

Title: Majorana Neutrinos with a look at the Sky

Date: May 26, 2015 10:00 AM

URL: <http://pirsa.org/15050105>

Abstract:

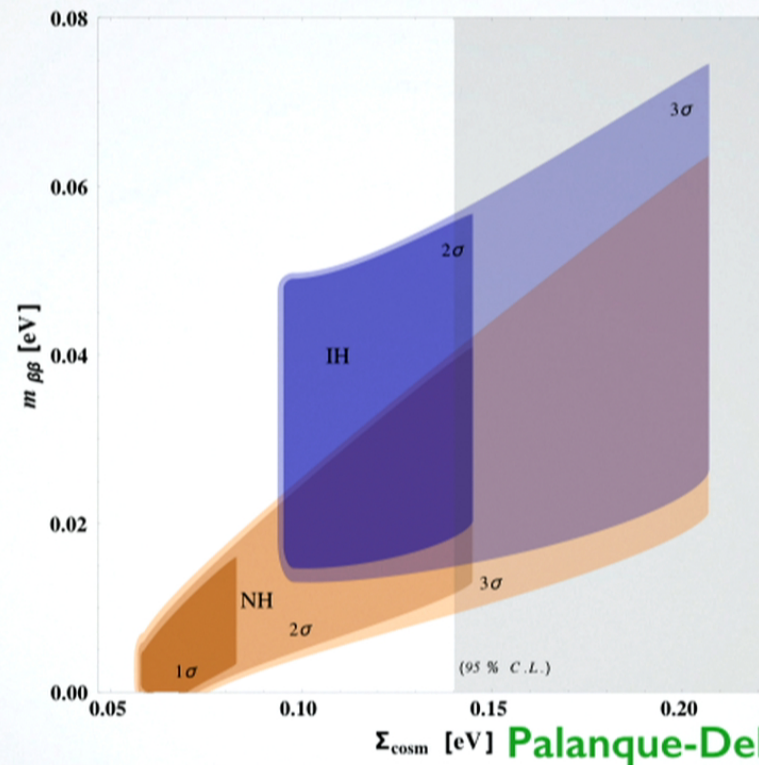
Massive neutral fermions can be realized in Nature as Majorana particles. Are neutrinos Majorana particles? Our most accepted theoretical prejudice can be verified by searching for neutrinoless double-beta decay. I will overview the current knowledge of the neutrino mass spectrum and discuss theoretical scenarios where cosmological data can contribute to resolve this challenging question. Some cosmological observables sensitive to neutrino masses are outlined.

WHERE ARE WE GOING?

Very aggressive limit by Lyman forest data from BOSS

Palanque-Delabrouille et al, 2014

If correct, very interesting situation if $0\nu\beta\beta$ is discovered soon



CONCLUSIONS

Discovery of neutrino masses by neutrino oscillations (flavor conversion) opens the possibility to test whether neutrinos are Majorana fermions or not.

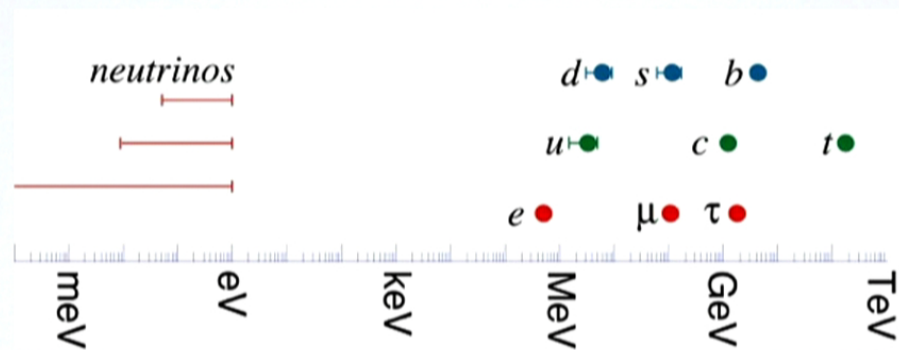
Lots of work ahead:

- neutrinoless double-beta decay experiments
- more precise matrix elements
- neutrino mass measurements (lab)
- neutrino mass measurements (cosmos)

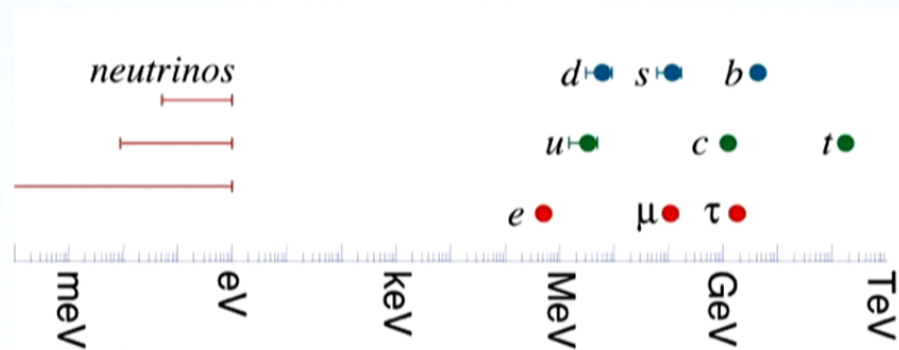
On the way, we may be surprised by finding BSM total lepton number violating currents.

PART I: MASS & MIX
PART II: MAJORANA
PART III: COSMIC ν

PART I: MASS & MIX



PART I: MASS & MIX

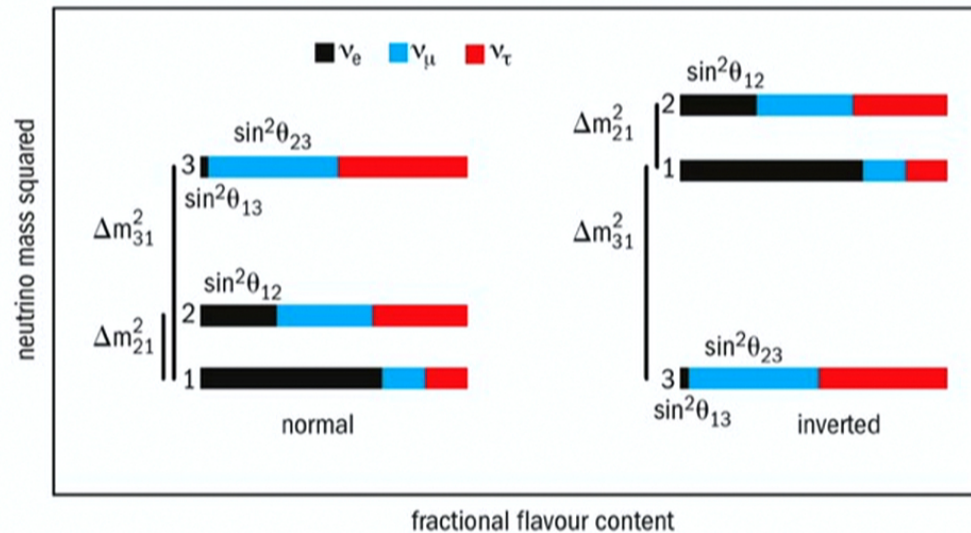


PHYSICS INSENSITIVE TO M

Physical parameters mass matrix: 3 masses, 3 angles, 1 phase

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

$$J_{CP}^{\max} = \cos \theta_{12} \sin \theta_{12} \cos \theta_{23} \sin \theta_{23} \cos^2 \theta_{13} \sin \theta_{13}$$

