Title: Towards new foundations of quantum theory from first principles and from quantum field theory

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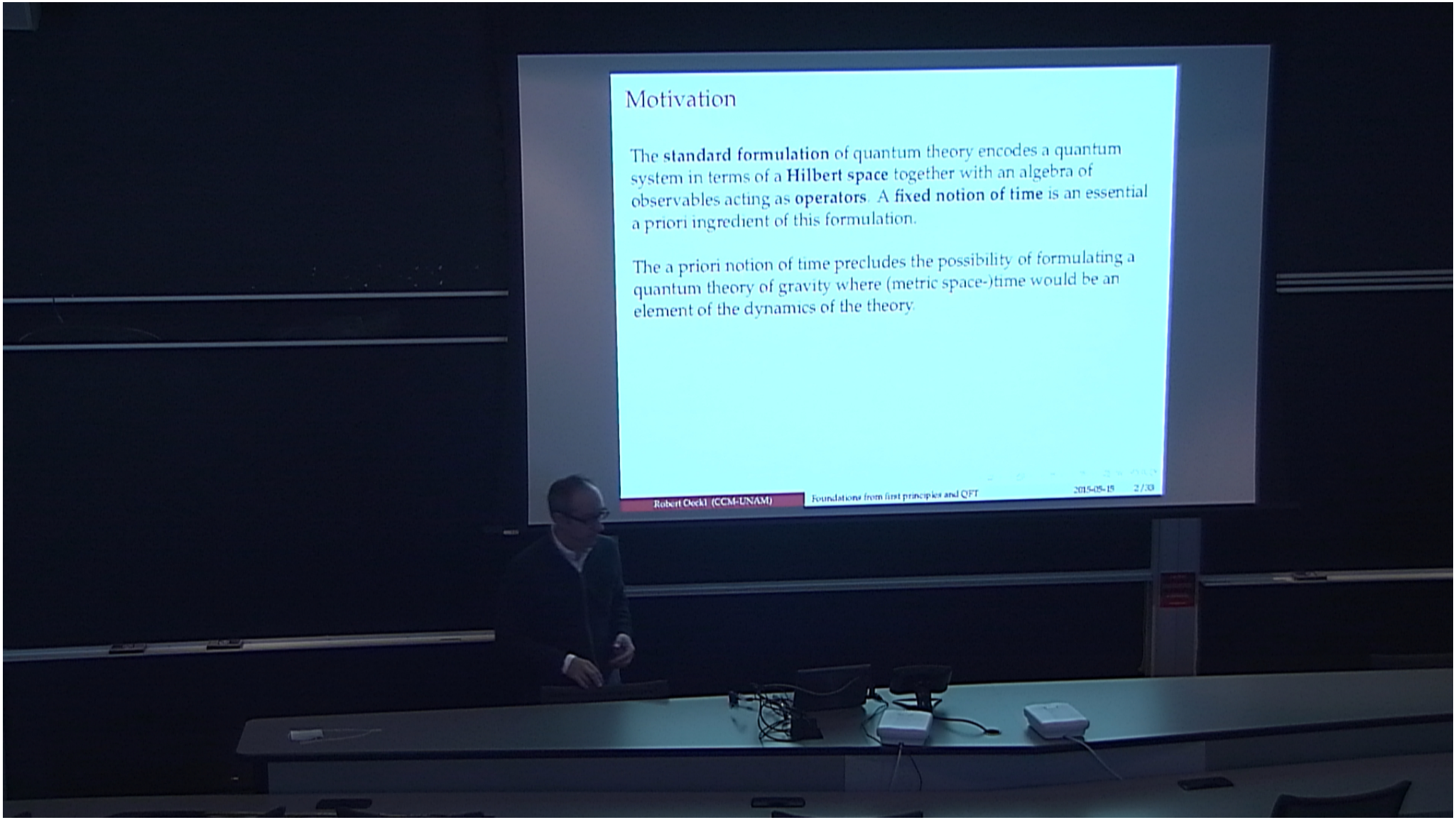
Abstract: As is well known, time plays a special role in the standard formulation of quantum theory, bringing the latter into severe conflict with the principles of general relativity. This suggests the existence of a more fundamental and (as it turns out) covariant and timeless formulation of quantum theory. A conservative way to look for such a formulation would be to start from quantum theory as we know it, taken in its experimentally most successful form of quantum field theory, and try to uncover structure in the formalism made for actual physical predictions. A radical way to look for such a formulation would be to forget the standard formulation, take only a few first principles (locality and operationalism turn out to be good ones) and try to construct things from there. Remarkably, approaches following these apparently opposite paths have recently been shown to converge in a single framework. In this talk I want to provide an overview of the current understanding of the resulting "positive formalism", its implications, and the paths that led to it. This includes relations to works of Witten and Segal in mathematical physics and of Aharonov, Hardy and others in quantum foundations.

Towards new foundations of quantum theory from first principles and from quantum field theory
Robert Oeckl
CCM-UNAM
Information Theoretic Foundations for Physics
Perimeter Institute
15 May 2015


Motivation
The standard formulation of quantum theory encodes a quantum system in terms of a Hilbert space together with an algebra of observables acting as operators. A fixed notion of time is an essential a priori ingredient of this formulation.
The a priori notion of time precludes the possibility of formulating a quantum theory of gravity where (metric space-)time would be an element of the dynamics of the theory.
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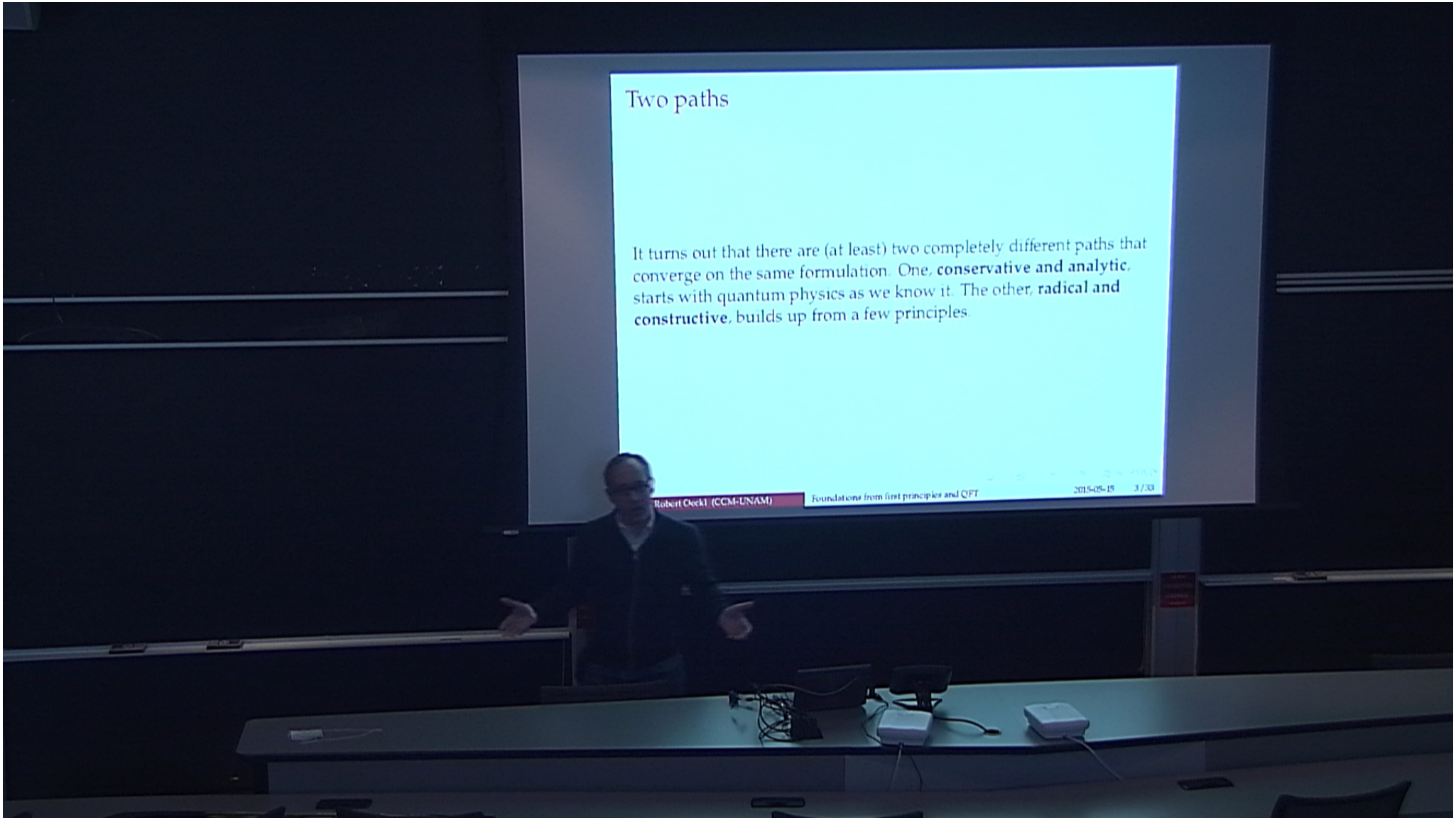
Two paths
It turns out that there are (at least) two completely different paths that converge on the same formulation. One, **conservative and analytic**, starts with quantum physics as we know it. The other, **radical and constructive**, builds up from a few principles.
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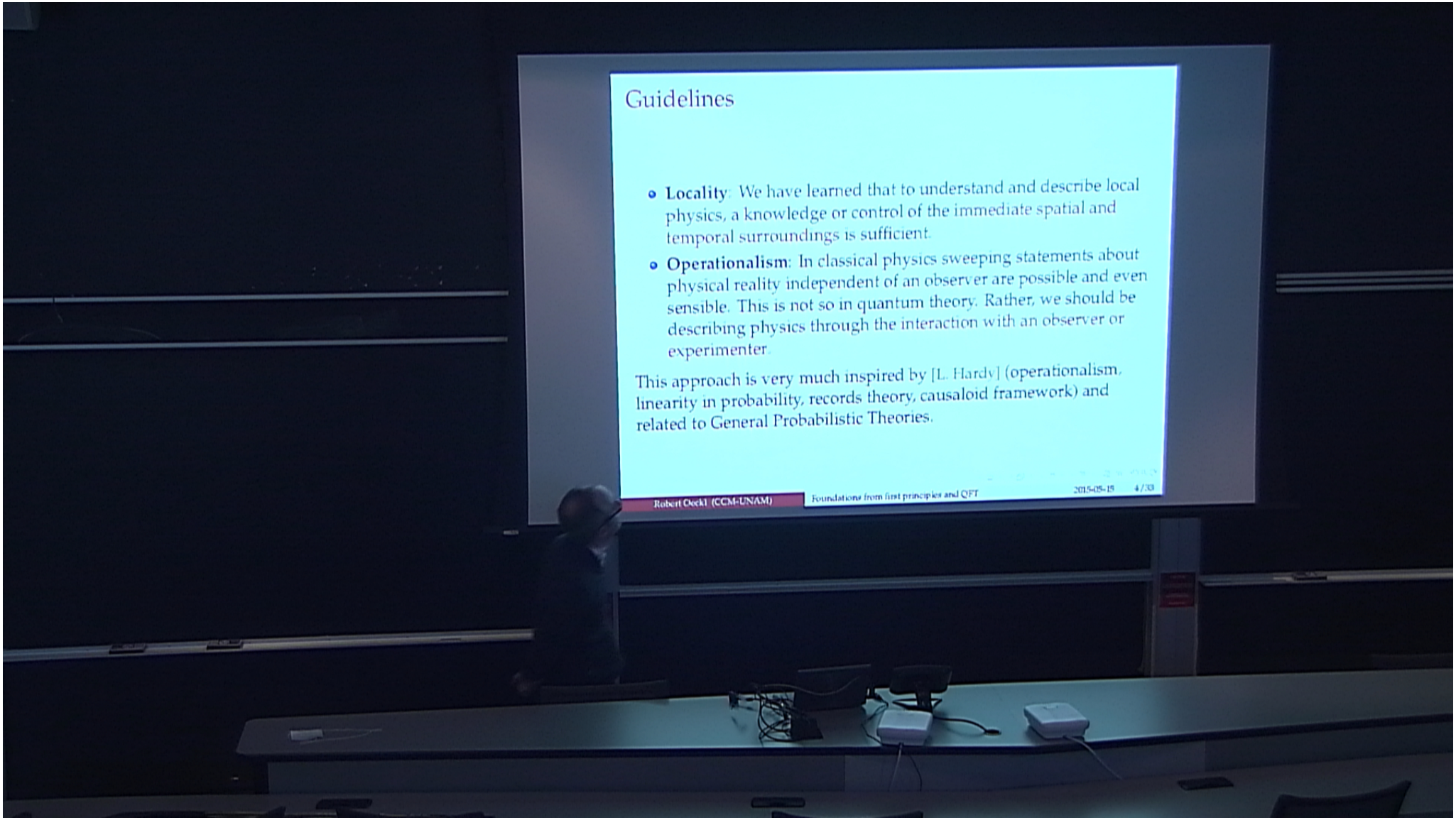
Guidelines
Locality: We have learned that to understand and describe local physics, a knowledge or control of the immediate spatial and temporal surroundings is sufficient.
Operationalism: In classical physics sweeping statements about physical reality independent of an observer are possible and even sensible. This is not so in quantum theory. Rather, we should be describing physics through the interaction with an observer or experimenter.
This approach is very much inspired by [L. Hardy] (operationalism, linearity in probability, records theory, causaloid framework) and related to General Probabilistic Theories.
