Title: Unruh-DeWitt detectors in RQI: from the basics to frontiers

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Abstract: <p>In relativistic quantum information (RQI) we study quantum information in relativistic systems to obtain more insights to both quantum and gravitational physics on the one hand, and to find new ideas for quantum information processing on the other. One of the popular models in RQI is the Unruh-DeWitt (UD) detector theory, in which localized objects, called detectors, are coupled to and moving in relativistc quantum fields. In this mini-course I will discuss the UD detector theory in detail, mainly on the nonperturbative methods and their applications to RQI.</p>

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Errata :

1. $$H = \frac{1}{\left|\frac{dz^0(\tau)}{d\tau}\right|}\left(\frac{P^2}{2m_0} + \frac{m_0}{2}\underline{\Omega_0^2 Q^2} - \lambda_0 \chi Q \phi_{\vec{z}(\tau)}\right) + \frac{1}{2}\int d^3x \left[(\pi^{\vec{x}})^2 + |\vec{\nabla}\phi_{\vec{x}}|^2 \right.$$

2. Derivative Coupling theory in $\mathbb{M}^2$

$$S = -\int d^2x \, \frac{1}{2}\partial_\mu\phi \, \partial^\mu\phi + \frac{m_0}{2}\int d\tau \left[\dot{Q}^2 - \Omega_0^2 Q^2\right] + \underbrace{\lambda \int d^2x \int d\tau \, \dot{Q}(\tau)\phi(x)\,\delta^2(x - z(\tau))}_{\text{or } \lambda\int d\tau \, Q \, \partial_\tau \phi(z(\tau))}$$

✓ * Canonical Quantization.

* (K, Δ)-representation

* Evolution of correlators

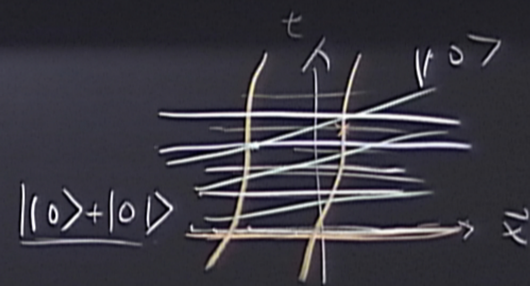
$$S = S_\phi + S_Q + S_{int}$$

$$= -\frac{1}{2}\int d^4x\, \partial_\mu\phi\, \partial^\mu\phi + \frac{m_0}{2}\int d\tau \left(\dot{Q}^2 - \Omega_0^2 Q^2\right)$$

$$+ \lambda \int d^4x \int d\tau\, \chi(\tau)\, Q(\tau)\, \phi(x)\, \delta^4(x - Z(\tau))$$

$$+\lambda\int d^4x\int d\tau\,\chi(\tau)\,Q(\tau)\,\phi(x)\,\delta^4(x-Z(\tau))$$

[1] Choose a foliation



[2] Equal-time

$$[\hat{Q}(\tau(t)),\ \hat{P}(\tau(t))] = i\hbar$$

$$[\hat{\Phi}_{\vec{x}}(t),\ \hat{\pi}_{\vec{x}'}(t)] = i\hbar\,\delta^3(\vec{x}-\vec{x}')$$

$$\hat{H}[\hat{Q},\hat{P},\hat{\pi},\hat{\Phi}]$$

Heisenberg e.o.m $\quad \partial_t\hat{Q} = \frac{i}{\hbar}[\hat{Q},\hat{H}]\ \ldots$

$$\Rightarrow \begin{cases} \partial_t^2 \hat{Q}(t) + \Omega_0^2 \hat{Q}(\tau) = \frac{\lambda_0}{m_0} \chi(t) \hat{\Phi}(z^0(\tau), \vec{z}(\tau)) \\ (\partial_t^2 - \nabla^2) \hat{\Phi}_x(t) = \lambda_0 \int d\tau \, \chi(\tau) \hat{Q}(\tau) \delta^4(x - z(\tau)) \end{cases}$$

[3] Schrödinger picture

$$(i\hbar \partial_t - \hat{H}) \Psi[Q, \phi_{\vec{x}}; t] = 0$$

$$\hat{H}[\hat{Q} \to Q, \hat{\phi}_{\vec{x}} \to \phi_{\vec{x}}, \hat{P} \to \frac{\hbar}{i}\partial_Q, \hat{\pi}_{\vec{x}} \to \frac{\hbar}{i}\frac{\delta}{\delta \phi_{\vec{x}}}]$$