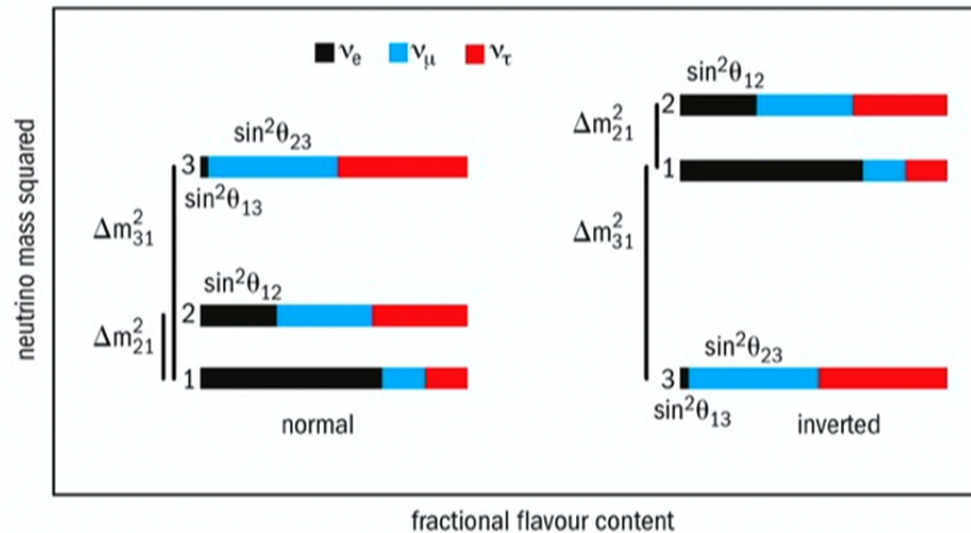


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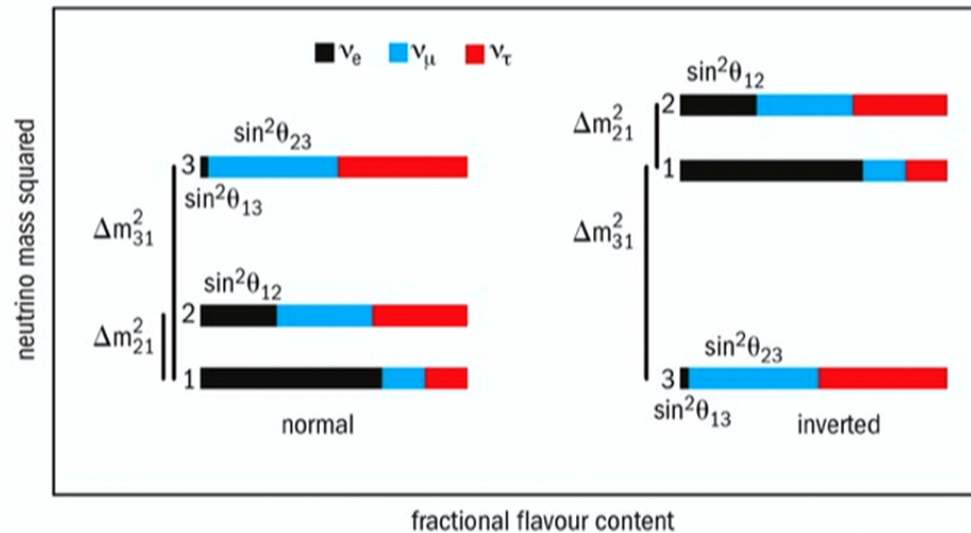


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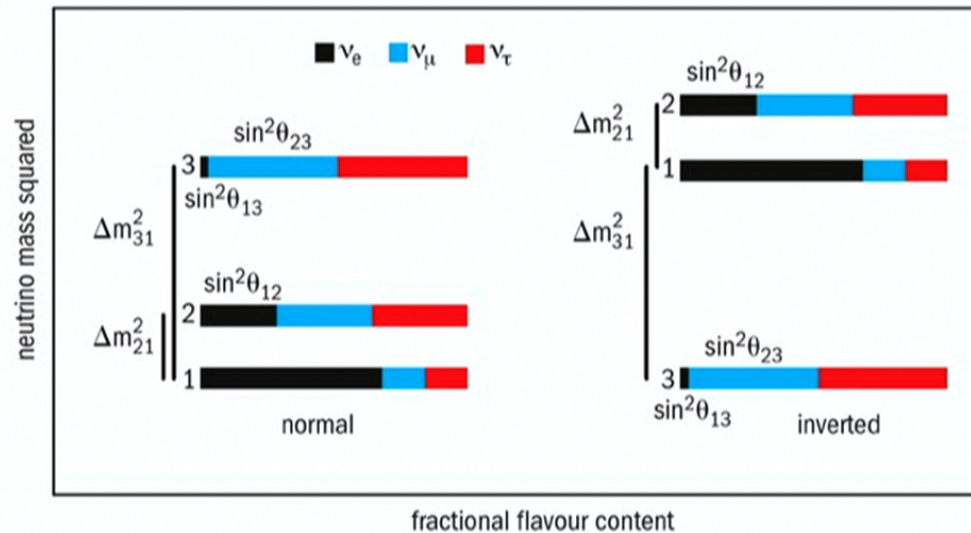


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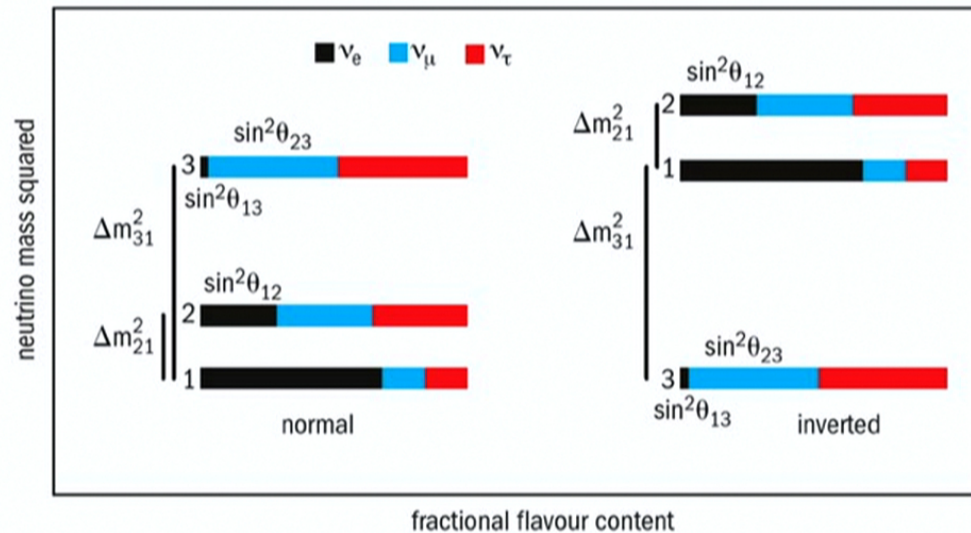


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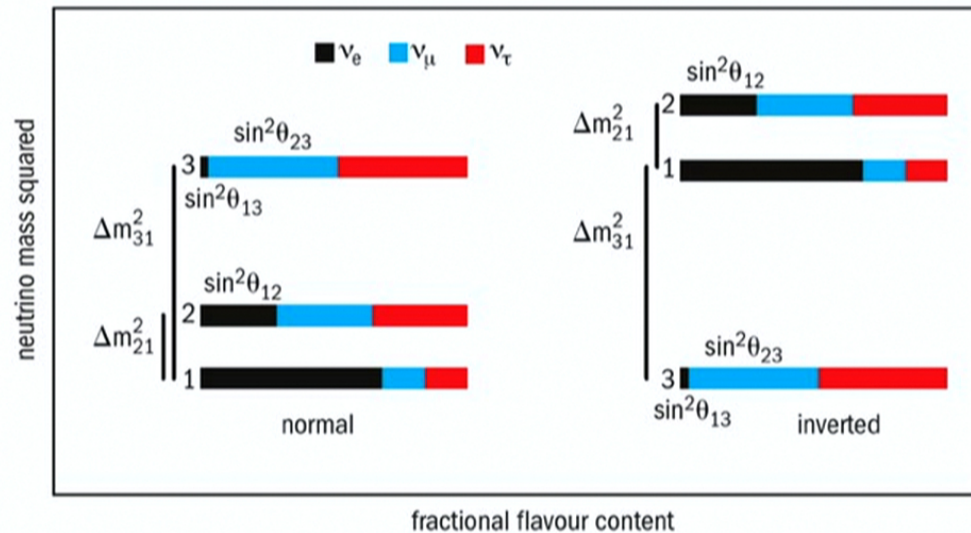


