Title: Theories of heat as inspiration for electrodynamics: From Kelvin to QFT

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Abstract: Perhaps the first use of the mathematical theory of heat to develop another theory was Thomsonâ€™s use of Fourierâ€™s equations to formulate equations for electrostatics in the 1840s. After extracting a lesson from this historical case, I will fast forward more than a century to examine the relationship between classical statistical mechanics and QFT that is induced by analytic continuation. While there is no doubt that this mathematical relationship has been heuristically useful in guiding developments in both statistical mechanics and QFT, this is a case in which the physical interpretation of the mathematics does not carry over from one theory to the other.

Questions raised during the conference

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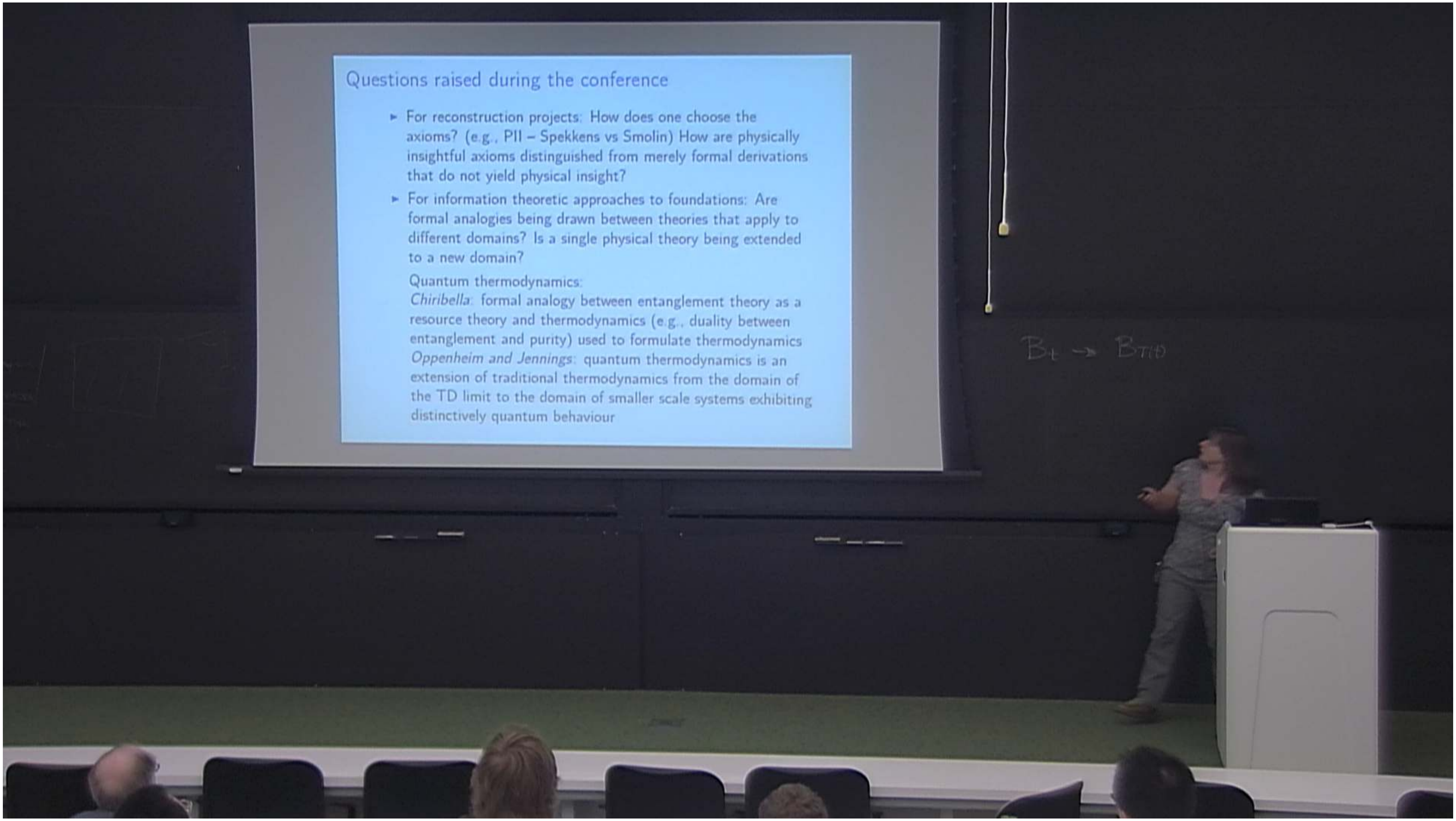

