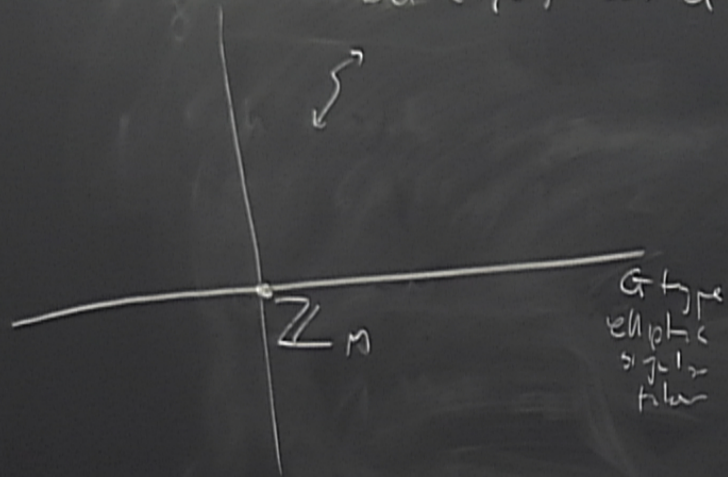
SO 8 $\rightsquigarrow E_6 \rightsquigarrow E_7 \rightsquigarrow E_8$

$h = (1, b, b^{-1}) \qquad b \in \mathbb{Z}_{Mk}$

$g h^N \sim (a, 1, a^{-1}) \qquad \mathbb{Z}_k \qquad z_i \rightsquigarrow z_i^k$

M MS , $\mathbb{C}^2/\Gamma_G$

6d (1,0) $\rightsquigarrow$ $G \times G$ flavor symmetry



SO 8

$\rightsquigarrow E_6 \rightsquigarrow E_7 \rightsquigarrow E_8$

$g = (a, a^{-1}, 1) \qquad a \in \mathbb{Z}_{2,3,4,6} \equiv k$

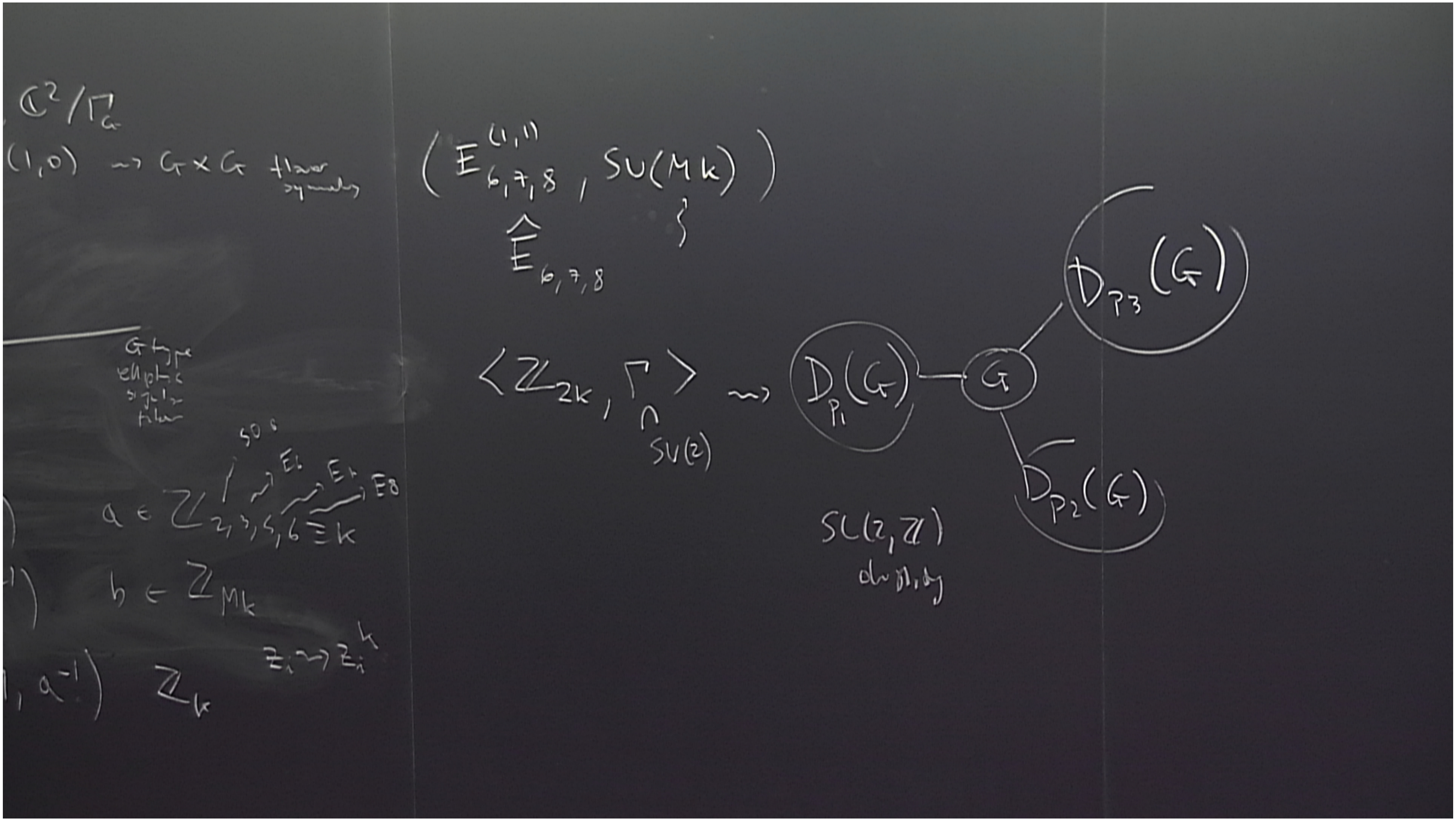
$h = (1, b, b^{-1}) \qquad b \in \mathbb{Z}_{Mk}$