Density mtx. $\rho(Q, \phi_{\vec{x}}; Q', \phi'_{\vec{x}}; t) = \Psi[Q, \phi_{\vec{x}}; x] \Psi^*[Q', \phi'_{\vec{x}}; x]$

$$\Rightarrow \begin{cases} \partial_t \hat{Q}(t) + \Omega \hat{Q}(t) = \frac{\lambda_0}{m_0}\chi \\ (\partial_t^2 - \nabla^2)\hat{\Phi}_x(t) = \lambda_0 \int d\tau\, \chi(\tau)\, \hat{Q}(\tau)\, \delta^4(x - z(\tau)) \end{cases}$$

[3] Schrödinger picture

$$(i\hbar\partial_t - \hat{H})\Psi[Q, \phi_x; t] = 0$$

$$\hat{H}[\hat{Q} \to Q,\ \hat{\phi}_x \to \phi_x,\ \hat{P} \to \frac{\hbar}{i}\partial_Q,\ \hat{\pi}_x \to \frac{\hbar}{i}\frac{\delta}{\delta\phi_x}]$$

Density mtx. $\rho(Q, \phi_x; Q', \phi'_x; t) = \Psi[Q, \phi_x; t]\, \Psi^*[Q', \phi'_x; t]$

Reduced state of detector: $\rho_R(Q, Q'; t) = \mathrm{Tr}_\phi \rho = \int \mathcal{D}\phi\, \Psi[Q, \phi; t]\, \Psi^*[Q', \phi; t]$

$$+ \int d^3x\, Q\, G^{QX}(t)\, \phi_x \}$$

* (K, Δ)-representation

[Unruh, Zurek PRD 40 ('89), 1071]

\- Wigner fn rep. $W(\vec{P}, \vec{Q}, t) = \int \underbrace{\mathcal{D}\Delta}_{\sim \prod_j \frac{d\Delta_j}{2\pi\hbar}}\, e^{\frac{i}{\hbar}\vec{P}\cdot\vec{\Delta}}\, \rho\left(\vec{Q} - \frac{\vec{\Delta}}{2}, \vec{Q} + \frac{\vec{\Delta}}{2}; t\right)$

$$1 = \mathrm{Tr}\,\rho = \int \mathcal{D}P\, \mathcal{D}Q\, W(\vec{P}, \vec{Q}; t)$$

Density mtx: $\rho(Q,\phi_{\vec{x}}; Q',\phi'_{\vec{x}}; t) = \Psi[Q,\phi_{\vec{x}}; t]\,\Psi^*[Q',\phi'_{\vec{x}}; t]$

Reduced state of detector: $\rho_R(Q,Q';t) = \mathrm{Tr}_\phi \rho = \int \mathcal{D}\phi\; \Psi[Q,\phi;t]\,\Psi^*[Q',\phi;t]$

— Wigner fn rep: $W(P,Q;t) = \int \underbrace{\mathcal{D}\Delta}_{\sim \prod_j \frac{d\Delta_j}{2\pi\hbar}}\, e^{\cdots}\, \rho(Q-\frac{\Delta}{2}, Q+\frac{\Delta}{2}; t)$

$$1 = \mathrm{Tr}\rho = \int \mathcal{D}P\,\mathcal{D}Q\; W(\vec{P},\vec{Q};t)$$

— (K,Δ)-rep: $\rho(\vec{K},\vec{\Delta};t) = \int \mathcal{D}Q\, e^{\frac{i}{\hbar}\vec{K}\cdot\vec{Q}}\, \rho(\vec{Q}-\frac{\vec{\Delta}}{2}, \vec{Q}+\frac{\vec{\Delta}}{2}; t)$

$$\Rightarrow \mathrm{Tr}\rho = \int \mathcal{D}Q\; \rho(\vec{Q},\vec{Q};t) = \rho(\vec{K},\vec{\Delta};t)\Big|_{\vec{K}=\vec{\Delta}=\vec{0}}$$

Density mtx: $\rho(Q,\phi_{\vec{x}}; Q', \phi'_{\vec{x}}; t) = \Psi[Q,\phi_{\vec{x}}; t]\,\Psi^*[Q',\phi'_{\vec{x}}; t]$

