Title: The second laws of quantum thermodynamics

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Abstract:

# The 2nd laws of Quantum Thermodynamics

**J. Oppenheim (UCL)**

Brandao, Horodeck, Ng, J.O., Wehner PNAS (2014)
quant-ph/1305.5278

Horodeck:, J.O.; Nature Communications (2013)
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Brandao, Horodeck, J.O., Renes, Spekkens, PRL (2013)
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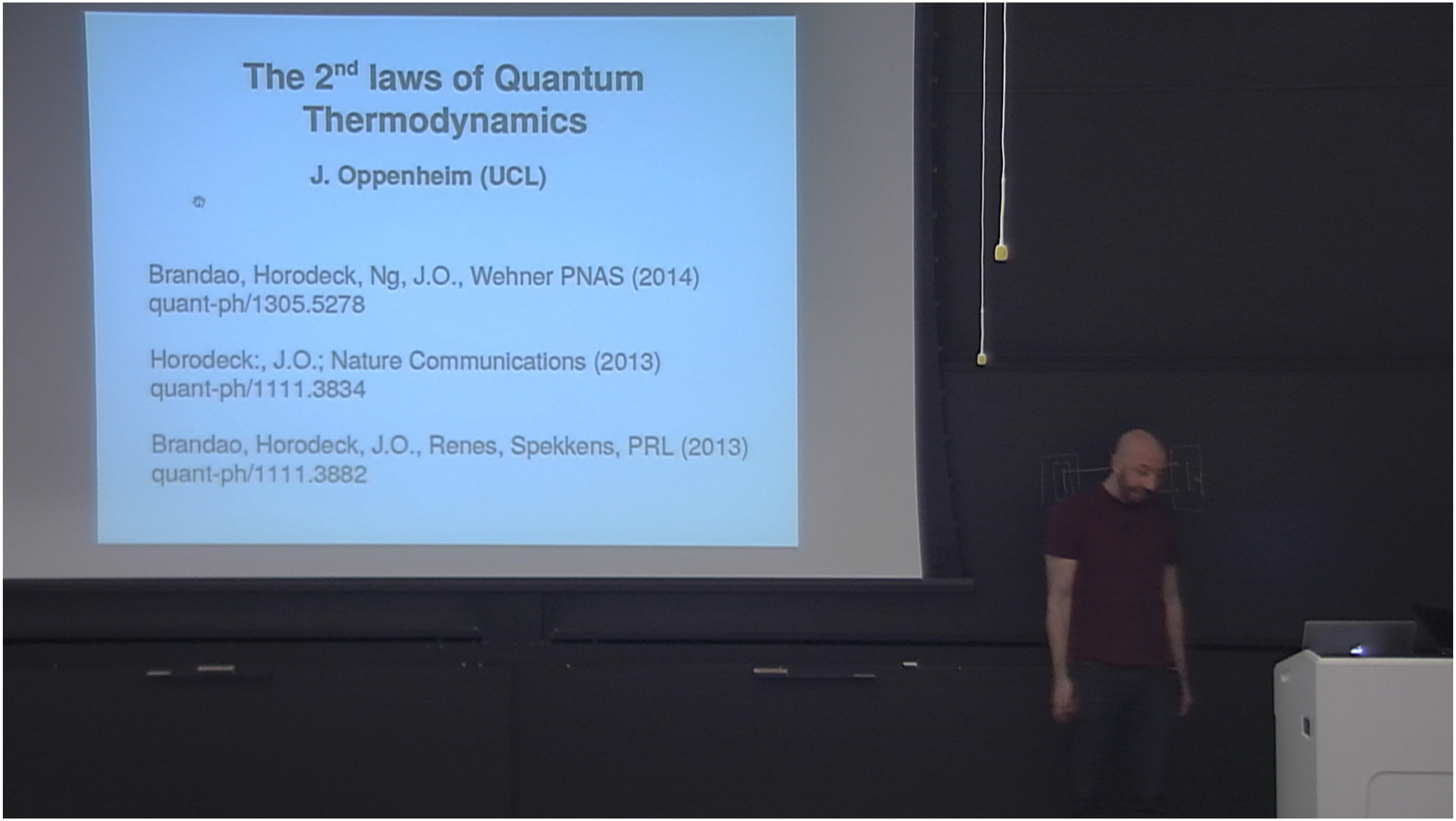
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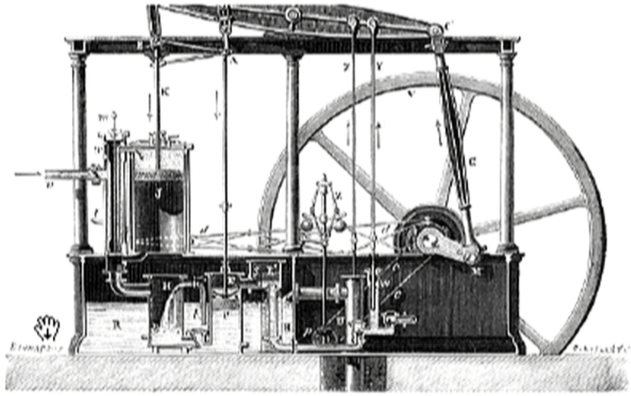
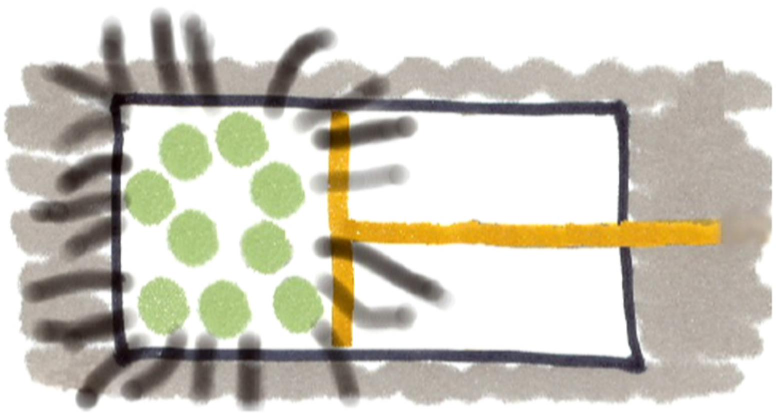