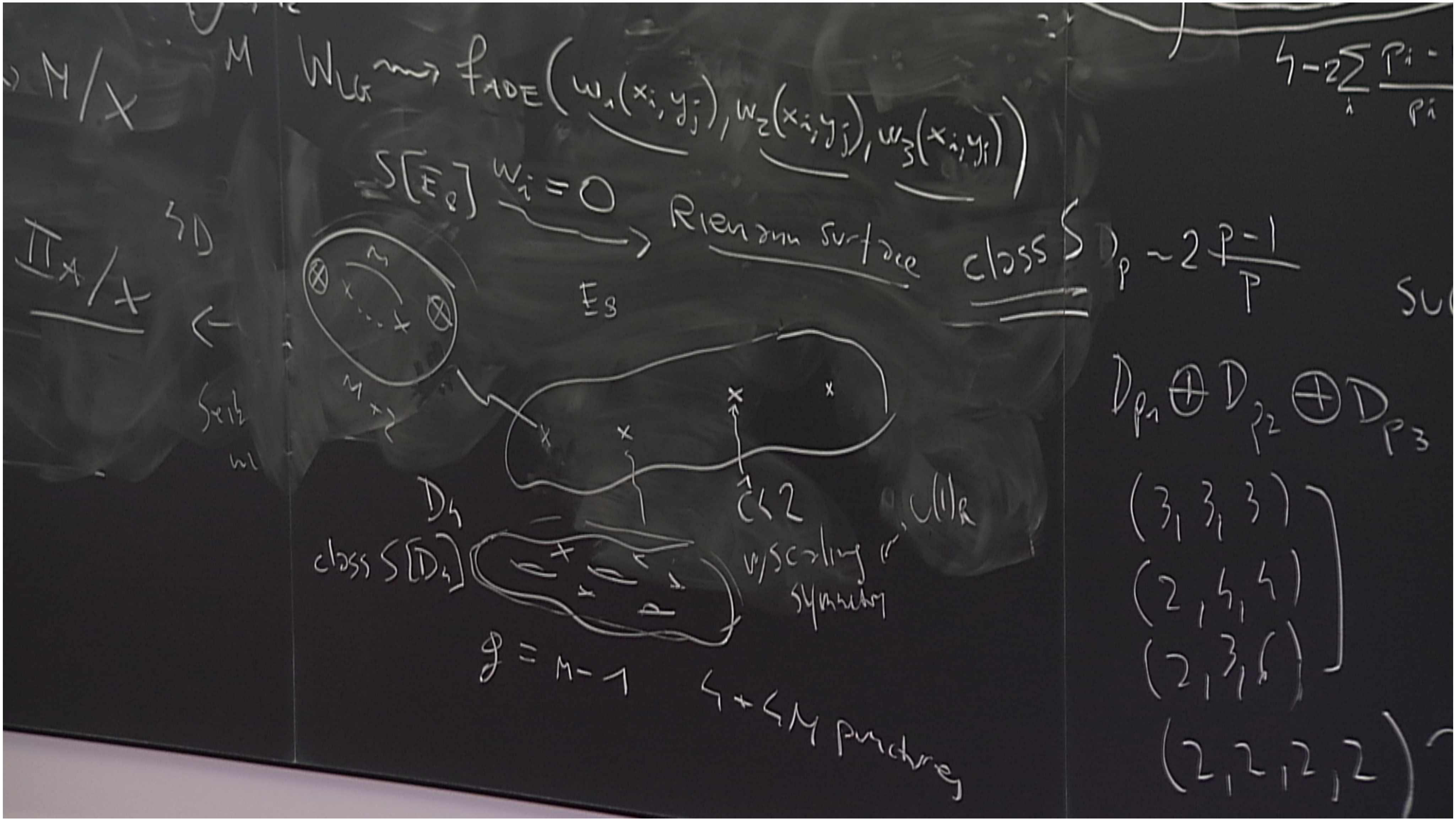
$gh^n \sim (a, 1, a^{-1}) \qquad \mathbb{Z}_k \qquad z_i \rightsquigarrow z_i^k$