Reduced state of detector: $\rho_R(Q,Q';t) = \mathrm{Tr}_\phi \rho = \int \mathcal{D}\phi\, \Psi[Q,\phi;t]\,\Psi^*[Q',\phi;t]$

(K, Δ)-rep: $\rho(\vec{K}, \vec{\Delta}; t) = \int \mathcal{D}Q\, e^{\frac{i}{\hbar}\vec{K}\cdot\vec{Q}}\,\rho\left(\vec{Q} - \frac{\vec{\Delta}}{2}, \vec{Q} + \frac{\vec{\Delta}}{2}; t\right)$

[1] $\mathrm{Tr}\rho = \int \mathcal{D}Q\,\rho(\vec{Q},\vec{Q};t) = \rho(\vec{K},\vec{\Delta};t)\Big|_{\vec{K}=\vec{\Delta}=\vec{0}}$

[2] Reduced State $\rho_R(K_Q,\Delta_Q;t) = \mathrm{Tr}_\phi\rho = \rho(K_Q,\vec{K}_\phi,\Delta_Q,\vec{\Delta}_\phi)\Big|_{\vec{K}_\phi=\vec{\Delta}_\phi=\vec{0}}$

[3] Generating fn of symmetrized correlator

$$\langle Q_i Q_j\rangle \equiv \langle\underbrace{\{Q_i(t),Q_j(t)\}}_{=\frac{1}{2}(Q_iQ_j+Q_jQ_i)}\rangle = \int\mathcal{D}Q\, Q_i Q_j\,\rho(\vec{Q},\vec{Q};t)$$

Density mtx: $\rho(Q, \phi_{\vec{x}}; Q', \phi'_{\vec{x}}; t) = \Psi[Q, \phi_{\vec{x}}; t]\, \Psi^*[Q', \phi'_{\vec{x}}; t]$

Reduced state of detector: $\rho_R(Q, Q'; t) = \mathrm{Tr}_\phi \rho = \int \mathcal{D}\phi\, \Psi[Q, \phi; t]\, \Psi^*[Q', \phi; t]$

$1 = \mathrm{Tr}\rho = \int \mathcal{D}P\, \mathcal{D}Q\, W(\vec{P}, \vec{Q}; t) \sim \prod_j \frac{d\Delta_j}{2\pi}$

– (K, Δ)-rep.: $\rho(\vec{K}, \vec{\Delta}; t) = \int \mathcal{D}Q\, e^{\frac{i}{\hbar}\vec{K}\cdot\vec{Q}} \rho\left(\vec{Q} - \frac{\vec{\Delta}}{2}, \vec{Q} + \frac{\vec{\Delta}}{2}; t\right)$

$\Rightarrow \mathrm{Tr}\rho = \int \mathcal{D}Q\, \rho(\vec{Q}, \vec{Q}; t) = \rho(\vec{K}, \vec{\Delta}; t)\Big|_{\vec{K} = \vec{\Delta} = \vec{0}}$

$\Big| K_\phi = \Delta_\phi = 0$

[3]. Generating fn of symmetrized correlator.

$\langle Q_i, Q_j \rangle \equiv \langle \{ Q_i(t), Q_j(t) \} \rangle = \int \mathcal{D}Q\, Q_i Q_j\, \rho(\vec{Q}, \vec{Q}; t) = \left[\frac{\hbar \delta}{i \delta K_i} \frac{\hbar \delta}{i \delta K_j} \rho(\vec{K}, \vec{\Delta}; t) \right]_{\vec{K} = \vec{\Delta} = 0}$

$\frac{1}{2}(Q_i Q_j + Q_j Q_i)$

$$\langle P_j \rangle = \left[\frac{i\hbar\,\delta}{\delta \Delta_j} \rho(\vec{K},\vec{\Delta};t) \right]_{\vec{K}=\vec{\Delta}=\vec{0}}$$

$\boxed{4}$ Normalization

Gaussian state: $\rho(K,\Delta;t) = 1 \cdot \exp -\frac{1}{2\hbar}\{K\langle Q^2(t)\rangle K - 2K\langle Q,P\rangle\Delta + \Delta\langle P^2\rangle\Delta\} - \frac{i}{\hbar}K\langle Q\rangle + \frac{i}{\hbar}\Delta\langle P\rangle\}$