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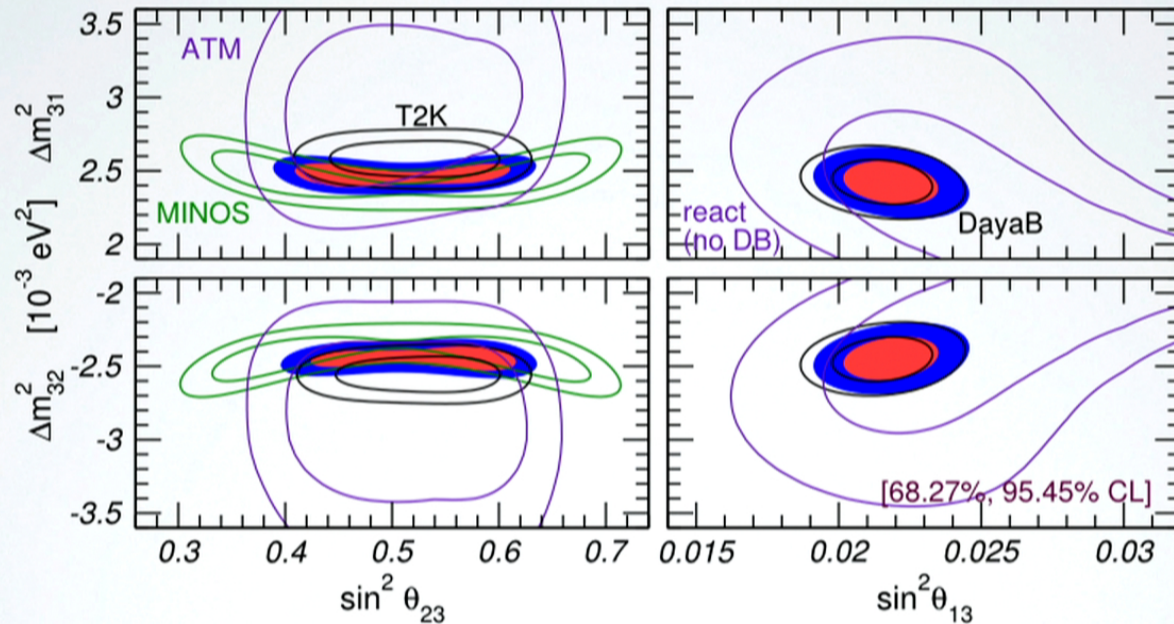
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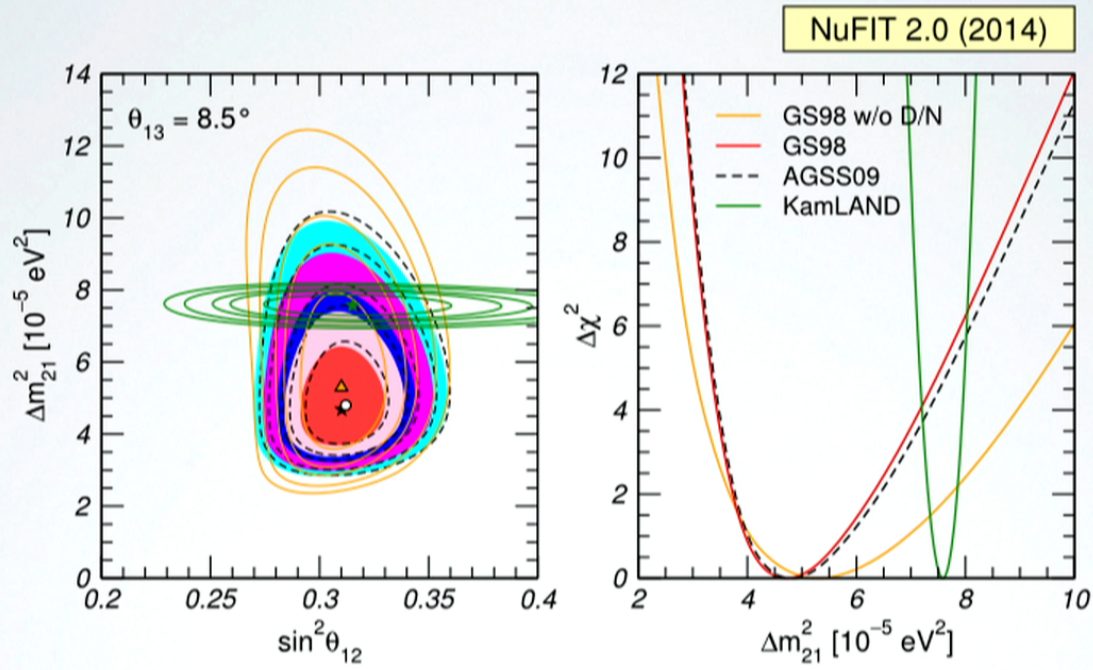
LARGER MASS SPLITTING

NuFIT 2.0 (2014)



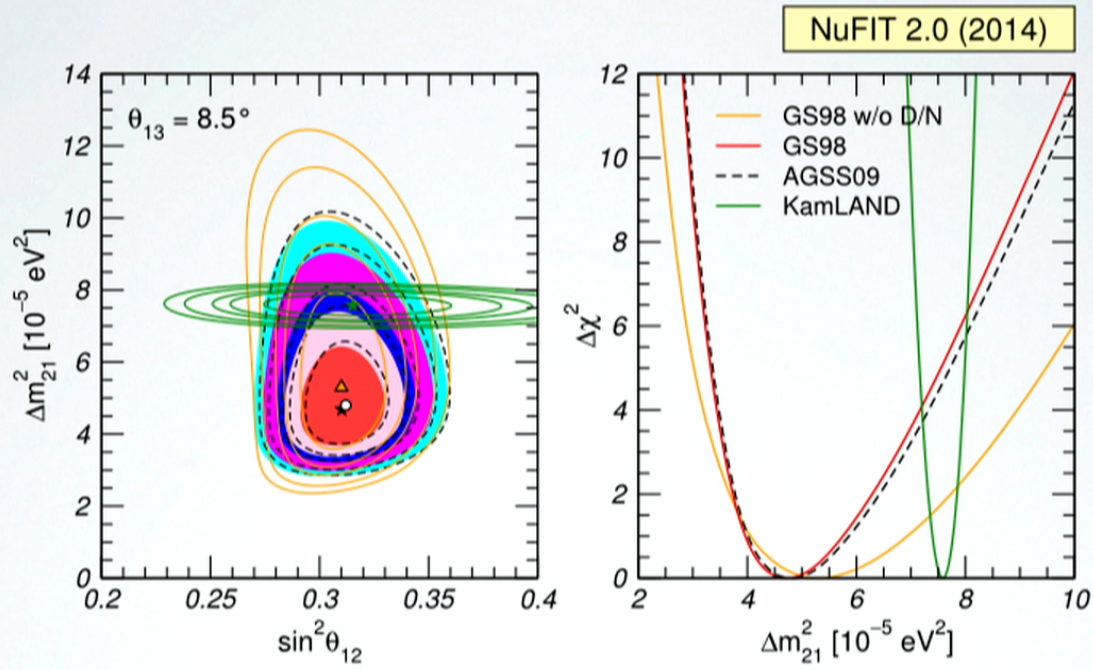
Atmospheric + accelerator + reactor neutrino data: compatible.
Larger mass splitting, one large + one small mixing angles
Appearance in LBL + reactor: weak hints to CP phase

SMALLER MASS SPLITTING



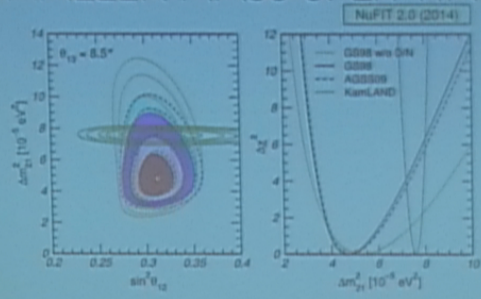
Solar neutrinos + long baseline reactor data: Compatible
Smaller mass splitting, large mixing angle
2-sigma tension in mass splitting (DN data vs reactor spectrum)