$$\left(E^{(1,1)}_{6,7,8} \, , \, SU(Mk) \right)$$

$\hat{E}_{6,7,8}$

$$\langle \mathbb{Z}_{2k}, \Gamma \rangle$$


$\mathbb{C}^2/\Gamma_G$
$(1,0) \rightsquigarrow G \times G$ flavor symmetry
$\left(E^{(1,1)}_{6,7,8} \,, SU(Mk) \right)$
$\hat{E}_{6,7,8}$
G type elliptic singular fiber
$\langle \mathbb{Z}_{2k}, \Gamma \rangle \rightsquigarrow$
$\cap$
$SU(2)$
$D_{p_1}(G)$
G
$D_{p_2}(G)$
$D_{p_3}(G)$
$a \in \mathbb{Z}_{2,3,4,6} \equiv k$
E_6
E_7
E_8
$b \in \mathbb{Z}_{Mk}$
$z_i \rightsquigarrow z_i^k$
$\mathbb{Z}_k$
$SL(2,\mathbb{Z})$
duality


M/X
$W_{LG} \rightsquigarrow f_{ADE}(w_1(x_i,y_j), w_2(x_i,y_j), w_3(x_i,y_i))$
$S[E_8]$ $W_i = 0$
Riemann surface
class S
E_8
D_4
class $S[D_4]$
$g = n-1$
$4+4n$ punctures
$U(1)_R$
w/ scaling symmetry
$D_p \sim 2\frac{p-1}{p}$
$D_{p_1} \oplus D_{p_2} \oplus D_{p_3}$
(3, 3, 3)
(2, 4, 4)
(2, 3, 6)
(2, 2, 2, 2)

M5 , $\mathbb{C}^2/\Gamma_G$

6d (1,0) $\rightsquigarrow$ $G \times G$ flavor symmetry

$\mathbb{Z}_M$

G type elliptic singular fiber

$(\ ,a^{-1},1)$ $\quad a \in \mathbb{Z}_{2,3,5,6} \equiv k$

SO8, E6, E7, E8

$(\ ,b,b^{-1})$ $\quad b \in \mathbb{Z}_{Mk}$

$z_i \rightsquigarrow z_i^k$

$(a;1,a^{-1})$ $\quad \mathbb{Z}_k$

$\left(E^{(1,1)}_{6,7,8},\ SU(Mk)\right)$

$\hat{E}_{6,7,8}$

$\langle \mathbb{Z}_{2k}, \Gamma \rangle \rightsquigarrow$ $D_{p_1}(G)$ — G — $D_{p_3}(G)$, $D_{p_2}(G)$

$\Gamma \subset SU(2)$

not orbifold
M M5s $\mathbb{C}^2/\mathbb{Z}_N$

$SL(2,\mathbb{Z})$ duality