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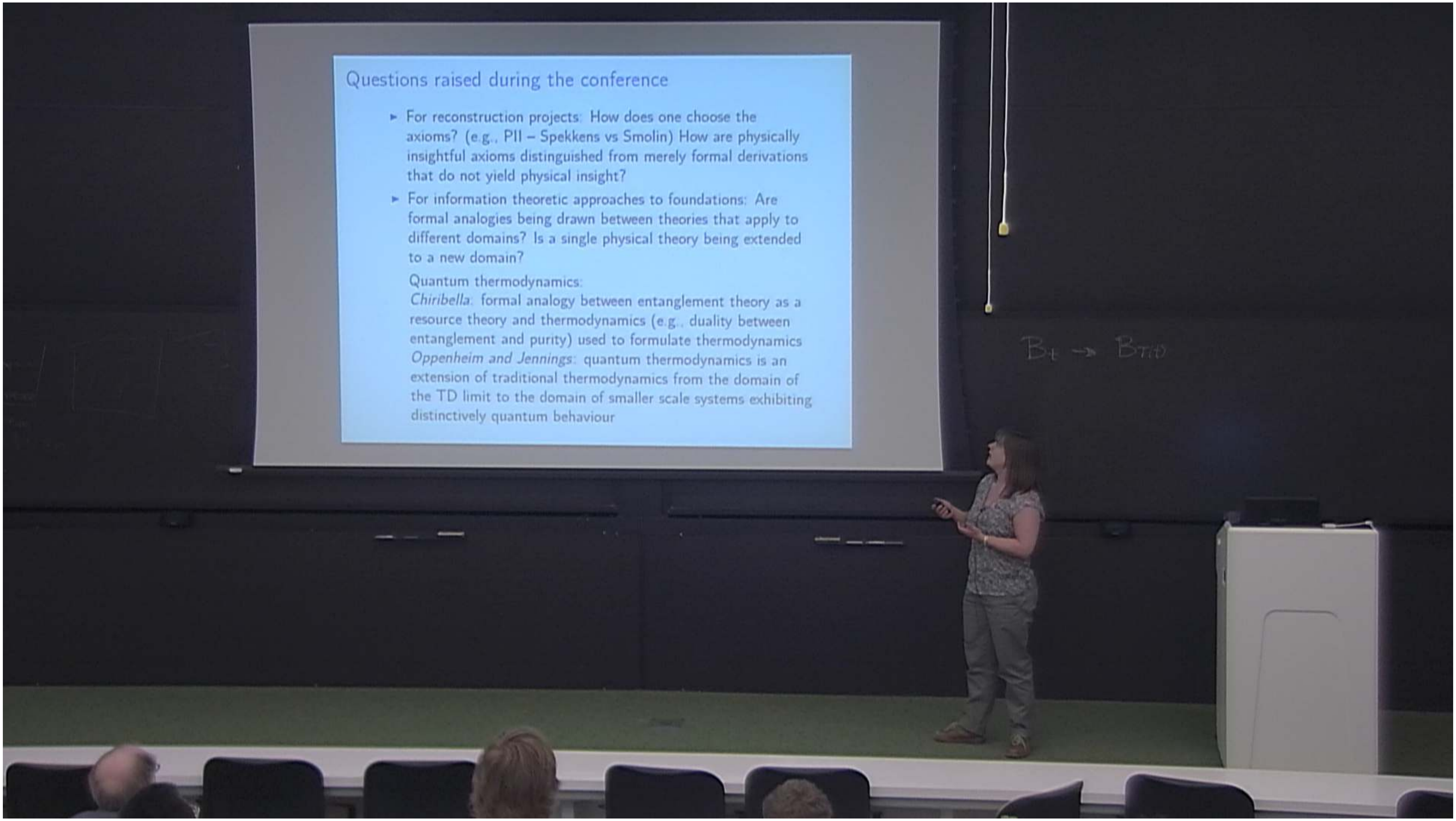
For information theoretic approaches to foundations: Are formal analogies being drawn between theories that apply to different domains? Is a single physical theory being extended to a new domain?
Quantum thermodynamics:
Chiribella: formal analogy between entanglement theory as a resource theory and thermodynamics (e.g., duality between entanglement and purity) used to formulate thermodynamics
Oppenheim and Jennings: quantum thermodynamics is an extension of traditional thermodynamics from the domain of the TD limit to the domain of smaller scale