Title: Timeless configuration space and the emergence of classical behavior for closed systems

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Abstract: <p>In this talk, I will explore a timeless interpretation of quantum mechanics of closed systems, solely in terms of path integrals in non-relativistic timeless configuration space. What prompts a fresh look at the foundational problems in this context, is the advent of multiple gravitational models in which Lorentz symmetry is only emergent. In this setting, I propose a new understanding of records as certain relations between two configurations, the recorded one and the record-holding one. These relations are formalized through a factorization of the amplitude kernel, which forbids unwanted 'recoherence' of branches. On this basis, I show that in simple cases the Born rule is consistent with counting the relative density of observers with the same records. Furthermore, unlike what occurs in consistent histories, in this context there is indeed a preferred notion of coarse-grainings: those centered around piece-wise classical paths in configuration space (with a certain radius). Thus, this new understanding claims to resolve aspects of the measurement problem which are still deemed controversial in the standard approaches (but which probably leaves others open...).</p>

Timeless quantum mechanics in configuration space: an outsider view

Henrique Gomes

Perimeter Institute for Theoretical Physics

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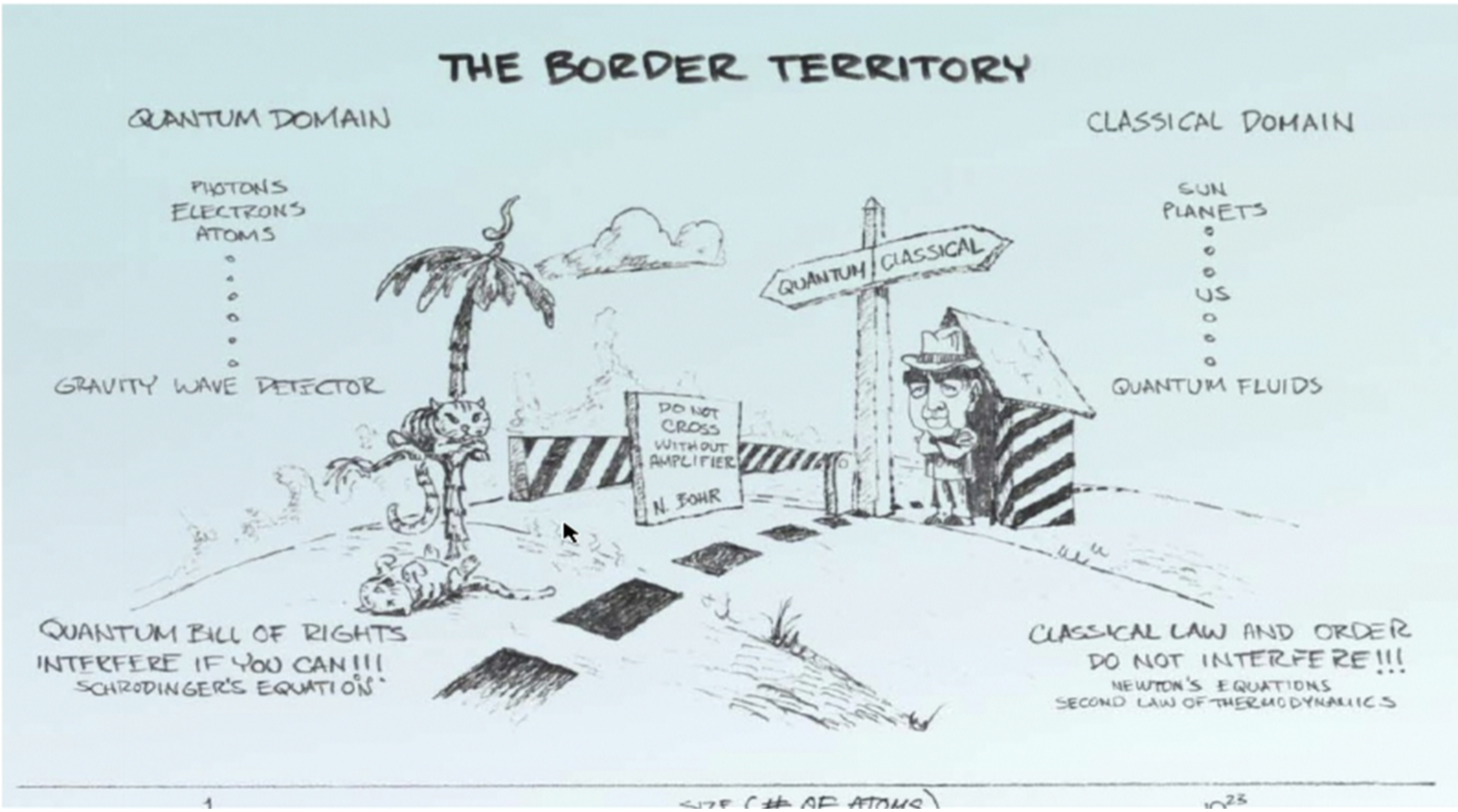
The Bohrder territory



The classical/quantum divide

$$\hat{H}|\psi\rangle = i\hbar \frac{d}{dt}|\psi\rangle \quad (1)$$

Universe described by $|\psi\rangle$ evolves into state containing many alternatives (not seen to coexist in our experience).

Root of our unease with quantum theory: clash between the principle of superposition – reflected in the linearity of equation (1) – and everyday classical reality.

This is one aspect of the *measurement problem*.

For Bohr, classical apparatus measuring the quantum system was a *necessity*, and quantum theory was not universal.

Outline

- Brief intro to space-time consistent histories and some of its shortcomings.
- Timeless (quantum) 'dynamics' and Jacobi metrics.
- The meat: quantum mechanics and timeless (non-relativistic) configuration space.

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- The meat: quantum mechanics and timeless (non-relativistic) configuration space.

  Main features:
  - No superpositions: only interference between coarse-grainings.
  - Dynamical notion of locality (even in Diff-invariant context).
  - Operational definition of records in this context: no re-coherence.
  - Preferred notion of coarse-grainings (maximally extremal).

Consistent histories

Consistent histories [Griffiths, Gell-Mann and Hartle, Omnes]

Condition that allows probabilities to be assigned to various alternative histories of a system such that the probabilities for each history obey the rules of classical probability while being consistent with the Schrödinger equation. The *absence of quantum mechanical interference between histories* is the sufficient condition.

Let $\{P_{\alpha_k}(t_k)\}$ be a set of orthogonal projection operators. For example, α_k could be the position interval an electron might arrive at a screen at time t_k.

The projectors are taken to be exhaustive and exclusive:

$$\sum_{\alpha_k} P_{\alpha_k}(t_k) = 1 \ , \quad P_{\alpha_k}(t_k) P_{\alpha'_k}(t_k) = \delta_{\alpha_k \alpha'_k} P_{\alpha_k}(t_k)$$

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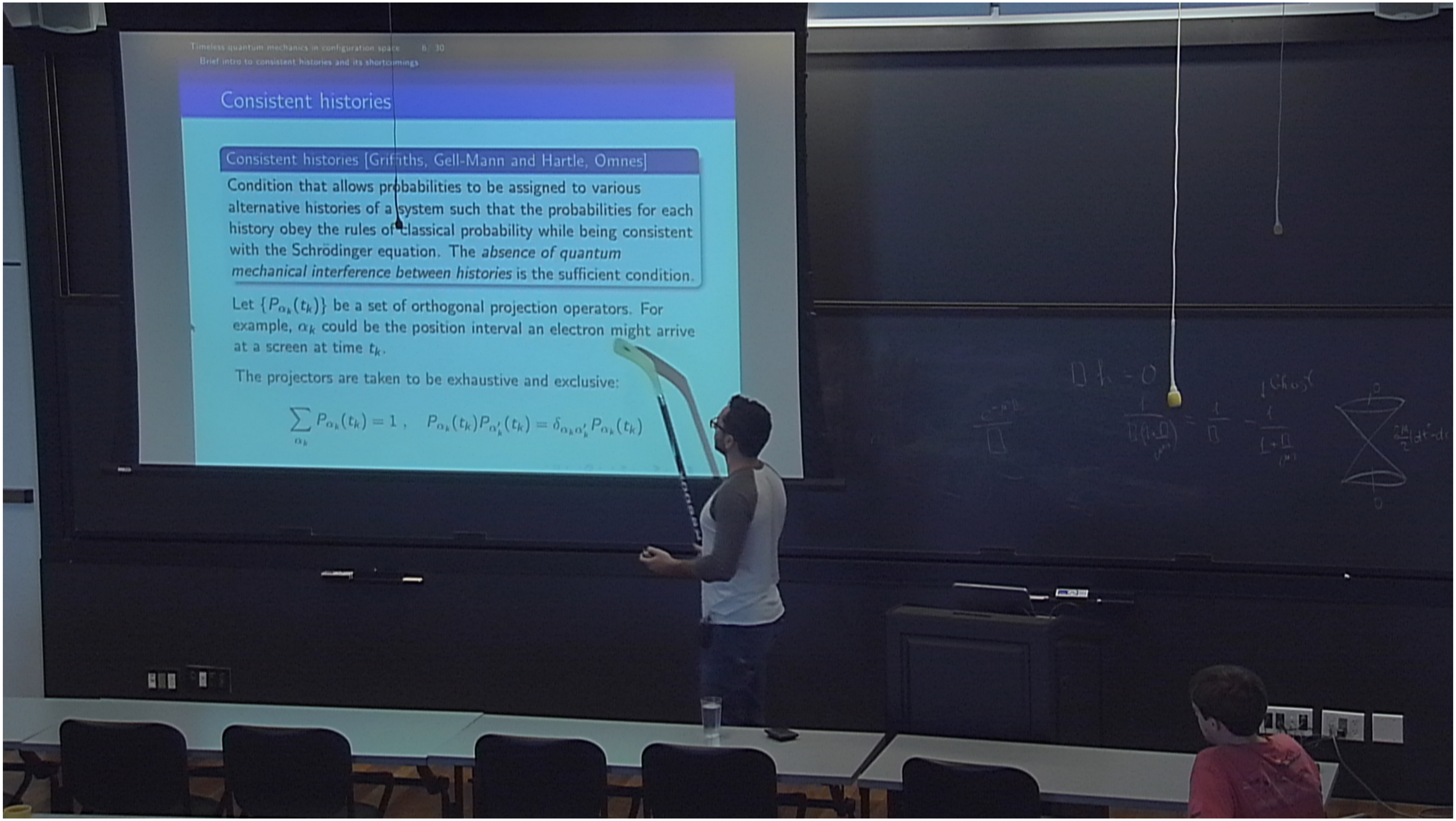
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Decoherence in consistent histories

The *decoherence functional* is a measure of interference between pairs of histories:

$$D(\alpha, \alpha') := \mathrm{tr}(C_\alpha \rho C_\alpha^\dagger)$$

For pure initial states: $\langle \psi_\alpha | \psi_{\alpha'} \rangle$, where $|\psi_\alpha\rangle = C_\alpha |\psi\rangle$.

In path integral language:

$$D(\alpha, \alpha') = \int \mathcal{D}x \mathcal{D}y \, e^{i(S[x(t)] - S[y(t)])/\hbar} \rho(x_o, y_o)$$

If $\Re(D(\alpha, \alpha')) \approx 0$ for $\alpha \neq \alpha'$, assign probabilities to histories:

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Limitations of decoherence and consistent histories

Decoherence requires a separation of ‘system and environment’. Not straightforwardly applicable to the entire Universe (also problems with microscopic Universe [Riedel]).

Consistent histories aims to resolve that by a formulation that dispenses with the separation, and focuses on interference between different histories.

A remaining issue with consistent histories is that one must still ‘choose’ the coarse-grained sets (or the histories).