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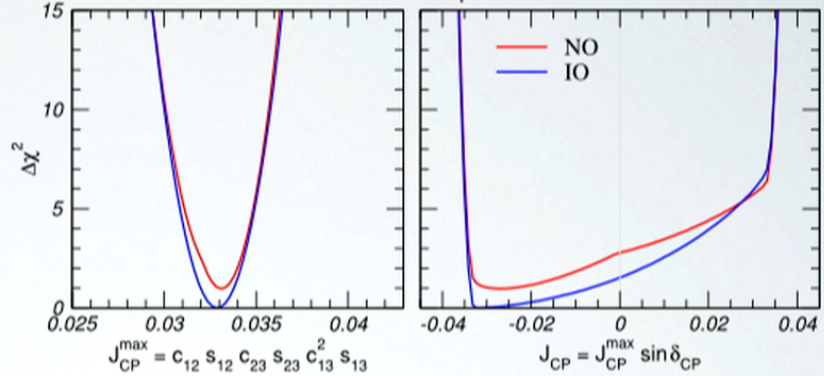
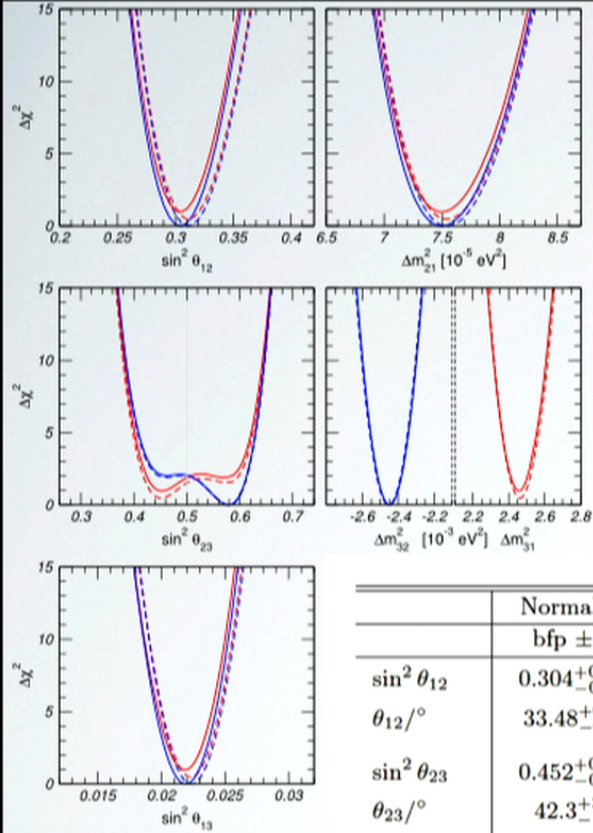
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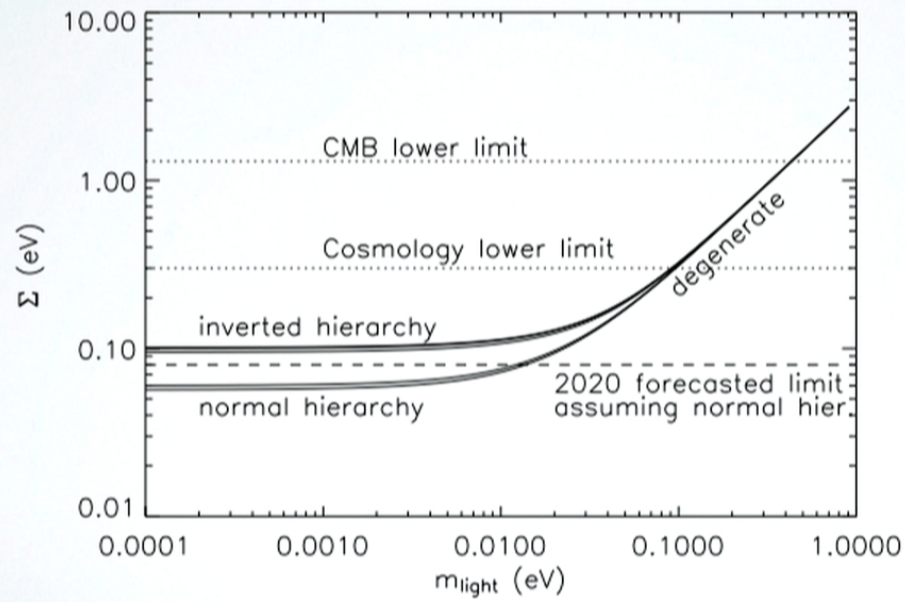
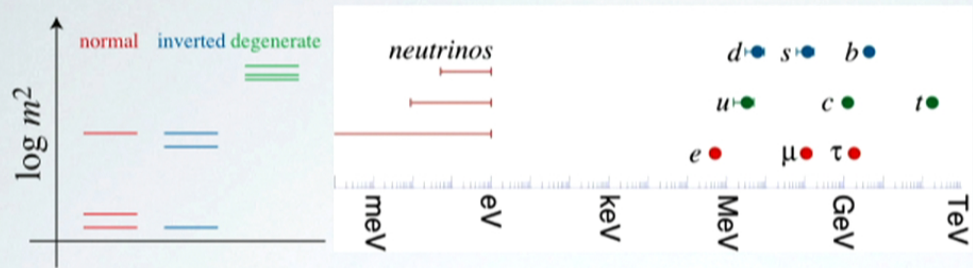
NuFIT 2.0 (2014)



Gonzalez-Garcia et al, 2014

	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering 3σ range
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \rightarrow 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \rightarrow 0.344$	$0.270 \rightarrow 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \rightarrow 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \rightarrow 35.91$	$31.29 \rightarrow 35.91$
$\sin^2 \theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \rightarrow 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \rightarrow 0.644$	$0.385 \rightarrow 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \rightarrow 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \rightarrow 53.3$	$38.3 \rightarrow 53.3$
$\sin^2 \theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \rightarrow 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \rightarrow 0.0251$	$0.0188 \rightarrow 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \rightarrow 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \rightarrow 9.11$	$7.87 \rightarrow 9.11$
$\delta_{CP}/^\circ$	306^{+39}_{-70}	$0 \rightarrow 360$	254^{+63}_{-62}	$0 \rightarrow 360$	$0 \rightarrow 360$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \rightarrow 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \rightarrow 8.09$	$7.02 \rightarrow 8.09$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \rightarrow +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \rightarrow -2.307$	$[+2.325 \rightarrow +2.599]$ $[-2.590 \rightarrow -2.307]$

IN SUMMARY:



HOW

are generated these patterns of masses and mixings?

Strategy: lepton-quark differences may be simpler to explain than detailed pattern of masses and mixings.

SM is defined by the gauge symmetries and the field content. Anything allowed by the symmetries occurs in Nature. Because of neutrino masses, we add right handed neutrinos. Following our prejudice, the Majorana mass term should exist.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - m_D \bar{\nu}_R \nu_L + \text{h.c.} + 1/2 m_R \bar{\nu}_R^T C^\dagger \nu_R + \text{h.c.}$$

Majorana mass term, Majorana neutrinos and Lepton Number Violation are connected.

PART II: MAJORANA

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

Majorana (sub. by Fermi), 1937

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di DIRAC ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

MAJORANA MASS TERM

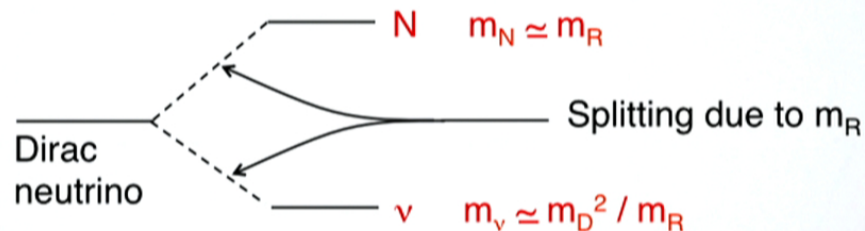
Neutrinos: Real Majorana spinors of mass m

$\underline{\nu}_R$ and $\underline{\nu}_R^c$

are not mass eigenstates. Therefore, the Majorana mass term induces mixing of the two states, i.e., the mass eigenstate is a combination of both (neutrino and antineutrino).