Fig. 59. — Machine à balancier de Watt.

1st wave
Carnot (1824)
Joule (1843)
Kelvin (1849)
Clausius (1854)



2nd wave
(stat mech)
Maxwell (1871)
Boltzman (1875)
Gibbs (1876)

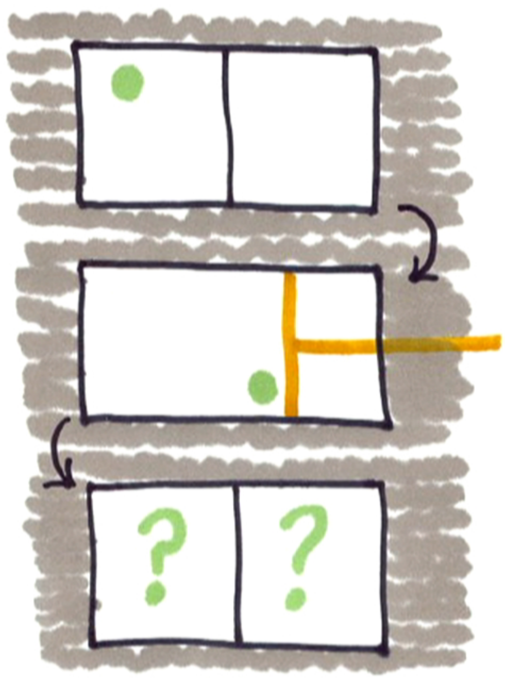
# Thermodynamics as information

Maxwell

Szilard

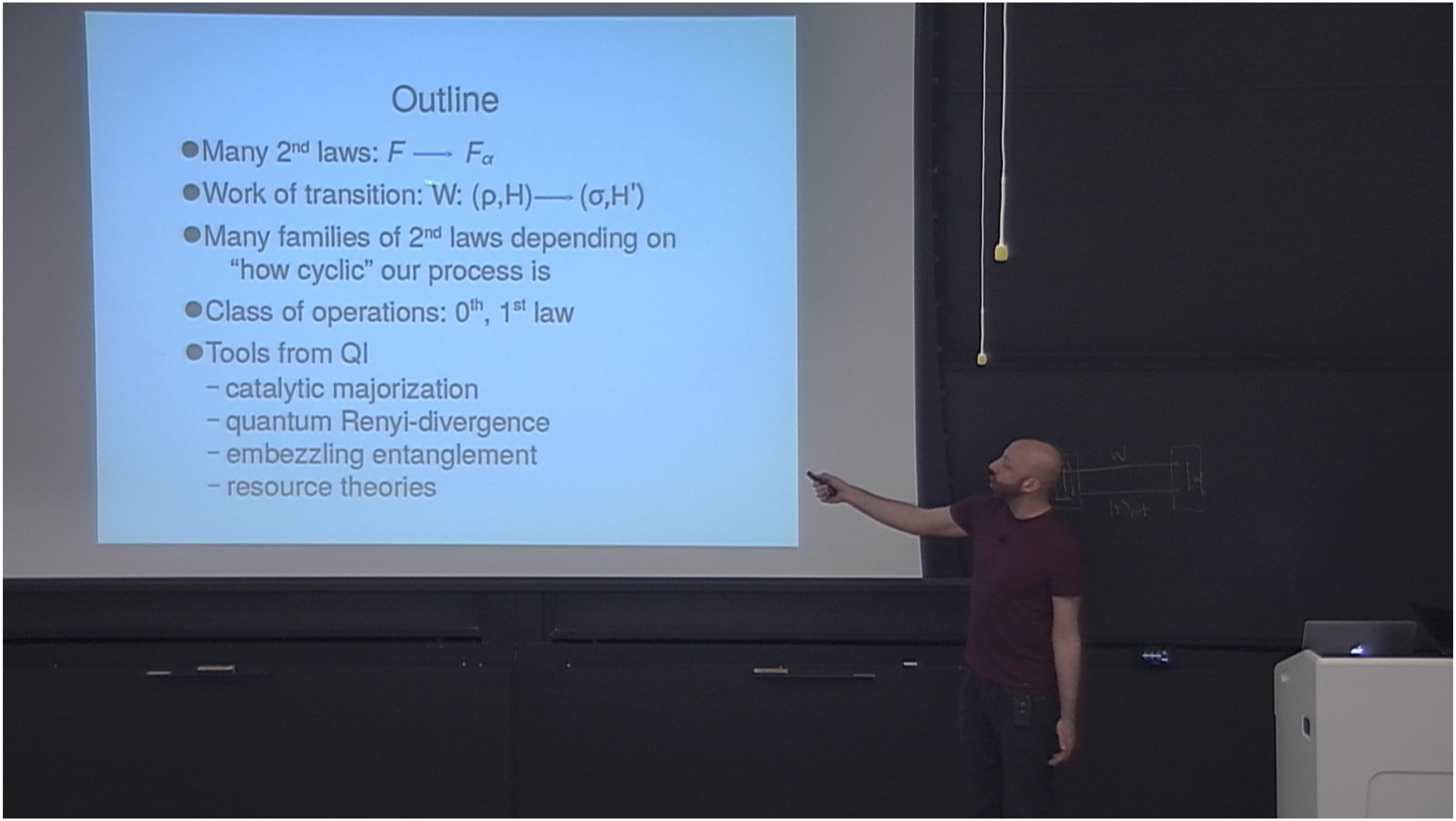
Landauer

Bennett



# Outline

- Many $2^{nd}$ laws: $F \longrightarrow F_\alpha$
- Work of transition: W: $(\rho,H) \longrightarrow (\sigma,H')$
- Many families of $2^{nd}$ laws depending on "how cyclic" our process is
- Class of operations: $0^{th}$, $1^{st}$ law
- Tools from QI
  - catalytic majorization
  - quantum Renyi-divergence
  - embezzling entanglement
  - resource theories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# Quantum Thermodynamics

- **Gibbs state with full information**
  - Gemmer, Michel, Mahler (2005), Popescu, Short, Winter (2006)
- **Meaning of Negative Entropy, conditional erasure**
  - Del Rio et al. (2011), Faist et al. (2013)
- **Smallest possible fridges**
  - Linden, Popescu, Skrzypczyk (2010)
- **Deterministic tranformations**
  - HHO (2003), Dahlston et al. (2010), HO (2011), Aaberg (2011), Egloff et al. (2012)!
- **Average work extraction**
  - Brandao et. al. (2011), Skrzypczyk et. al. (2013)