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The Wheeler DeWitt equation

Canonical quantization of GR yield:

$$\hat{H}\Psi = (-\Delta + U)\Psi = 0$$

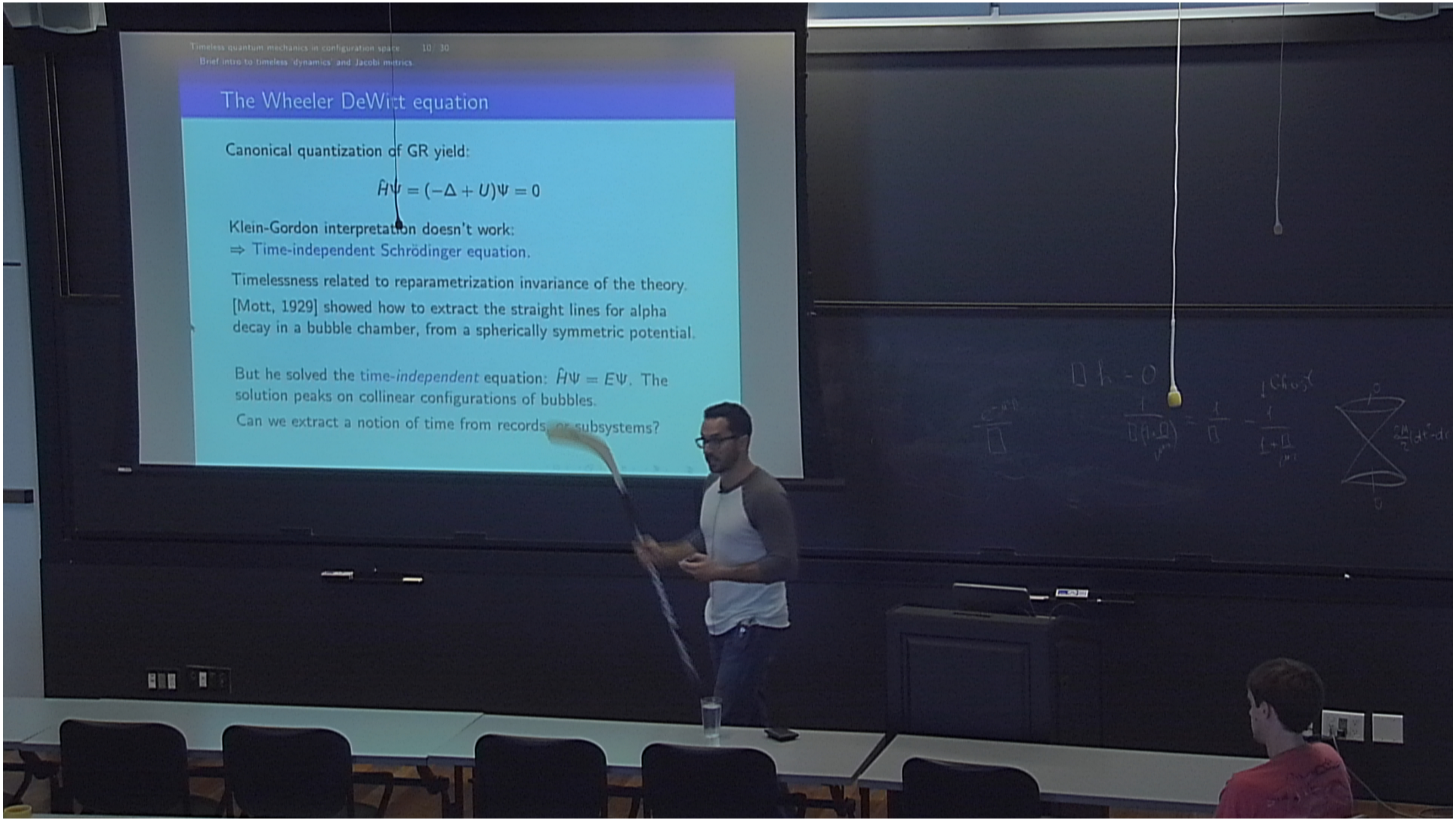
Klein-Gordon interpretation doesn't work:
$\Rightarrow$ Time-independent Schrödinger equation.

Timelessness related to reparametrization invariance of the theory.

[Mott, 1929] showed how to extract the straight lines for alpha decay in a bubble chamber, from a spherically symmetric potential.

But he solved the time-*independent* equation: $\hat{H}\Psi = E\Psi$. The solution peaks on collinear configurations of bubbles.

Can we extract a notion of time from records, or subsystems?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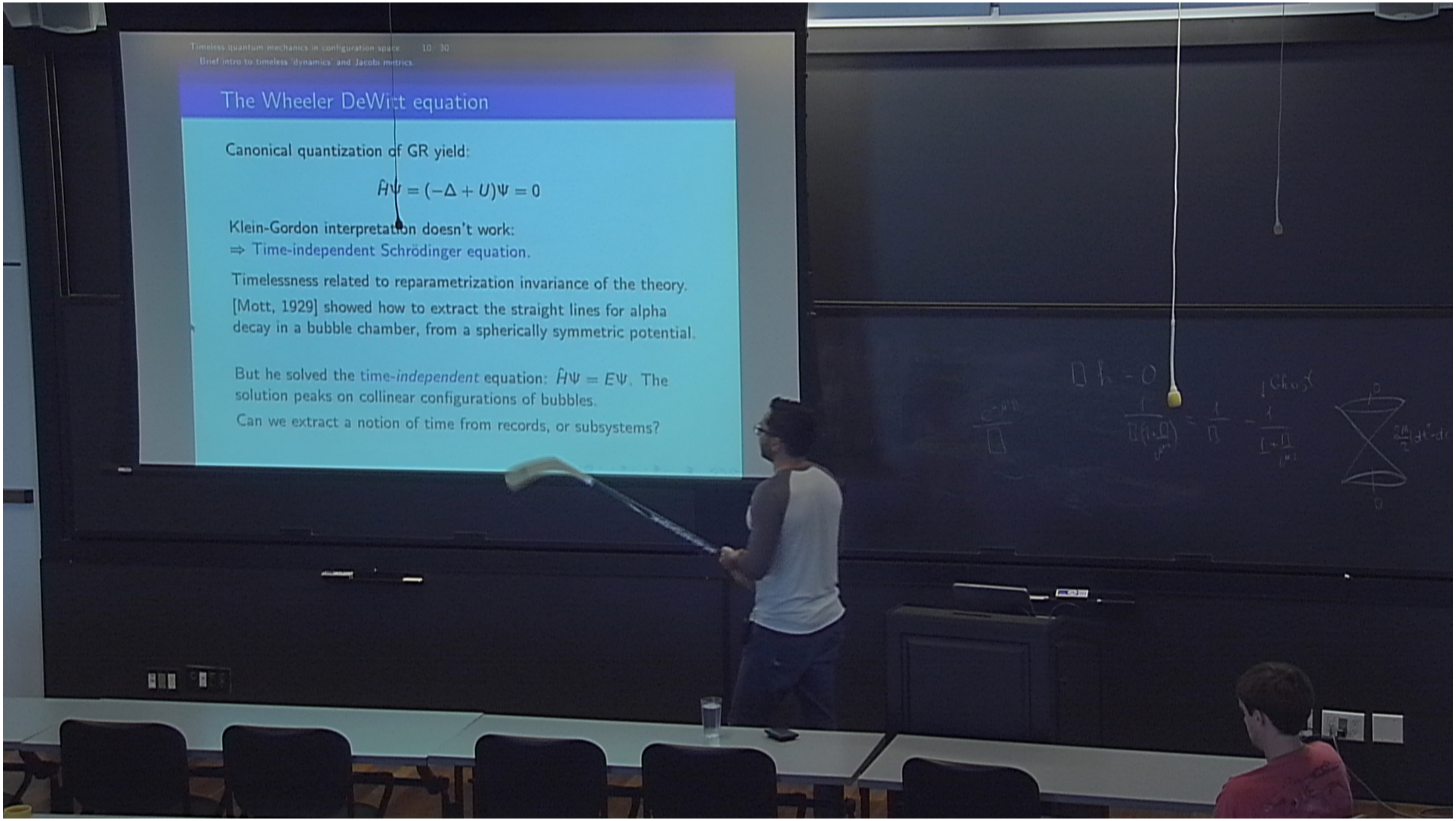
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The many approaches for extracting time.

Yes, one can, in many different approaches. Let me focus on three branches (not exclusive):

- **Internal time**: [Barbour, Kuchař, etc] argue that global reparametrization invariance should not be associated with a gauge freedom: one of the variables can be a 'hidden time' of the theory (e.g. density of scalar field).
- **Emergent time**: Born-Oppenheimer approximation for subsystems: $\Psi = \psi(H)\chi(H, L)$ for heavy and light dofs. WKB approximation and Hamilton-Jacobi relation yield an effectively time dependent equation [Kiefer, Anderson, etc.]
- **Timelessness at face value**: supplant 'becoming' with 'being'. History, or dynamics, apparent notions constructed from the instant. [Page, Halliwell, Barbour, etc]

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Example of timeless to timeful: Barbour-Bertotti model

Relational, reparametrization invariant model:

$$S_{BB} = \int_{\lambda_i}^{\lambda_f} d\lambda \sqrt{T(\lambda)(E - V(q^i))}$$

where T and V are the kinetic and potential energies. EOM:

$$\sqrt{\frac{E-V}{T}} \frac{d}{d\lambda} \left(\sqrt{\frac{E-V}{T}} \frac{dq^i}{d\lambda} \right) = -g^{ij} \frac{\partial V}{\partial q^i}$$

One can recover the usual Newtonian EOM by using 'ephemeris time' (arclength):

$$d\tau_{BB} = d\lambda \sqrt{\frac{T}{E-V}} \Rightarrow \frac{d^2 q^i}{d\tau_{BB}^2} = -g^{ij} \frac{\partial V}{\partial q^i}$$

Jacobi (action) metric

If action can be written as

$$S = \int (T - V) dt$$

for system defined in a configuration space with Riemannian structure $(\mathcal{M}, g)$, extremal trajectories of S with energy $E = T + V$ coincide with the extremals of the length functional on configuration space: $L = \int ds$, defined for $(\mathcal{M}, h)$, where h is the 'on-shell action metric', conformally related to g by

$$h = 2(E - V)g$$

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Geometrodynamical analogues

A geometrodynamical analogue to the relation BB$\leftrightarrow$ Newtonian exists, for $E \sim 2\Lambda$ [Bairlein-Sharpe-Wheeler]:

$$S = \int dt \int d^3x \sqrt{g}\sqrt{T(R - 2\Lambda)}$$

with local experienced time defined by

$$N = \sqrt{\frac{T}{R - 2\Lambda}}$$

where $T = G^{ijkl}\dot{g}_{ij}\dot{g}_{kl}$. Problem for geometric interpretation:

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In defense of configuration space

"Indeed, all measurements of quantum mechanical systems could be made to reduce eventually to position and time measurements (e.g., the position of a needle on a meter or time of flight of a particle). Because of this possibility a theory formulated in terms of position measurements is complete enough to describe all phenomena." - Feynman and Hibbs

I would add that even the reading of a clock is abstracted from a position measurement; we often identify it with a given configuration of that common contraption on the wall.