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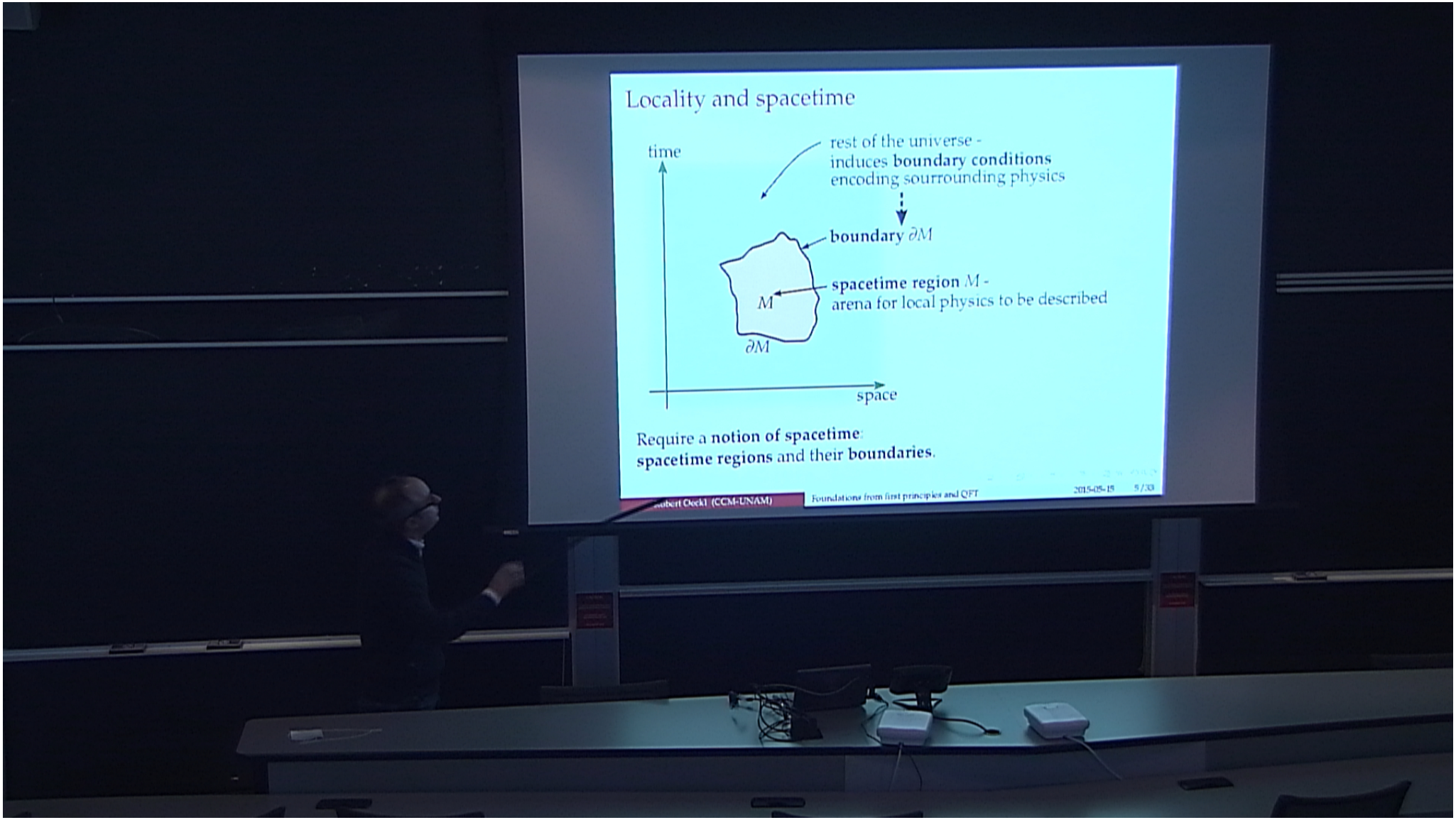
Locality and spacetime
time
rest of the universe -
induces boundary conditions
encoding sourrounding physics
boundary ∂M
spacetime region M -
arena for local physics to be described
M
∂M
space
Require a notion of spacetime:
spacetime regions and their boundaries.
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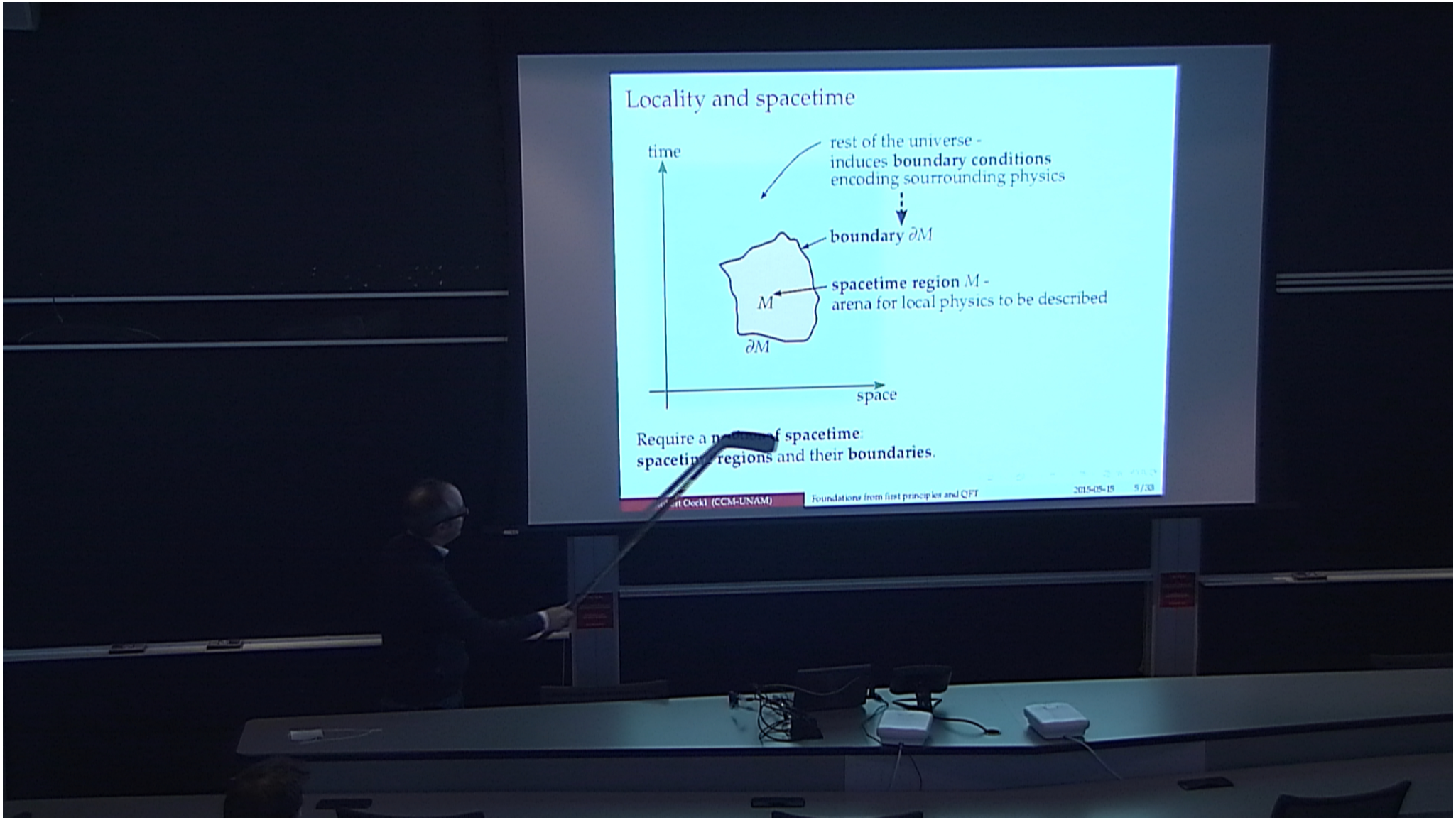
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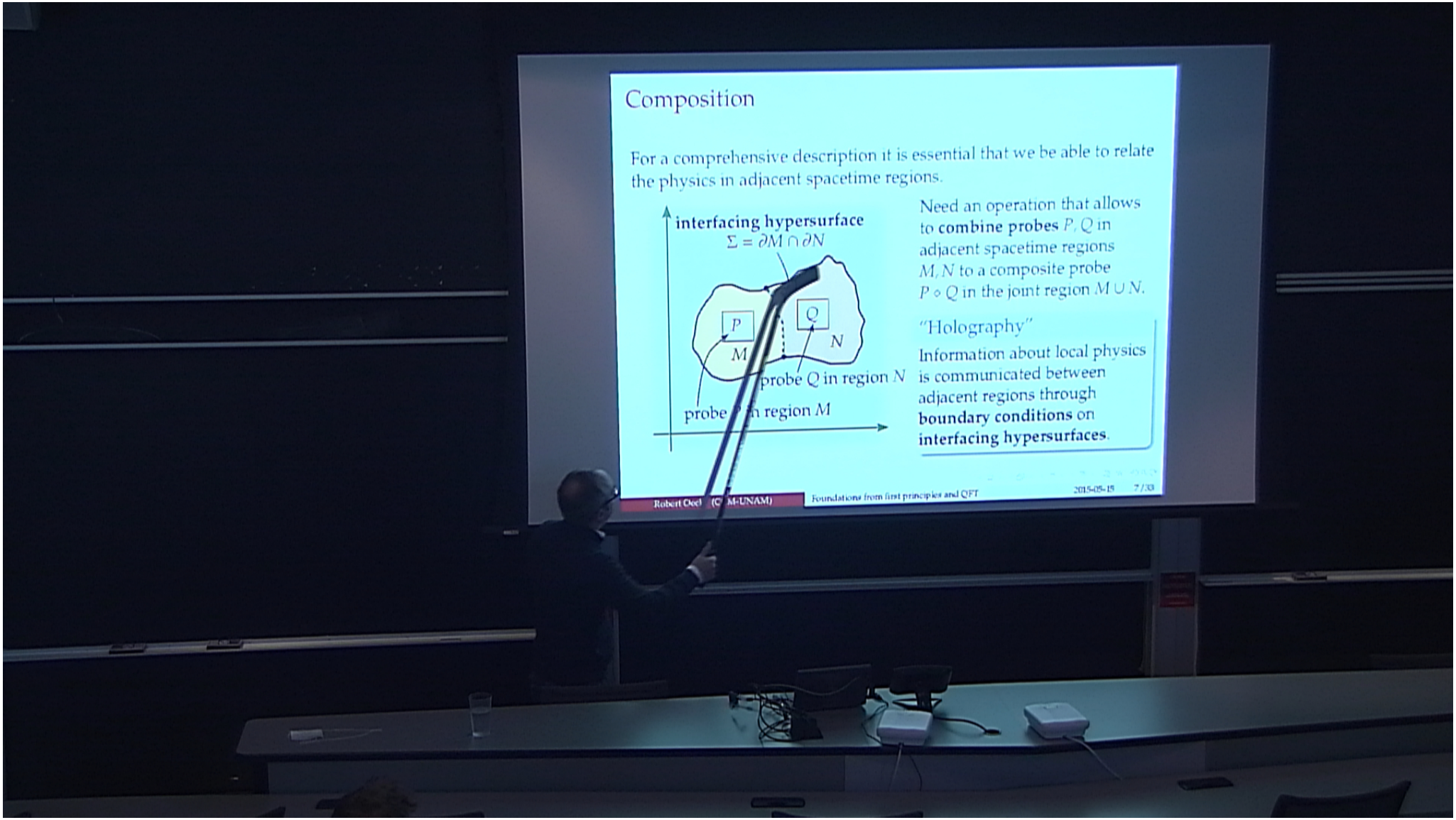
Composition
For a comprehensive description it is essential that we be able to relate the physics in adjacent spacetime regions.
interfacing hypersurface
$\Sigma = \partial M \cap \partial N$
P
Q
M
N
probe Q in region N
probe P in region M
Need an operation that allows to **combine probes** P, Q in adjacent spacetime regions M, N to a composite probe $P \circ Q$ in the joint region $M \cup N$.
"Holography"
Information about local physics is communicated between adjacent regions through **boundary conditions** on **interfacing hypersurfaces**.