systems exhibiting distinctively quantum behaviour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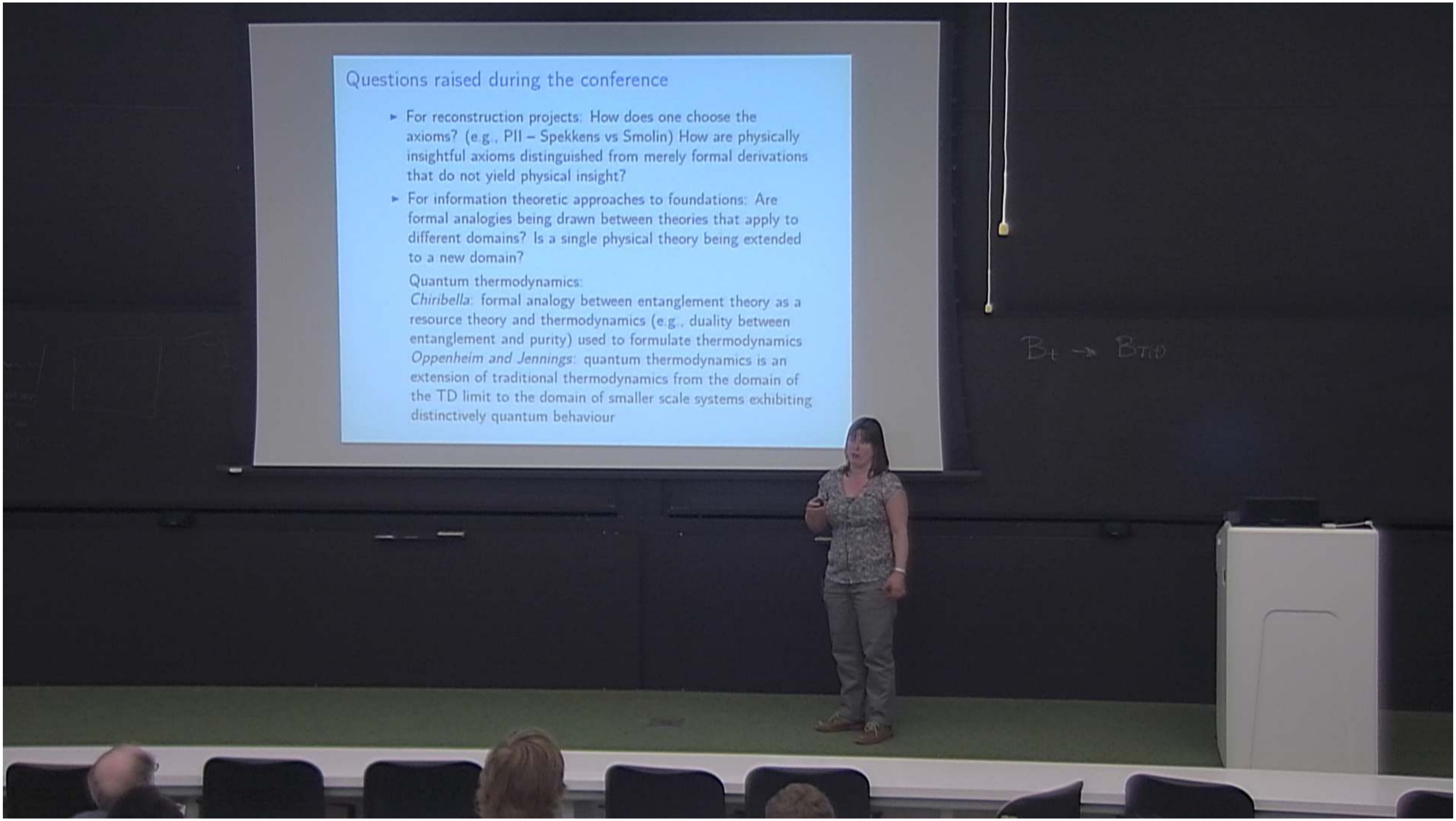
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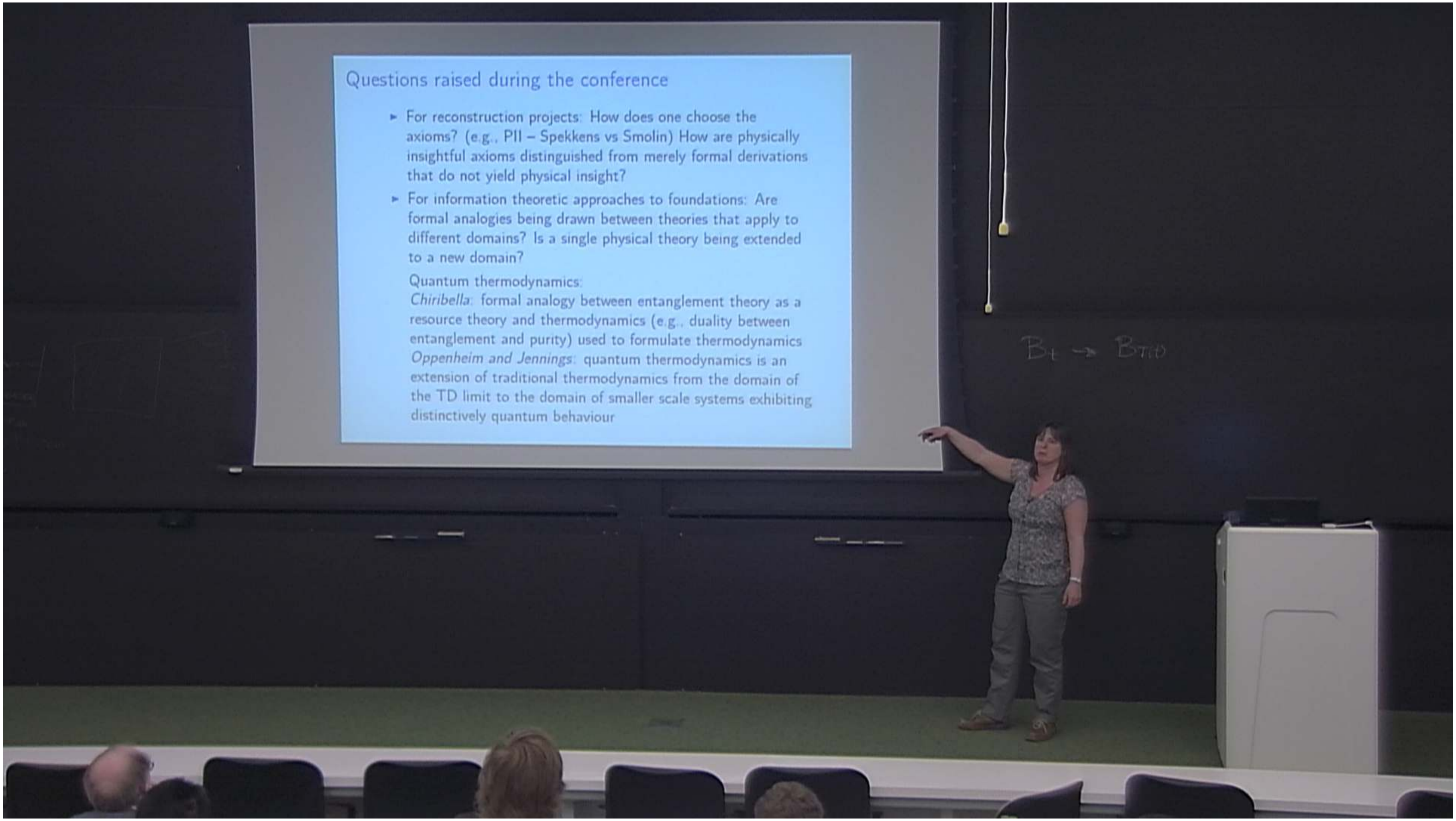
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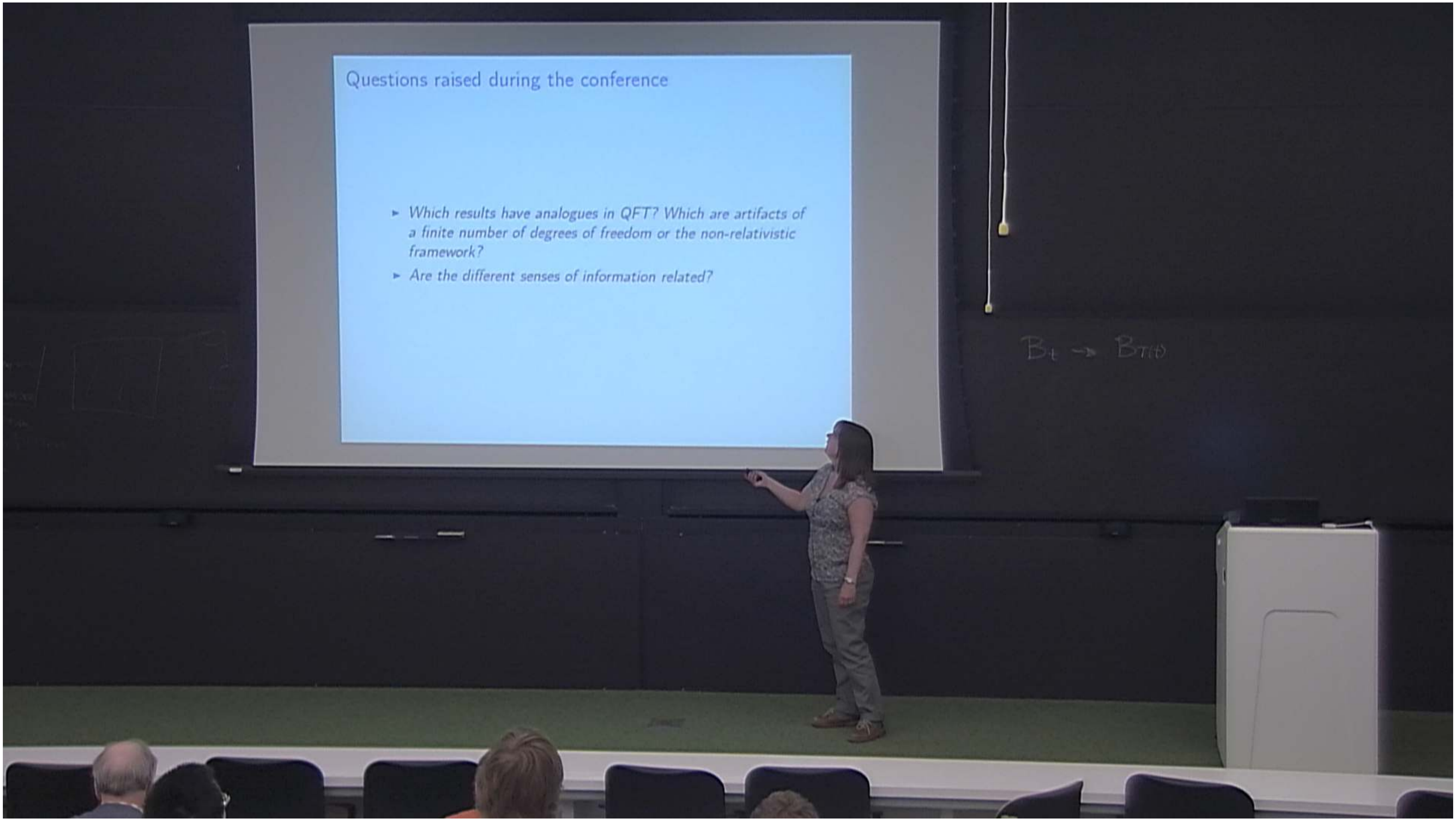
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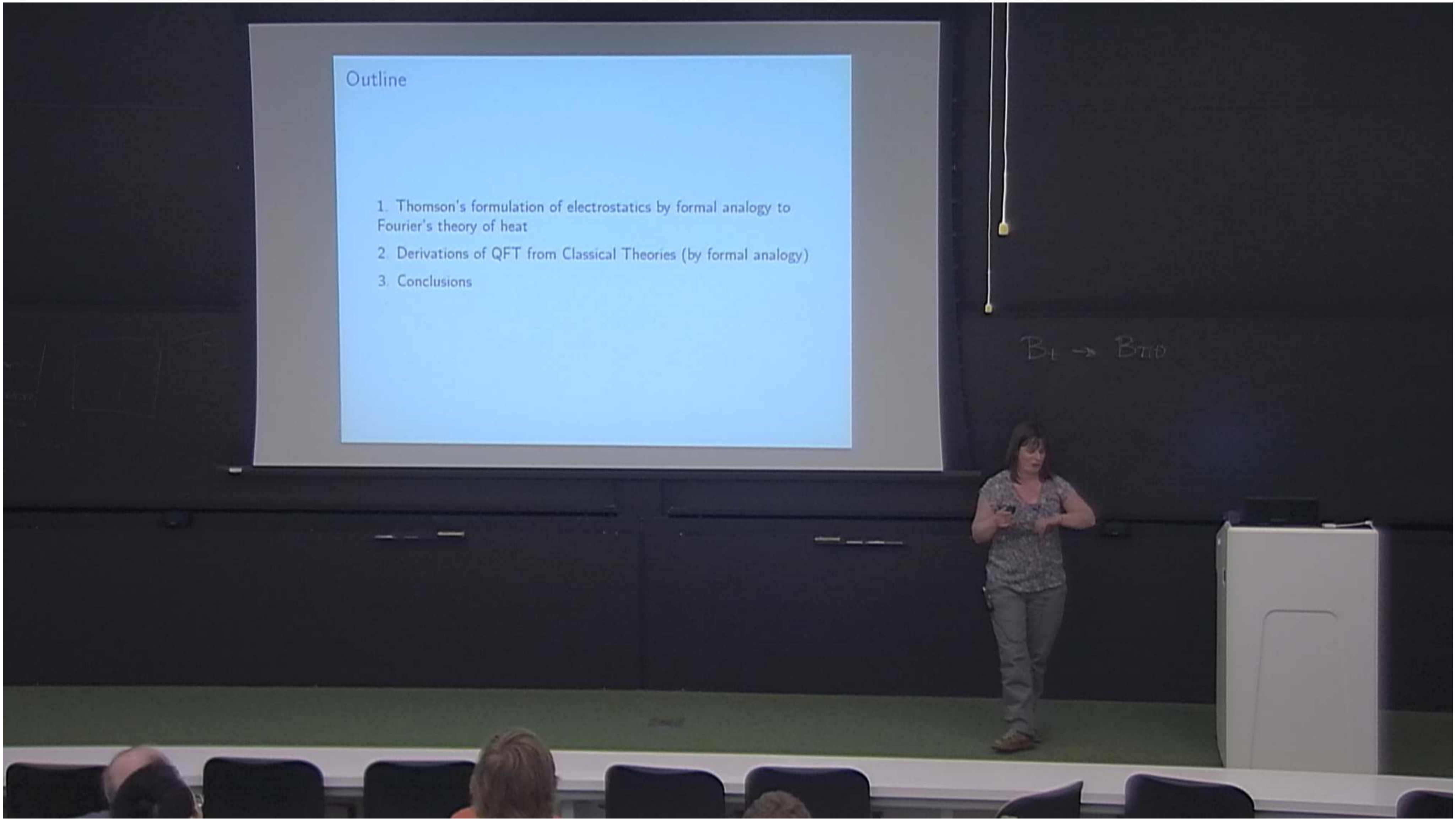
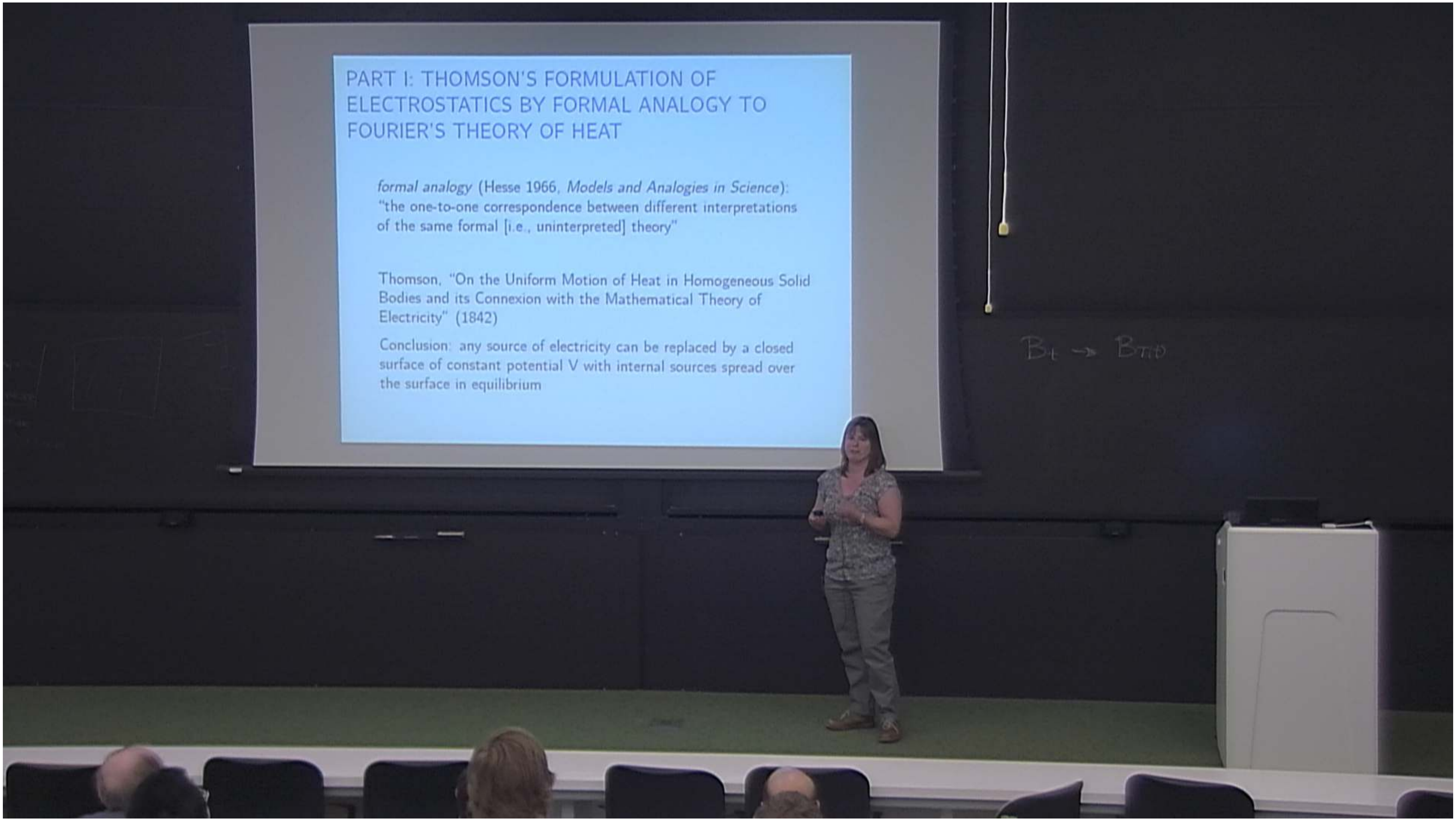
Which results have analogues in QFT? Which are artifacts of a finite number of degrees of freedom or the non-relativistic framework?
Are the different senses of information related?