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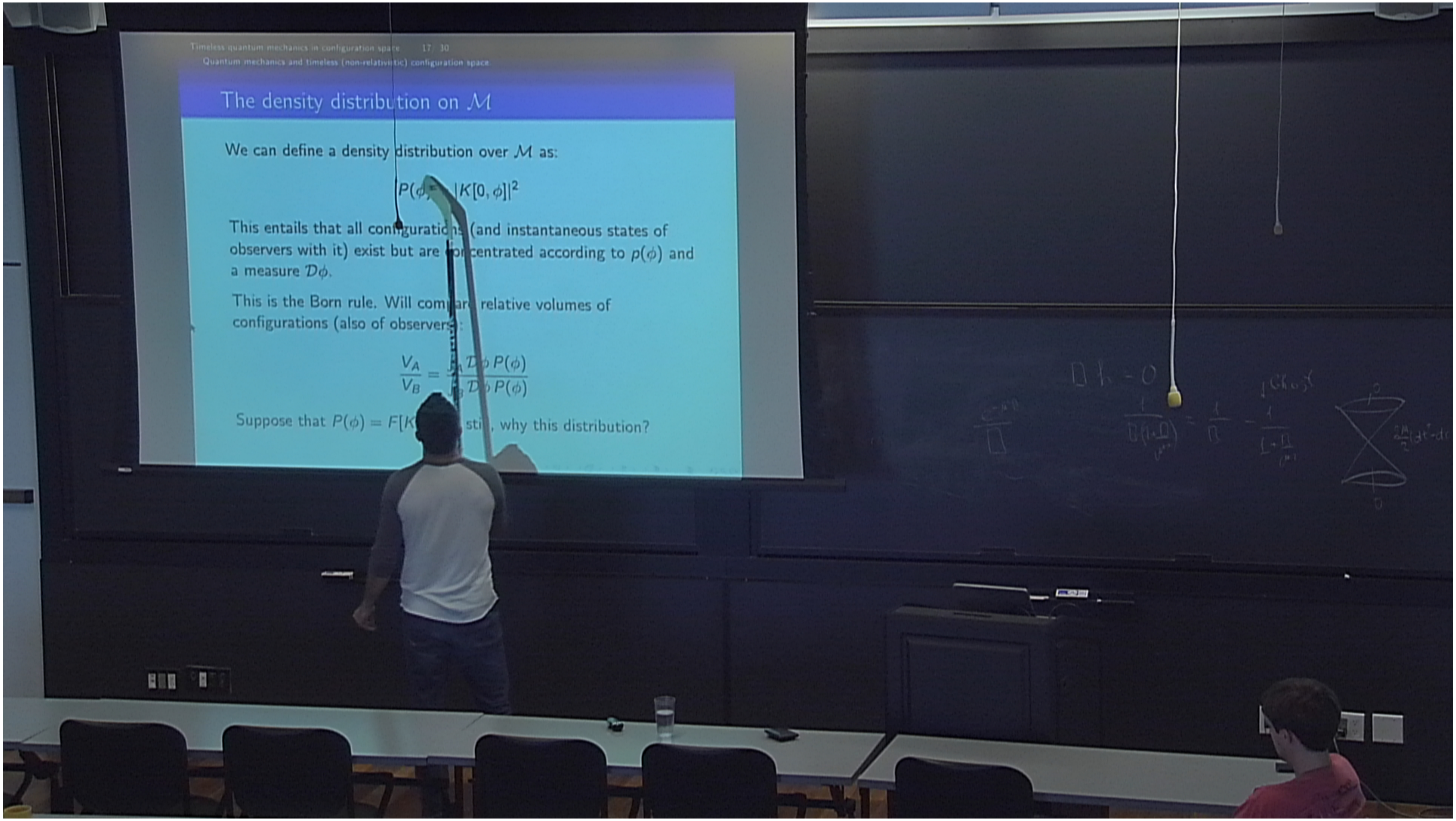
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$$\frac{V_A}{V_B} = \frac{\int_A \mathcal{D}\phi \, P(\phi)}{\int_B \mathcal{D}\phi \, P(\phi)}$$

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Semi-classical approximation

For extremal paths $\gamma_{\rm cl}$, semi-classical approximation:

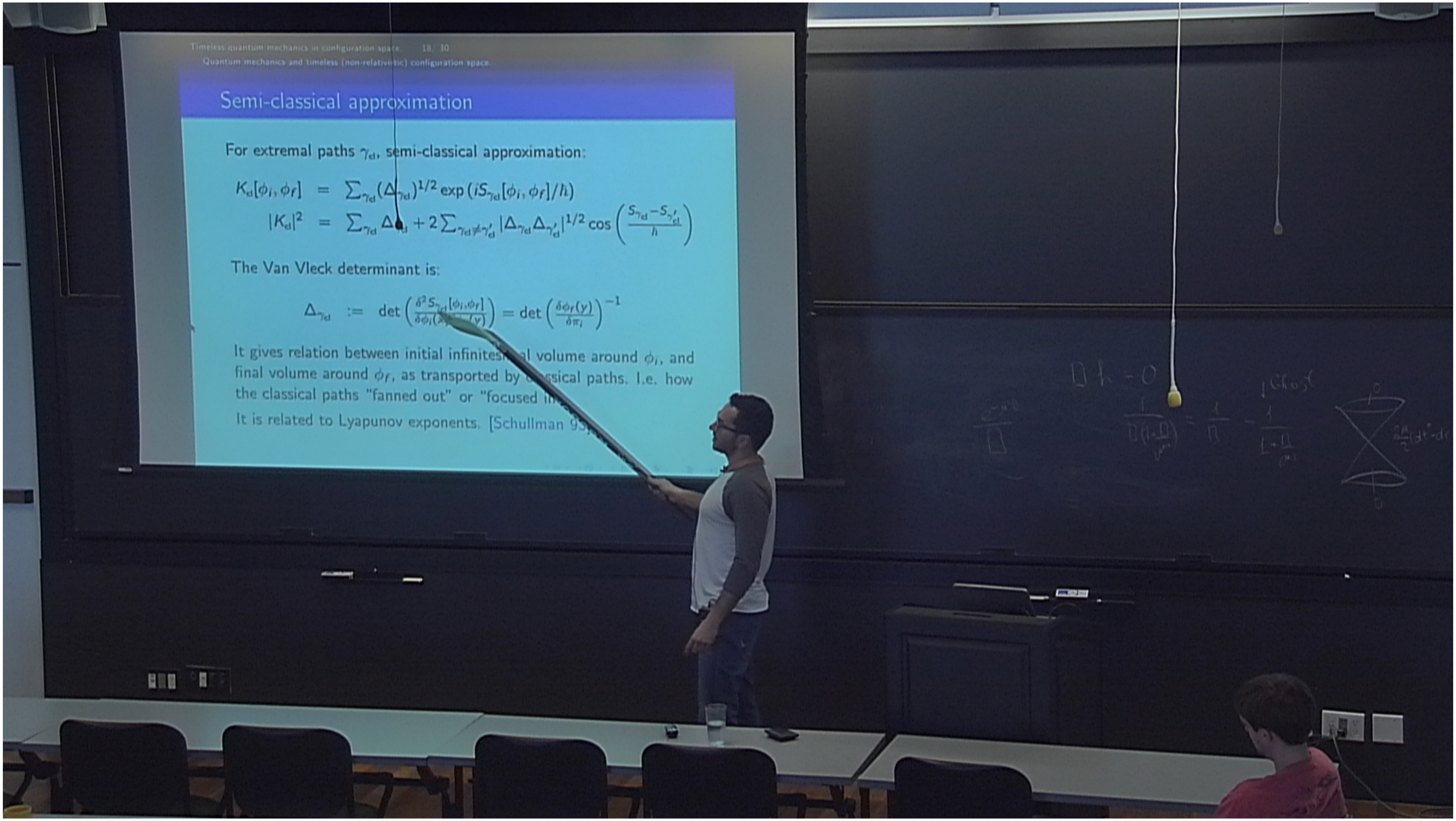
$$\begin{aligned} K_{\rm cl}[\phi_i,\phi_f] &= \sum_{\gamma_{\rm cl}}(\Delta_{\gamma_{\rm cl}})^{1/2}\exp\left(iS_{\gamma_{\rm cl}}[\phi_i,\phi_f]/\hbar\right) \\ |K_{\rm cl}|^2 &= \sum_{\gamma_{\rm cl}}\Delta_{\gamma_{\rm cl}} + 2\sum_{\gamma_{\rm cl}\neq\gamma'_{\rm cl}}|\Delta_{\gamma_{\rm cl}}\Delta_{\gamma'_{\rm cl}}|^{1/2}\cos\left(\frac{S_{\gamma_{\rm cl}}-S_{\gamma'_{\rm cl}}}{\hbar}\right) \end{aligned}$$

The Van Vleck determinant is:

$$\Delta_{\gamma_{\rm cl}} := \det\left(\frac{\delta^2 S_{\gamma_{\rm cl}}[\phi_i,\phi_f]}{\delta\phi_i(x)\delta\phi_f(y)}\right) = \det\left(\frac{\delta\phi_f(y)}{\delta\pi_i}\right)^{-1}$$

It gives relation between initial infinitesimal volume around ϕ_i, and final volume around ϕ_f, as transported by classical paths. I.e. how the classical paths "fanned out" or "focused in".

It is related to Lyapunov exponents. [Schullman 93]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Interpretation of the Born rule in relative densities

For two distinct final configurations, ϕ_1 and ϕ_2, the 'classically propagated relative densities of configurations' is:

$$\frac{d\rho_1}{d\rho_2} = \frac{\Delta_1}{\Delta_2}$$

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Born rule is a natural extension of this property for a functional

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Coarse-grainings in configuration space

A bundle ν_γ of paths around a curve $\gamma \in \Gamma(\phi_i, \phi_f)$ are those paths contained in some open set around γ.

Coarse-grained histories

A set of coarse–grained histories $\{C_\alpha \subset \Gamma(\phi_i, \phi_f), \alpha \in I\}$ between two configurations ϕ_i and ϕ_f will be said to be exclusive and exhaustive to order ϵ if i) $C_\alpha \cap C_{\alpha'} = \emptyset$ for $\alpha \neq \alpha'$ (exclusive) ii) $\nu_\gamma \in C_\alpha$, such that iii)

$$K(\phi_i, \phi_f) = \sum_\alpha K_\alpha + \mathcal{O}(\epsilon) = \sum_\alpha \int_{C_\alpha} \mathcal{D}\gamma(t) e^{i(S[\gamma(t)]/\hbar)} + \mathcal{O}(\epsilon) \quad (2)$$

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i.e. the total transition amplitude from ϕ_i to ϕ_f is approximated to order ϵ (in the absolute value sense).