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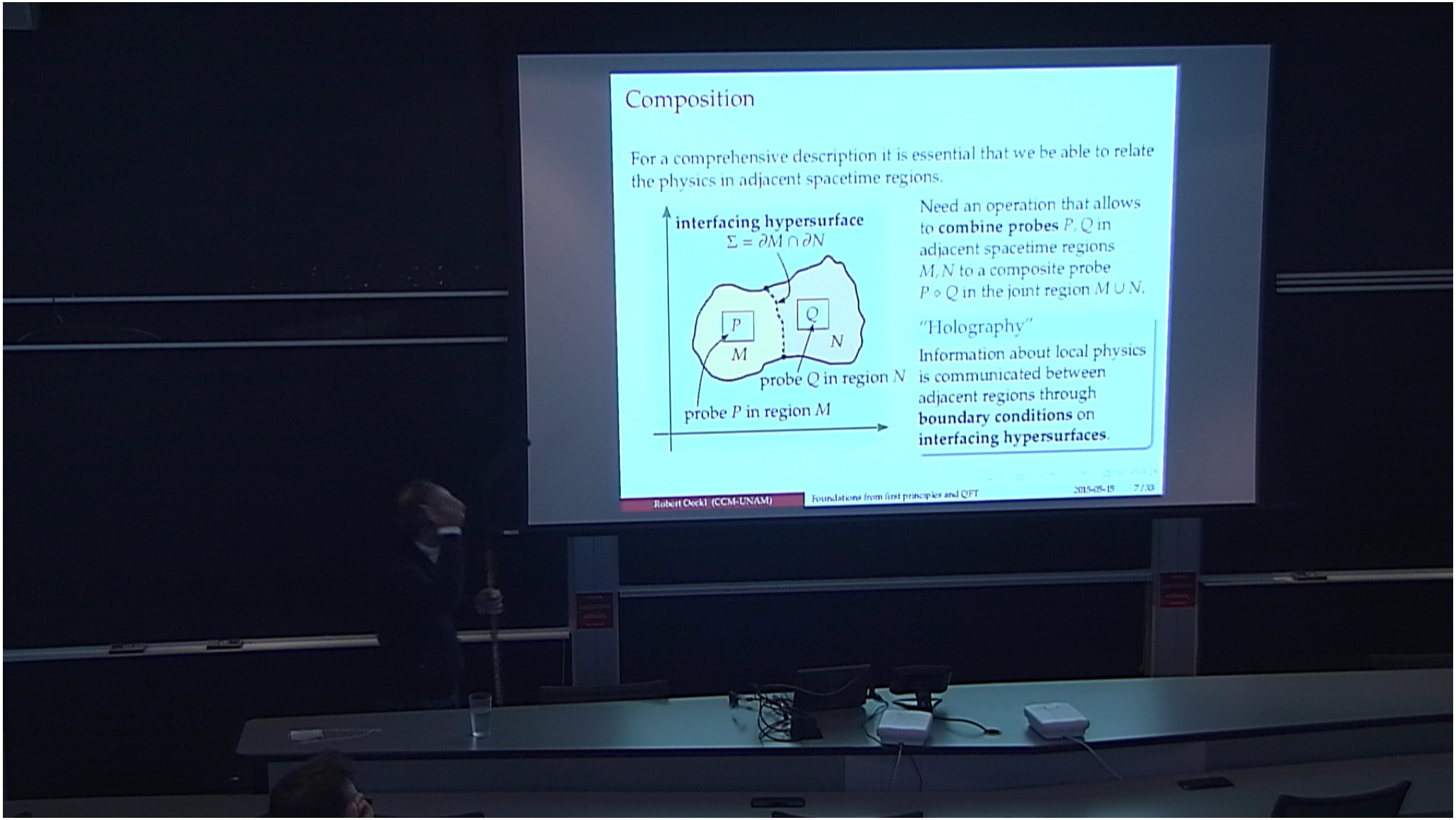
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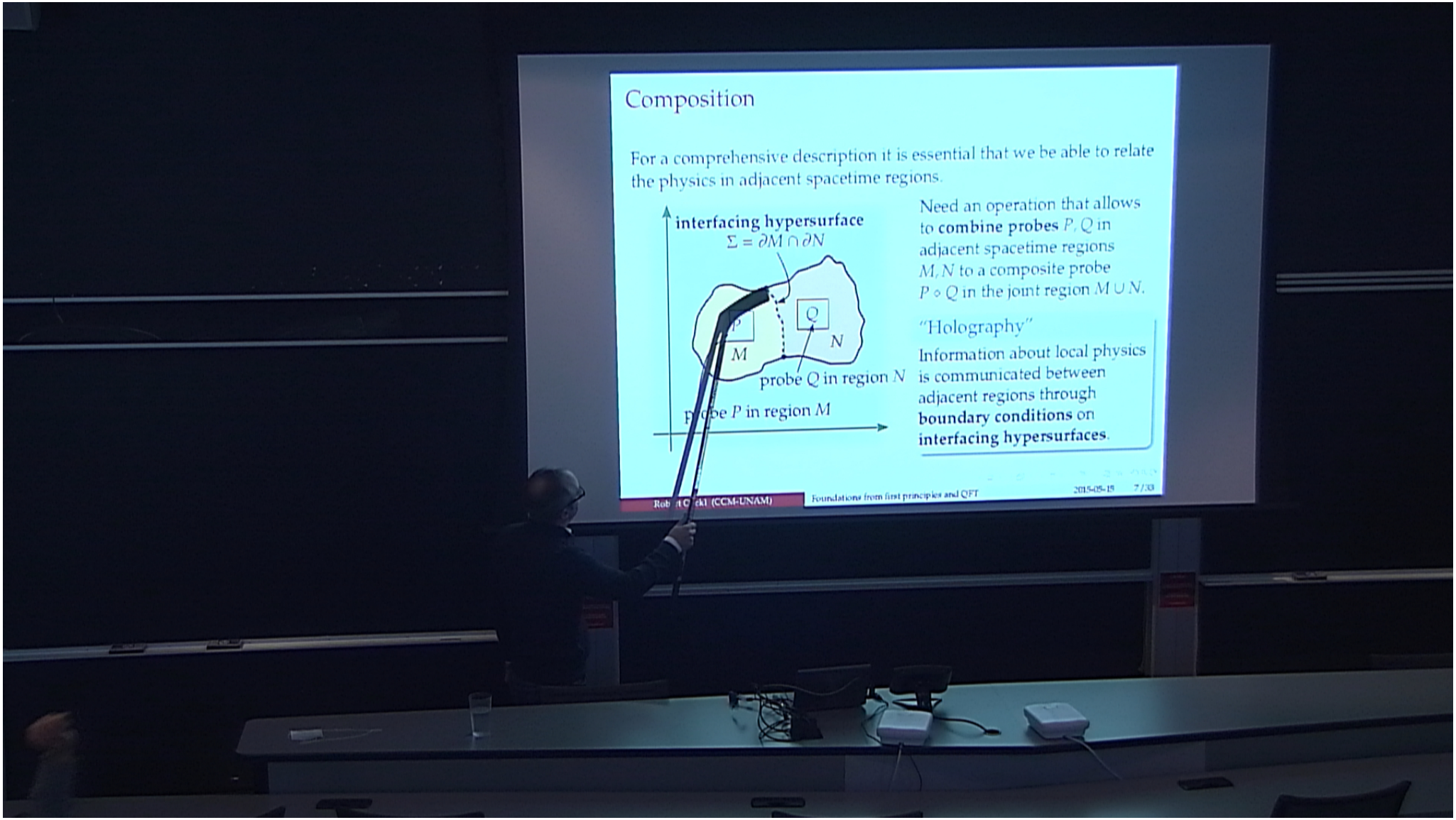
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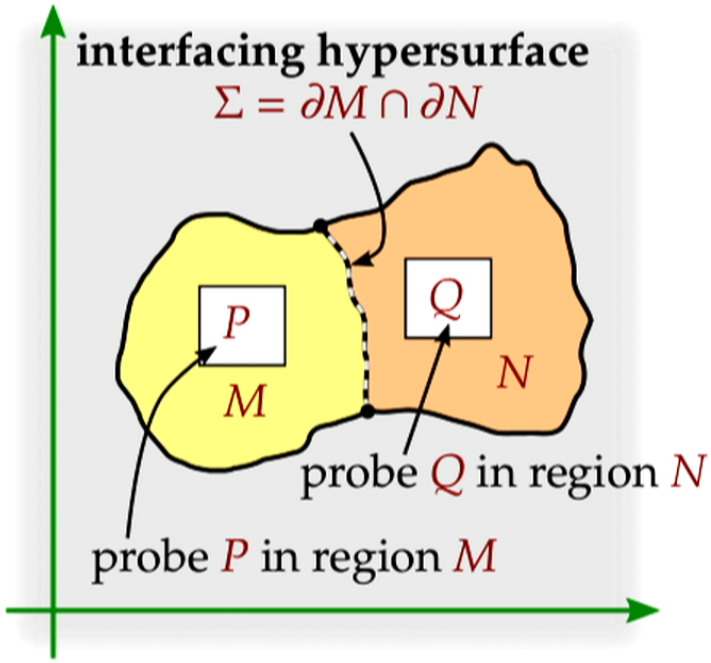

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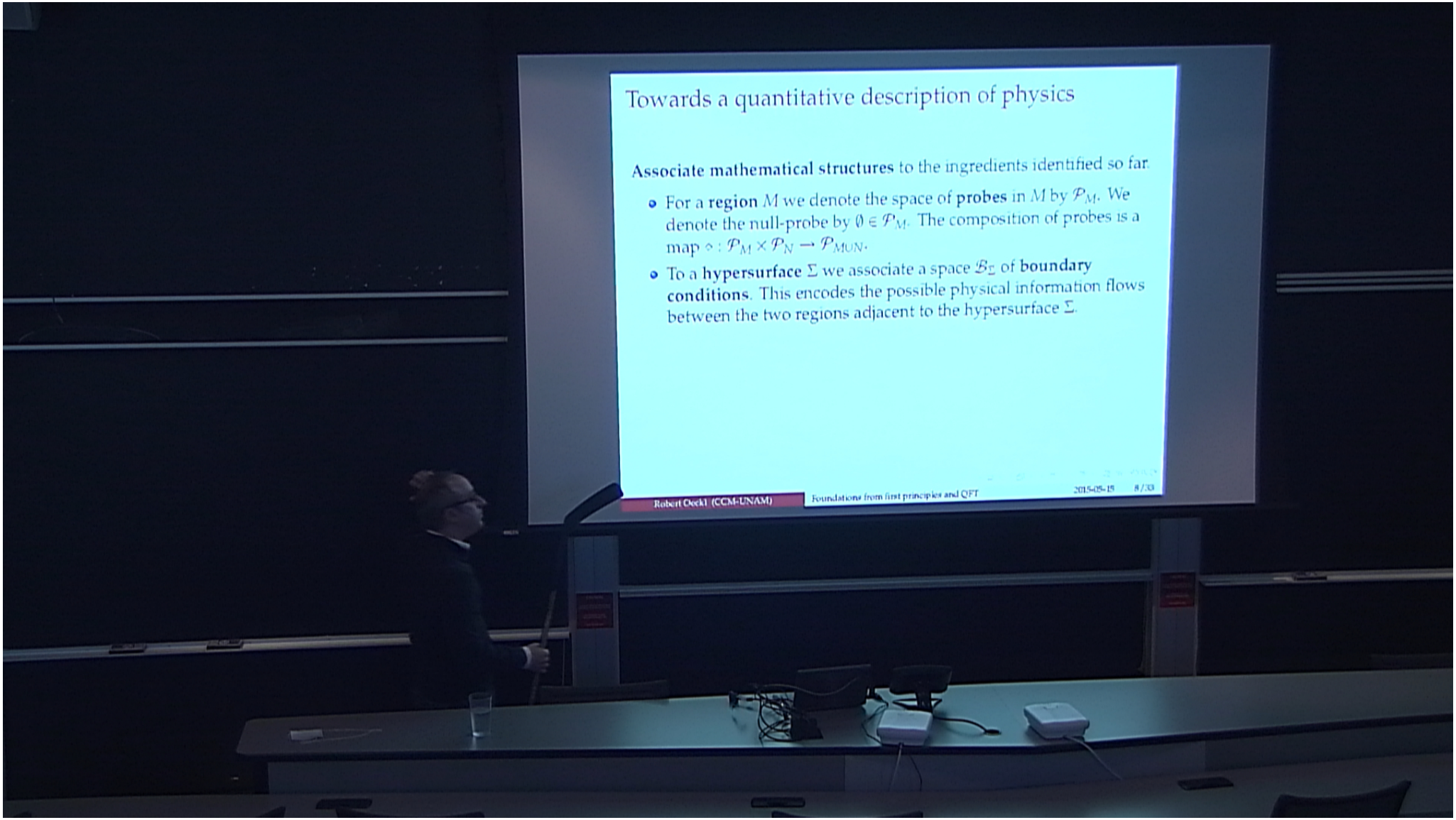
Towards a quantitative description of physics
Associate mathematical structures to the ingredients identified so far.
For a region M we denote the space of probes in M by $\mathcal{P}_M$. We denote the null-probe by $\emptyset \in \mathcal{P}_M$. The composition of probes is a map $\diamond : \mathcal{P}_M \times \mathcal{P}_N \to \mathcal{P}_{M \cup N}$.
To a hypersurface Σ we associate a space $\mathcal{B}_\Sigma$ of boundary conditions. This encodes the possible physical information flows between the two regions adjacent to the hypersurface Σ.
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Towards a quantitative description of physics

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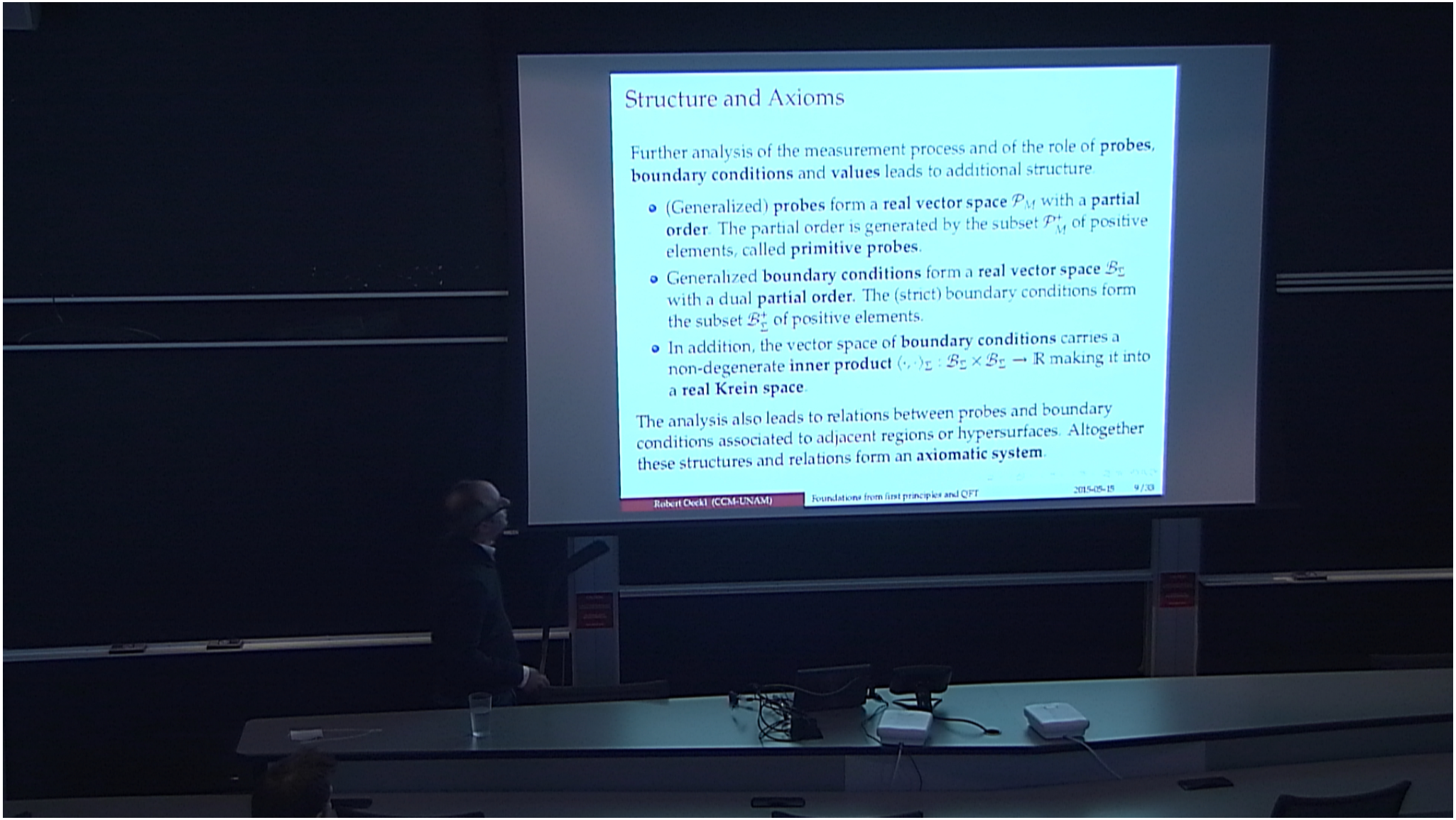
- To a **hypersurface** Σ we associate a space $\mathcal{B}_\Sigma$ of **boundary conditions**. This encodes the possible physical information flows between the two regions adjacent to the hypersurface Σ.
- To a **probe** P in a **spacetime region** M with **boundary condition** $b \in \mathcal{B}_{\partial M}$ we associate a **value**. We shall take this to be a **real number** and denote it by $(P, b)_M$. It encodes a property of the local physics in the interior as detected by the probe and subject to the boundary condition. Formally, $(\cdot, \cdot)_M : \mathcal{P}_M \times \mathcal{B}_{\partial M} \to \mathbb{R}$.


Structure and Axioms
Further analysis of the measurement process and of the role of **probes**, **boundary conditions** and **values** leads to additional structure.
(Generalized) **probes** form a **real vector space** $\mathcal{P}_M$ with a **partial order**. The partial order is generated by the subset $\mathcal{P}_M^+$ of positive elements, called **primitive probes**.
Generalized **boundary conditions** form a **real vector space** $\mathcal{B}_\Sigma$ with a dual **partial order**. The (strict) boundary conditions form the subset $\mathcal{B}_\Sigma^+$ of positive elements.
In addition, the vector space of **boundary conditions** carries a non-degenerate **inner product** $\langle \cdot, \cdot \rangle_\Sigma : \mathcal{B}_\Sigma \times \mathcal{B}_\Sigma \to \mathbb{R}$ making it into a **real Krein space**.