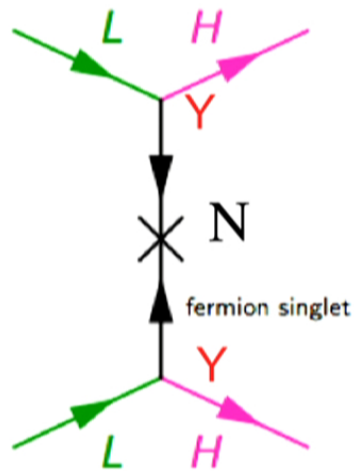
The Majorana mass term (non protected by SM symmetries) splits a Dirac neutrino into two Majorana neutrinos

$$\mathcal{L} = \mathcal{L}_{SM} - 1/2 (\underline{\nu}_L^c \underline{\nu}_R) [(0 \ m_D)(m_D \ m_R)] (\underline{\nu}_L \ \underline{\nu}_R^c)^T + \text{h.c.}$$



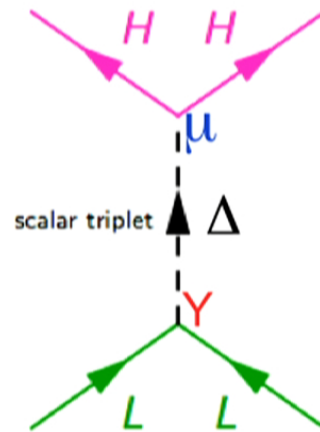
The Seesaw models

- Three types of models yield the Weinberg operator at tree level



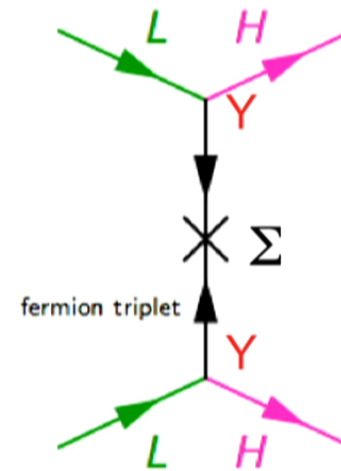
Type I

$$m_\nu \sim v^2 Y_N^T \frac{1}{M_N} Y_N$$



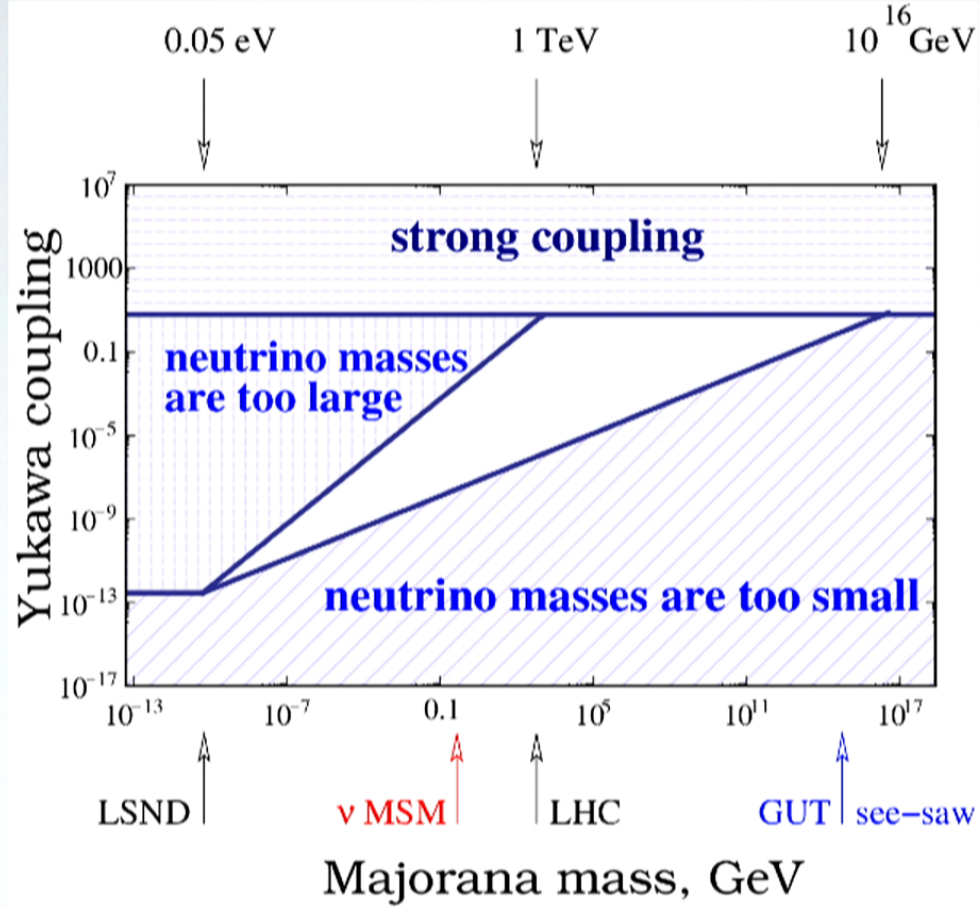
Type II

$$m_\nu \sim v^2 Y_\Delta \frac{\mu}{M_\Delta^2}$$

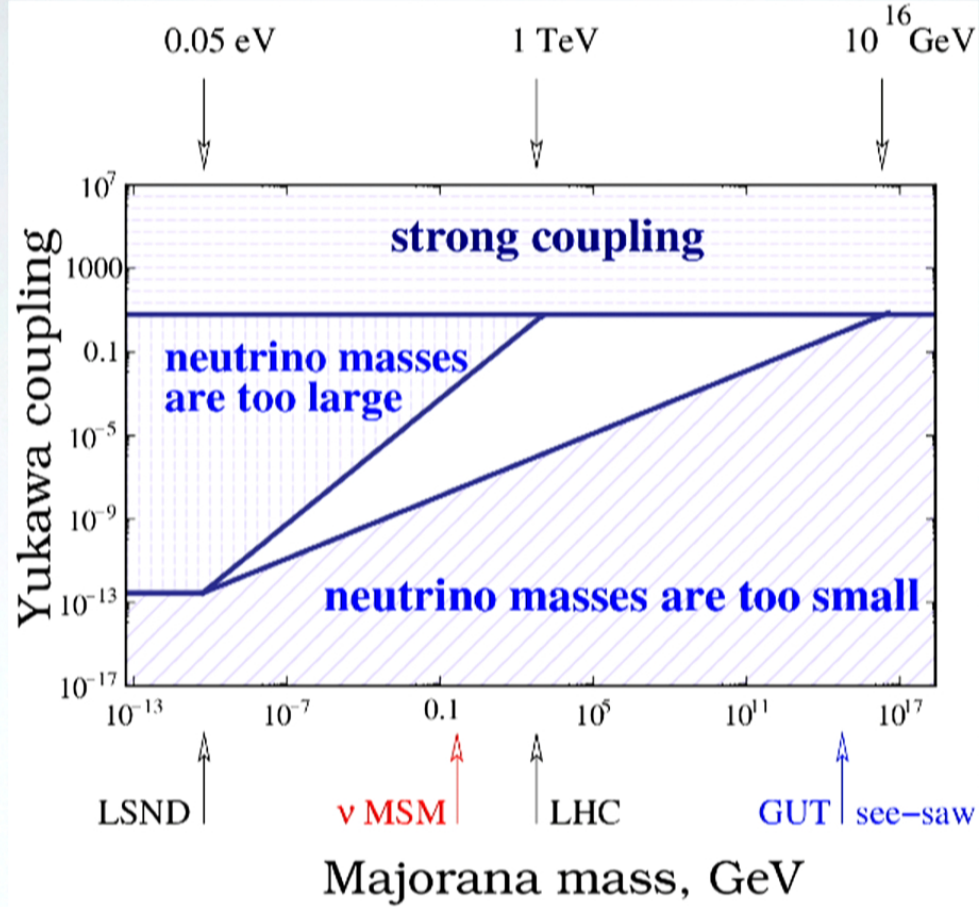


Type III

$$m_\nu \sim v^2 Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$



Majorana mass is unknown. GUT scale ?



Majorana mass is unknown. GUT scale ?

HOW MANY STATES AND PARAMETERS? MINIMAL MODELS

Most general (renormalizable) Lagrangian compatible with SM gauge symmetries:

$$\mathcal{L} = \mathcal{L}_{SM} - \sum_{i=1}^{n_R} \bar{l}_L^\alpha Y^{\alpha i} \tilde{\Phi} \nu_R^i - \sum_{i,j=1}^{n_R} \frac{1}{2} \bar{\nu}_R^{ic} M_N^{ij} \nu_R^j + h.c.$$

$Y: 3 \times n_R$

$M_N: n_R \times n_R$

Hernandez et al, 2011

Number of Physical Parameters

n_R	L_i	# zero modes	# masses	# angles	# CP phases	
1	-	2	2	2	0	→ 3+1 minimal
	+1	2	1	2	0	→ 1 Dirac
2	-	1	4	4	3	→ 3+2 minimal
	(+1,+1)	1	2	3	1	→ 2 Dirac
	(+1,-1)	3	1	3	1	
3	-	0	6	6	6	
	(+1,+1,+1)	0	3	3	1	→ 3 Dirac
	(+1,-1,+1)	2	2	6	4	
	(+1,-1,-1)	4	1	4	1	