Piece-wise extremal coarse-grainings

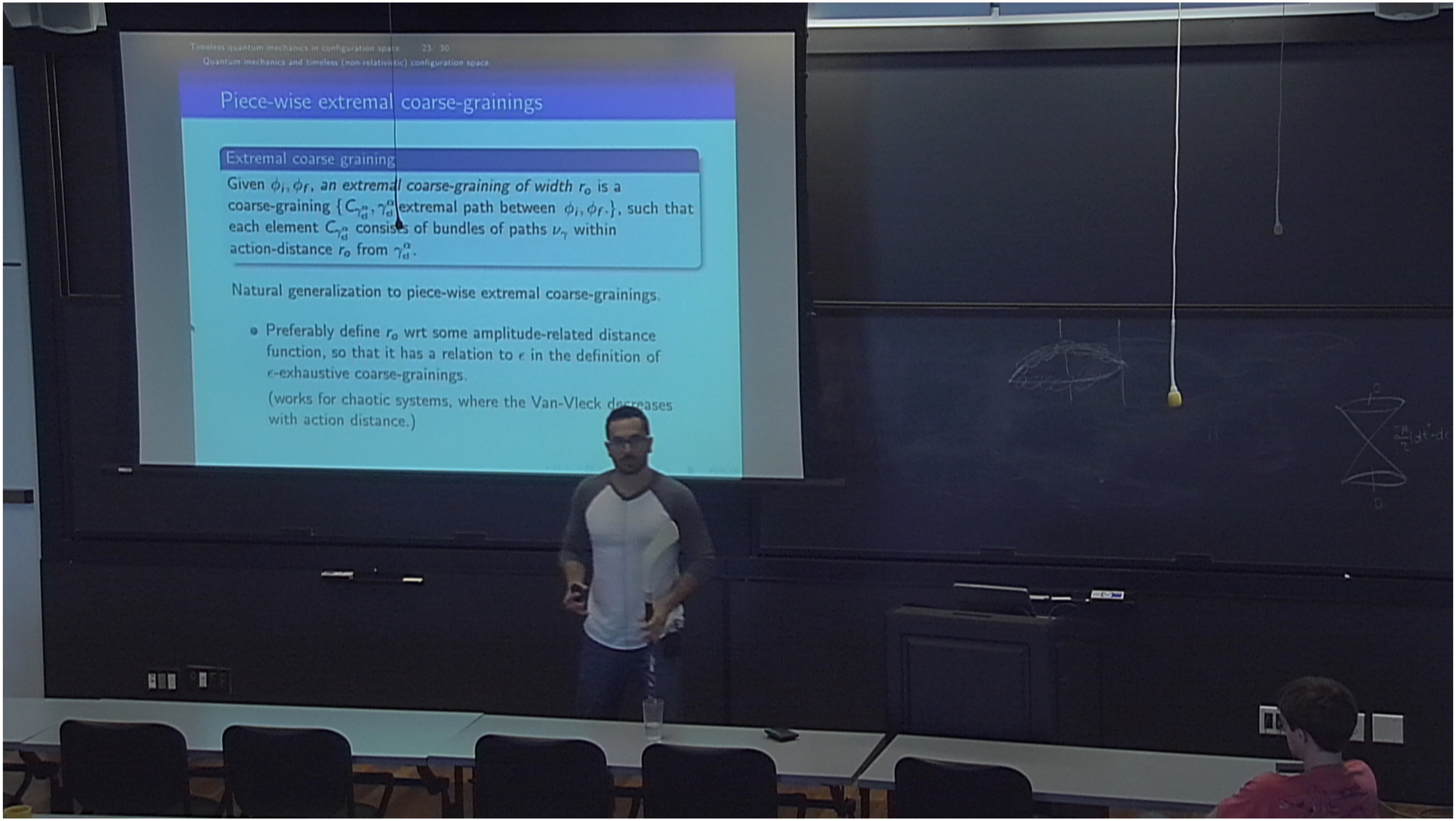
Extremal coarse graining

Given ϕ_i, ϕ_f, *an extremal coarse-graining of width* r_o is a coarse-graining $\{C_{\gamma^\alpha_{\rm cl}}, \gamma^\alpha_{\rm cl}$ extremal path between $\phi_i, \phi_f.\}$, such that each element $C_{\gamma^\alpha_{\rm cl}}$ consists of bundles of paths ν_γ within action-distance r_o from $\gamma^\alpha_{\rm cl}$.

Natural generalization to piece-wise extremal coarse-grainings.

- Preferably define r_o wrt some amplitude-related distance function, so that it has a relation to ϵ in the definition of ϵ-exhaustive coarse-grainings.

  (works for chaotic systems, where the Van-Vleck decreases with action distance.)


Timeless quantum mechanics in configuration space 23/ 30
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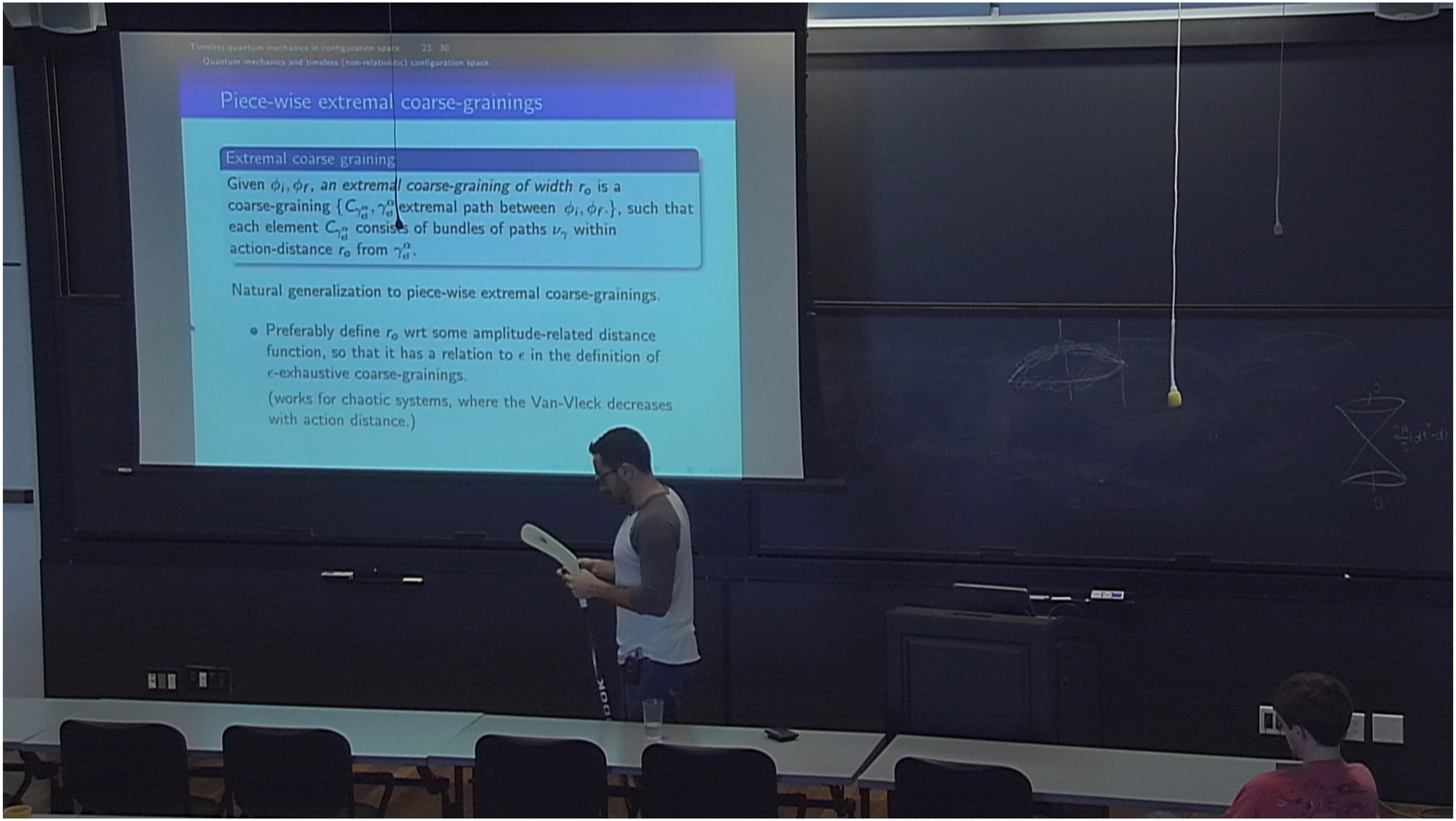
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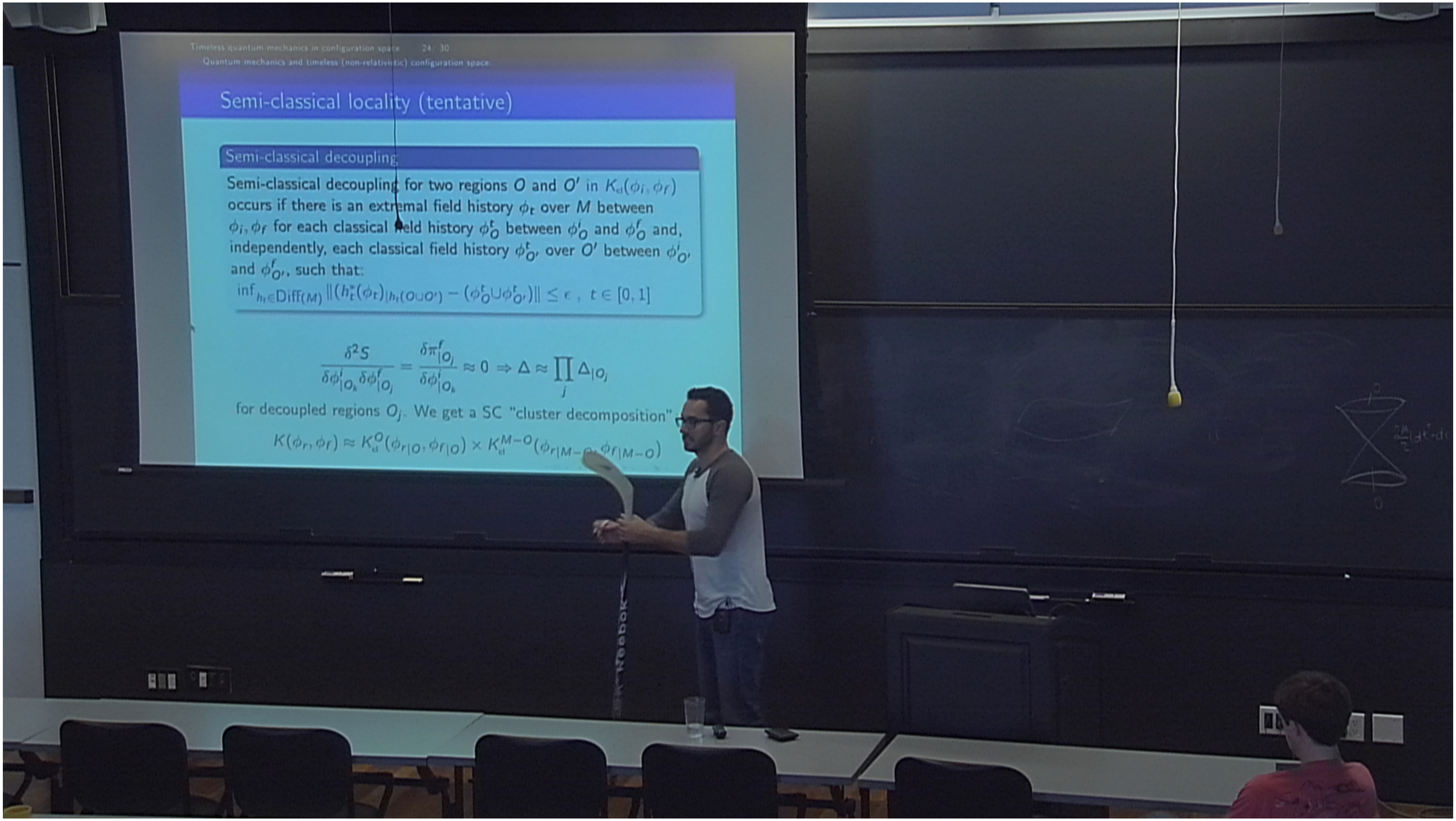
Timeless quantum mechanics in configuration space 24 / 30
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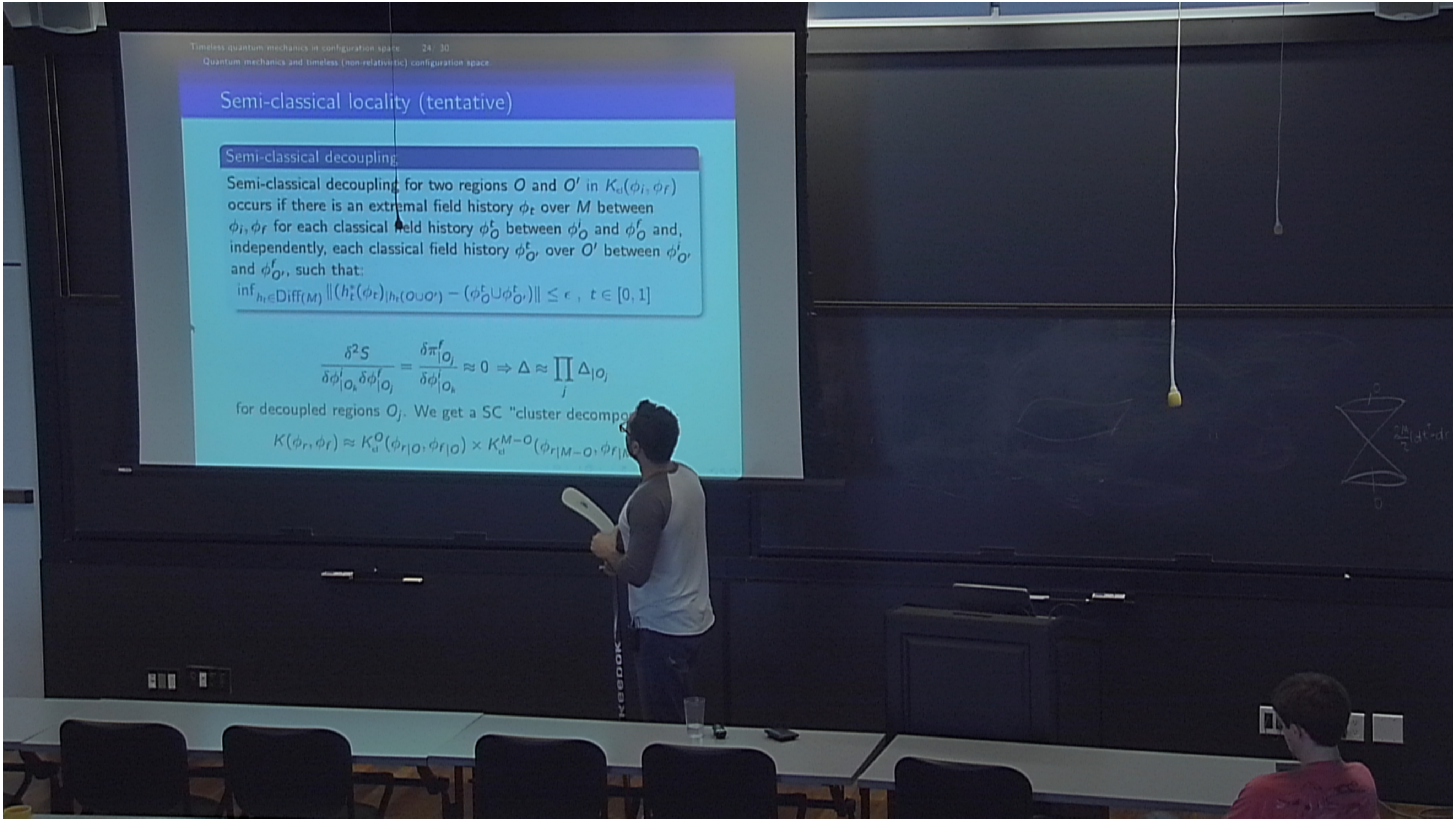
$\inf_{h_t \in \mathrm{Diff}(M)} \| (h_t^* (\phi_t))_{|h_t(O \cup O')} - (\phi^t_O \cup \phi^t_{O'}) \| \leq \epsilon \,, \ t \in [0,1]$
$\frac{\delta^2 S}{\delta \phi^i_{|O_k} \delta \phi^f_{|O_j}} = \frac{\delta \pi^f_{|O_j}}{\delta \phi^i_{|O_k}} \approx 0 \Rightarrow \Delta \approx \prod_j \Delta_{|O_j}$
for decoupled regions O_j. We get a SC "cluster decomposition"
$K(\phi_r, \phi_f) \approx K^O_{cl}(\phi_{r|O}, \phi_{f|O}) \times K^{M-O}_{cl}(\phi_{r|M-O}, \phi_{f|M-O})$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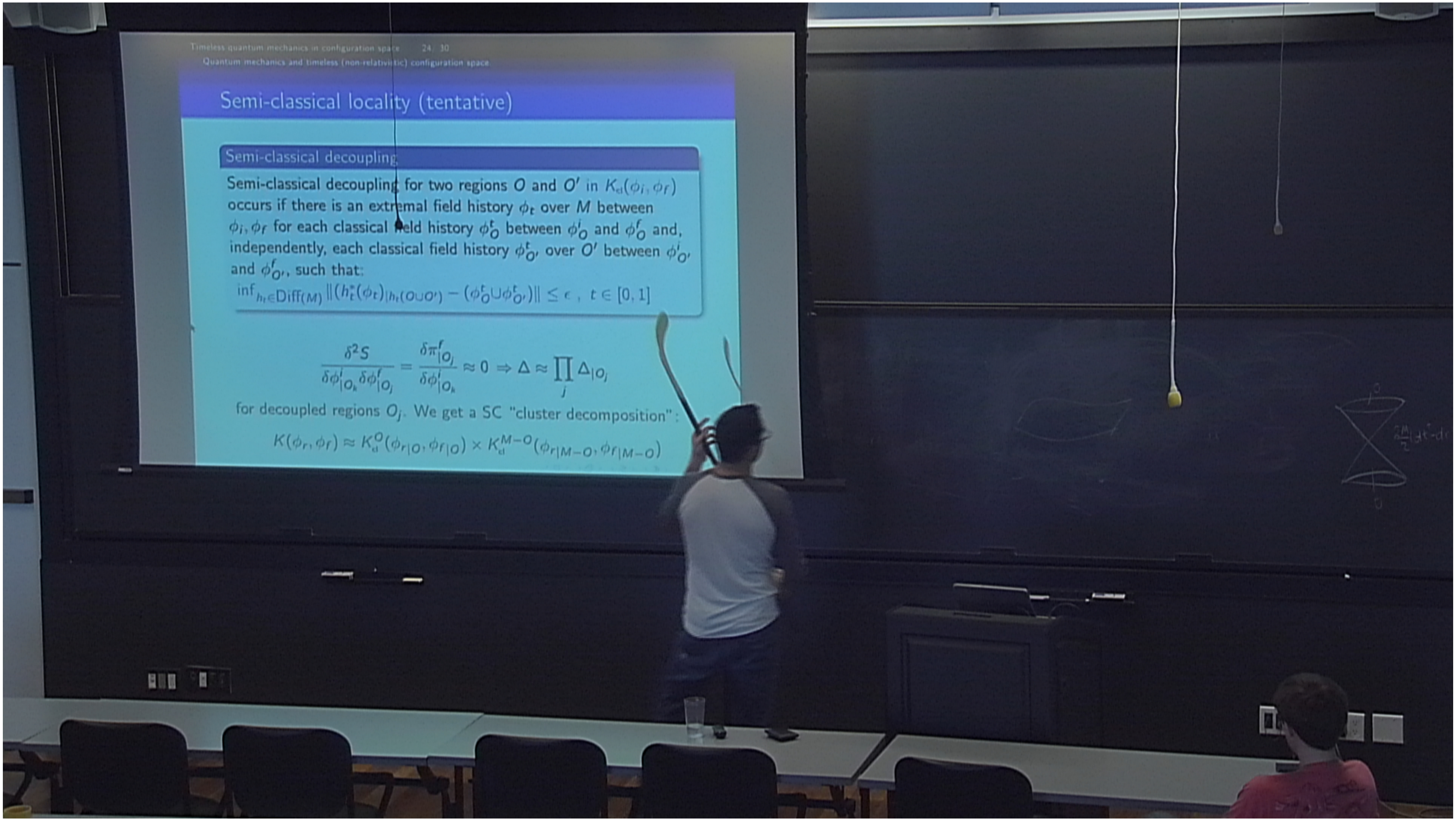