- **Non-ideal heat baths, correlations, entanglement**
  - Reeb, Wolf (2013); Gallego et. al. (2013), Hovhannisyan et. al. (2013), Mueller et. al. (2014)
- **Thermalisation times**
- **Micro-engines & machines**
  - Scovil & Schultz-Dubois (1959), Howard (1997)
  - Rousselet et al. (1994), Faucheux et al. (1995), Scully (2002)
- **Pioneering works**
  - Ruch and Mead (75), Janzig et. al. (2000)

Its called thermodynamics because we take the thermodynamic limit!

System size number of particles $\longrightarrow \infty$

Thermodynamics in the opposite extreme.
Finite size (micro, nano) and/or quantum
Deterministic laws

# Quantum thermodynamics

- No thermodynamic limit
- Coherences in energy eigenbasis
- More precise control
- Rigorous theory

# 3 laws of thermodynamics

0) If $R_1$ is in equilibrium with $R_2$ and $R_3$ then $R_2$ is in equilibrium with $R_3$

*1)* $dE = dQ - dW$ (energy conservation)

2) Heat can never pass from a colder body to a warmer body without some other change occuring. – Clausius

3) One can never attain $S(\rho) = C$ in a finite number of steps

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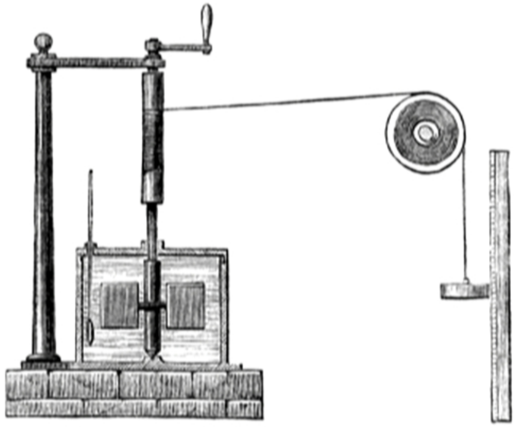
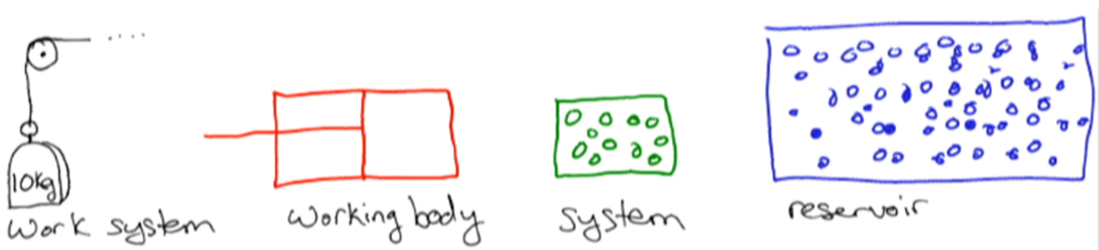