$\boxed{5}$ Projection $\hat{\mathbb{P}}_Q = |q\rangle_Q\langle q|$

$\rho_Q(Q,Q') = \langle Q|\hat{\mathbb{P}}_Q|Q'\rangle \longrightarrow \rho_Q(K_Q,\Delta_Q)$

Write $\rho'(K_Q,\Delta_Q,\vec{K}_\phi,\vec{\Delta}_\phi) = \rho_Q(K_Q,\Delta_Q)\int \frac{d\bar{K}_Q}{2\pi\hbar}\, d\bar{\Delta}_Q\, \rho_Q^*(\bar{K}_Q,\bar{\Delta}_Q)\,\rho(\bar{K}_Q,\bar{\Delta}_Q,K_\phi,\Delta_\phi;t)$

CAUTION

III mode fns.

$$\hat{Q}(\tau) = \xi^{Q}(\tau)\hat{Q}_{I} + \xi^{P}(\tau)\hat{P}_{I} + \int d^3x \left(\xi^{\phi}(\vec{x},\tau)\hat{\phi}^{I}_{\vec{x}} + \xi^{\pi}(\vec{x},\tau)\hat{\pi}^{I}_{\vec{x}} \right)$$

↑initial

$$= \sqrt{\frac{\hbar}{2\Omega_r m_0}}\left[\xi^{a}(\tau)\hat{a}_I + \xi^{a*}(\tau)\hat{a}^{\dagger}_I\right] + \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega}}\left[\xi^{\vec{k}}(\tau)\hat{b}^{I}_{\vec{k}} + \xi^{\vec{k}*}(\tau)\hat{b}^{I\dagger}_{\vec{k}}\right]$$

$$\hat{\phi}_{\vec{x}}(t) = \quad " \quad \phi^{a}_{\vec{x}}(\tau) \quad " \quad \phi^{a*}_{\vec{x}}(\tau) \quad " \quad " \quad \phi^{\vec{k}}_{\vec{x}}(t) \quad " \quad \phi^{\vec{k}*}_{\vec{x}}(t)$$

$$\exists.\quad \partial_\tau^2 \xi^{\Xi}(\tau) + \Omega_0^2 \xi^{\Xi}(\tau) = \frac{\lambda_0}{m_0}\chi(t)\,\phi^{\Xi}_{\vec{z}(\tau)}(z^0(\tau)) \qquad \Xi = a, \{\vec{k}\}$$

$$\phi^{\bar{x}}_{\vec{x}}(t) = \phi^{\bar{x}(0)}_{\vec{x}}(t) + \lambda_0 \int d\tau \, G_{ret}(x - z(\tau))\, \chi(\tau)\, q^{\bar{x}}(\tau)$$

$$\left\{ \begin{array}{l} \phi^{q(0)}_{\vec{x}} = 0, \quad \phi^{\vec{k}}_{\vec{x}}(t) = e^{-i\omega t + i\vec{k}\cdot\vec{x}} \\ G_{ret}(x - x') = \dfrac{1}{4\pi}\, \delta(\sigma)\, \theta(t - t') \end{array} \right.$$

← Assuming all modes have been included

$$\sigma = \frac{-1}{2}(x_\mu - x'_\mu)(x^\mu - x'^\mu)$$

Synge world fn.

$$\mathrm{Im} \, \langle [\phi_{\vec{x}}(t), \phi_{\vec{x}'}(t')] \rangle$$

$$\sim \int \frac{d^3k}{(2\pi)^3} \left(\cdots \right)$$

$$\phi^{\gamma}_{\vec{x}}(t) = \phi^{\gamma(0)}_{\vec{x}}(t) + \lambda_0 \int d\tau \, G_{ret}(x - z(\tau)) \, \chi(\tau) \, q^{\gamma}(\tau)$$

$$\left\{ \begin{array}{l} \phi^{q(0)}_{\vec{x}} = 0, \quad \phi^{\vec{k}}_{\vec{x}}(t) = e^{-i\omega t + i\vec{k}\cdot\vec{x}} \\ G_{ret}(x - x') = \frac{1}{4\pi} \delta(\sigma) \, \theta(t - t') \end{array} \right.$$