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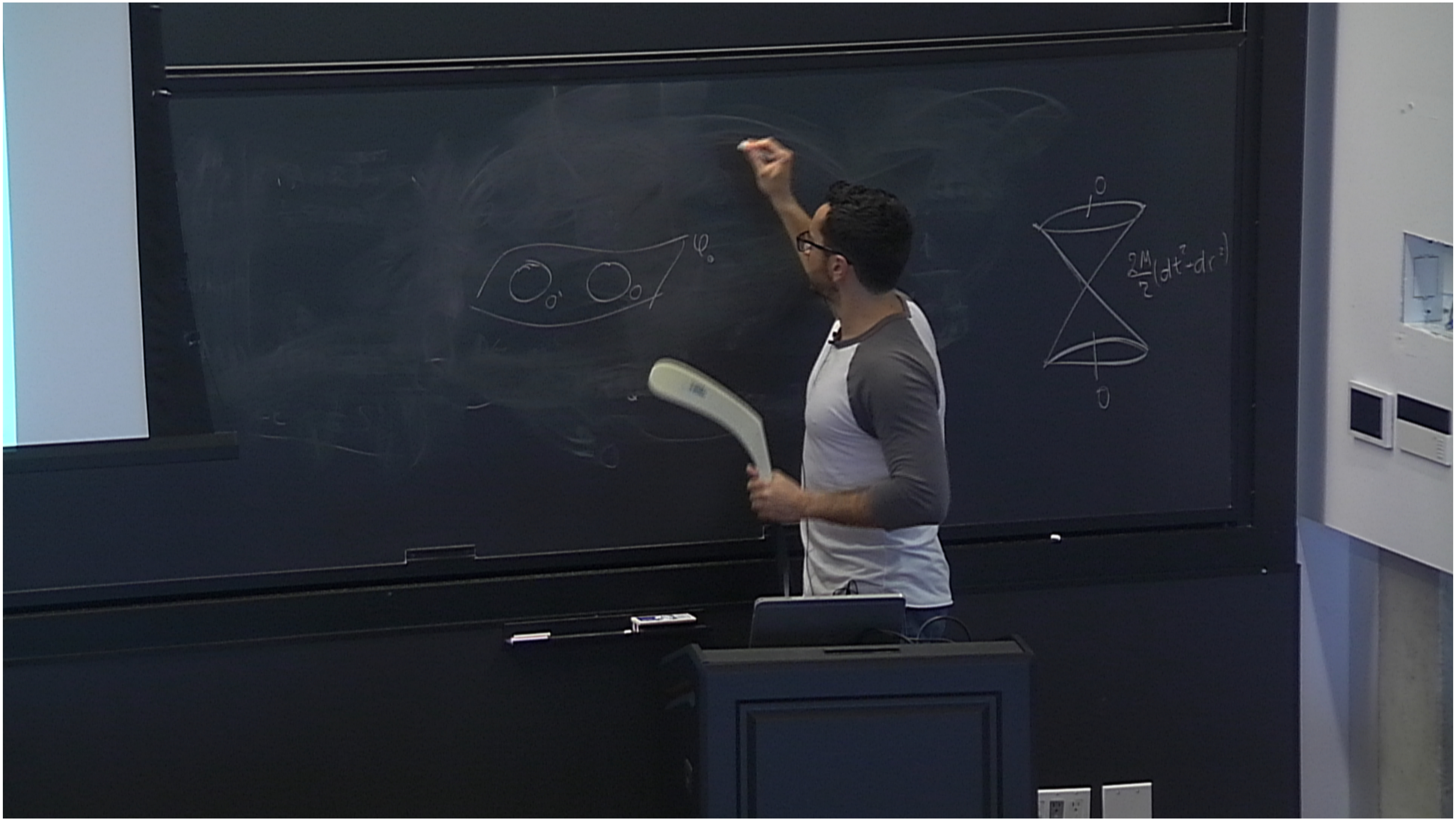
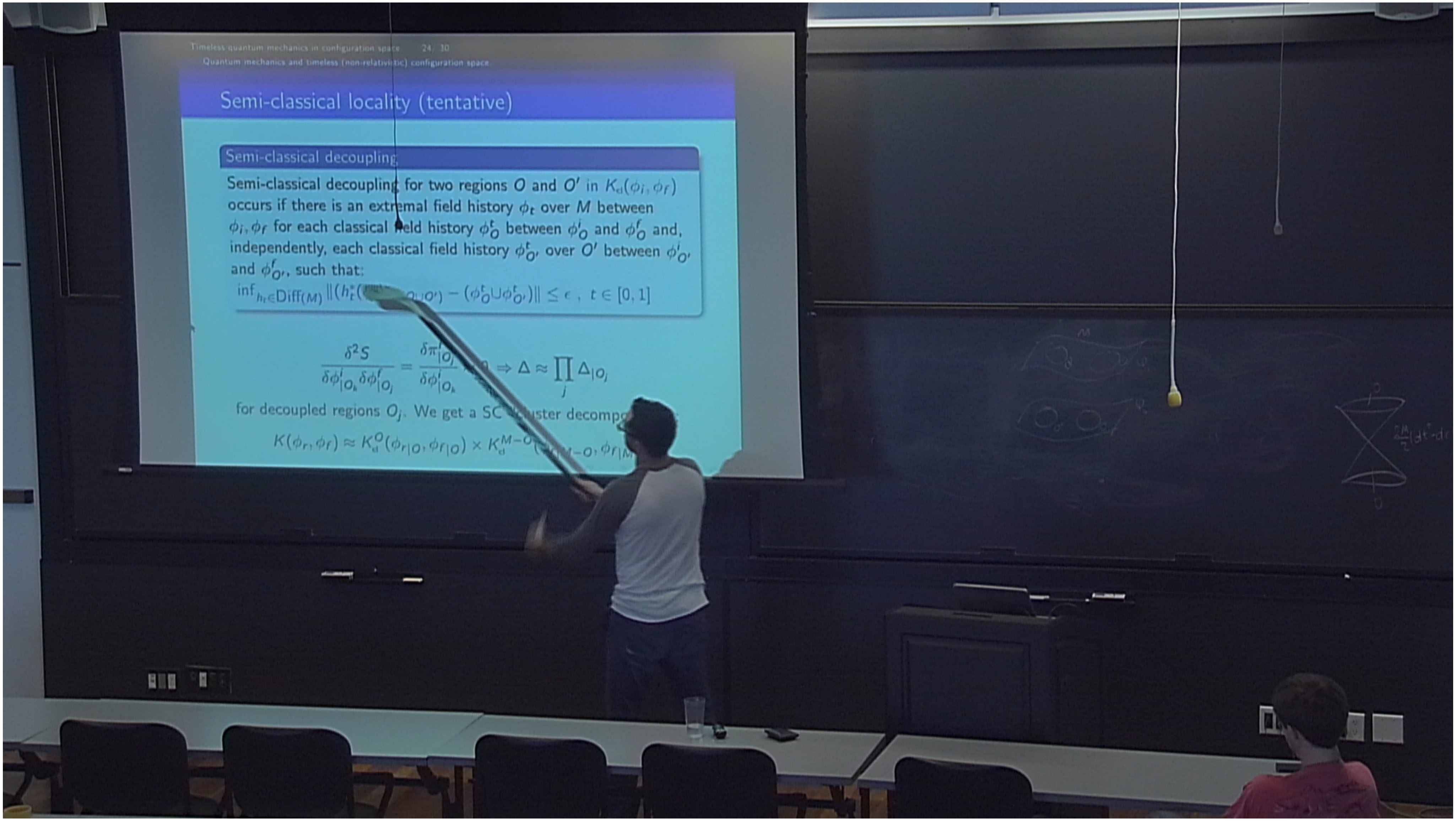