Outline
1. Thomson's formulation of electrostatics by formal analogy to Fourier's theory of heat
2. Derivations of QFT from Classical Theories (by formal analogy)
3. Conclusions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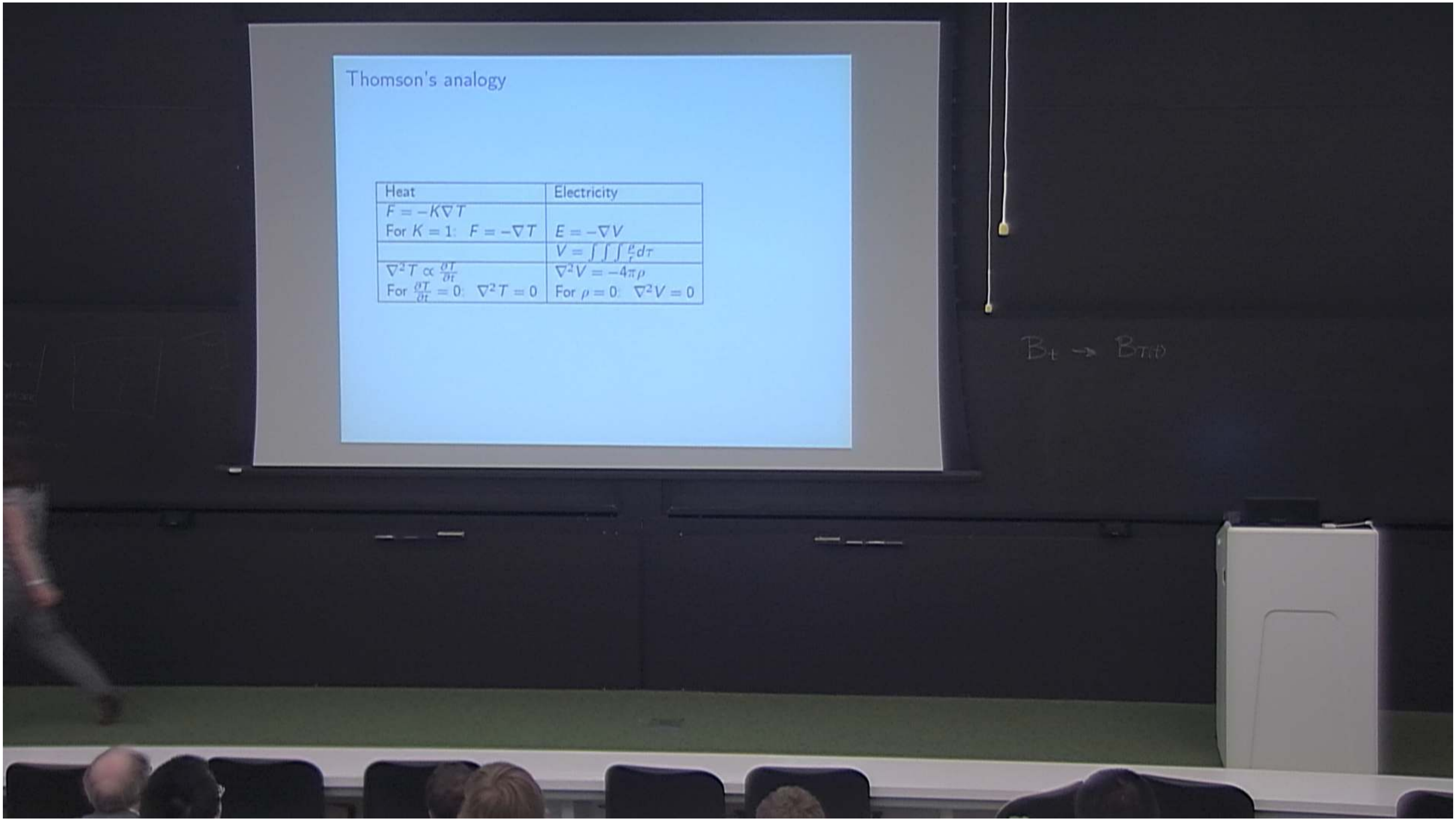
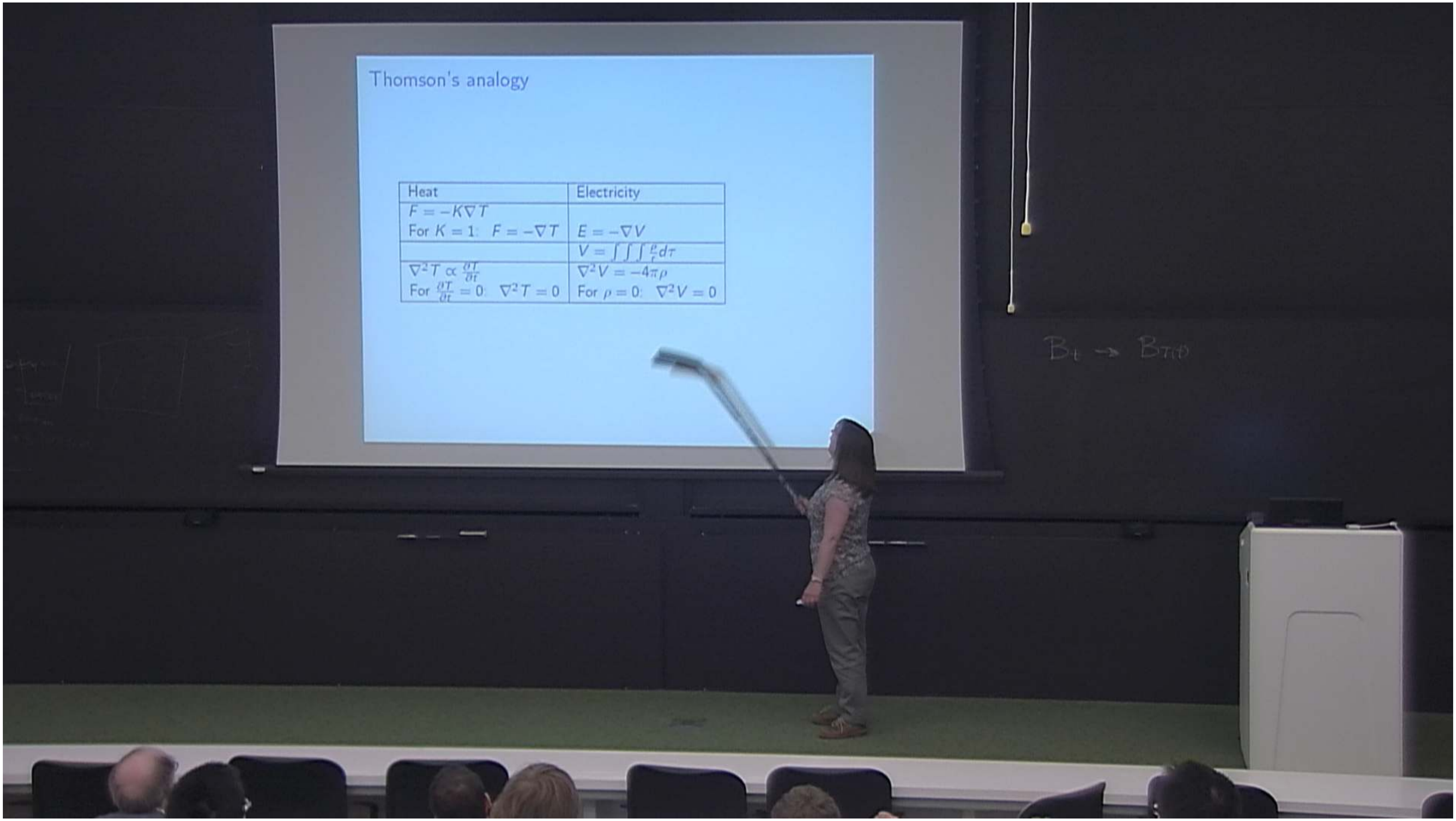