The analysis also leads to relations between probes and boundary conditions associated to adjacent regions or hypersurfaces. Altogether these structures and relations form an **axiomatic system**.
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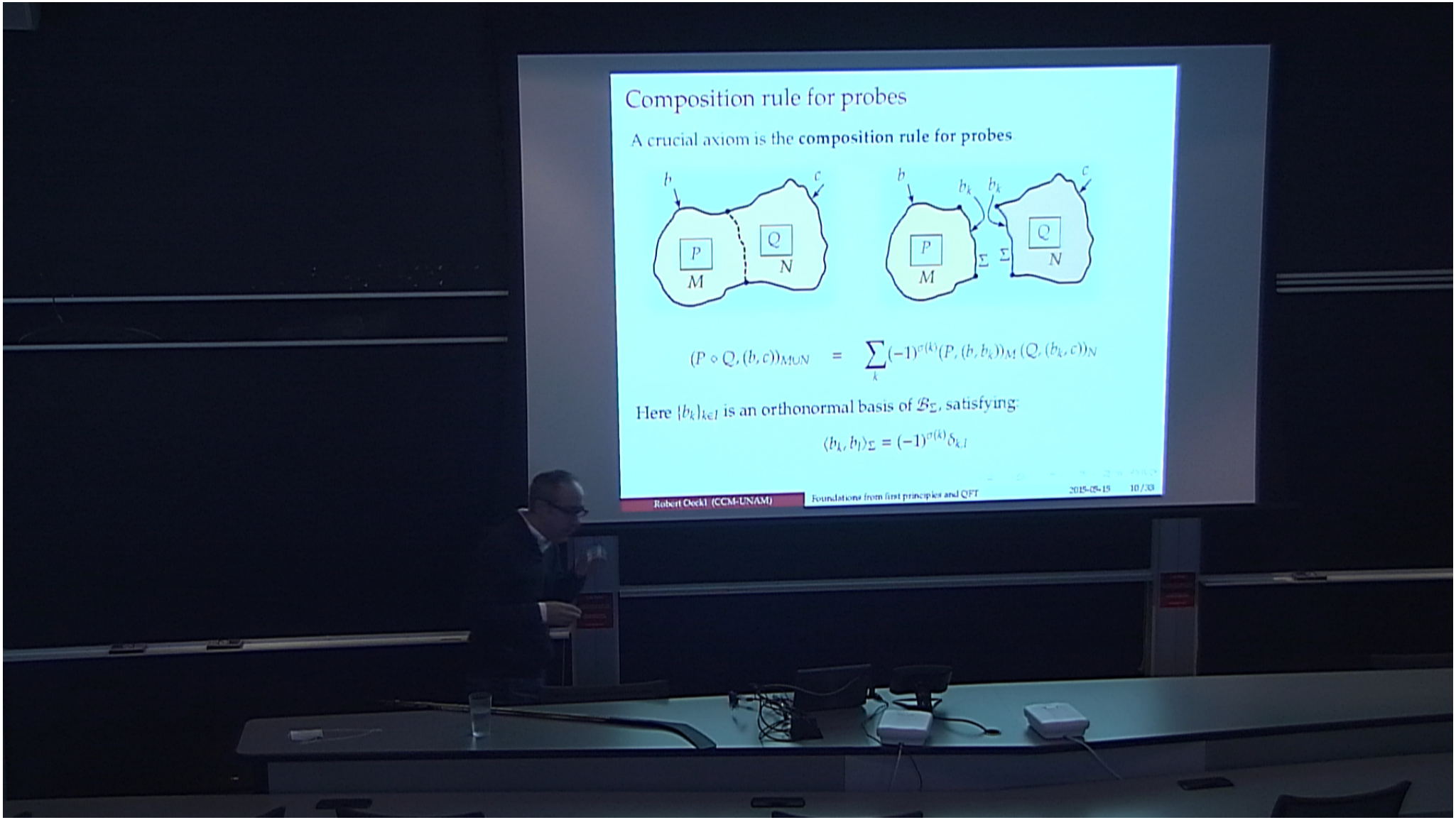
Composition rule for probes
A crucial axiom is the composition rule for probes
b
c
P
Q
M
N
$(P \circ Q, (b, c))_{M \cup N} = \sum_k (-1)^{\sigma(k)} (P, (b, b_k))_M \, (Q, (b_k, c))_N$
Here $\{b_k\}_{k \in I}$ is an orthonormal basis of $\mathcal{B}_\Sigma$, satisfying
$\langle b_k, b_l \rangle_\Sigma = (-1)^{\sigma(k)} \delta_{k,l}$
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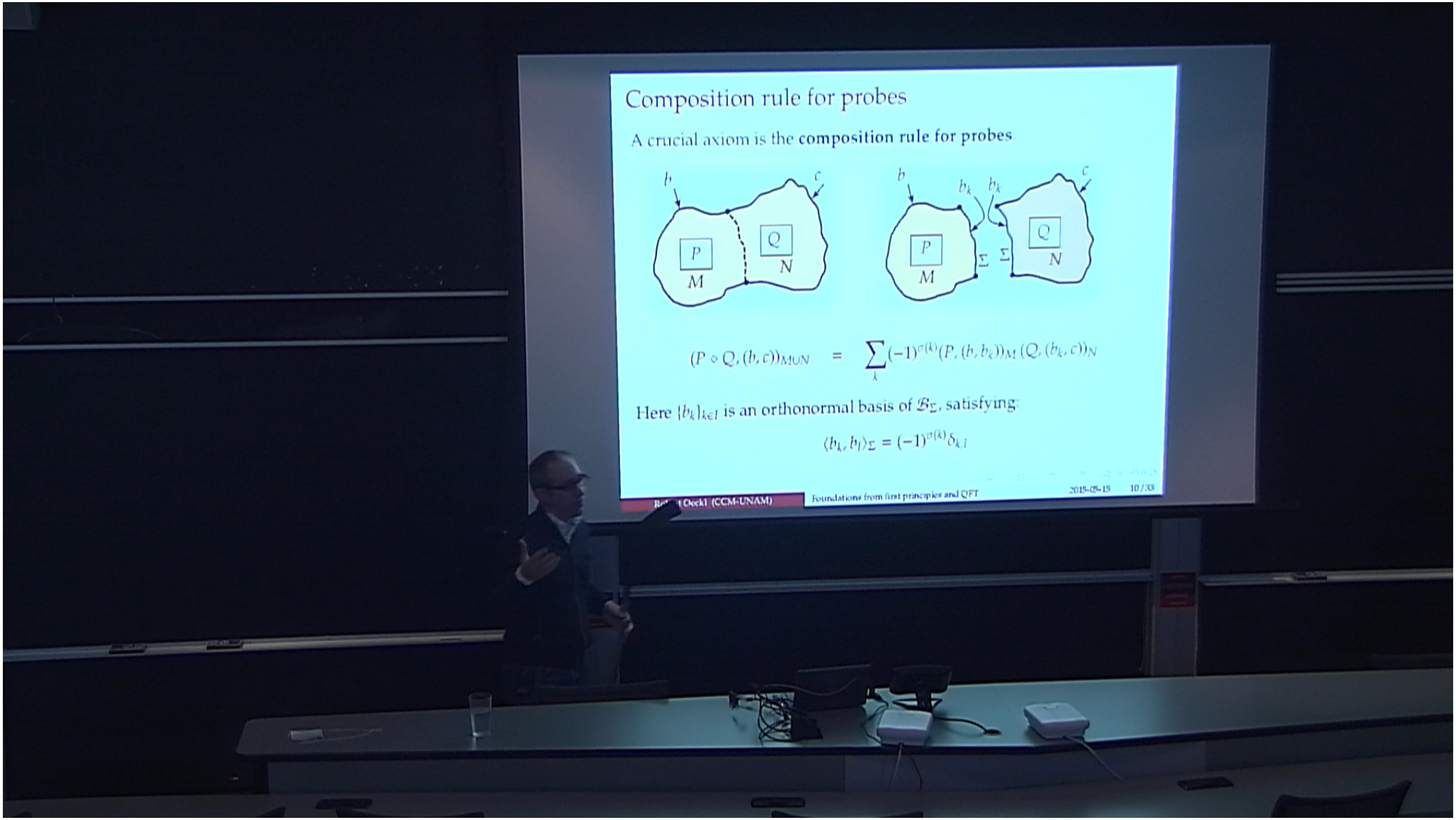
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b
c
P
Q
M
N
b
b_k
b_k
c
P
Q
M
N
Σ
Σ
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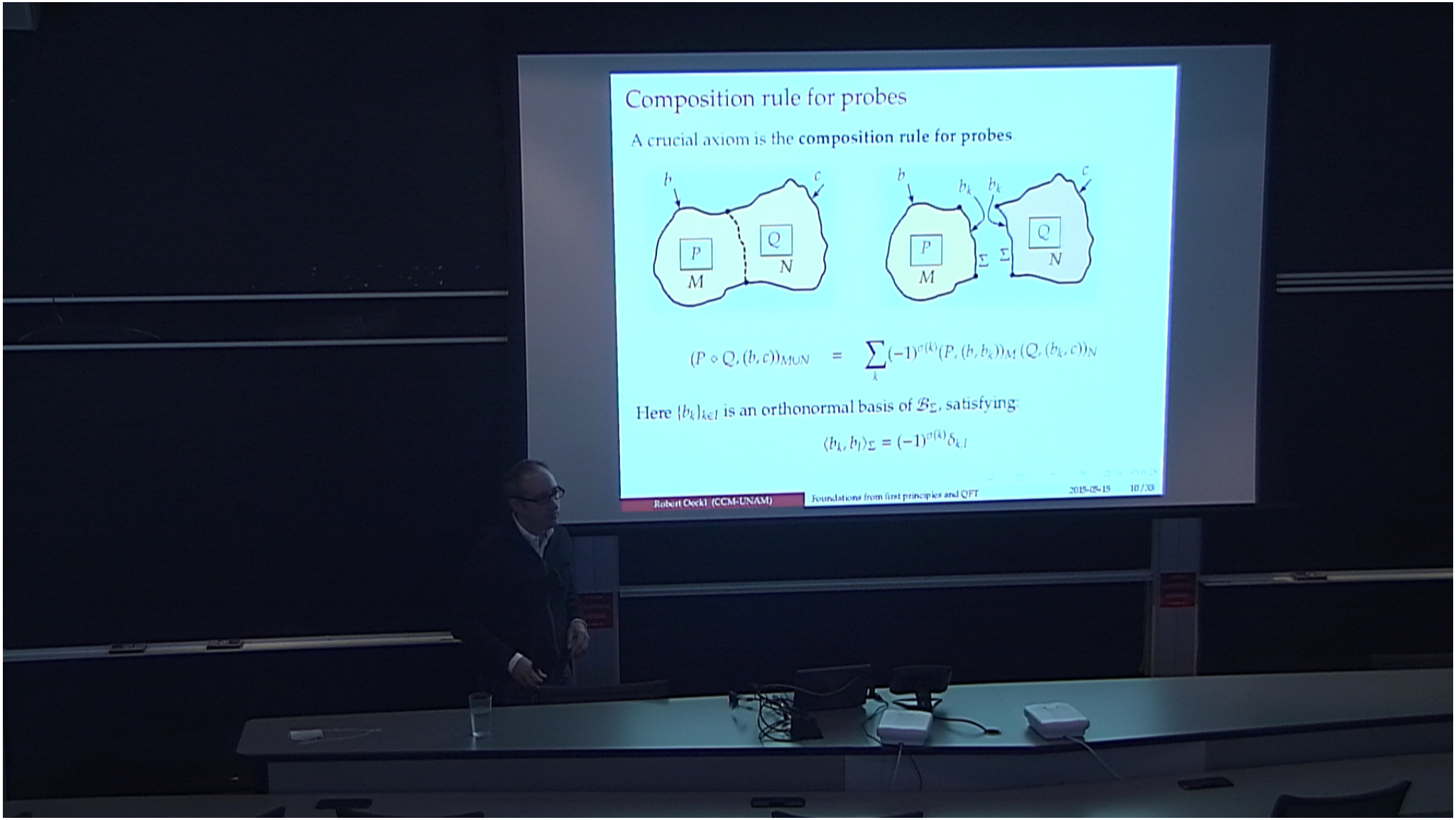
Composition rule for probes
A crucial axiom is the composition rule for probes
b
c
P
M
Q
N
b
b_k
b_k
c
P
M
Σ
Σ
Q
N
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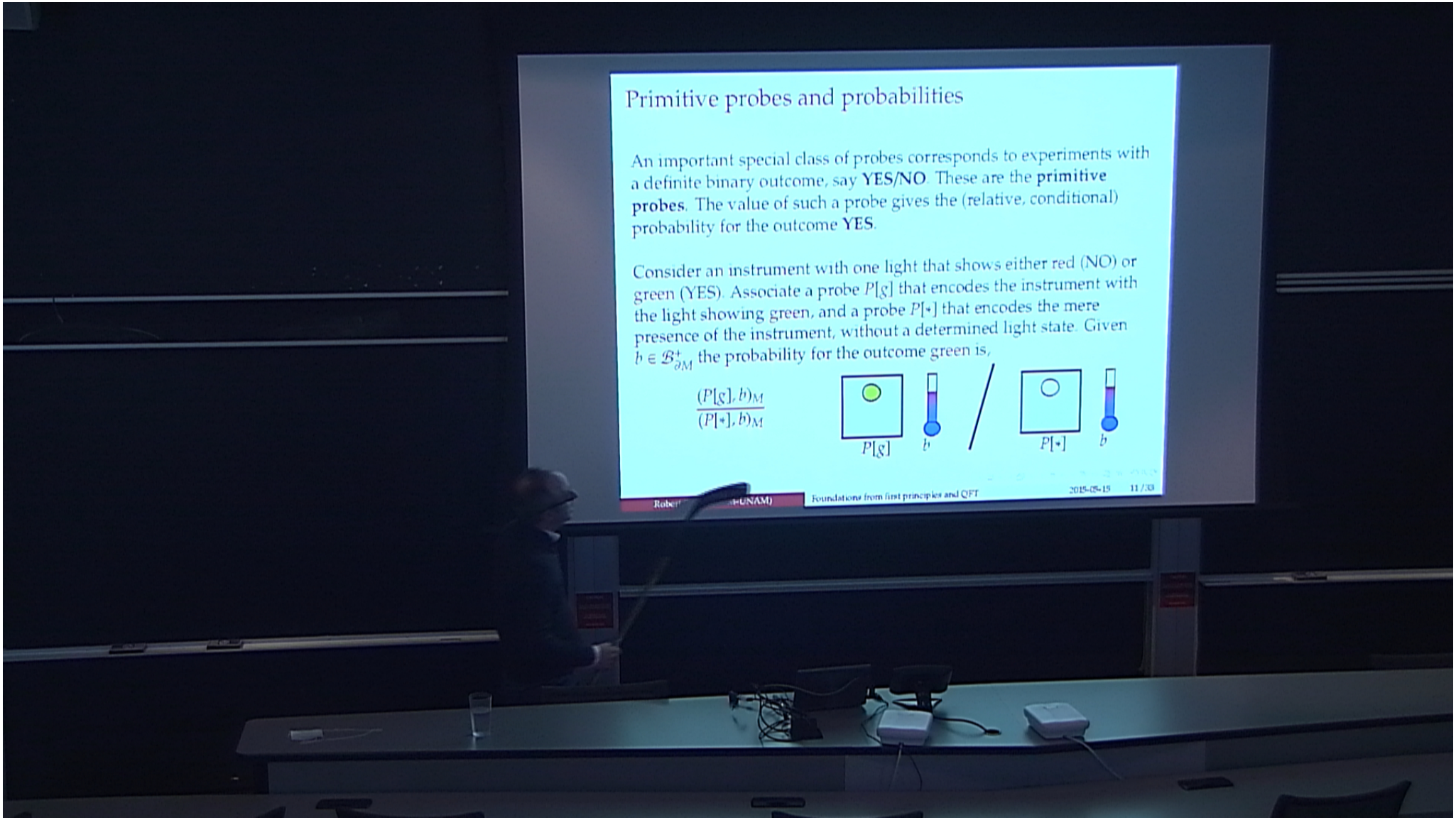
Primitive probes and probabilities
An important special class of probes corresponds to experiments with a definite binary outcome, say YES/NO. These are the primitive probes. The value of such a probe gives the (relative, conditional) probability for the outcome YES.