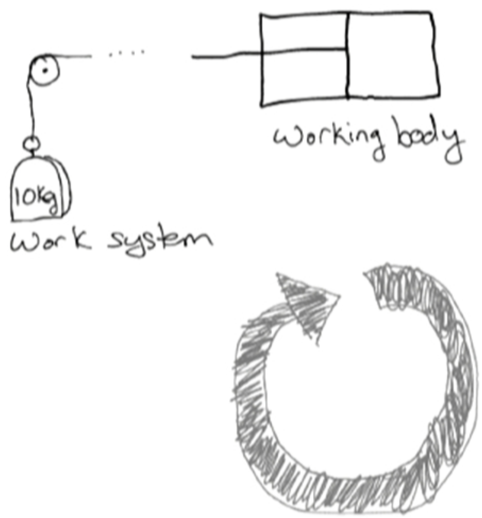
Joule (1843)

Not a consequence, but part of the class of operations

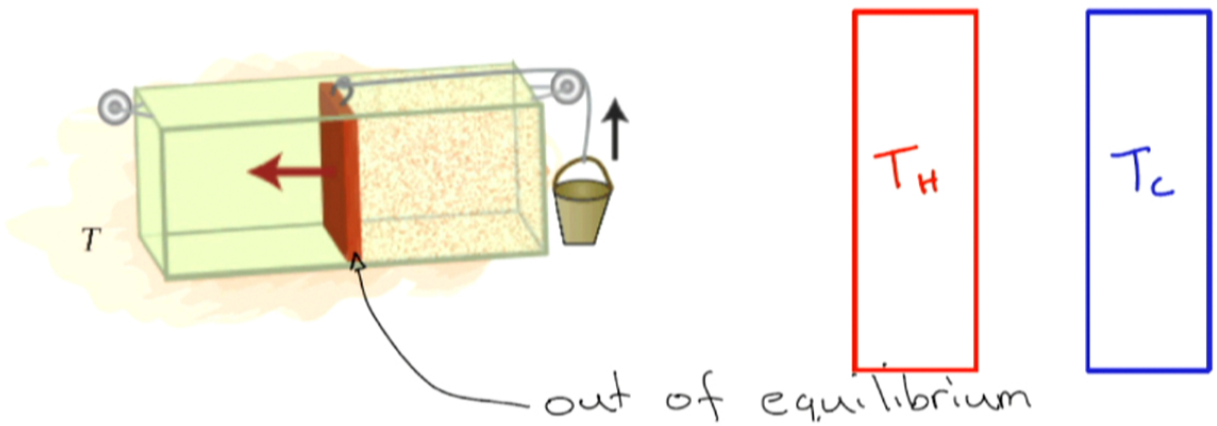


# The second law

Heat can never pass from a colder body to a warmer body without some other change occurring – Clausius




# Free Energy




$$F = E - TS$$

$$W_{gain} = F(\rho_{initial}) - F(\rho_{final})$$

$$\rho_{initial} \rightarrow \rho_{final} \quad \text{only if} \quad \Delta F \geq 0$$

# Catalytic Thermal Operations $\Lambda_\tau$

- $(\rho_s, H_s)$

  $\rho_s$ = **resource**

  $H_s$ = Hamiltonian

- adding **free states** $\tau_R$
- work system W
- borrowing ancillas (working body) and returning them in the "same" state $\sigma_c$
- energy conserving unitaries $U$
  (**1st law**) $[U, H_s + H_w + H_R + H_c] = 0$
- tracing out (trash)

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# Thermal Operations


or in the micro - regime

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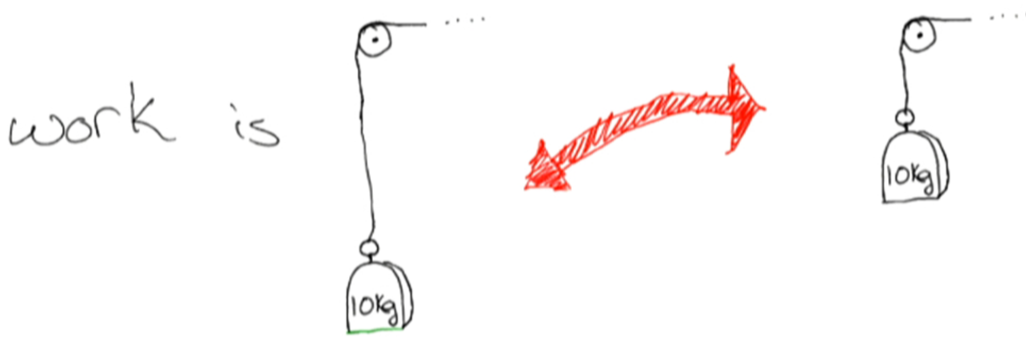



or in the micro - regime

# 1 law of quantum thermodynamics

**Class of operations**

0) The only free state $\tau$ which doesn't enable arbitrary transitions is the Gibbs state
1) Energy conserving unitaries