Thomson's analogy
Heat
Electricity
$F = -K\nabla T$
For $K = 1$: $F = -\nabla T$
$E = -\nabla V$
$V = \iiint \frac{\rho}{r} d\tau$
$\nabla^2 T \propto \frac{\partial T}{\partial t}$
$\nabla^2 V = -4\pi\rho$
For $\frac{\partial T}{\partial t} = 0$: $\nabla^2 T = 0$
For $\rho = 0$: $\nabla^2 V = 0$



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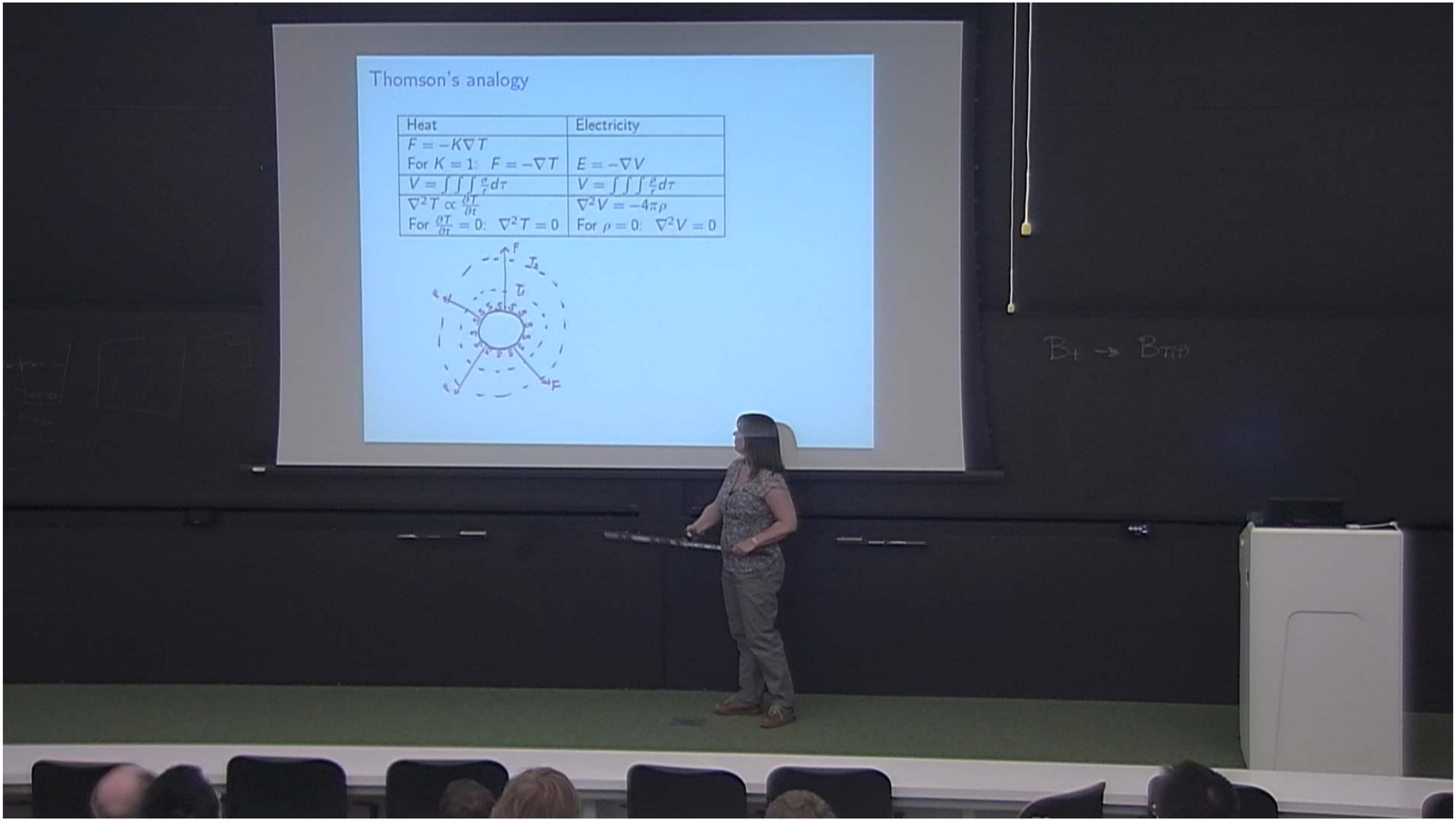
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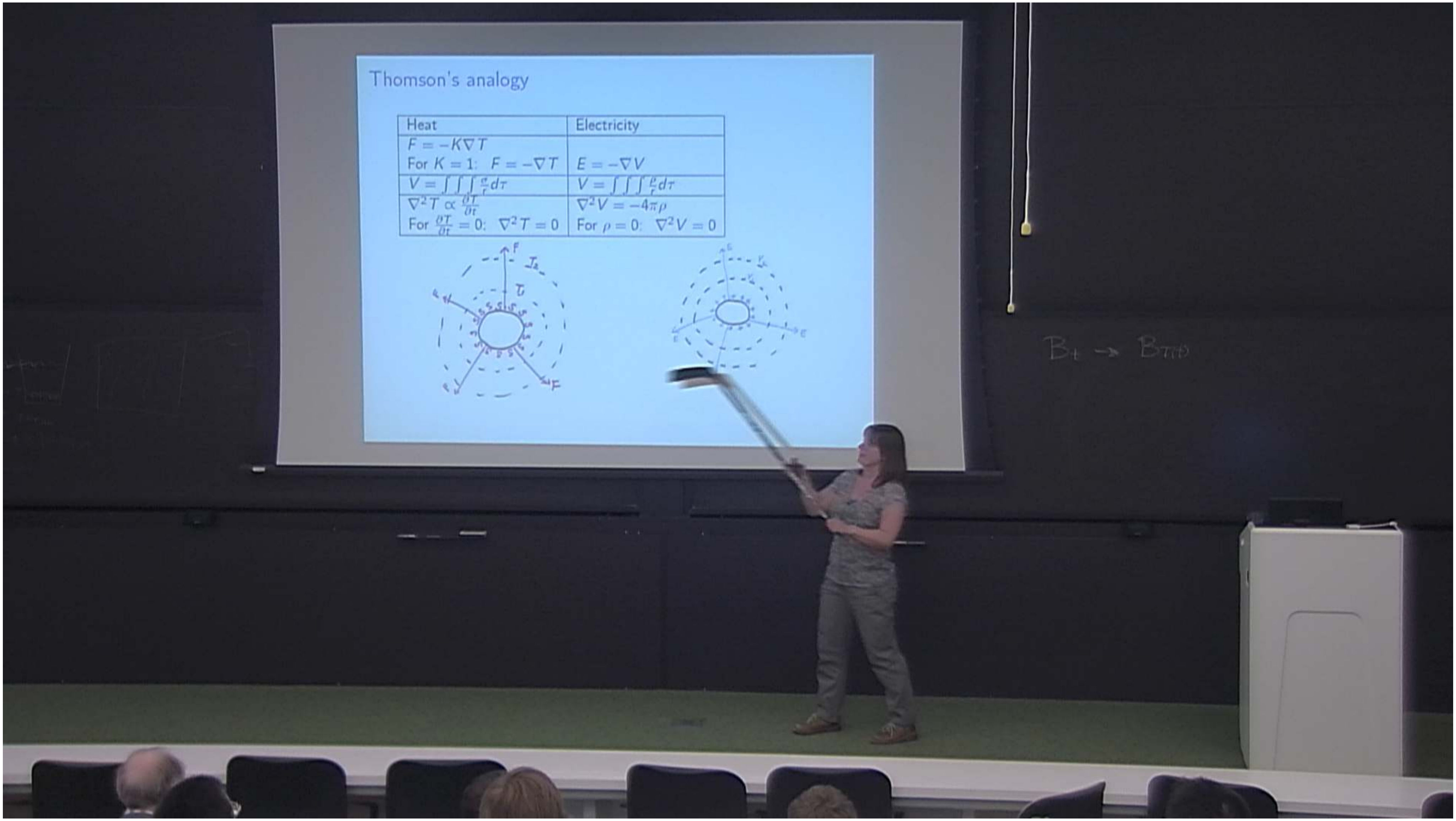
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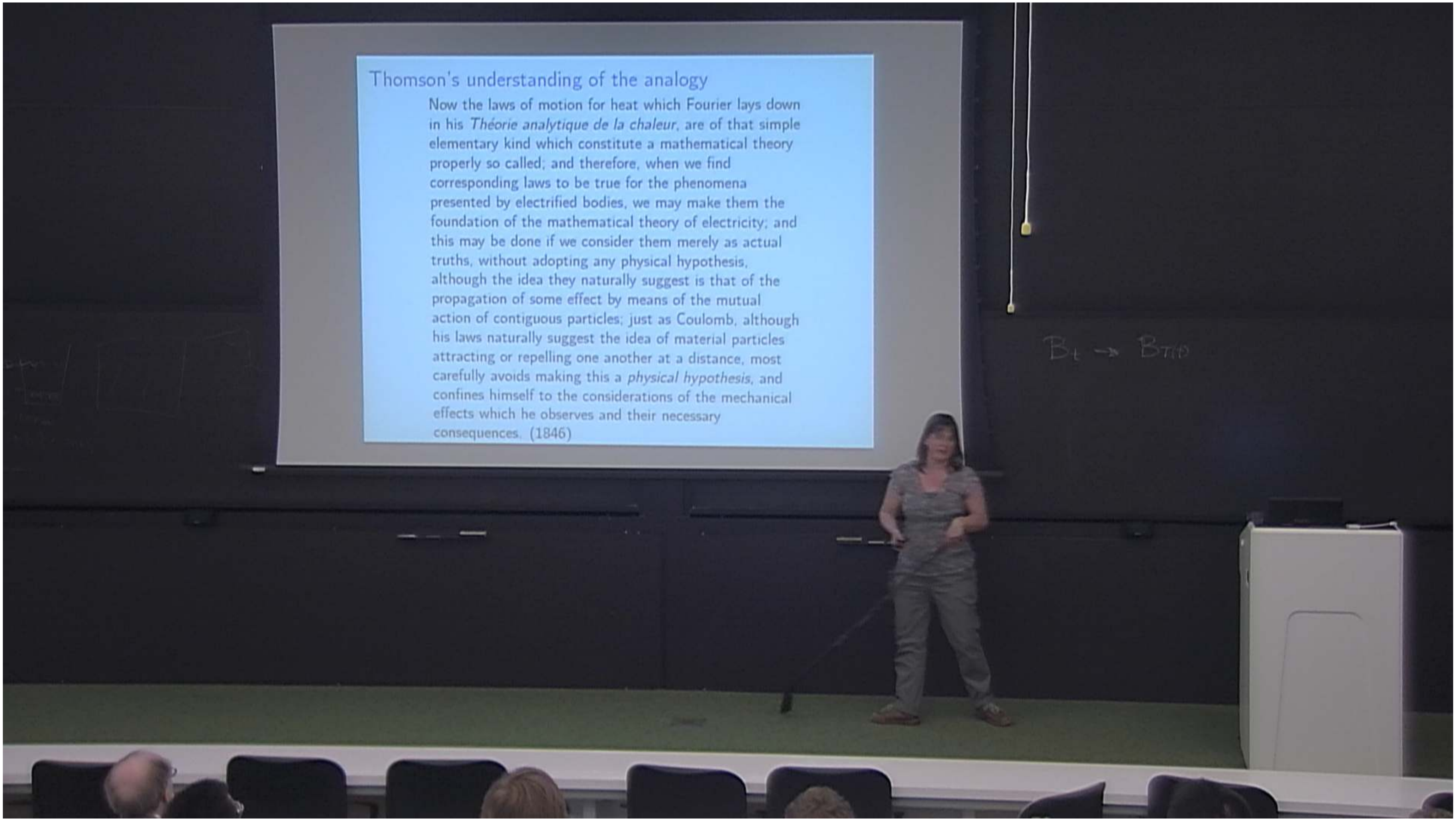
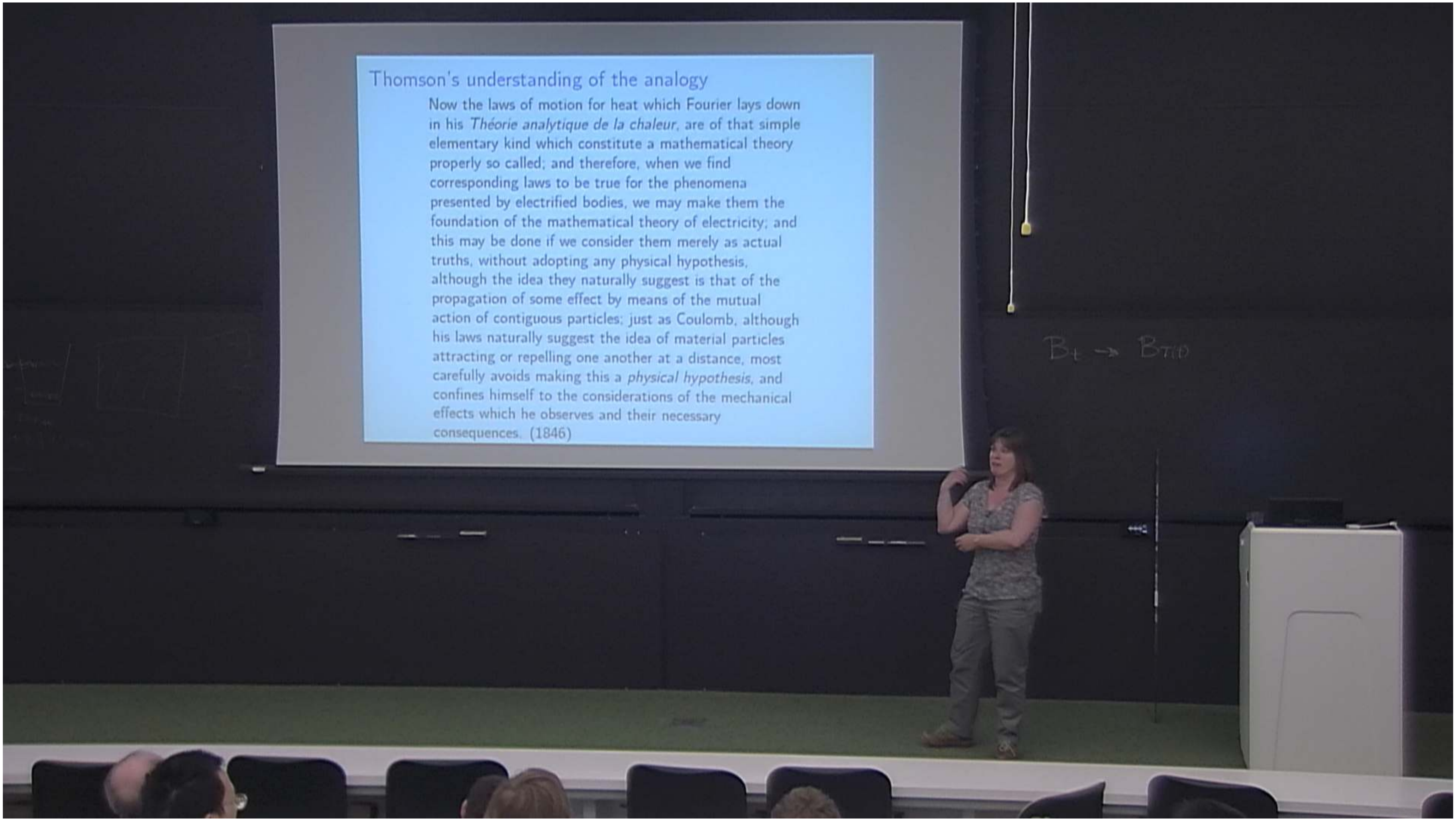
E
E