Consider an instrument with one light that shows either red (NO) or green (YES). Associate a probe $P[g]$ that encodes the instrument with the light showing green, and a probe $P[*]$ that encodes the mere presence of the instrument, without a determined light state. Given $b \in \mathcal{B}^+_{\partial M}$ the probability for the outcome green is,
$\frac{(P[g], b)_M}{(P[*], b)_M}$
$P[g]$
b
$P[*]$
b
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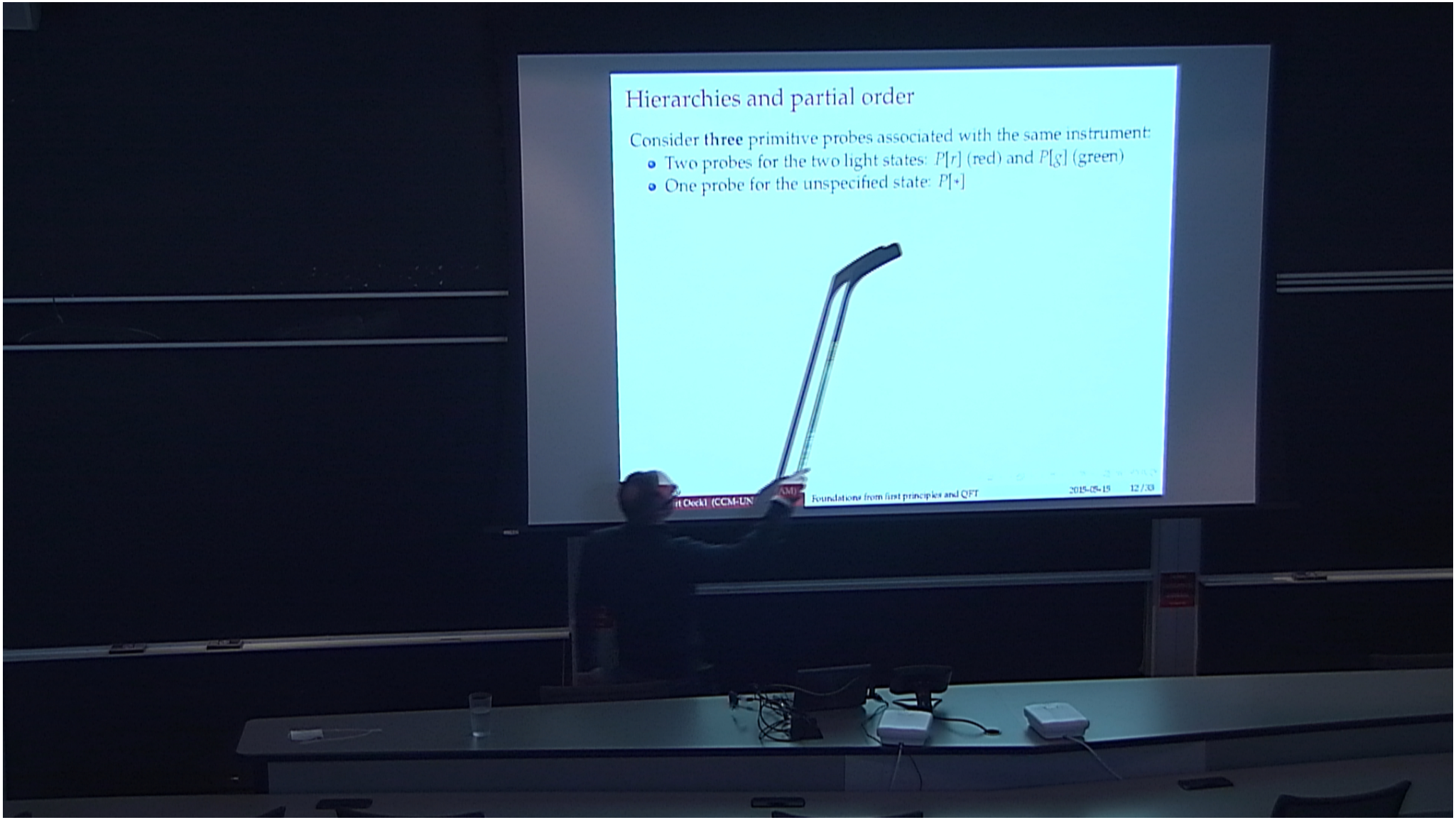
Hierarchies and partial order
Consider three primitive probes associated with the same instrument:
Two probes for the two light states: P[r] (red) and P[g] (green)
One probe for the unspecified state: P[*]
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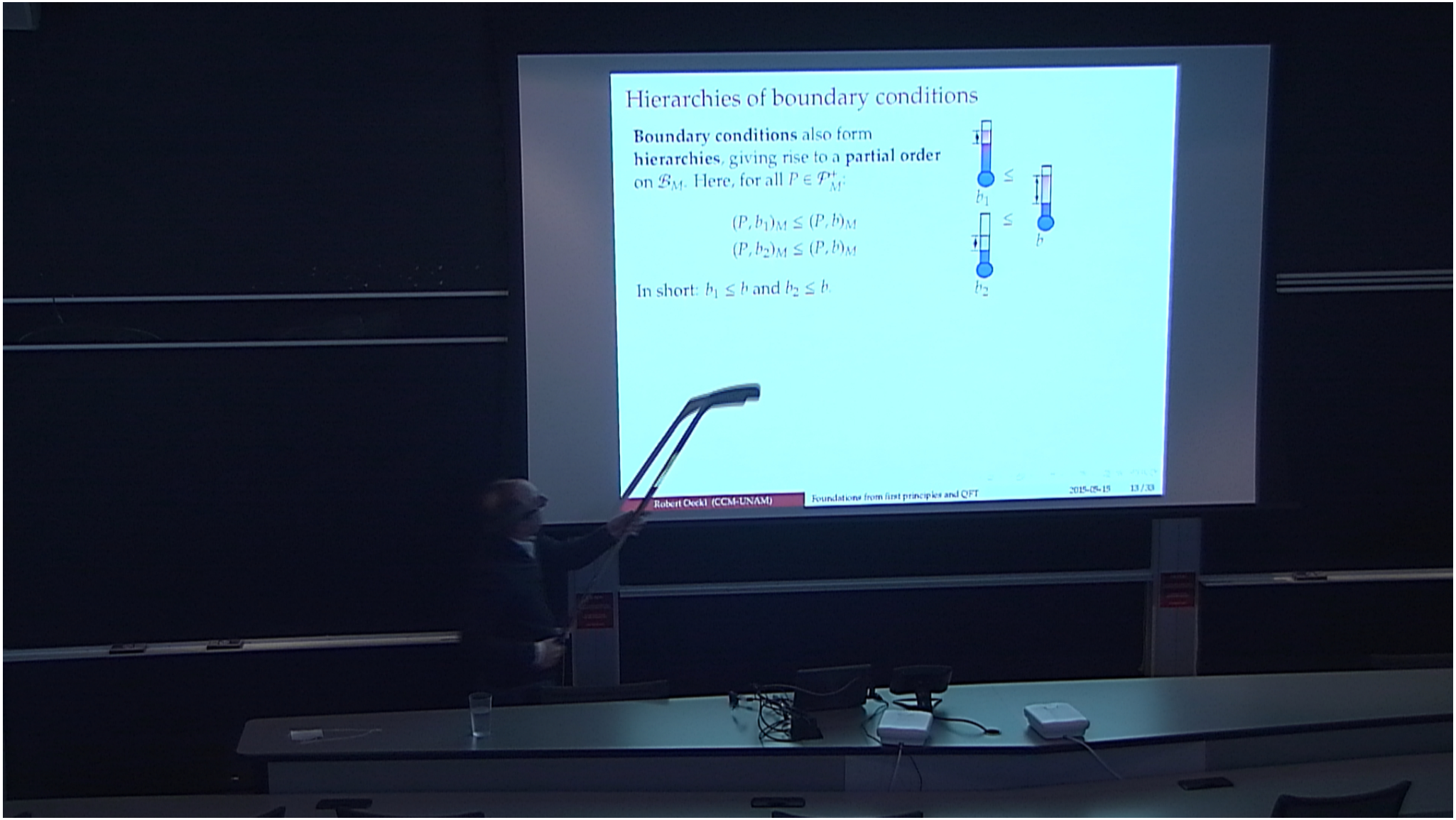
Hierarchies of boundary conditions
Boundary conditions also form **hierarchies**, giving rise to a **partial order** on $\mathcal{B}_M$. Here, for all $P \in \mathcal{P}_M^+$:
$(P, b_1)_M \leq (P, b)_M$
$(P, b_2)_M \leq (P, b)_M$
In short: $b_1 \leq b$ and $b_2 \leq b$
b_1
$\leq$
b
$\leq$
b_2
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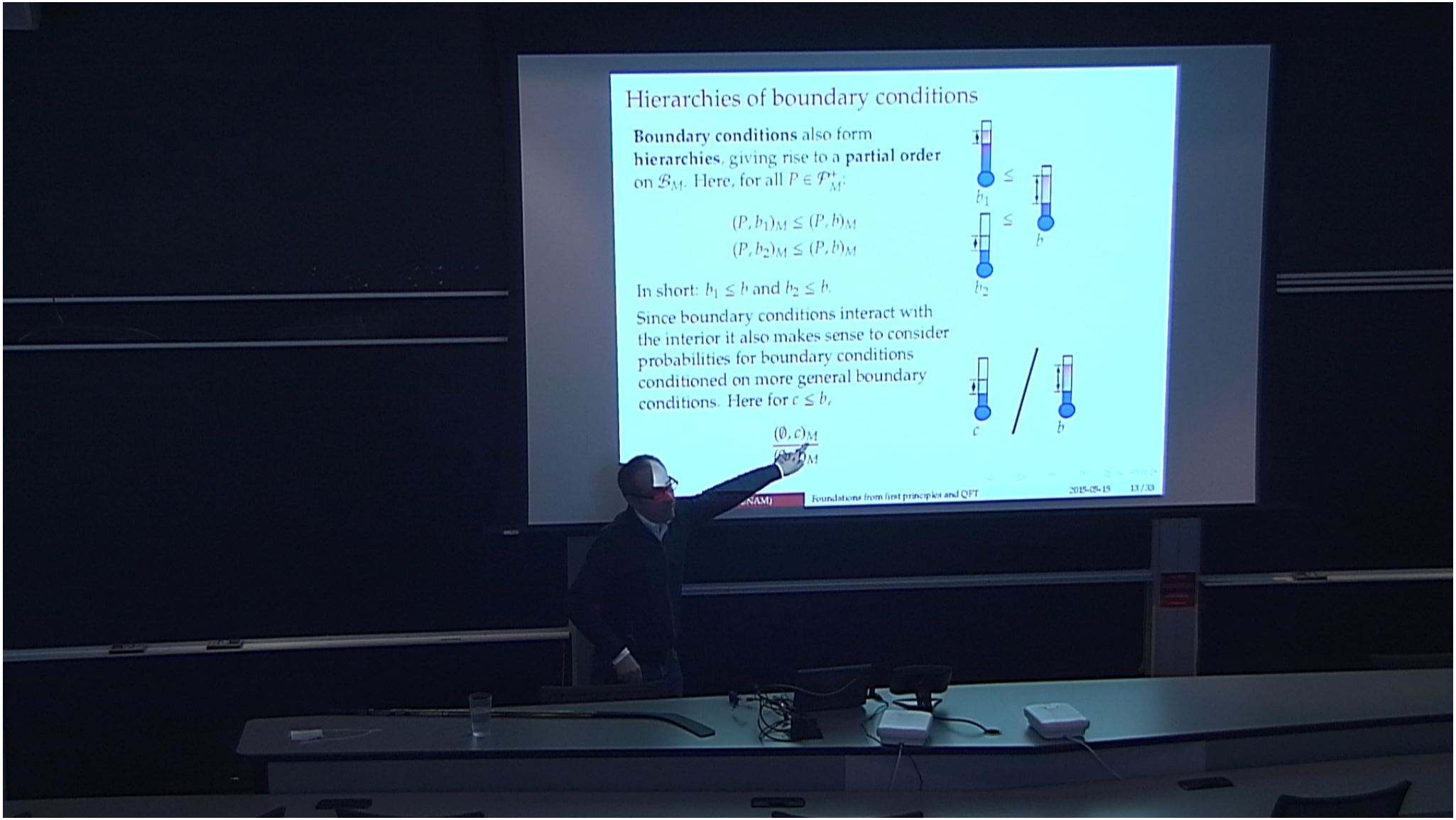
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In short: $b_1 \leq b$ and $b_2 \leq b$.