**State Transitions**

2) [cyclic]* $\rho_s$ must get closer to $\tau_s$ in terms of free energy type distances $F_\alpha(\rho_s \,||\, \tau_s)$ $\alpha \geq 0$

2') [single system] $\rho_s$ must get closer to $\tau_s$ in terms thermo-majorisation

*depends on how cyclic

# Zeroeth Law

*After decohering in the energy eigenbasis, one can extract* work from many copies of any state which is not passive

$(p_i \leq p_j$ iff $E_i \geq E_j)$

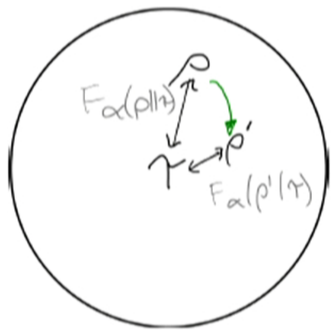
*[just swap levels i and j, while raising the weight, and repeat over many blocks]*

Many copies of any state except the thermal state results in a state which is not passive after decohering (Pusz and Woronowicz (78), Brandao et. al. 2011)

This gives us an equivalence class, of allowed free states labelled by ($\tau_\beta$ , $H_R$)

Any other free state allows arbitrary transitions.

# The second laws (psuedo-classical)




Thermal monotones:

$$F_\alpha(\rho||\tau) = \frac{kT}{1-\alpha}\log\, tr\rho^\alpha\tau^{1-\alpha} - kT\log Z \quad \alpha \geq 0$$

$$F_1(\rho||\tau) = F(\rho)$$

- Ordinary 2nd law is one of many
- In macroscopic limit, weak interactions, all $F_\alpha \simeq F$
- For $\rho$ block diagonal, 2nd law is iff

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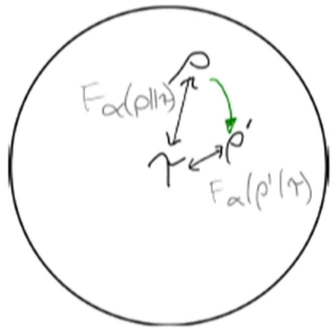


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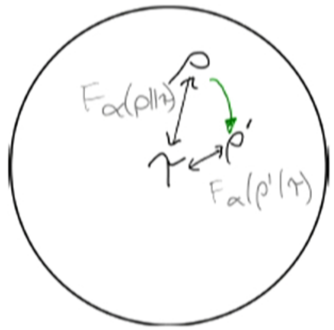


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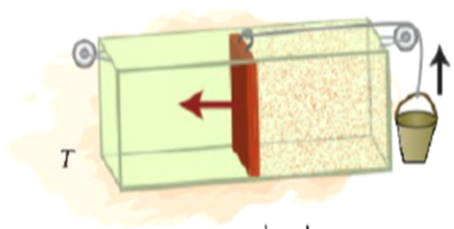
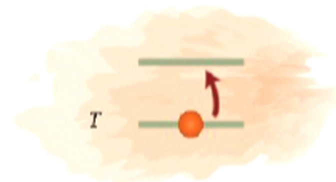
Macro