← Assuming all modes have been included

$$\sigma = \frac{-1}{2}(x_\mu - x'_\mu)(x^\mu - x'^\mu)$$

Synge world fn.

$$\mathrm{Im} \, \langle [\phi_{\vec{x}}(t), \phi_{\vec{x}'}(t')] \rangle$$

$$\sim \int \frac{d^3k}{(2\pi)^3} (\cdots)$$

[2] Renormalization

$$\hat{G}_{ret}(x-x') \equiv \frac{1}{4\pi}\sqrt{\frac{8}{\pi}}\Lambda^2 e^{-2\Lambda^4\sigma^2}\theta(t-t') \xrightarrow{\Lambda\to\infty} G_{ret}$$

$$\Rightarrow \phi^{\mathbf{x}}_{\vec{z}(\tau)}(z^0(\tau)) \sim \lambda_0\int d\tau\, \hat{G}_{ret}(x-z(\tau))\,\chi(\tau)\,\xi^{\mathbf{x}}(\tau)$$

$$\approx \frac{\lambda_0}{4\pi}\left[\Lambda\varsigma\,\xi^{\mathbf{x}}(\tau) - \partial_\tau\xi^{\mathbf{x}}(\tau) + O(\Lambda^{-1})\right]$$

ς: Some const

$\Lambda \gg 1$

$$① \Rightarrow \partial_\tau^2\xi^{\mathbf{x}}(\tau) + \Omega_0^2\xi^{\mathbf{x}}(\tau) \approx \frac{\lambda_0}{m_0}\chi(t)\left[\varphi^{\mathbf{x}(0)}_{\vec{z}(\tau)}(z^0(\tau)) + \frac{\lambda_0}{4\pi}\Lambda\varsigma\,\xi^{\mathbf{x}}(\tau) - \frac{\lambda_0}{4\pi}\partial_\tau\xi^{\mathbf{x}}(\tau)\right]$$

$\Lambda \gg 1$

Some const

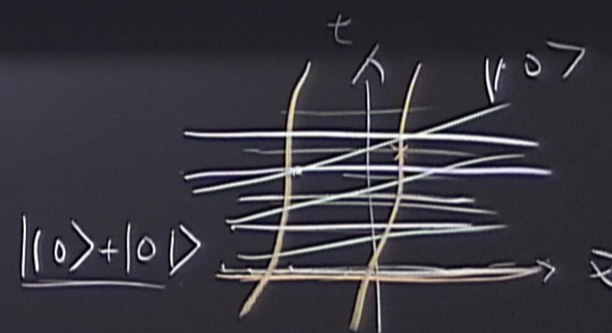
① $\Rightarrow$ $\partial_\tau^2 \delta z^{\Delta}(\tau) + \Omega_0^2\, \delta z^{\Delta}(\tau) \approx \frac{\lambda}{m_0}\chi(t)\left[\varphi^{\Delta(0)}_{\vec{z}(\tau)}(z^\mu(\tau)) + \frac{\lambda_0}{4\pi}\Lambda\varsigma\, \delta z^{\Delta}(\tau) - \frac{\lambda_0}{4\pi}\partial_\tau \delta z^{\Delta}(\tau)\right]$

Assume sudden-switch $\chi(t) = \Theta(t)$

$$\left(\partial_\tau^2 + 2\gamma\partial_\tau + \Omega_r^2\right)\delta z^{\Delta}(\tau) = \frac{\lambda_0}{m_0}\varphi^{\Delta(0)}_{\vec{z}(\tau)}(z^\mu(\tau))$$

$\gamma = \frac{\lambda_0^2}{8\pi m_0}$, $\Omega_r^2 = \Omega_0^2 - \frac{\lambda_0^2}{4\pi m_0}\Lambda\varsigma$

[1] Choose a foliation



[2] Equal-time

$[\hat{Q}(\tau(t)), \hat{P}(\tau(t))] = i\hbar$

$[\hat{\Phi}_{\vec{x}}(t), \hat{\pi}_{\vec{x}'}(t)] = i\hbar\,\delta^3(\vec{x} - \vec{x}')$