Complexity ↓
↑ predictivity

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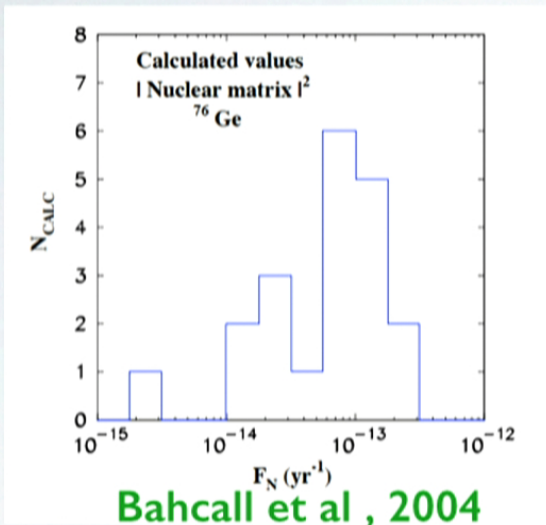
Hernandez et al, 2011

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DIFFICULTIES: CONTROL TH. SYSTEMATICS



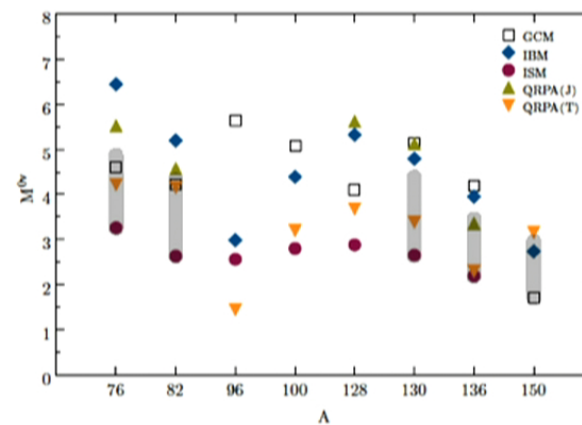
New methods and revisited old methods had improved, but still large errors

Gomez-Cadenas et al , 2001 I

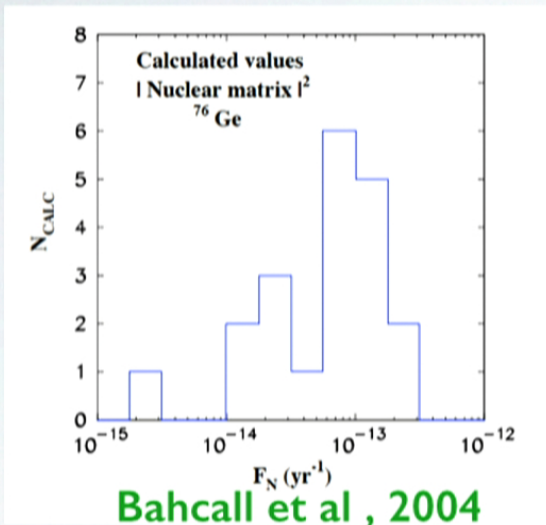
$0\nu\beta\beta: nn \ ppe^-e^-$ without neutrinos

$$|\langle m^{vee} \rangle| = \frac{m_e}{(T_{1/2} F_N)^{1/2}}$$

$$F_N = G^{0\nu} |M^{0\nu}_f - (g_A/g_V)^2 M^{0\nu}_{GT}|^2$$



DIFFICULTIES: CONTROL TH. SYSTEMATICS



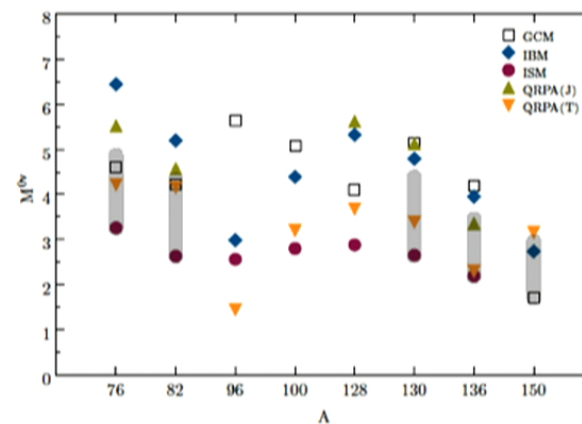
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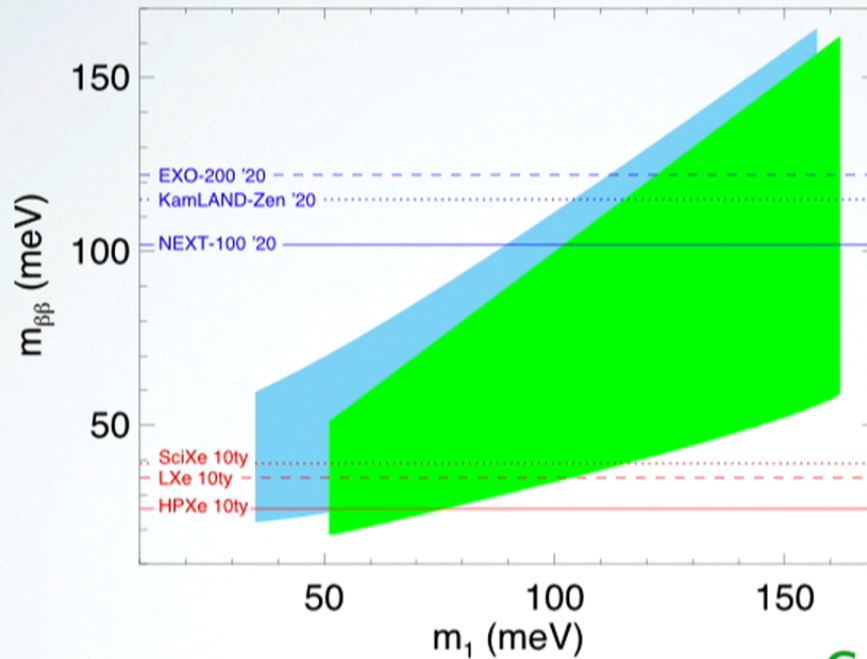


$0\nu\beta\beta$

$U_{ei} \bar{\nu}_i$ is emitted [RH + O{ m_i/E }LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\text{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta} \quad |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}|$$



For illustration,
assumed claim of
neutrino mass
detection by
cluster counts

Gomez-Cadenas et al , 2013

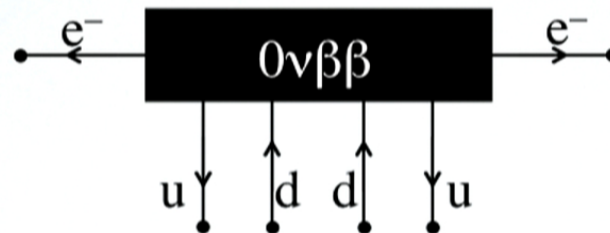
Assumed: light Majorana neutrinos source $0\nu\beta\beta$

BSM (heavy nus, LR, ...) may also contribute to the signal.

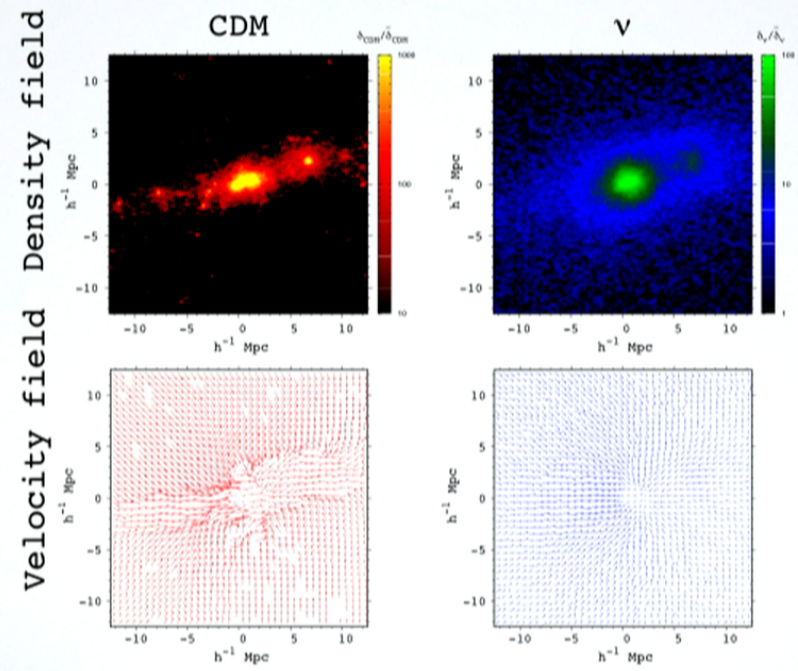
THEOREM FOR SM (AND BSM) GAUGETH.

Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a **Majorana mass term**:

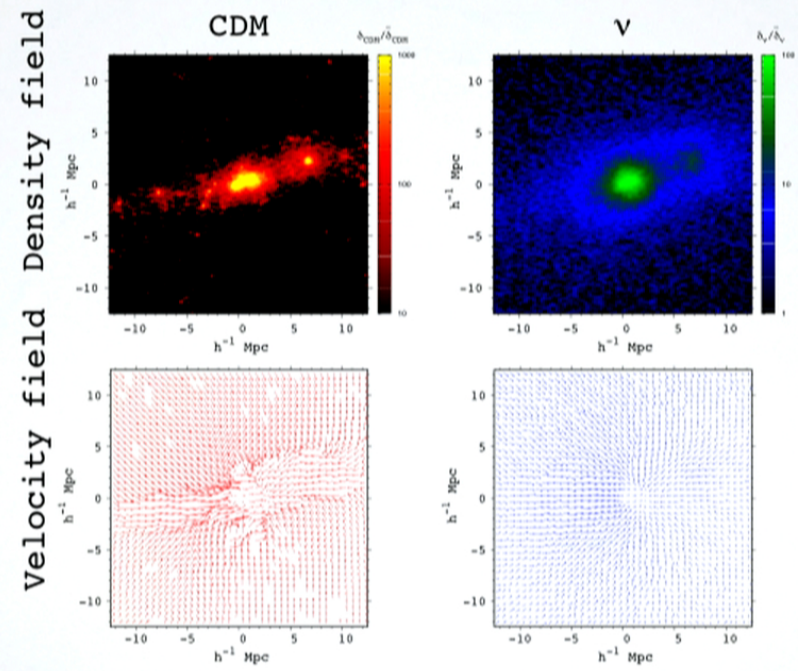
Schechter and Valle, 1982



PART III: COSMIC $\sqrt{\quad}$



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DIRECT NEUTRINO MASS MEASUREMENTS

Methods:

Spectroscopic: single beta decay, neutrino mass is in the phase factor for the antineutrino. Sensitivity depends on the fraction of events close to the end-point. KATRIN (tritium). Goal: 0.6 eV

Calorimetric: Electron capture decays with low Q-value. Same neutrino phase space factor appears, steeper Q dependence. HOLMES, ECHO, NUMECS (^{163}Ho). Goal: close to Katrin

Frequency techniques: Beta decay. Emission of radiation by the electron in a cyclotron. Project 8 ($^{83\text{m}}\text{Kr}$). Goal: Test method

Neutrino mass is $[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2]^{1/2}$

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CNB

Radiation density after neutrino decoupling and BBN ($T < m_e$)

$$f_\nu(p, T) = \frac{1}{e^{p/T} + 1}$$

$$\rho_r = \rho_\gamma + \rho_\nu = \frac{\pi^2}{15} T_\gamma^4 + 3 \times \frac{7}{8} \times \frac{\pi^2}{15} T_\nu^4 = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right] 3 \rho_\gamma$$

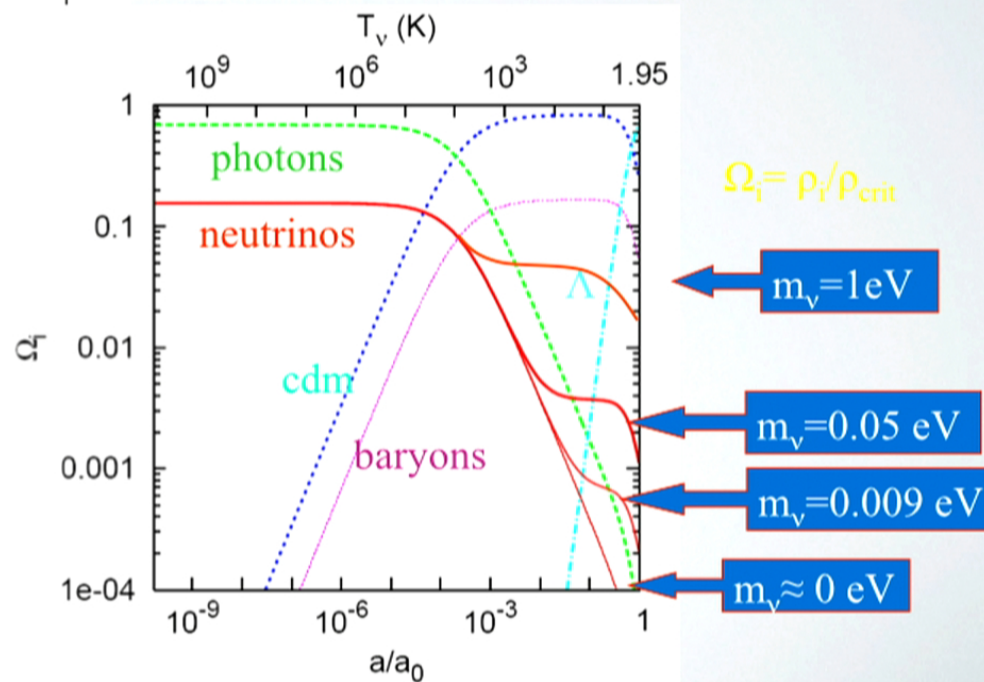
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Mangano et al, 2005

Today,
 339 cm^{-3}
 at 1.95 K
 (0.17 meV)

$$0.06 \text{ (0.1) eV} \lesssim \sum_i m_i \lesssim 6 \text{ eV}$$

$$0.0013 \text{ (0.0022)} \lesssim \Omega_\nu \lesssim 0.13$$



CNB

Radiation density after neutrino decoupling and BBN ($T < m_e$)