weight

Micro




$\omega, t$

~~W=F=E-TS~~

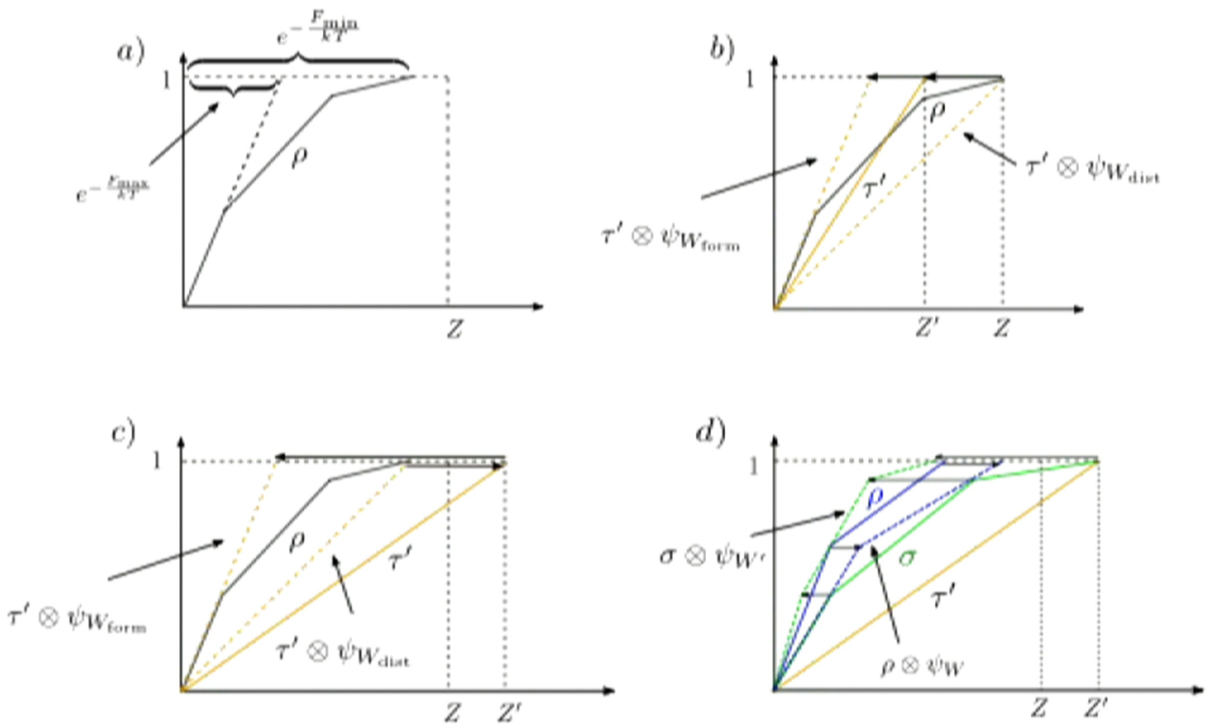
$F$ → $W_{form} = F_\infty$

$F$ → $W_{dist} = F_0$

$$F_\infty = kT \log \min\{\lambda : \rho \leq \lambda\tau\} - kT \log Z$$

$$F_0 = kT \log \sum h(\omega, g, E_i) e^{-\beta E_i}$$

# Work of transition: (ρ,H)→(σ,H')




$\Delta F$ — $W_{form}$ → $F_\infty(\rho||\tau)$ ⟶ Strengthens Dahlston et al. for $H = 0$

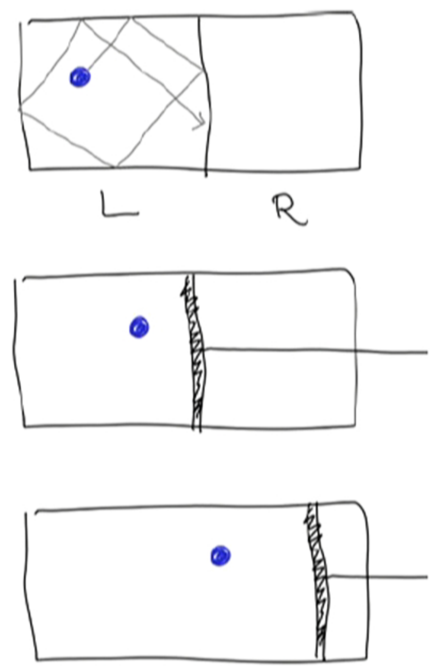
$\Delta F$ — $W_{dist}$ → $F_o(\omega||\tau)$

c.f. Ahlberg (2011)

