Title: Causal fermion systems from an information theoretic perspective

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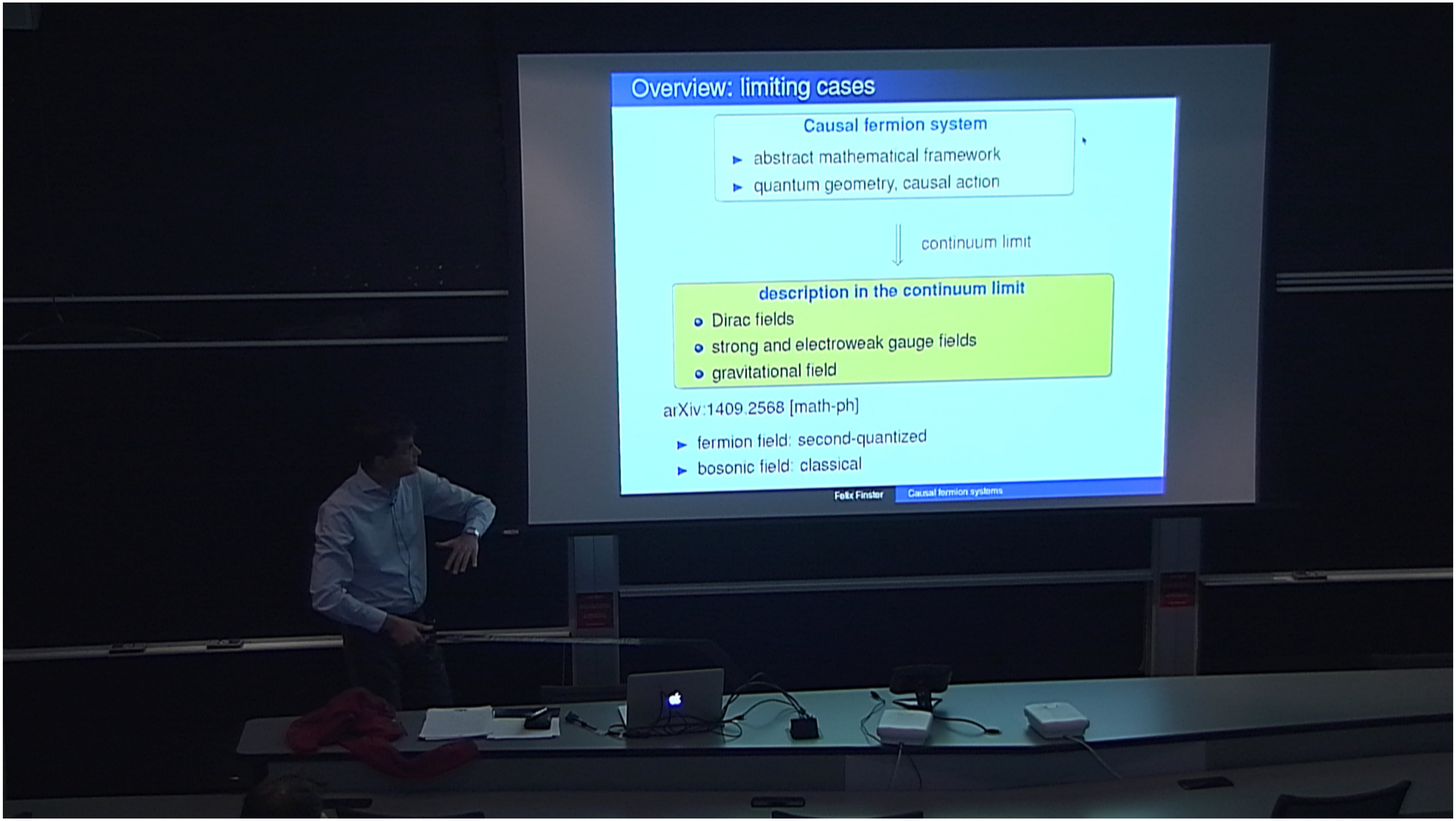
Abstract: The theory of causal fermion systems is an approach to describe fundamental physics. It gives quantum mechanics, general relativity and quantum field theory as limiting cases and is therefore a candidate for a unified physical theory. Instead of introducing physical objects on a preexisting space-time manifold, the general concept is to derive space-time as well as all the objects therein as secondary objects from the structures of an underlying causal fermion system. The dynamics of the system is described by the causal action principle.