φ₀
O₀ O₀
2M/r (dt² − dr²)
0
0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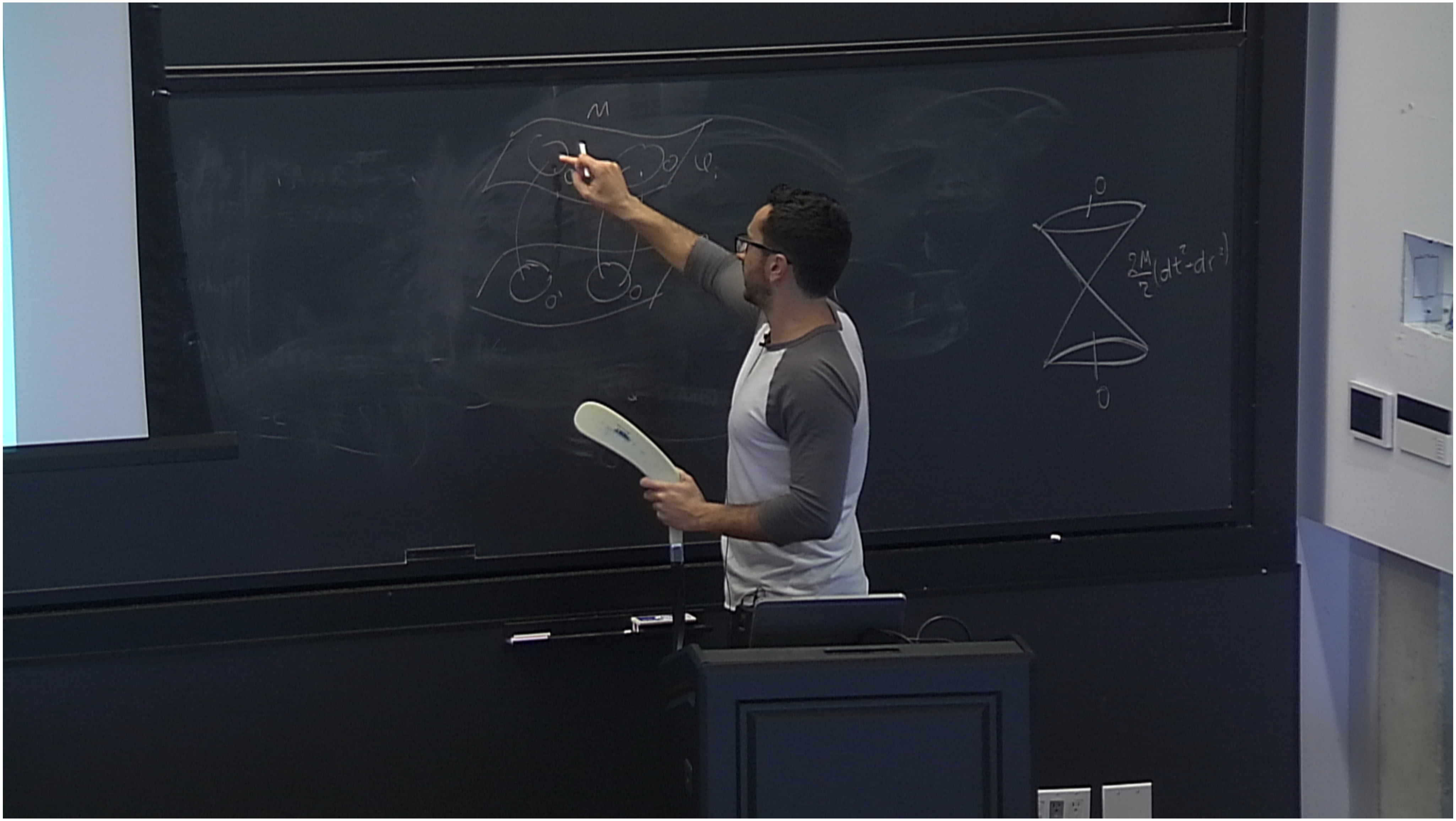
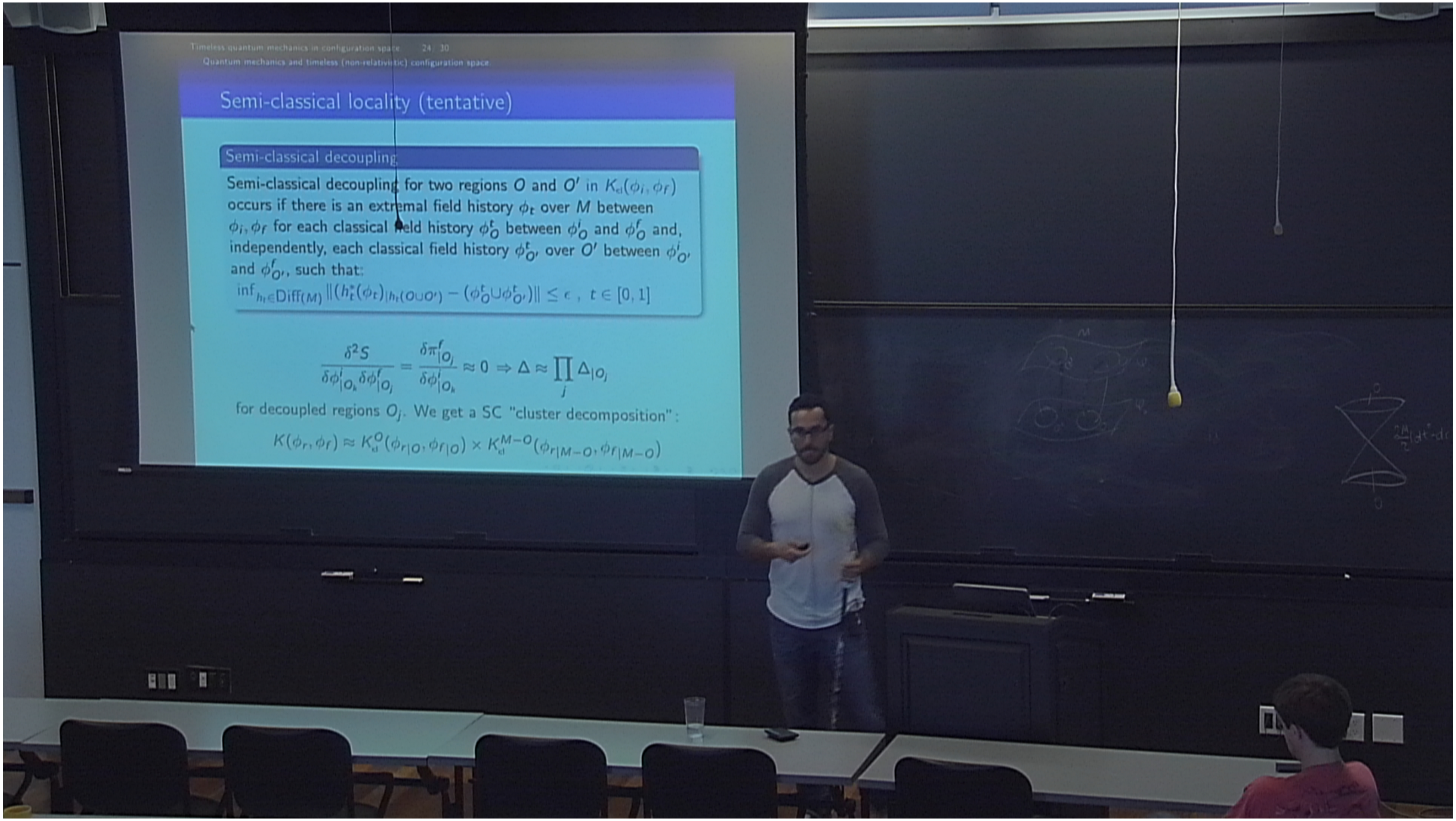

M
$\frac{2M}{r}(dt^2 - dr^2)$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Semi-classical records (tentative)

Records here are similar to projection operators in consistent histories, but are the same for all histories.

Semi-classical records

A field configuration ϕ contains a semi-classical record of ϕ_r if all the extremal coarse-grainings between 0 and ϕ include ϕ_r. If this occurs then:

$$K(0,\phi) \approx K(0,\phi_r)K(\phi_r,\phi)$$

This definition of records gives a Bayesian spin to the present approach of consistent histories.