Thomson's understanding of the analogy
Now the laws of motion for heat which Fourier lays down in his Théorie analytique de la chaleur, are of that simple elementary kind which constitute a mathematical theory properly so called; and therefore, when we find corresponding laws to be true for the phenomena presented by electrified bodies, we may make them the foundation of the mathematical theory of electricity; and this may be done if we consider them merely as actual truths, without adopting any physical hypothesis, although the idea they naturally suggest is that of the propagation of some effect by means of the mutual action of contiguous particles; just as Coulomb, although his laws naturally suggest the idea of material particles attracting or repelling one another at a distance, most carefully avoids making this a physical hypothesis, and confines himself to the considerations of the mechanical effects which he observes and their necessary consequences. (1846)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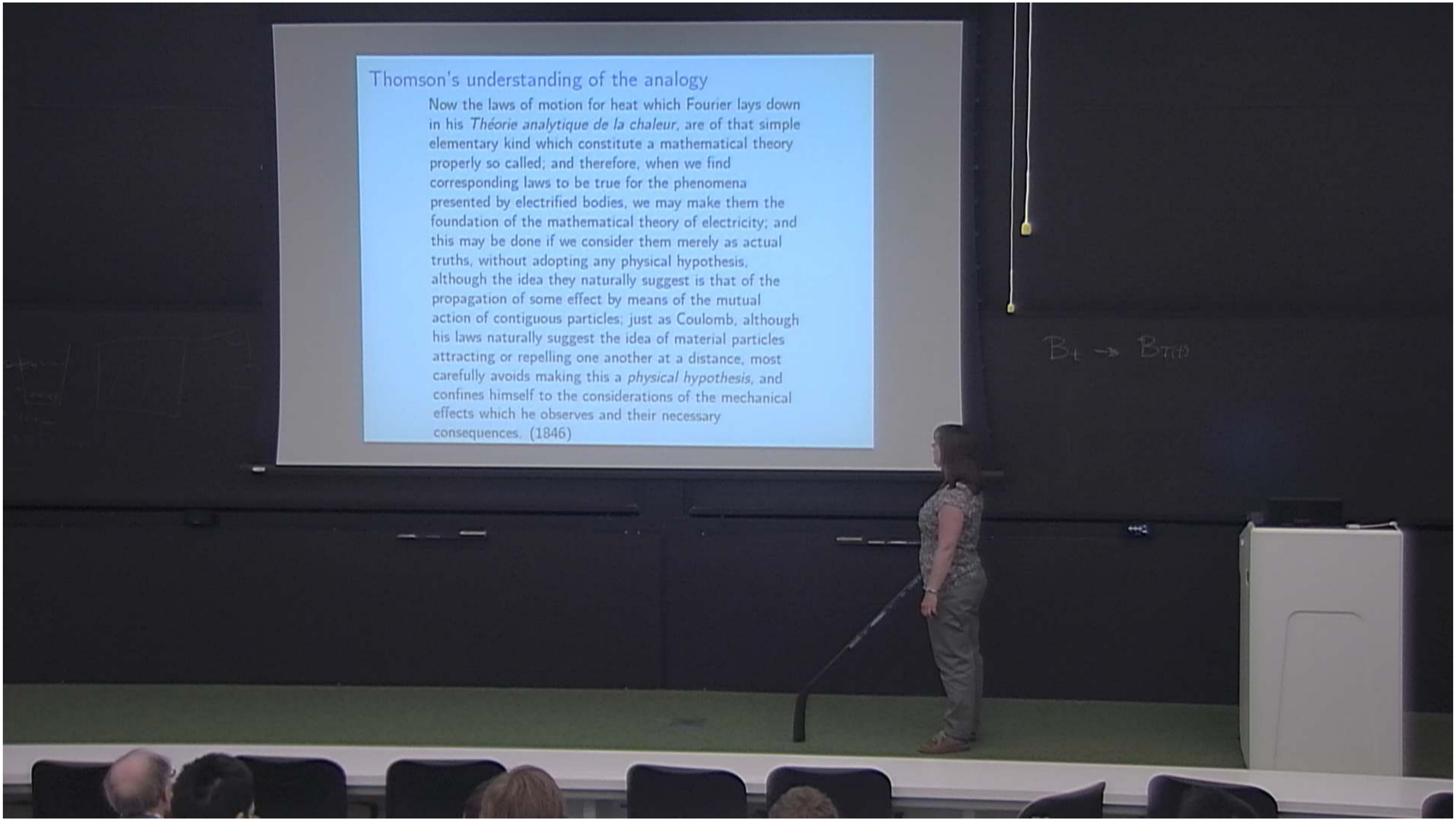
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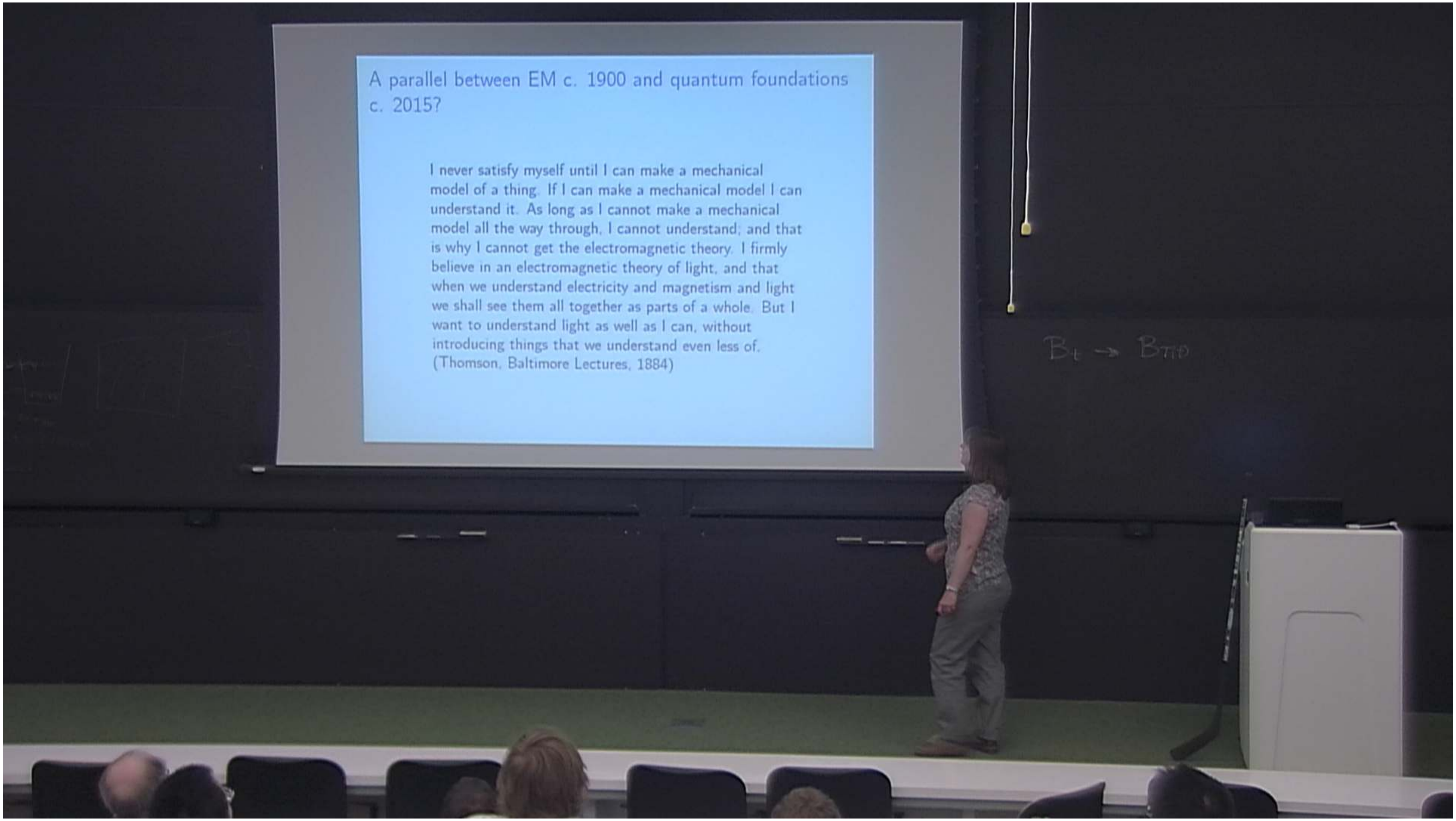
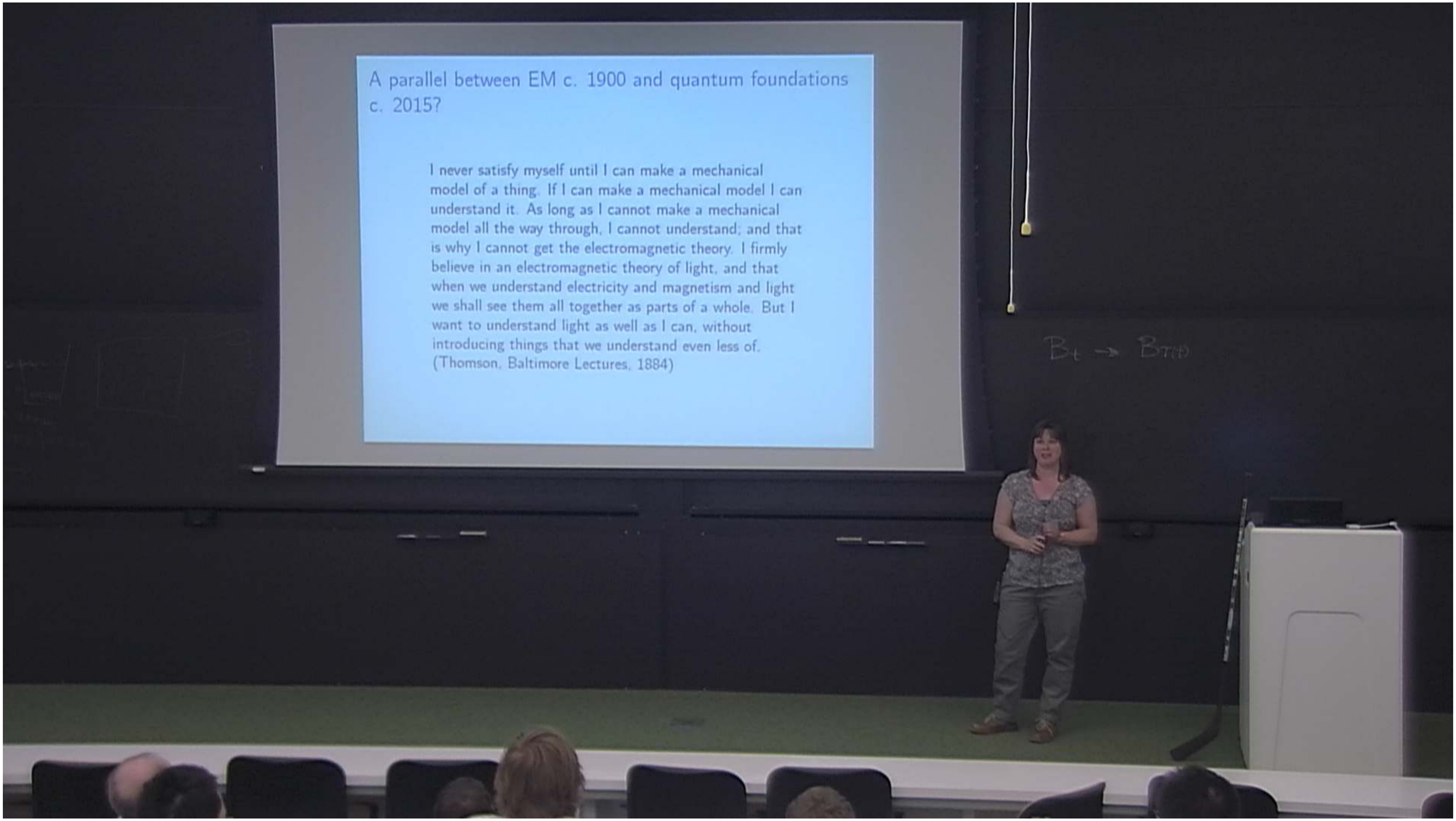