# Work distillation (H=0)

$p(L)=1$

$W_{dist}=kT\ln(2)$



# Work distillation

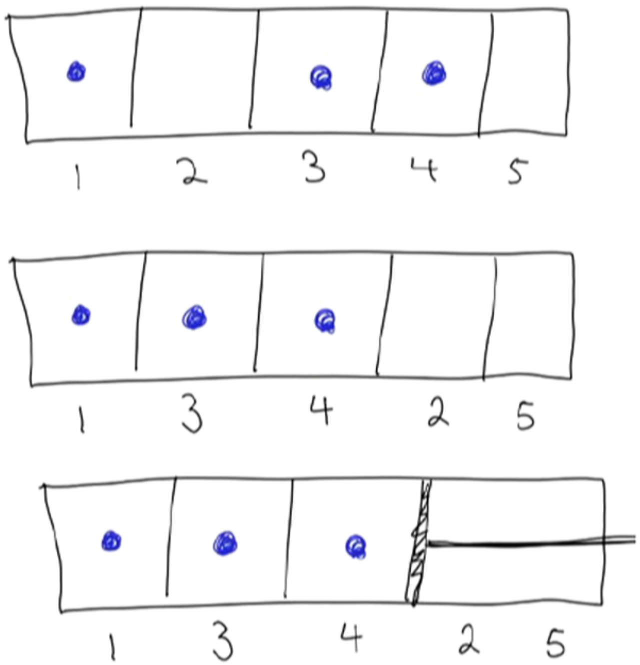
$p(1)=2/3$

$p(3)=1/6$

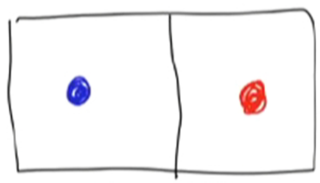
$p(4)=1/6$

$W_{dist}=kT[\ln(5)-\ln(3)]$

$=kT[\ln(d)-\ln(rank)]$



# Work of formation



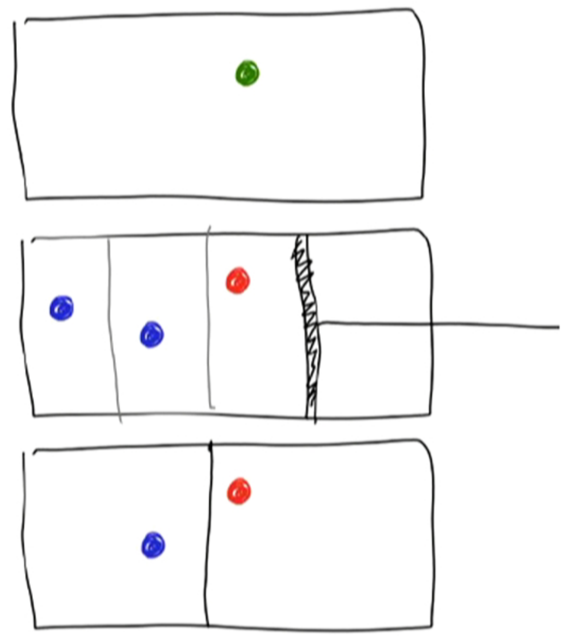
p(L)=2/3

p(R)=1/3

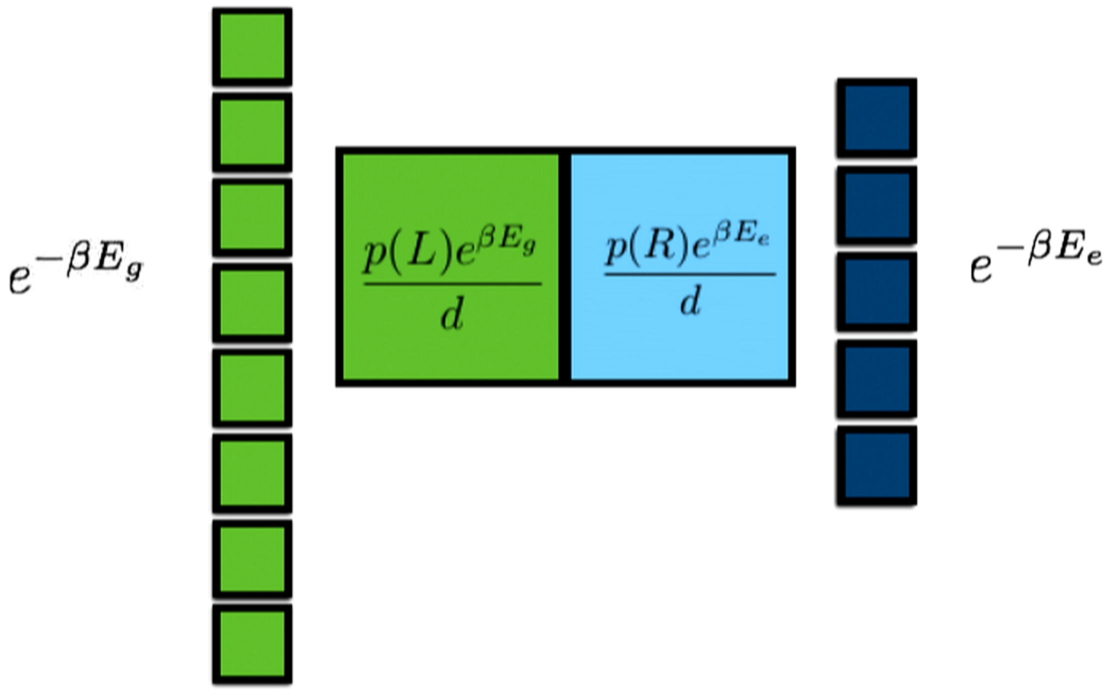
$W_{dist}$=0

$W_{form}$=kTln(2/3)

=kTln($p_{largest}$)



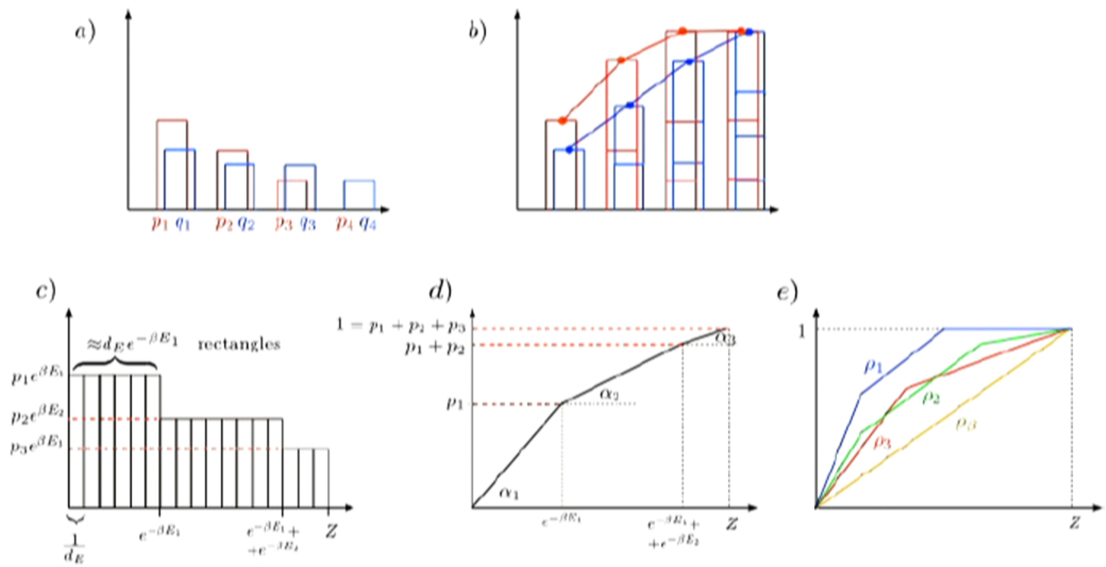
# Non-trivial Hamiltonian



# Quasi-classical 2nd laws

For degenerate energy levels

$x \xrightarrow{\text{N.O.}} y$ iff $x \succ y$ Horodecki JO (2003)