What one does is compare relative volumes of regions with the same records:

$$\frac{K(0,\phi_1)}{K(0,\phi_2)} \approx \frac{K(\phi_0,\phi_r)K(\phi_r,\phi_1)}{K(\phi_0,\phi_r)K(\phi_r,\phi_2)} = \frac{K(\phi_r,\phi_1)}{K(\phi_r,\phi_2)}$$

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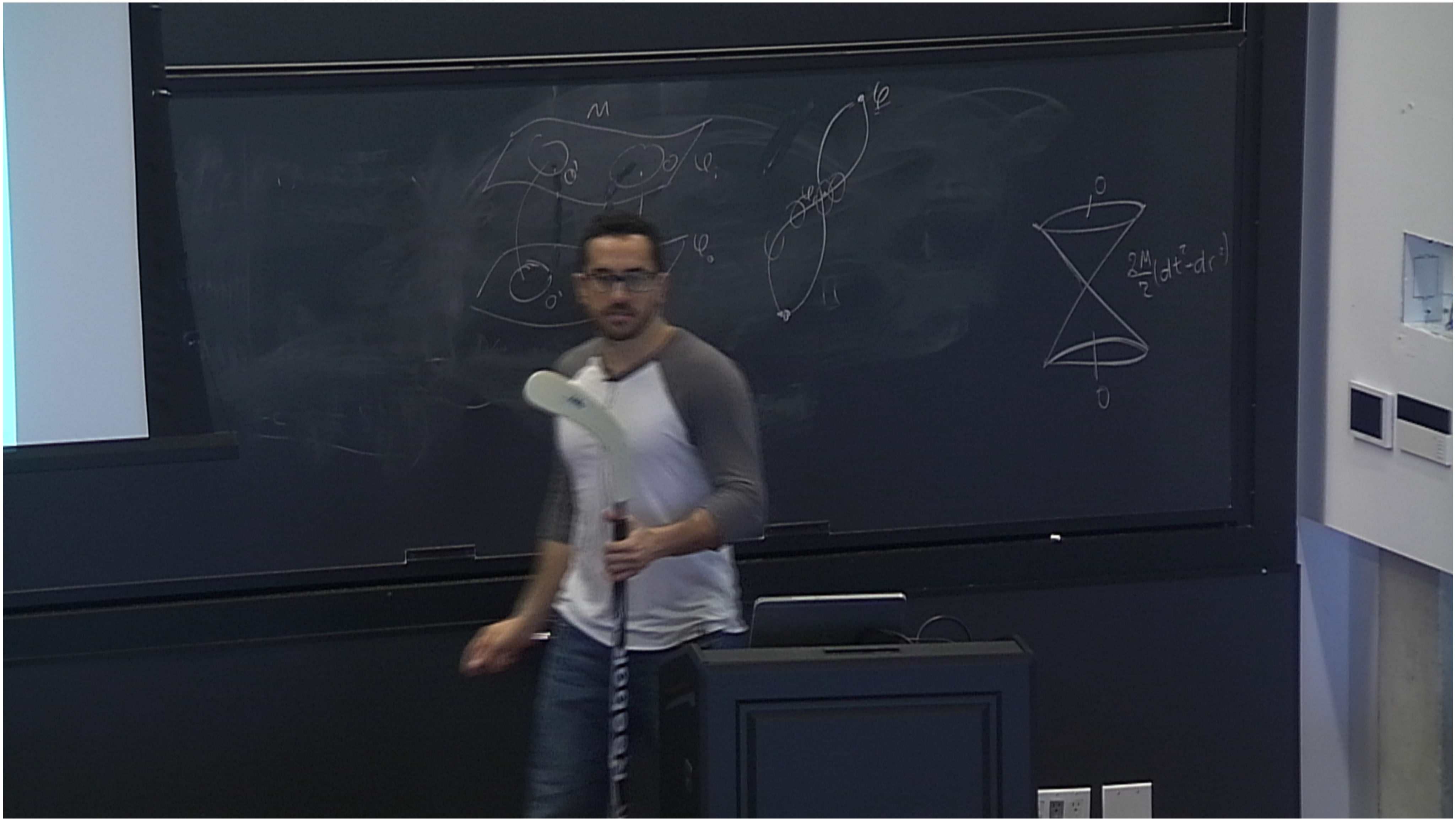
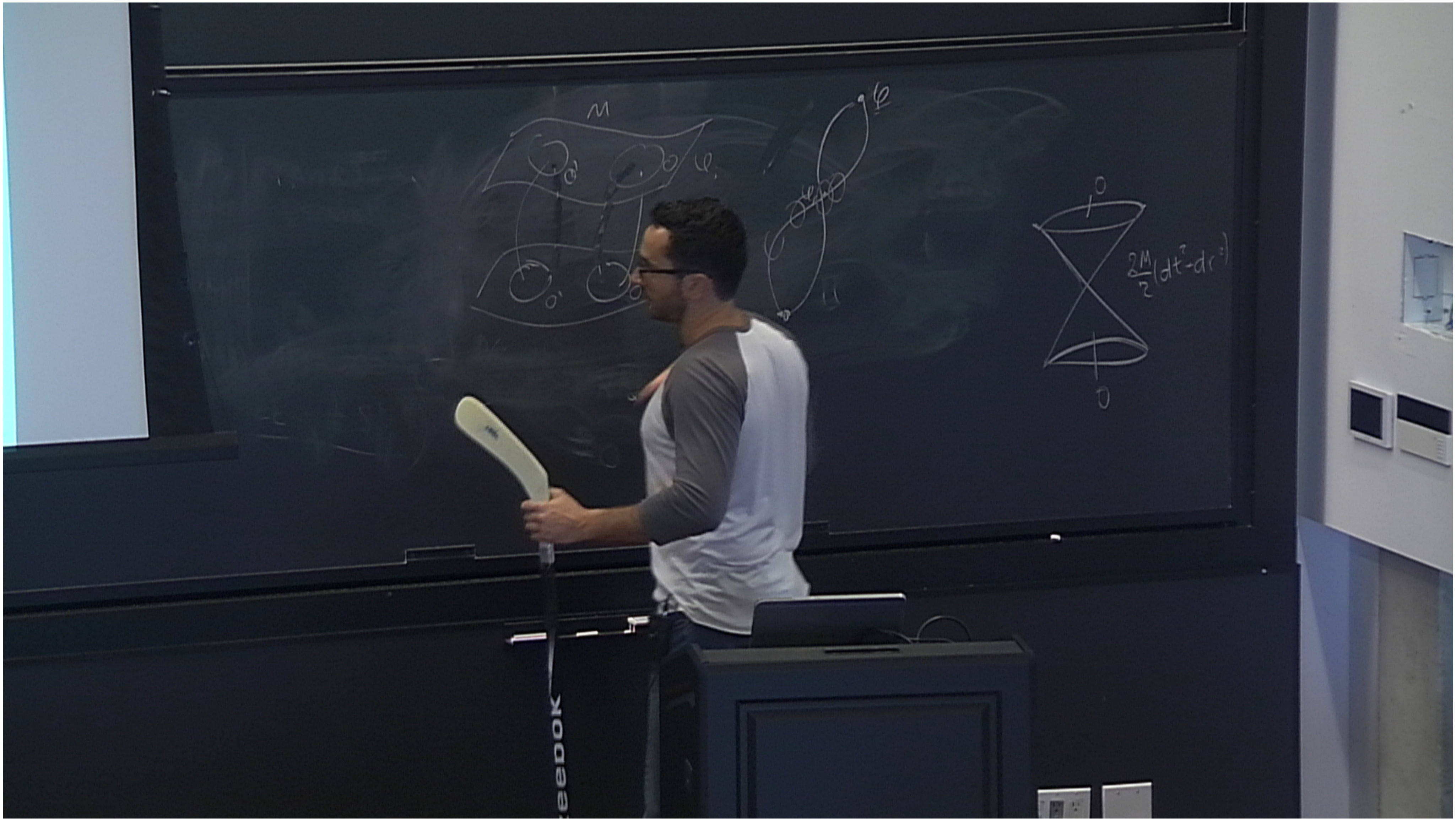
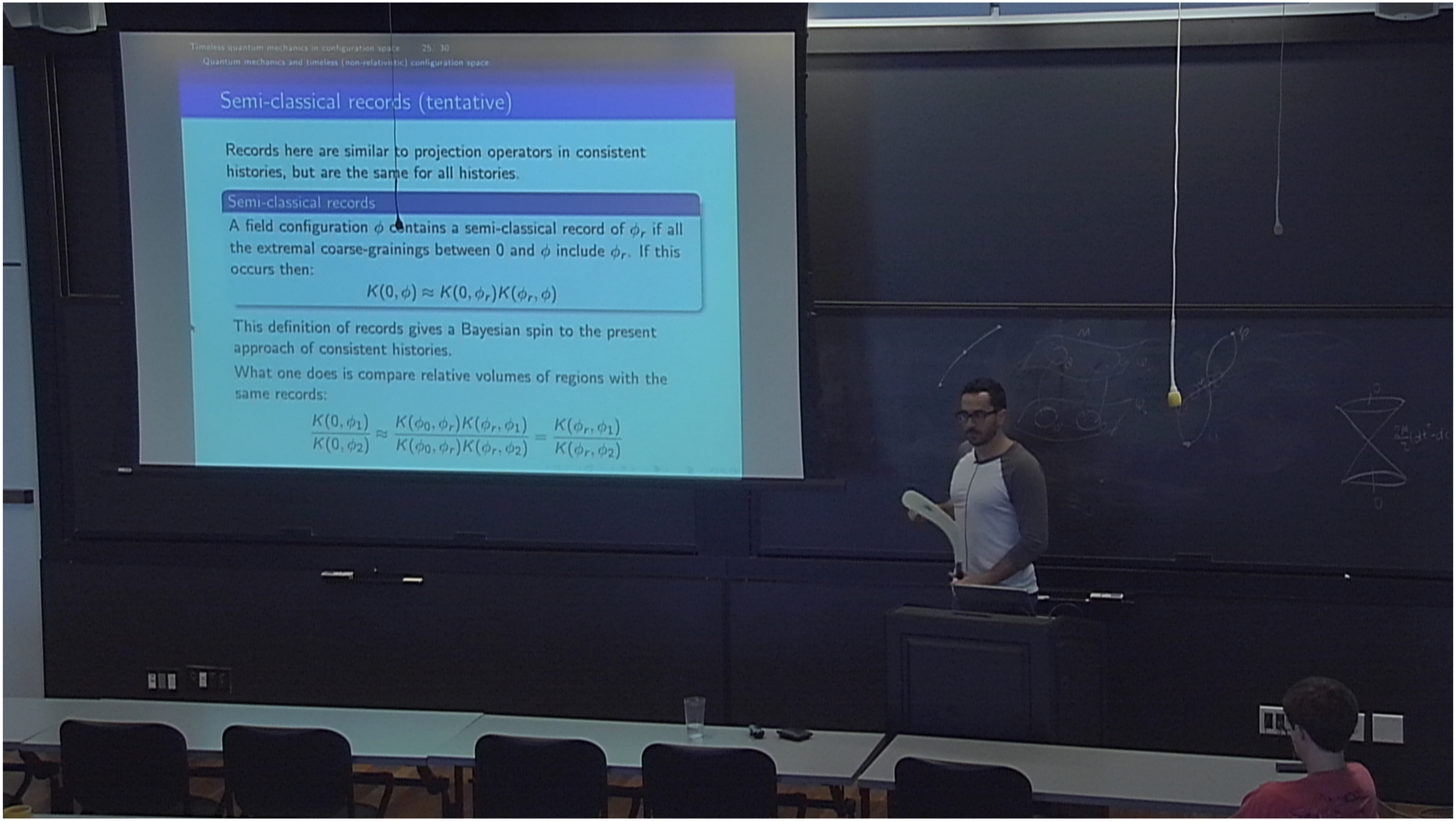