A parallel between EM c. 1900 and quantum foundations c. 2015?
I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through, I cannot understand; and that is why I cannot get the electromagnetic theory. I firmly believe in an electromagnetic theory of light, and that when we understand electricity and magnetism and light we shall see them all together as parts of a whole. But I want to understand light as well as I can, without introducing things that we understand even less of.
(Thomson, Baltimore Lectures, 1884)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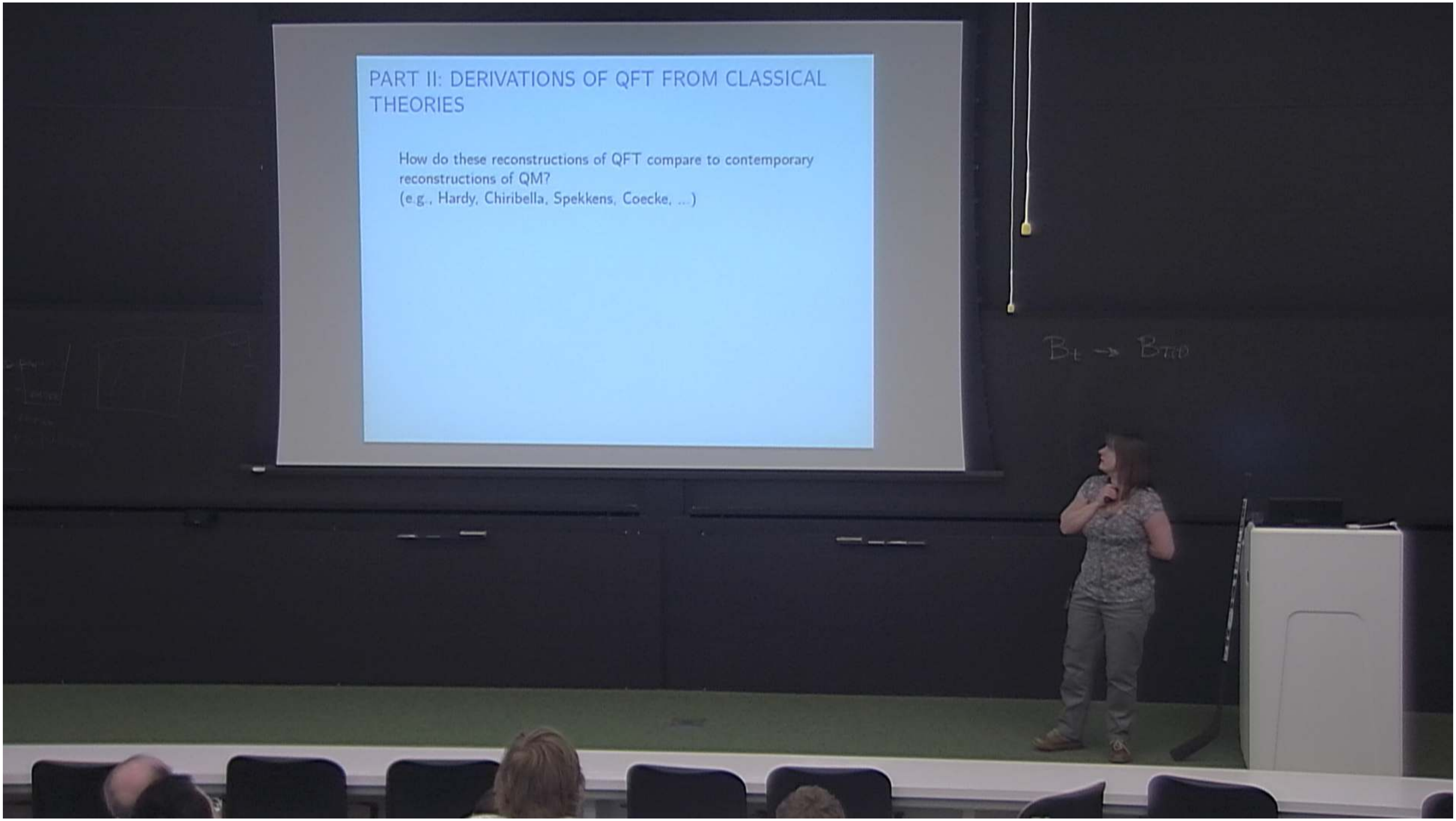
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Wightman's axioms for QFT (Streater and Wightman 1964)