Since boundary conditions interact with the interior it also makes sense to consider probabilities for boundary conditions conditioned on more general boundary conditions. Here for $c \leq b$,
$\frac{(\emptyset, c)_M}{(\emptyset, b)_M}$
b_1
$\leq$
b
b_2
$\leq$
c
b
NAM)
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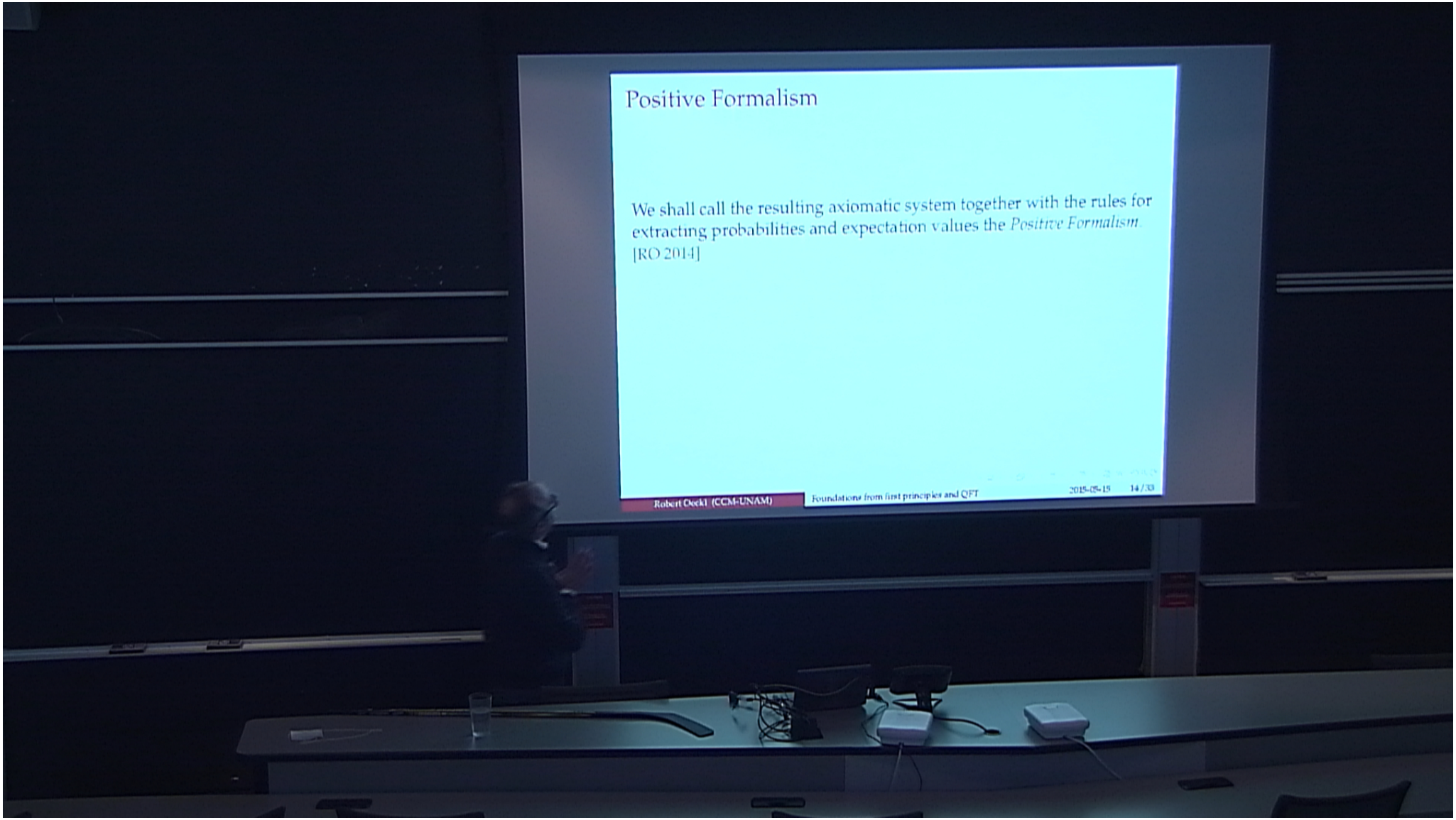
Positive Formalism
We shall call the resulting axiomatic system together with the rules for extracting probabilities and expectation values the *Positive Formalism*.
[RO 2014]
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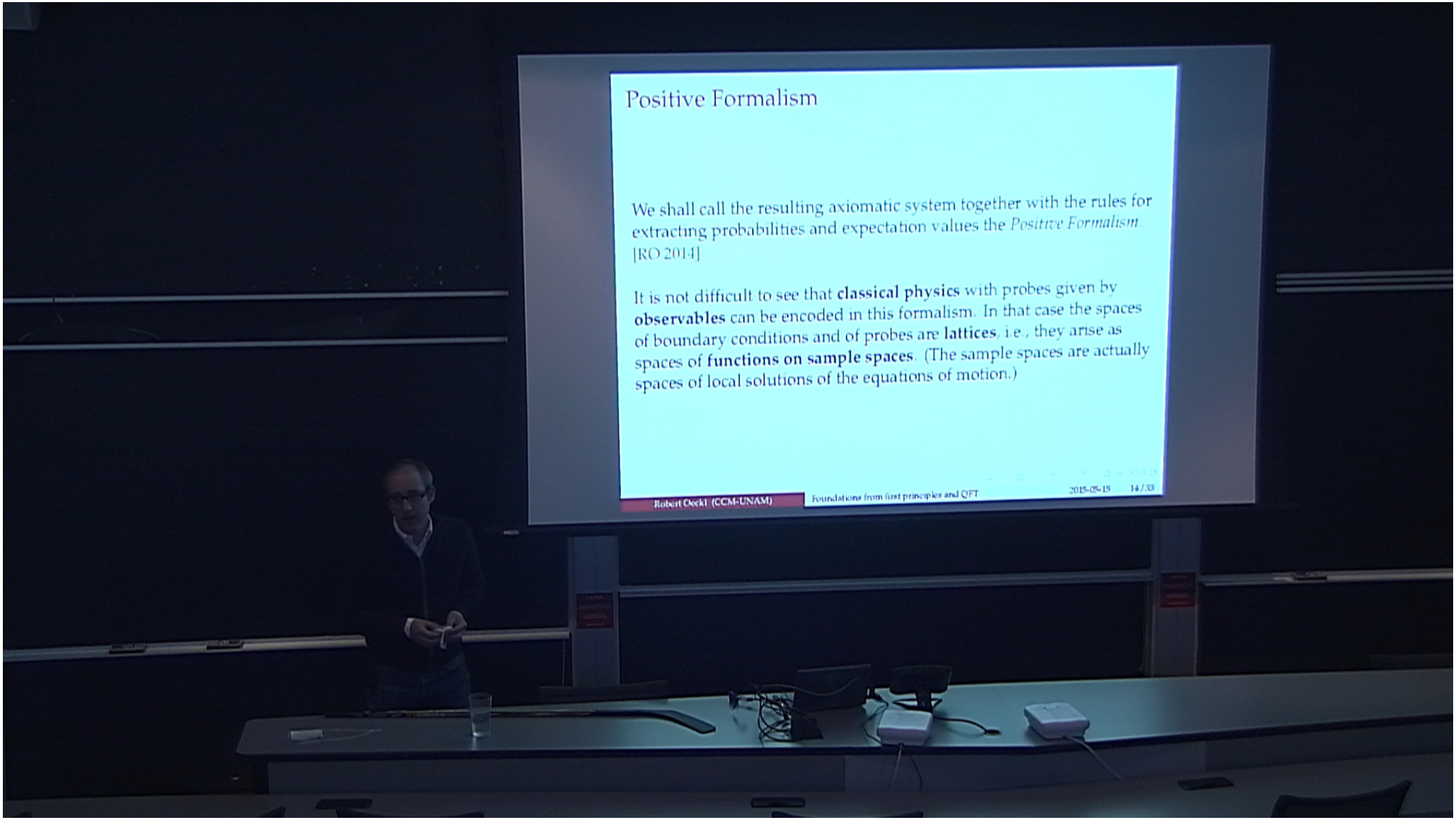
Positive Formalism
We shall call the resulting axiomatic system together with the rules for extracting probabilities and expectation values the *Positive Formalism*. [RO 2014]
It is not difficult to see that **classical physics** with probes given by **observables** can be encoded in this formalism. In that case the spaces of boundary conditions and of probes are **lattices**, i.e., they arise as spaces of **functions on sample spaces**. (The sample spaces are actually spaces of local solutions of the equations of motion.)
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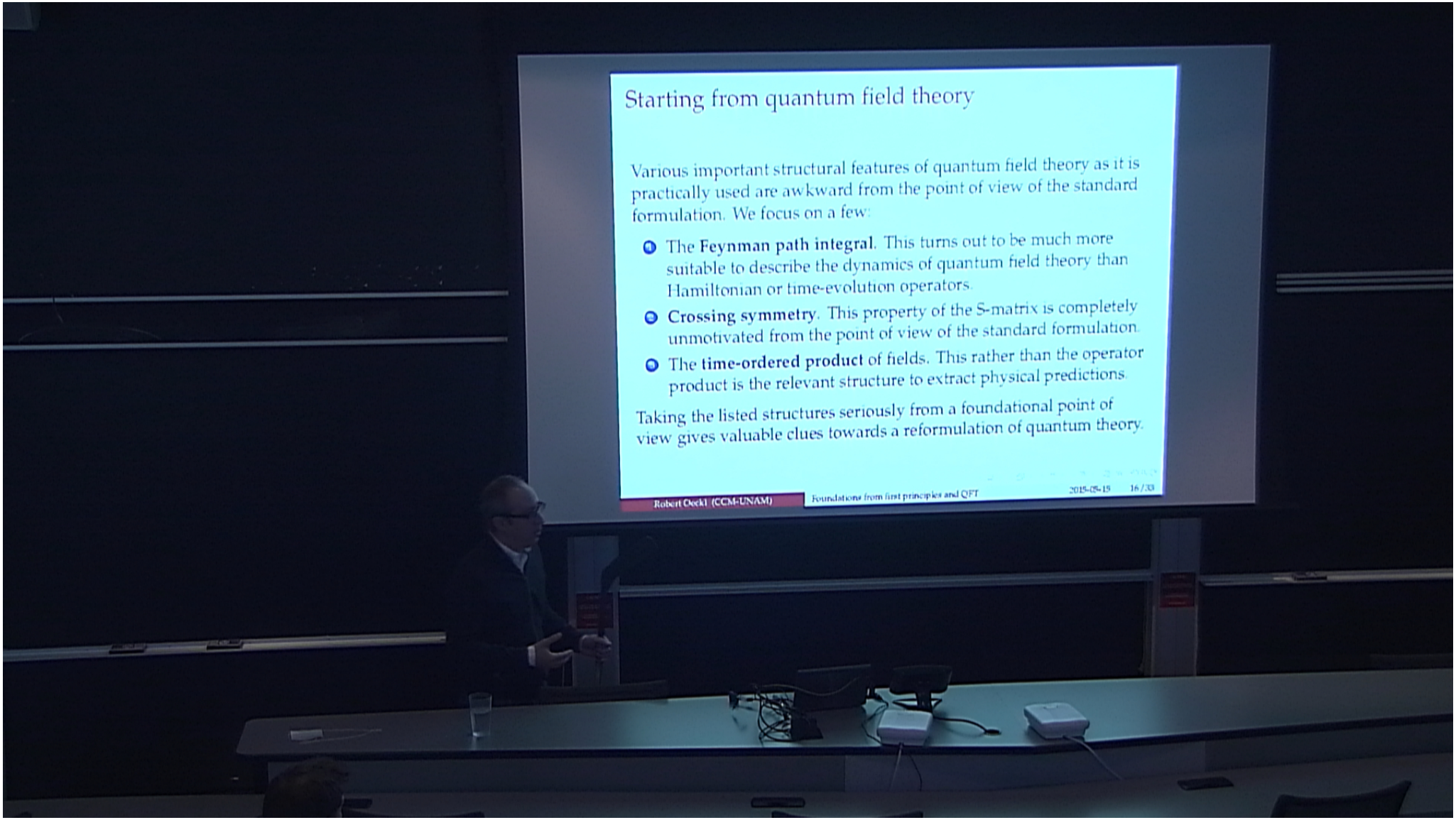
Starting from quantum field theory
Various important structural features of quantum field theory as it is practically used are awkward from the point of view of the standard formulation. We focus on a few:
1. The **Feynman path integral**. This turns out to be much more suitable to describe the dynamics of quantum field theory than Hamiltonian or time-evolution operators.
2. **Crossing symmetry**. This property of the S-matrix is completely unmotivated from the point of view of the standard formulation.
3. The **time-ordered product** of fields. This rather than the operator product is the relevant structure to extract physical predictions.
Taking the listed structures seriously from a foundational point of view gives valuable clues towards a reformulation of quantum theory.
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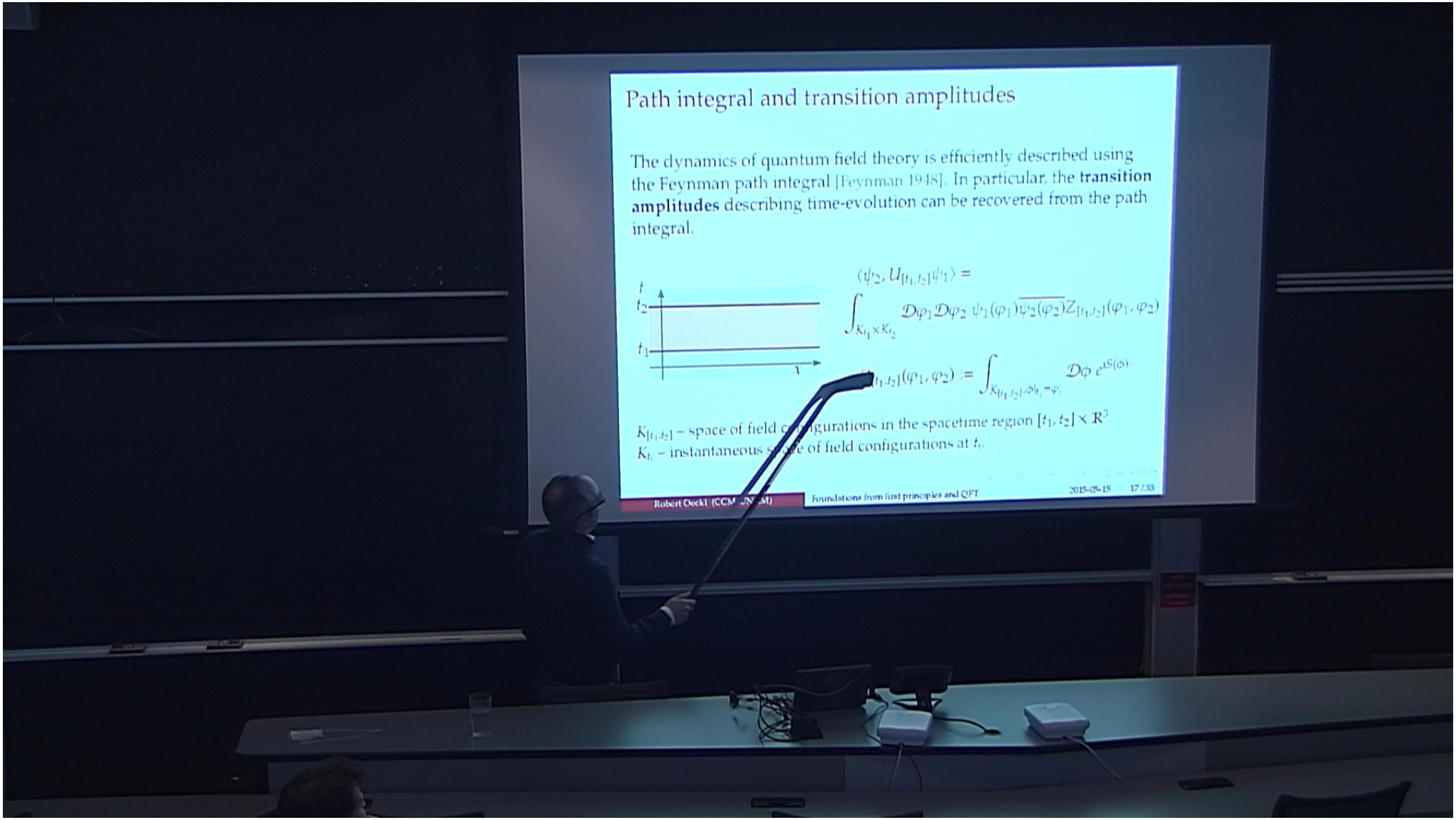
Path integral and transition amplitudes
The dynamics of quantum field theory is efficiently described using the Feynman path integral [Feynman 1948]. In particular, the **transition amplitudes** describing time-evolution can be recovered from the path integral.