M
φ_1
φ_0
$\frac{2M}{r}(dt^2-dr^2)$


M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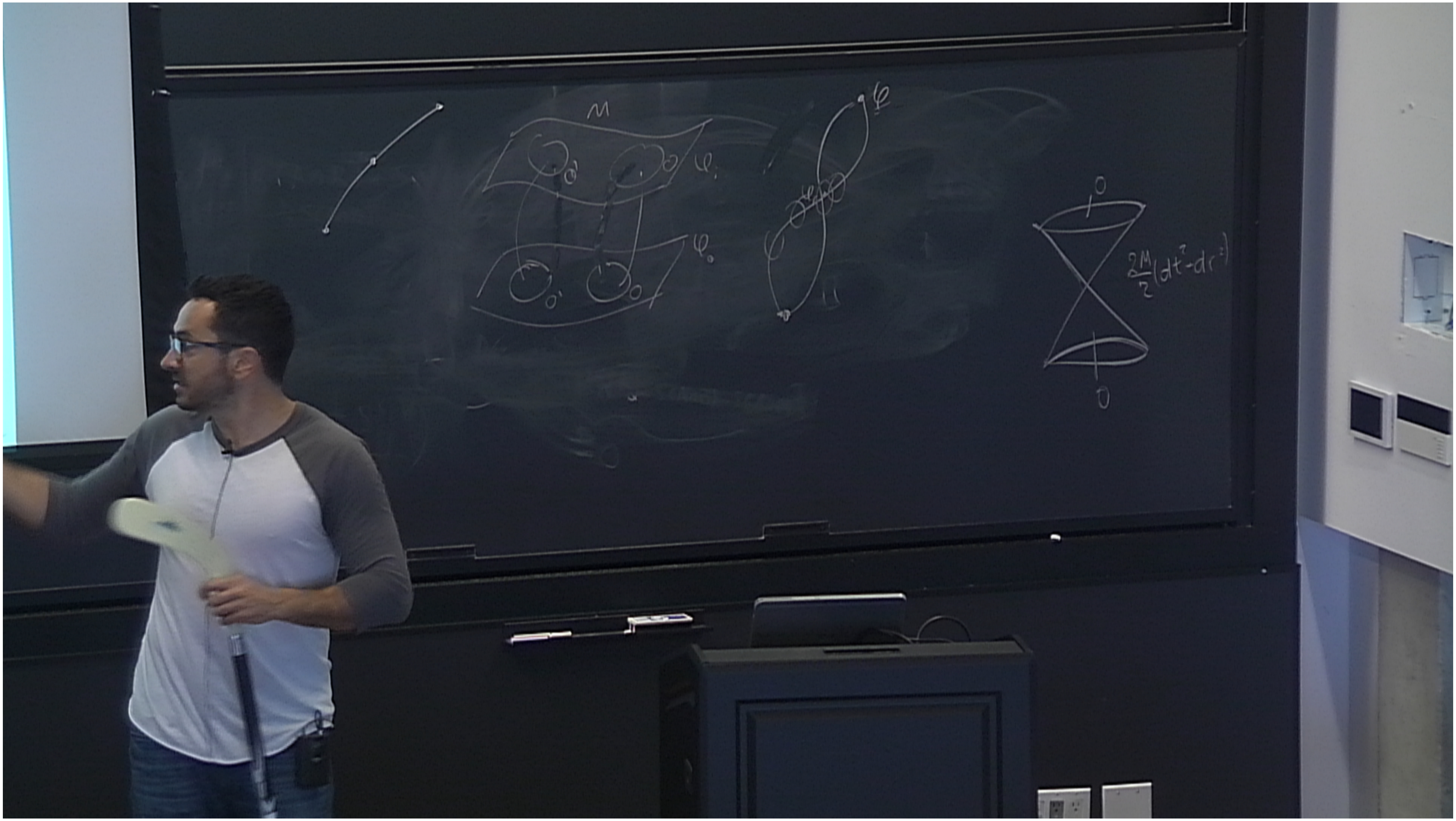
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Application to gravity

- Problem: non-relativistic description. Paths γ_1 and γ_2 that intersect in one foliation might not intersect in another.

  Requires a preferred notion of *simultaneity, to work in this simple form*. But Lorentz invariance recoverable in many ways.

- Einstein-Aether, Hořava, Shape Dynamics are such theories.

'Gravitationally induced decoherence'. Sure. In this respect nothing special about the gravitational field. Classical paths in configuration space may diverge rapidly due to non-linearity.

Repeat [Gooding and Unruh 14] calculation for self-gravitating interferometry.

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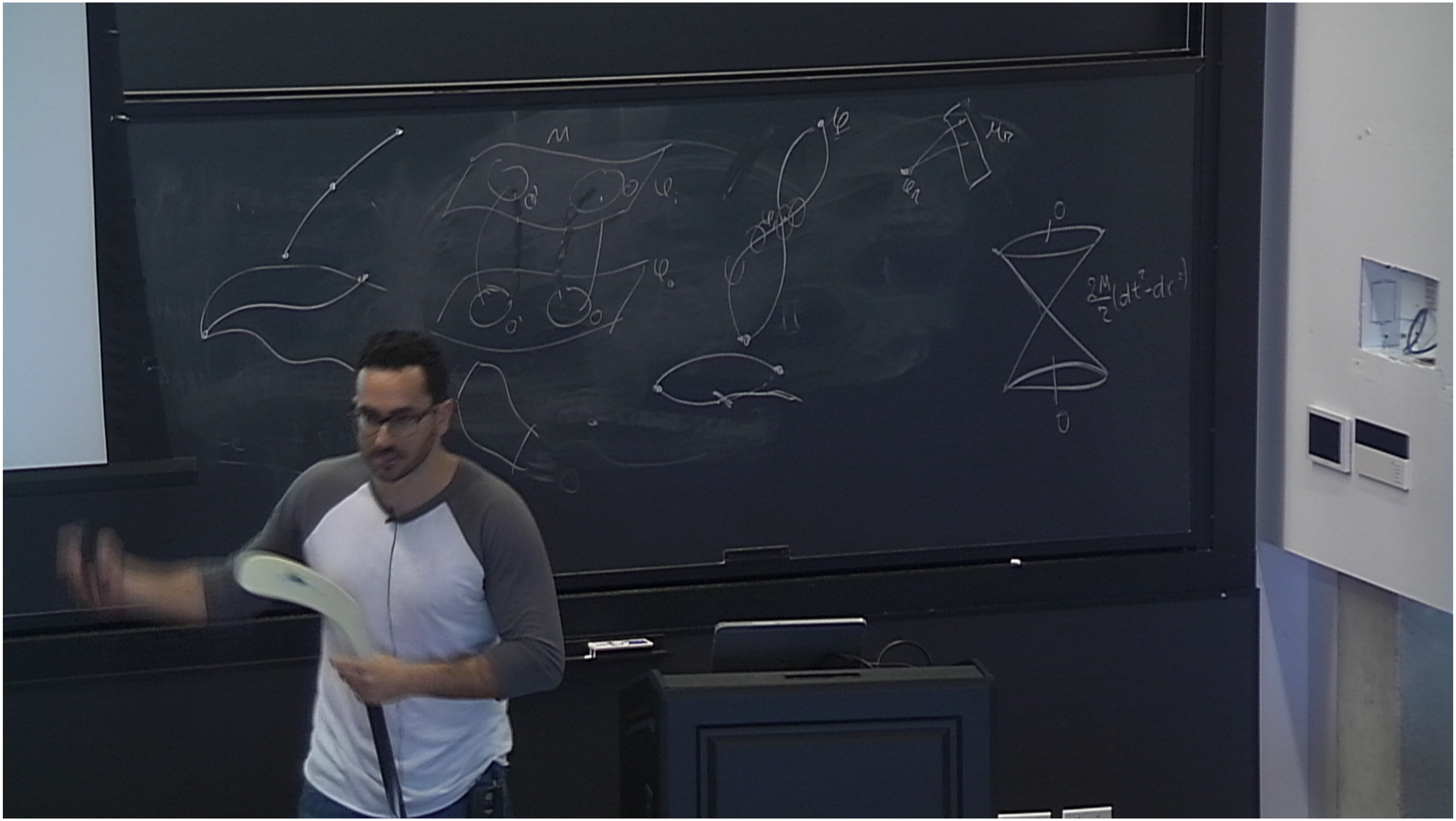
M
φ_i
φ_0
φ
$\frac{2M}{r}(dt^2-dr^2)$

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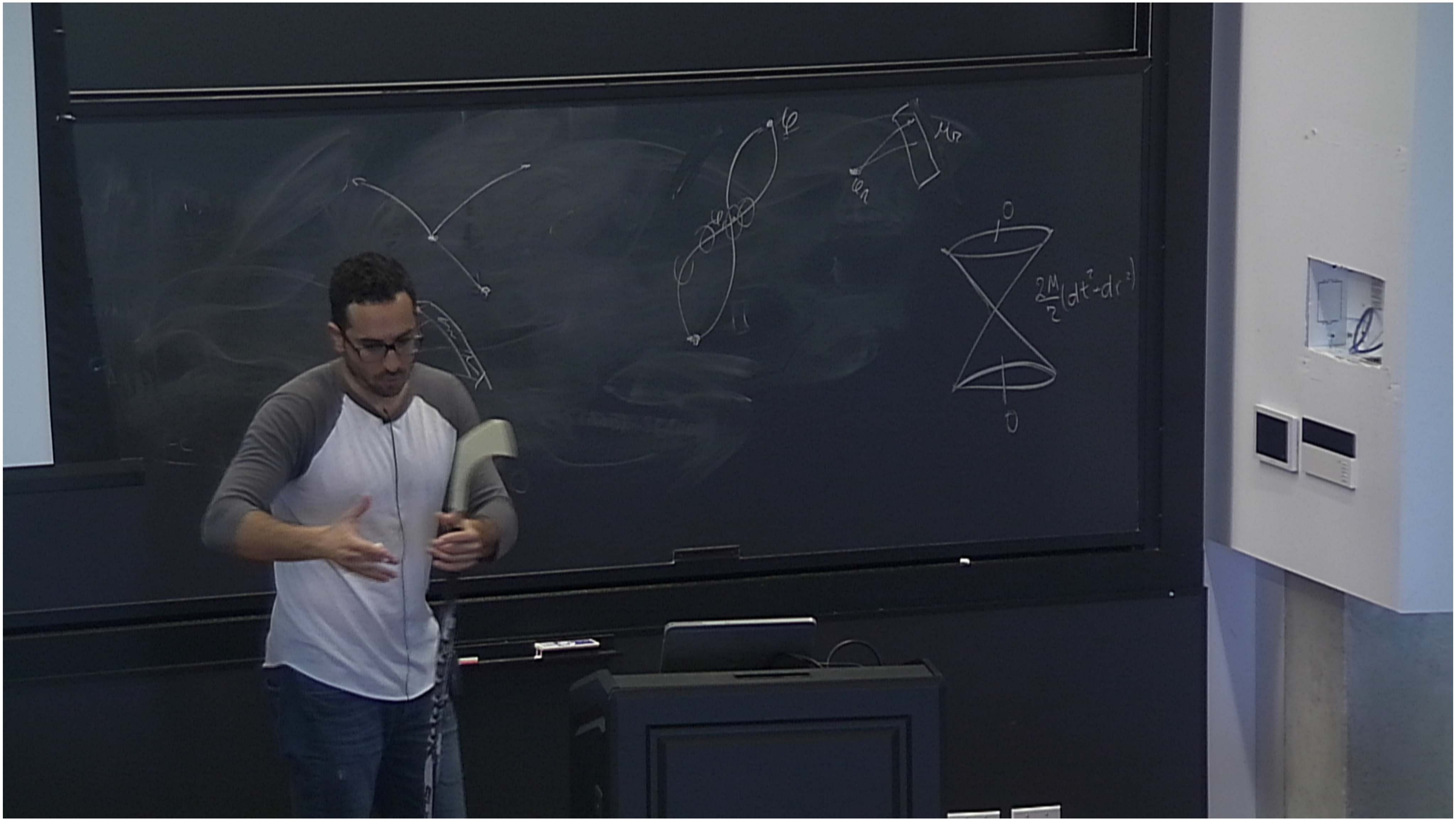
The heuristic emerging picture

- Similar to Everett-Wheeler, but here *all points* in configuration space 'simultaneously' exist.
- Each *configuration* is completely classical. The quantum character makes itself felt by interference between different histories.
    - If a brain state is entirely described by the configuration it is in, the question "why we observe a classical world?" has no meaning.
- No meaning to "quantum state of the Universe". Just relative probabilities on submanifolds of $\mathcal{M}$ based on records.
- Dynamical notion of locality (even in Diff-invariant context)?
- Definition of records disallows "re-coherence".
- Preferred notion of coarse-grainings (maximally extremal).


0
2M/r (dt²-dr²)
0