1. Hilbert space structure $\mathcal{H}$
2. Energy-momentum spectral condition (uniqueness of vacuum Ω)
3. Field operators $\phi(f)$ for $f \in S(\mathbf{R}^d)$ (cyclicity of vacuum Ω))
4. Relativistic covariance (Poincaré transformations $U(a, \Lambda)$)
5. Microcausality

Goal: to find models of the axioms $(\phi, \mathcal{H}, U, \Omega)$ for interacting systems

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Wick rotation (Dyson 1949)

Wick rotation: $t \to -it'$

A Wick rotation transforms Lorentz four-vector products into (negative) Euclidean four-vector products:

$$x^2 = t^2 - |\mathbf{x}|^2 \to x'^2 = -t'^2 - |\mathbf{x}|^2 \quad (1)$$

In perturbative QED, a useful mathematical trick for managing divergences.

Model construction strategy

Wick rotate the set of Wightman functions for a QFT to obtain a set of Schwinger functions:

$$S_n(x_1, x_2, \dots x_n) = \langle \Omega, \phi_M(it_1, \mathbf{x}_1) \cdots \phi_M(it_n, \mathbf{x}_n)\Omega \rangle \qquad (2)$$

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Use the Schwinger functions to define a (classical) Euclidean field theory:

$$S_n(x_1, x_2, \dots x_n) = \int \phi_E(x_1)\phi_E(x_2) \cdots \phi_E(x_n) d\mu(\phi_E) \qquad (3)$$

For scalar bose Euclidean FT:
$\phi_E(x)$: random variable valued distribution
μ: measure on the distribution space
x_i: point in d-dimensional Euclidean space

The reception of this Euclidean model construction strategy

Barry Simon: "until EFT changed my tune, I tended to think of probabilists as a priesthood who translated perfectly simple functional analytic ideas into a strange language that merely confused the uninitiated"

Formal analogies

In the same way that Thomson developed a formal analogy between Fourier's theory of heat and electrostatics, Osterwalder and Schrader developed a formal analogy between QFT and EFT.

i.e., Osterwalder and Schrader chose their axioms for EFT to guarantee that the analytically continued Schwinger functions had the analytic properties possessed by the Wightman functions

By the lights of the contemporary reconstruction of QM projects, does the Osterwalder-Schrader reconstruction theorem provide a physically insightful reconstruction of QFT from EFT?

My intuition: **NO**

Why not?

- (reconstruction has only been carried out for toy models)

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Why not?

- (reconstruction has only been carried out for toy models)
- (EFT is not a real physical theory)
- The Osterwalder-Schrader axioms are not a plausible set of physical principles for QFT (i.e., do not improve upon the Wightman axioms). QFT is a relativistic theory; O-S axioms describe a theory on Euclidean space. In contrast, for example, Hardy's (2001) or (2011) axioms are intended to be an improvement upon standard axioms for QT and to replace the standard axioms.

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- The O-S reconstruction theorem does not illuminate the relationship between EFT and QFT in the same manner that formulating theories in the operational probability theory framework illuminates the relationship between QT and CPT. Analytic continuation obscures the physical relationship between the theories.

  The OPT framework offers a general mathematical framework in which axioms for QT and CPT can be formulated; the mathematical frameworks in which the O-S and Wightman axioms get formulated are different.

An opposing view?

From Glimm and Jaffe, *Quantum Physics: A Functional Integral Point of View*

> The unity of mathematical structures for problems of diverse origin in physics should be no surprise. For classical physics it is provided, for example, by a common mathematical formalism based on the wave equation and Laplace's equation. **The unity transcends mathematical structure and encompasses basic phenomena as well.** Thus particle physicists, nuclear physicists, and condensed matter physicists have considered similar scientific problems from complementary points of view. (emphasis added)

How do contemporary reconstruction of QM projects compare to Thomson's formal analogy?

Both OPT and Thomson present two physical theories in the same mathematical framework.

OPT characterizes the theories using different sets of mathematical axioms; Thomson characterizes the theories using the same set of mathematical axioms with different physical interpretations.

Conclusions

- The strategy of using formal analogies to a theory of heat as inspiration for formulating another theory predates quantum mechanics.
- While the Euclidean strategy for constructing models of Wightman QFTs does not achieve the goals of contemporary reconstruction projects for QT, some of the results may be illuminating.
  e.g., the formal relationship between QFT and EFT established by Wick rotation is based on relativity
- How to distinguish formal derivations from phyiscally insightful reconstructions of a theory? Comparing and contrasting to historical cases may be helpful.