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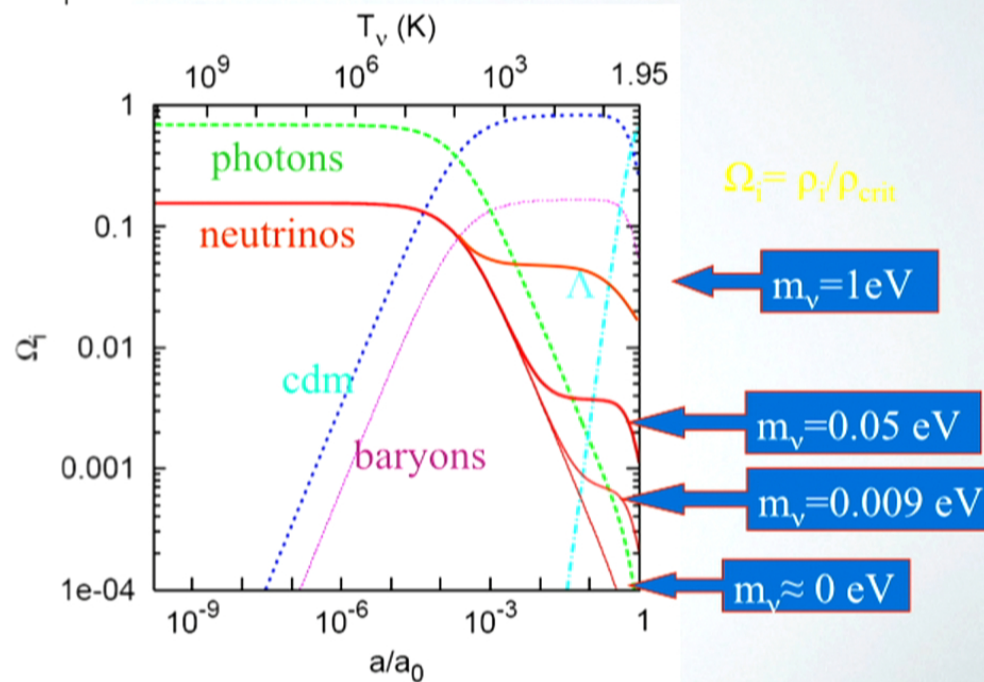
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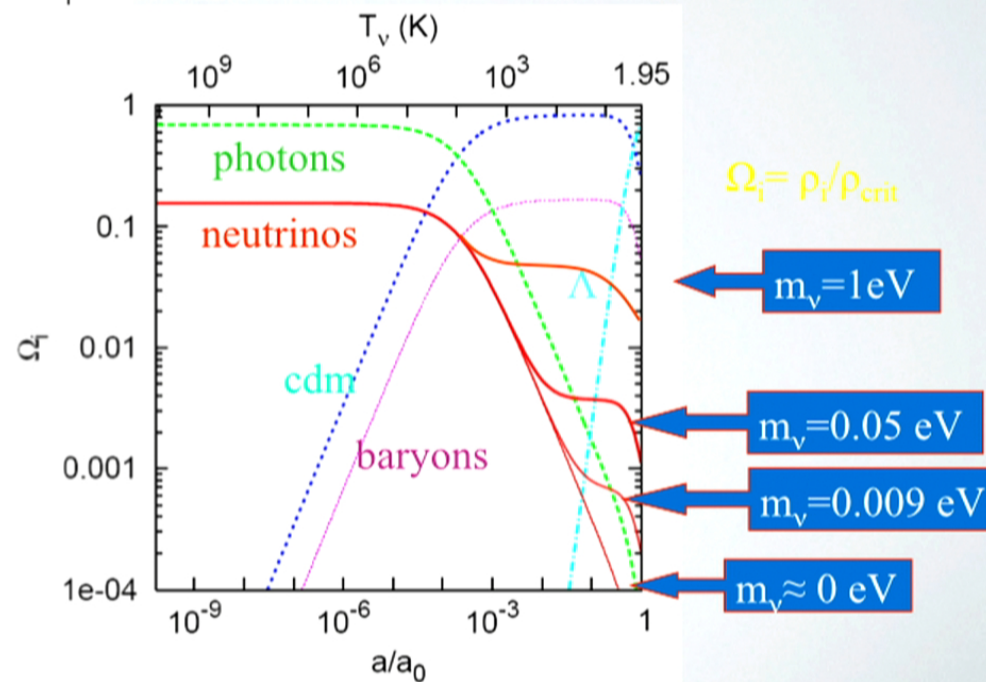
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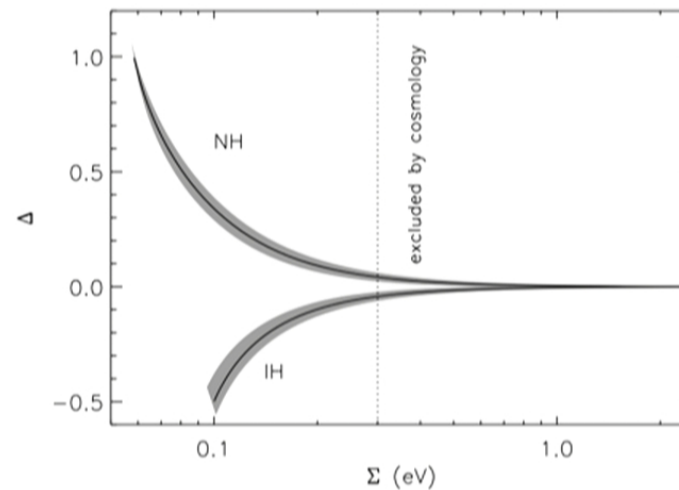
PARAMETERS IN ORDER OF RELEVANCE

Usually, only total mass is included in cosmological data analysis. We are coming close to the parameter region where data may be sensitive to more than one mass.

Neglect solar splitting is still a very good approximation.

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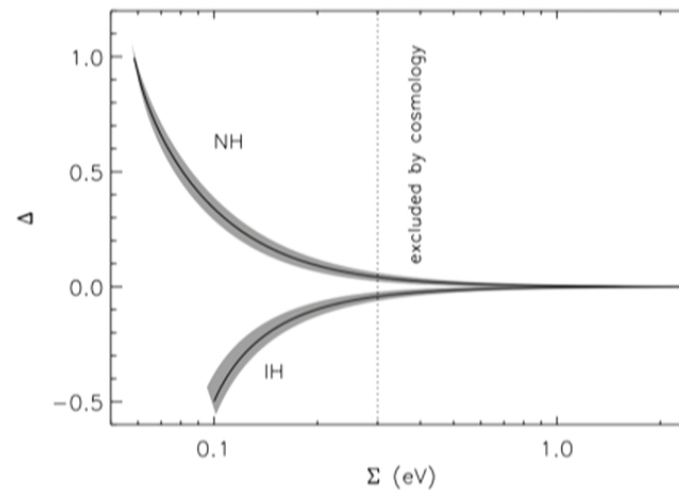
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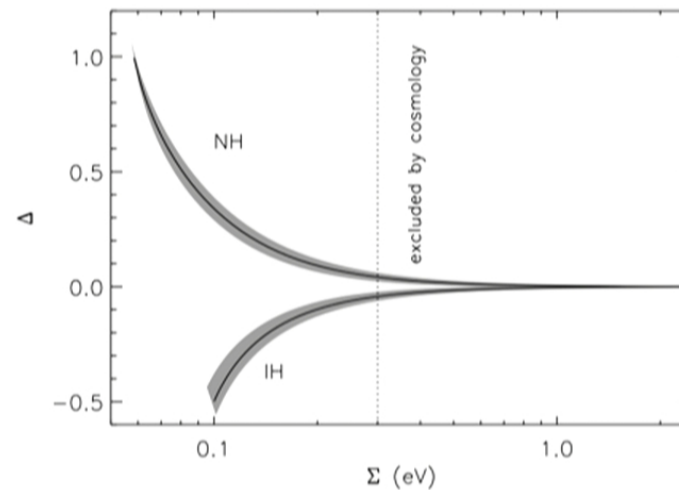
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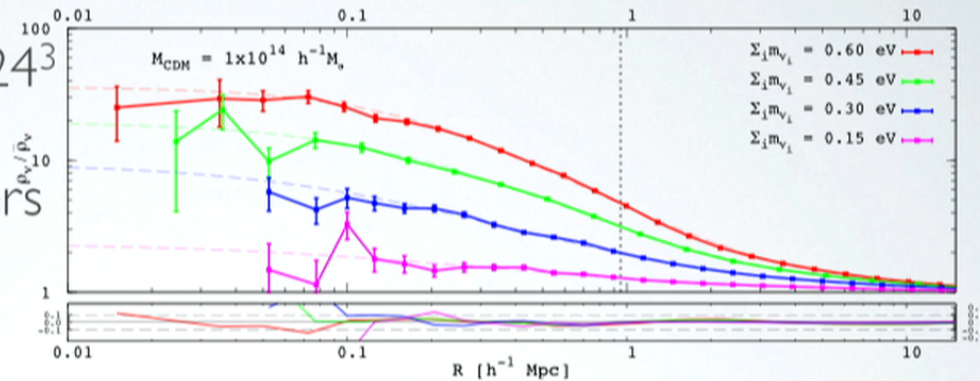
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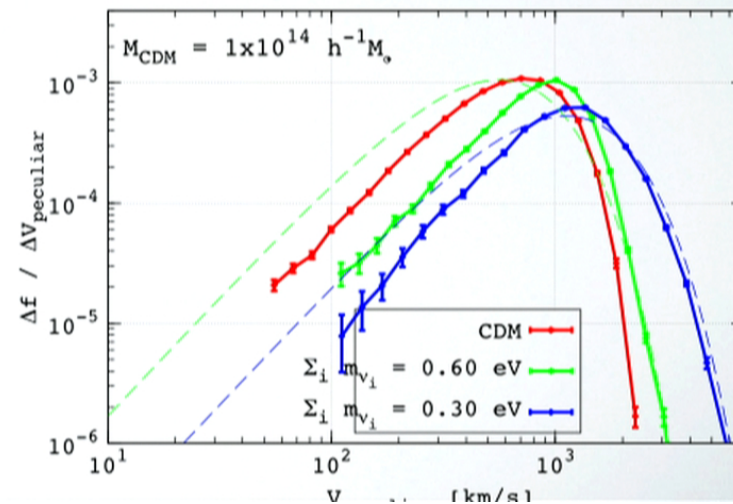
NONLINEAR REGIME

N-body simulations: Corrections to the standard results after including gravity
 Boxes with $512^3 \times 1024^3$ particles
 Overdensity in clusters of galaxies



Villaescusa-Navarro et al, 2013

Distorsion of the Fermi-Dirac distribution in clusters of galaxies.



POTENTIAL SIGNALS

Weak lensing and 21 cm
Neutrino wakes

Zhu et al, 2015
Zhu et al, 2014