$\hat{H}[\hat{Q}, \hat{P}, \hat{\pi}, \hat{\Phi}]$

$\Lambda \gg 1$

Some const

$\textcircled{1} \Rightarrow \quad \partial_\tau^2 \xi^{\delta}_{6}(\tau) + \Omega_0^2 \xi^{\delta}_{6}(\tau) \approx \frac{\lambda_0}{m_0}\chi(t)\left[\varphi^{\delta(0)}_{\vec{z}(\tau)}(z^0(\tau)) + \frac{\lambda_0}{4\pi}\Lambda\varsigma\, \xi^{\delta}_{6}(\tau) - \frac{\lambda_0}{4\pi}\partial_\tau \xi^{\delta}_{6}(\tau)\right]$

Assume sudden-switch $\chi(t) = \Theta(t)$

$$\left(\partial_\tau^2 + 2\gamma\partial_\tau + \Omega_r^2\right)\xi^{\delta}_{6}(\tau) = \frac{\lambda_0}{m_0}\varphi^{\delta(0)}_{\vec{z}(\tau)}(z^0(\tau))$$

$\gamma = \frac{\lambda_0^2}{8\pi m_0}$, $\quad \Omega_r^2 = \Omega_0^2 - \frac{\lambda_0^2}{4\pi m_0}\Lambda\varsigma$

$$\Rightarrow \quad \xi^{\delta}_{6}(\tau) = \xi^{\delta(0)}(\tau) + \frac{\lambda_0}{m_0}\int d\tau' \underbrace{\frac{1}{\Omega}K(\tau-\tau')}_{\text{Kernel}}\varphi^{\delta(0)}_{\vec{z}(\tau)}(z^0(\tau))$$

Kernel. $K \equiv e^{-\gamma(\tau-\tau')}\sin\Omega(\tau-\tau')$

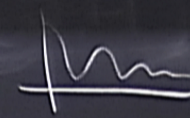
$\boxed{3}$ Suppose $|\Psi_I\rangle \equiv |E_0\rangle_Q \otimes |0_M\rangle_\phi$

$$\langle Q^2(\tau-\tau_I)\rangle = \lim_{\substack{\tau'\to\tau \\ \tau_I'\to\tau_I}} \langle\Psi_I|\frac{1}{2}\left(\hat{Q}(\tau)\hat{Q}(\tau') + \hat{Q}(\tau')\hat{Q}(\tau')\right)|\Psi_I\rangle$$

$\chi(t) = \Theta(t)$ $\qquad \frac{\lambda_0^2}{8\pi m_0}$ $\qquad \Omega_0^2 - \frac{\lambda_0^2}{4\pi m_0}\Lambda$

$= \langle Q(\tau-\tau_I)\rangle_a + \langle Q(\tau-\tau_I)\rangle_v$

$$\langle Q^2(\tau-\tau_I)\rangle_a = \frac{\hbar}{2\Omega_r m_0}\left|g^a(\tau-\tau_I)\right|^2 .$$



$$\underline{\langle Q^2(\tau-\tau_I)\rangle_v} = \int \frac{d^3k}{(2\pi)^3}\frac{\hbar}{2\omega}\left|g^{\vec{k}}(\tau)\right|^2 \longrightarrow \text{UV div}$$

(IR, too in (1+1) D)

$$= \int d\tau' d\tau'' \frac{\lambda_0^2}{m_0^2\Omega^2} K(\tau-\tau')K(\tau-\tau'') \times \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}\phi^{*\vec{k}(0)}(\tau')\phi^{\vec{k}(0)}(\tau'')$$

Λ_0 : time-scale of switch-on

Λ_1 : time-resolution of detector

$$\langle Q^2(\tau-\tau_I)\rangle_v = \int \frac{d^3k}{(2\pi)^3}\frac{\hbar}{2\omega}\left|\xi^{\vec{k}}(\tau)\right|^2 \longrightarrow \text{UV div}$$

(IR, too in (1+1) D)

$$= \int d\tau' d\tau'' \frac{\lambda_0^2}{m_0^2\Omega^2} K(\tau-\tau')K(\tau-\tau'') \times \int \frac{d^3k}{(2\pi)^3 2\omega}\phi^{*\vec{k}(0)}(\tau')\,\phi^{\vec{k}(0)}(\tau'')$$

$$= D^+(z(\tau'), z(\tau''))$$

Λ_0 : time-scale of switch-on

Λ_1 : time-resolution of detector

$\tau' \to \tau''$