t
t_2
t_1
$\langle \psi_2, U_{[t_1,t_2]}\psi_1 \rangle =$
$\int_{K_{t_1}\times K_{t_2}} \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \; \psi_1(\varphi_1)\overline{\psi_2(\varphi_2)} Z_{[t_1,t_2]}(\varphi_1,\varphi_2)$
$Z_{[t_1,t_2]}(\varphi_1,\varphi_2) = \int_{\mathcal{K}_{[t_1,t_2]},\phi|_{t_i}=\varphi_i} \mathcal{D}\phi \; e^{iS(\phi)}$
$K_{[t_1,t_2]}$ – space of field configurations in the spacetime region $[t_1,t_2]\times\mathbb{R}^3$
K_{t_i} – instantaneous space of field configurations at t_i
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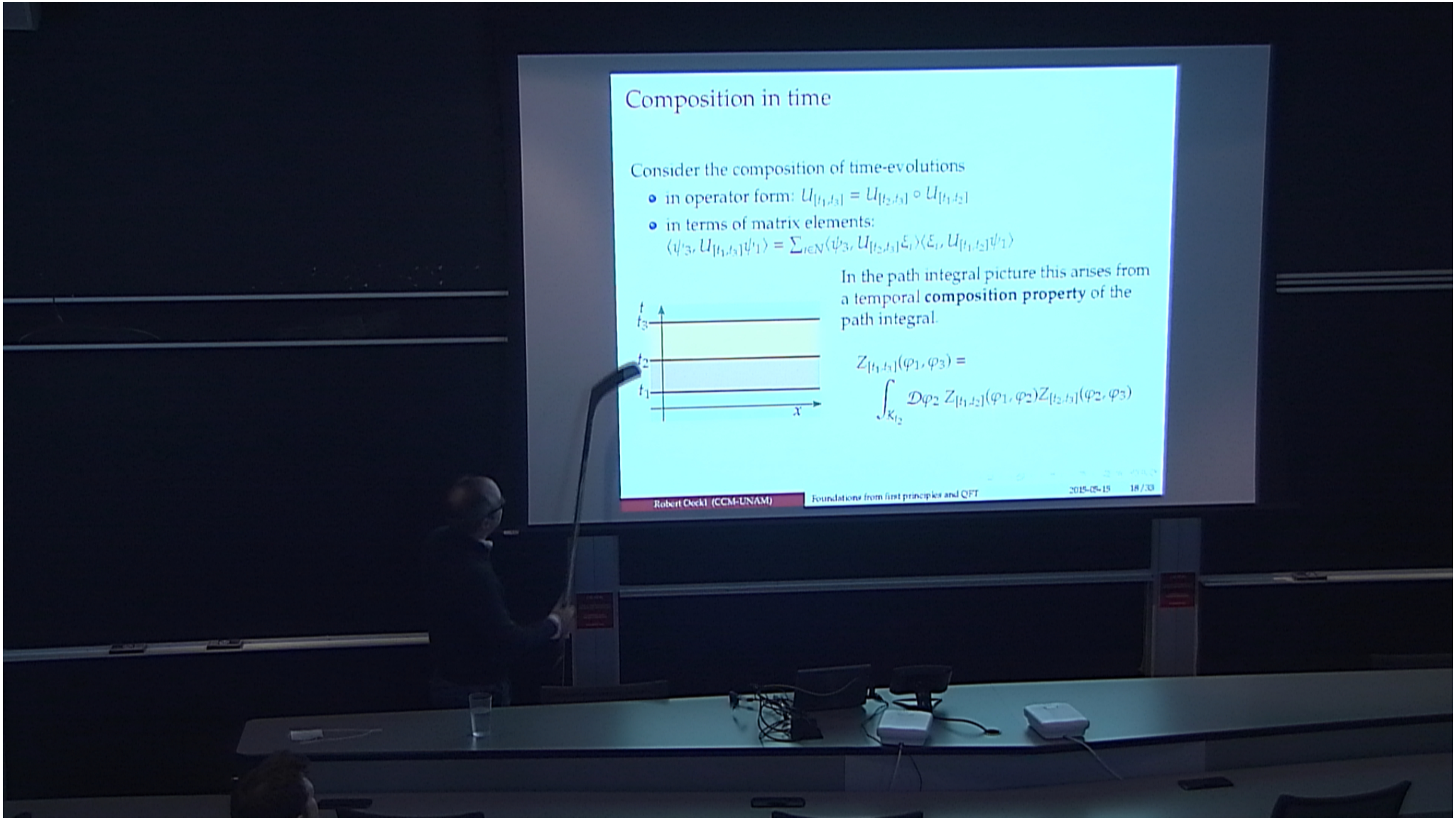
Composition in time
Consider the composition of time-evolutions
in operator form: $U_{[t_1,t_3]} = U_{[t_2,t_3]} \circ U_{[t_1,t_2]}$
in terms of matrix elements:
$\langle \psi'_3, U_{[t_1,t_3]} \psi'_1 \rangle = \sum_{i \in N} \langle \psi'_3, U_{[t_2,t_3]} \xi_i \rangle \langle \xi_i, U_{[t_1,t_2]} \psi'_1 \rangle$
In the path integral picture this arises from a temporal composition property of the path integral.
t
t_3
t_2
t_1
x
$Z_{[t_1,t_3]}(\varphi_1, \varphi_3) =$
$\int_{K_{t_2}} \mathcal{D}\varphi_2 \, Z_{[t_1,t_2]}(\varphi_1, \varphi_2) Z_{[t_2,t_3]}(\varphi_2, \varphi_3)$
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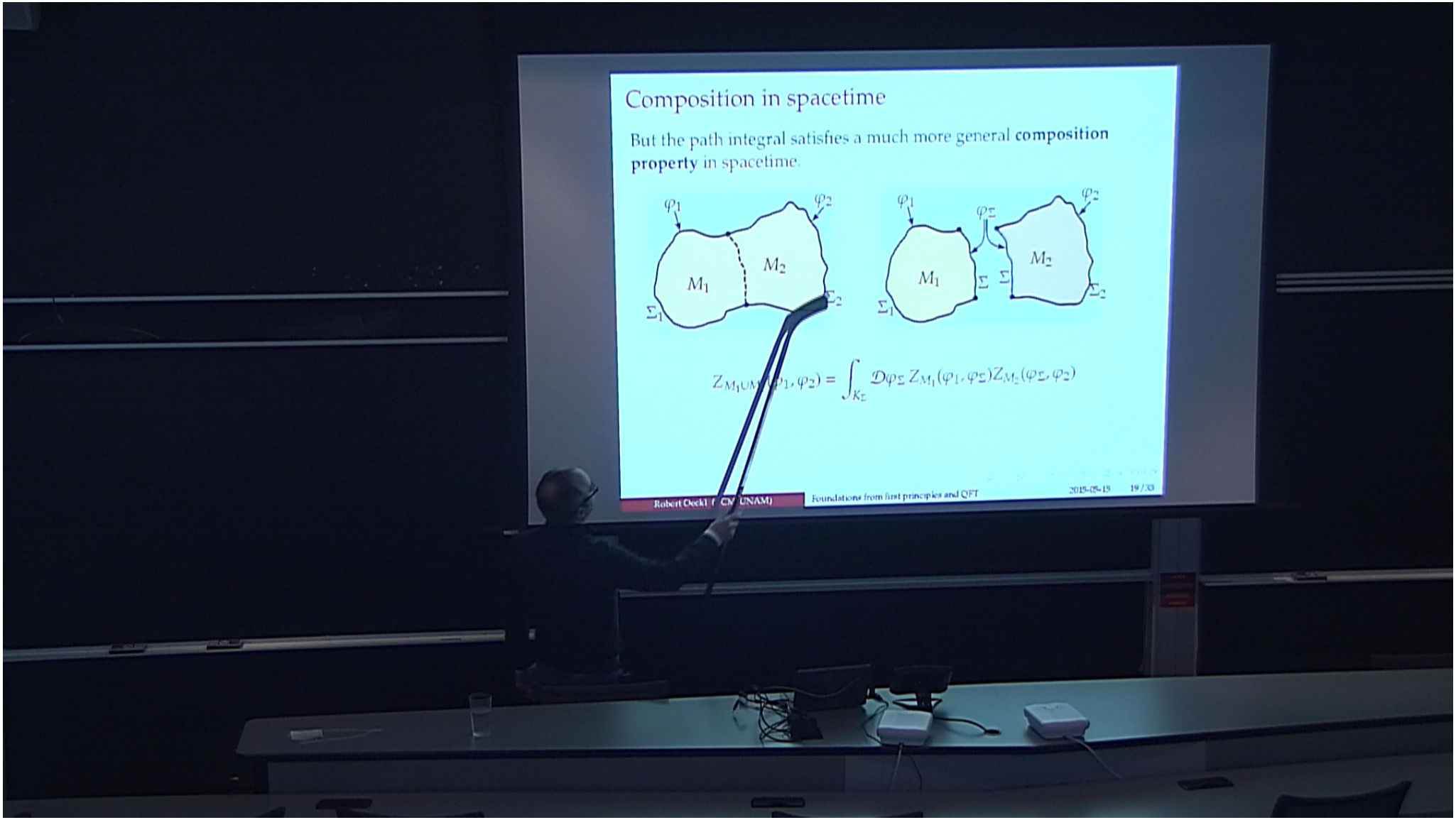
Composition in spacetime
But the path integral satisfies a much more general composition property in spacetime.
φ_1
φ_2
M_1
M_2
Σ_1
Σ_2
φ_Σ
Σ
$Z_{M_1 \cup M_2}(\varphi_1, \varphi_2) = \int_{K_\Sigma} \mathcal{D}\varphi_\Sigma \, Z_{M_1}(\varphi_1, \varphi_\Sigma) Z_{M_2}(\varphi_\Sigma, \varphi_2)$
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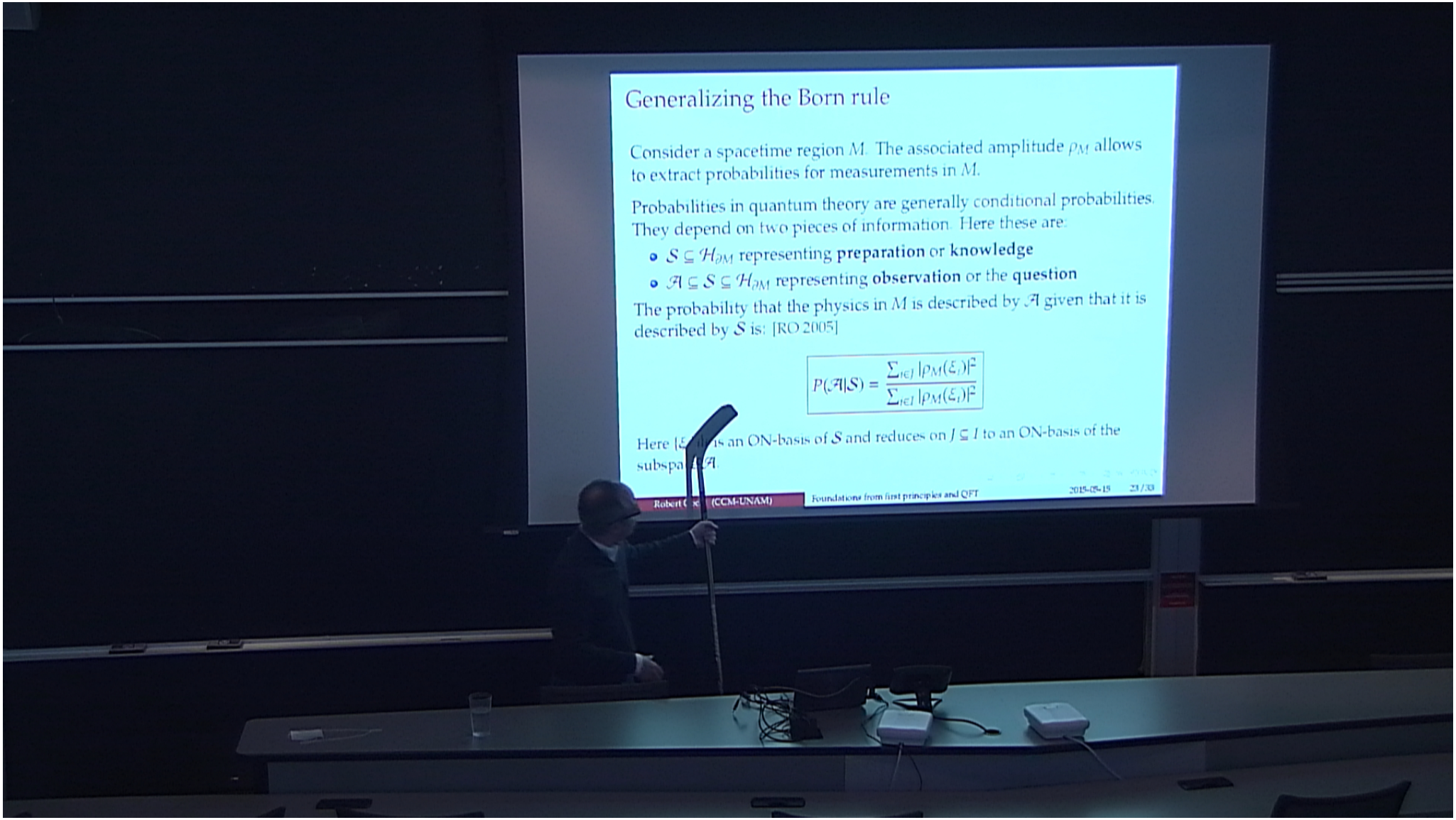
Generalizing the Born rule
Consider a spacetime region M. The associated amplitude ρ_M allows to extract probabilities for measurements in M.
Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:
$\mathcal{S} \subseteq \mathcal{H}_{\partial M}$ representing **preparation** or **knowledge**
$\mathcal{A} \subseteq \mathcal{S} \subseteq \mathcal{H}_{\partial M}$ representing **observation** or the **question**
The probability that the physics in M is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is: [RO 2005]
$$P(\mathcal{A}|\mathcal{S}) = \frac{\sum_{i\in J} |\rho_M(\xi_i)|^2}{\sum_{i\in I} |\rho_M(\xi_i)|^2}$$
Here $\{\xi$ is an ON-basis of $\mathcal{S}$ and reduces on $J \subseteq I$ to an ON-basis of the subspa $\mathcal{A}$.
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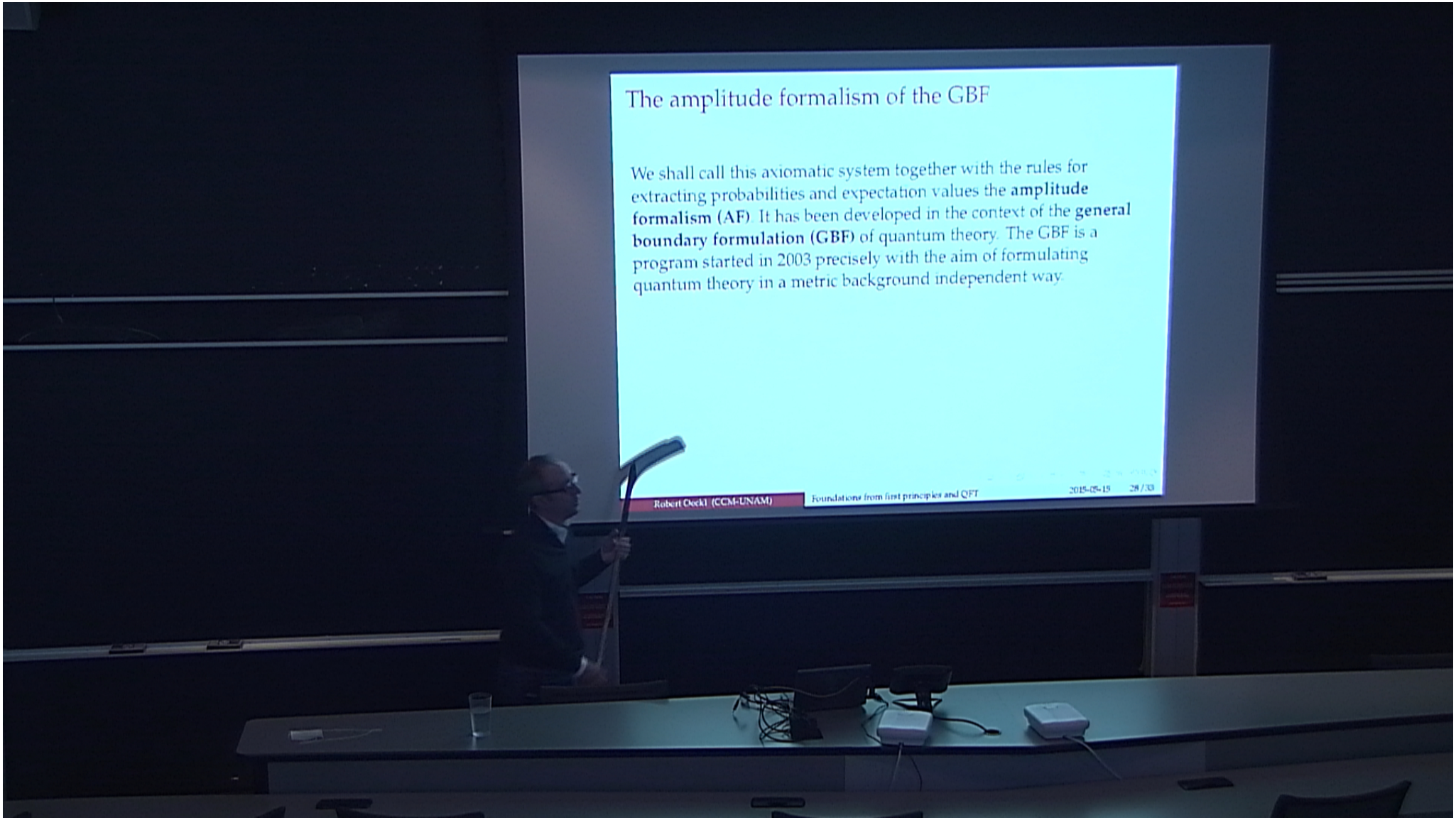
The amplitude formalism of the GBF
We shall call this axiomatic system together with the rules for extracting probabilities and expectation values the **amplitude formalism (AF)**. It has been developed in the context of the **general boundary formulation (GBF)** of quantum theory. The GBF is a program started in 2003 precisely with the aim of formulating quantum theory in a metric background independent way.
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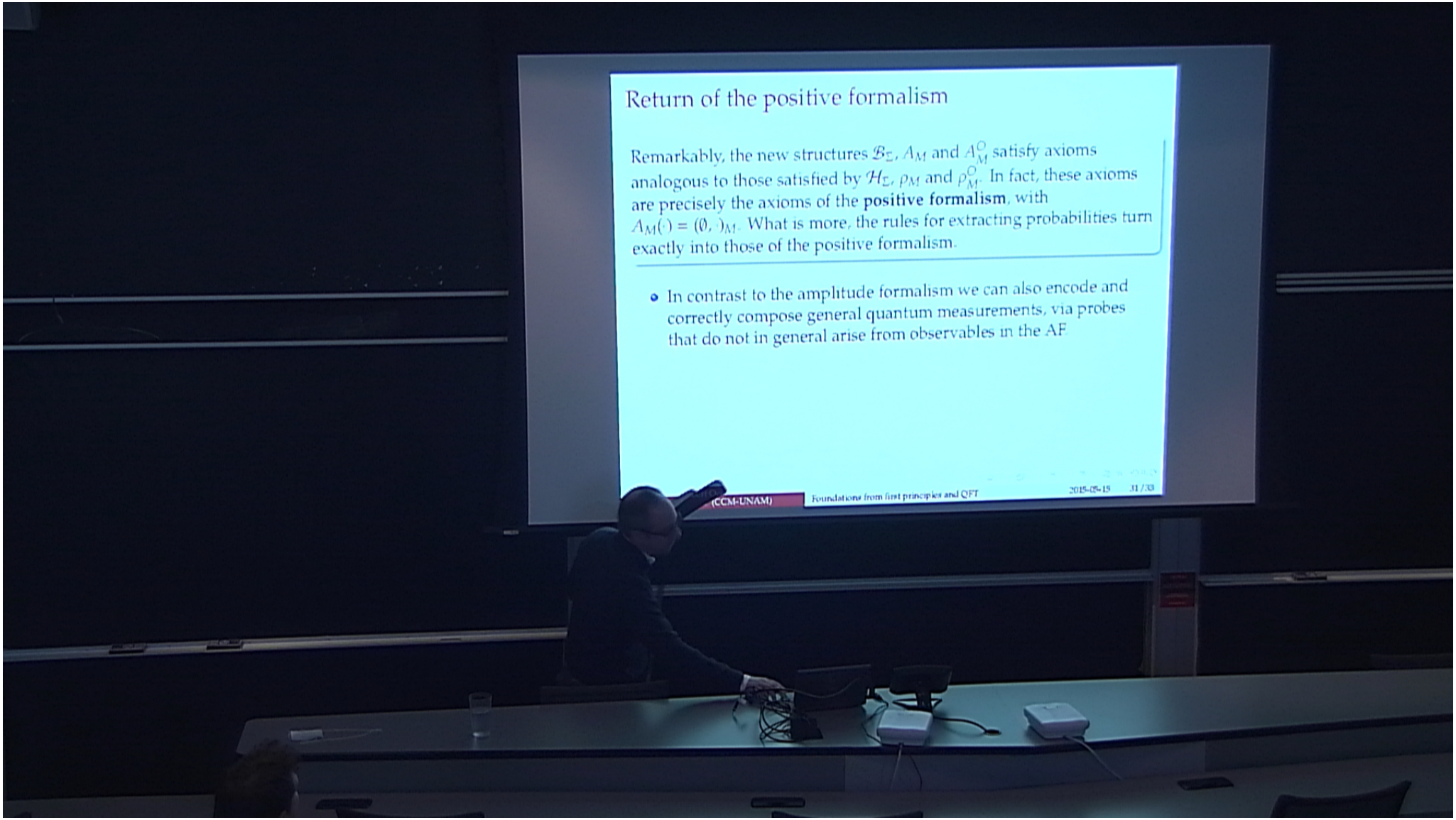
Return of the positive formalism
Remarkably, the new structures $\mathcal{B}_\Sigma$, A_M and A_M° satisfy axioms analogous to those satisfied by $\mathcal{H}_\Sigma$, ρ_M and ρ_M°. In fact, these axioms are precisely the axioms of the **positive formalism**, with $A_M(\cdot) = (\emptyset, \cdot)_M$. What is more, the rules for extracting probabilities turn exactly into those of the positive formalism.
In contrast to the amplitude formalism we can also encode and correctly compose general quantum measurements, via probes that do not in general arise from observables in the AF.
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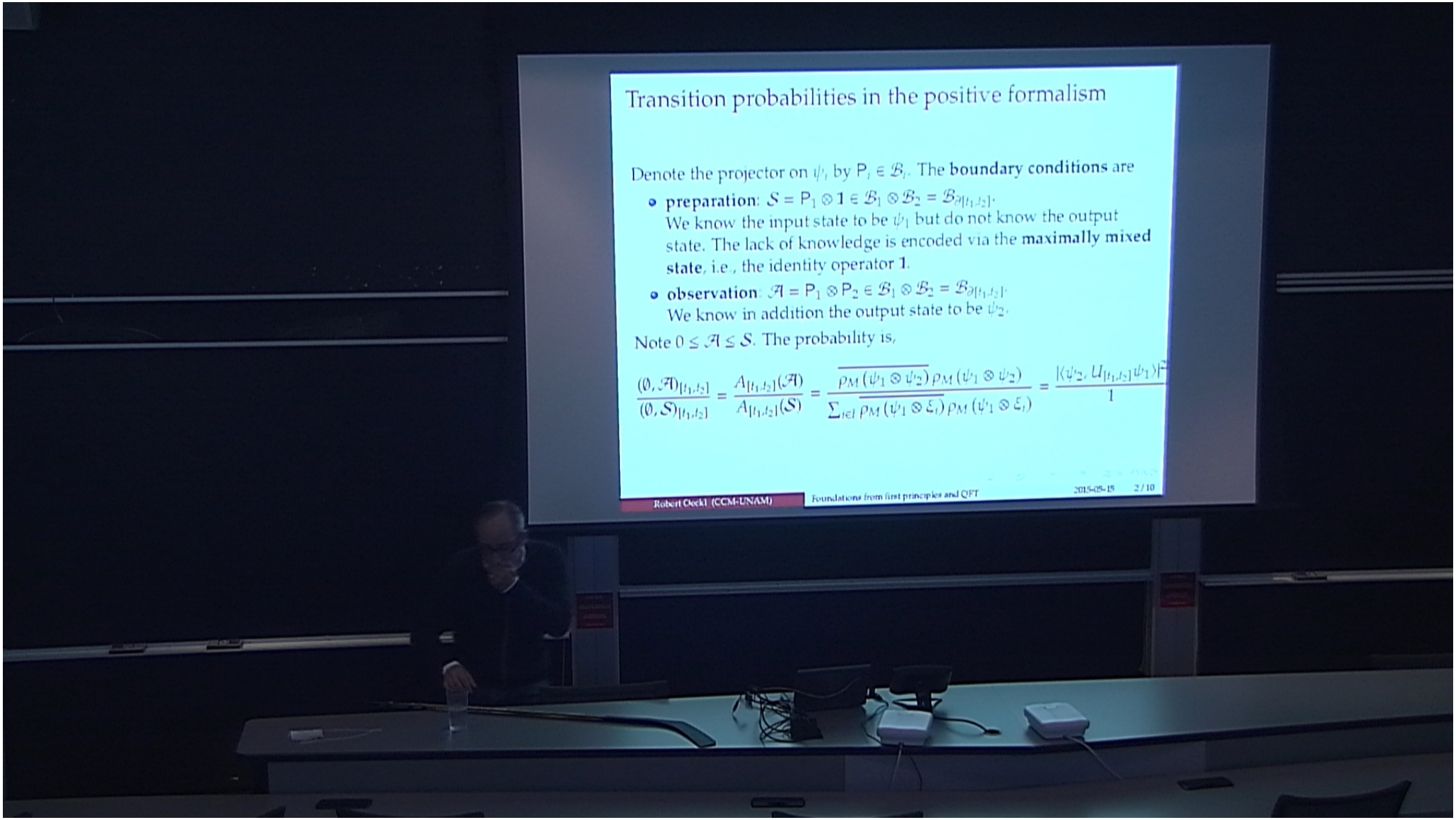
Transition probabilities in the positive formalism
Denote the projector on ψ_i by $P_i \in \mathcal{B}_i$. The **boundary conditions** are
preparation: $\mathcal{S} = P_1 \otimes \mathbf{1} \in \mathcal{B}_1 \otimes \mathcal{B}_2 = \mathcal{B}_{\partial[t_1,t_2]}$.
We know the input state to be ψ_1 but do not know the output state. The lack of knowledge is encoded via the **maximally mixed state**, i.e., the identity operator $\mathbf{1}$.
observation: $\mathcal{A} = P_1 \otimes P_2 \in \mathcal{B}_1 \otimes \mathcal{B}_2 = \mathcal{B}_{\partial[t_1,t_2]}$.
We know in addition the output state to be ψ_2.
Note $0 \leq \mathcal{A} \leq \mathcal{S}$. The probability is,
$$\frac{(\emptyset,\mathcal{A})_{[t_1,t_2]}}{(\emptyset,\mathcal{S})_{[t_1,t_2]}} = \frac{A_{[t_1,t_2]}(\mathcal{A})}{A_{[t_1,t_2]}(\mathcal{S})} = \frac{\overline{\rho_M(\psi_1 \otimes \psi_2)}\,\rho_M(\psi_1 \otimes \psi_2)}{\sum_{i\in I}\overline{\rho_M(\psi_1 \otimes \xi_i)}\,\rho_M(\psi_1 \otimes \xi_i)} = \frac{|\langle\psi_2, U_{[t_1,t_2]}\psi_1\rangle|^2}{1}$$
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