$P(E_1, g_1)e^{\beta E_1} \geq P(E_2, g_2)e^{\beta E_2} \geq \ldots \beta$ - ordered conjecture of Ruch & Mead (75)

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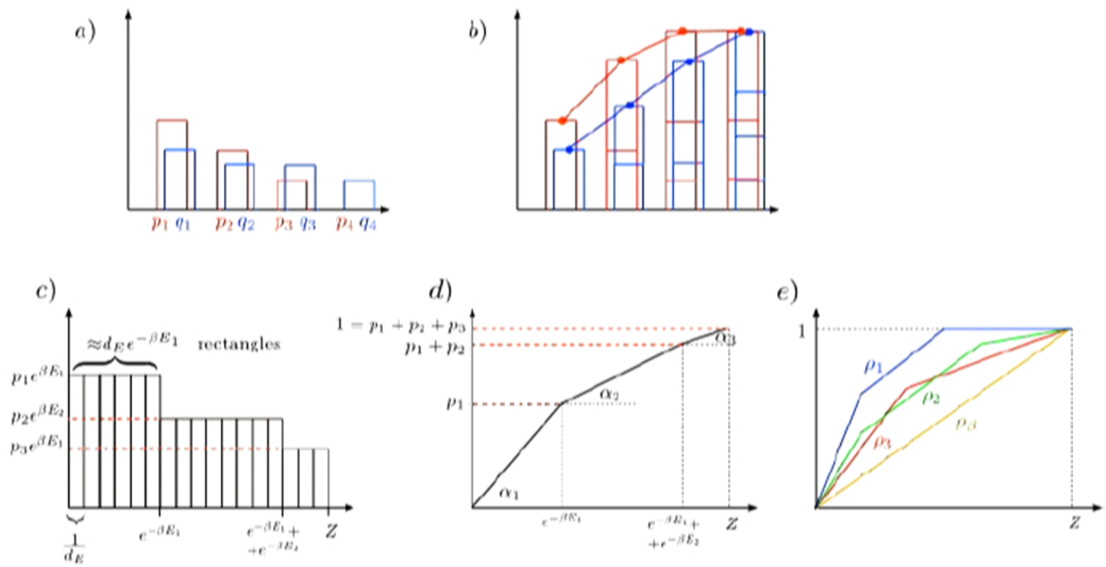


$P(E_1, g_1)e^{\beta E_1} \geq P(E_2, g_2)e^{\beta E_2} \geq \ldots \beta$ - ordered conjecture of Ruch & Mead (75)

# How cyclic?

$$\sigma_{in} \otimes \rho_S \rightarrow \sigma_{out} \otimes \rho'_s$$

$$|| \sigma_{in} - \sigma_{out} ||, \leq \epsilon$$

**Embezzlement**

$$\sigma_{in} = \sum_i \frac{1}{i} |i><i|$$ c.f. Van Dam, Hayden (2002)

$$||\sigma_{out} - \sigma_{in}||, \leq \epsilon$$

$$\sigma_{in} \otimes \frac{\mathbb{I}}{2} \longrightarrow \sigma_{out} \otimes |0><0|$$

# Catalytic transformations

Working body, clock, etc.

$x \xrightarrow{\text{C.N.O.}} y \qquad \exists Z \text{ s.t. } \quad x \otimes z > y \otimes z$

“trumping” e.g. Klimesh - Turgut

$$x = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\right) \qquad y = \left(\frac{4}{10}, \frac{4}{10}, \frac{1}{10}, \frac{1}{10}\right) \qquad z = \left(\frac{6}{10}, \frac{4}{10}\right)$$

$$x \not> y \qquad x \otimes z > y \otimes z$$

$$D_\alpha(x \,||\, \eta) \geq D_\alpha(y \,||\, \eta) \qquad \forall\, \alpha \in (-\infty, \infty)$$

$$\eta = \left(\frac{1}{k}, \ldots \frac{1}{k}\right)$$

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# 2$^{nd}$ laws : future directions

- **Laws for coherences**
  - Ćwikliński et. al., Lostaglio et. al.(x2), Narasumhachar [talk of D. Jennings]
- **Probabilistic transformations**
  - Perry, Alhambra, JO
- **Understanding catalysis**
  - Ng et. al., Mueller and Pastena
- **Restrictions on resources (e.g. time)**
  - Masanes, JO

# Conclusions

Laws of thermodynamics → 1st law

– Thermal Operations ($U_E$, $\tau$, $tr$)

$0^{th}$ : $\tau$ must be thermal for non-trivial theory

$2^{nd}$ * : $F_\alpha(\rho \,||\, \tau)$ must go down $0 \leq \alpha$

* : embezzlement

Quasi-classical states: $2^{nd}$ laws are also sufficient

Many free energies ⟶ irreversibility

Limitations due to finite size, quantumness

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