I will give a non-technical introduction, with an emphasis on conceptual issues related to information theory.


Overview: limiting cases
Causal fermion system
abstract mathematical framework
quantum geometry, causal action
continuum limit
description in the continuum limit
Dirac fields
strong and electroweak gauge fields
gravitational field
arXiv:1409.2568 [math-ph]
fermion field: second-quantized
bosonic field: classical
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Overview: limiting cases

Causal fermion system

$\Downarrow$ continuum limit

description in the continuum limit

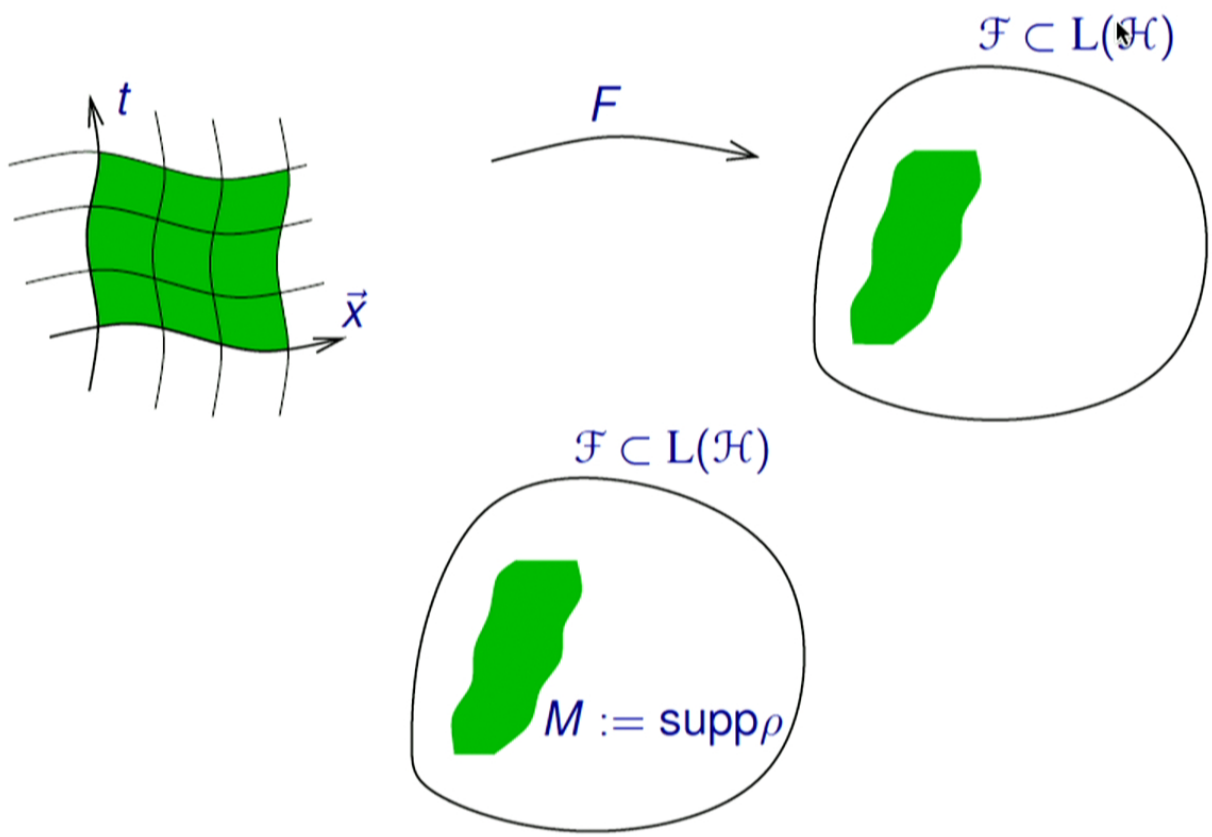
$\Downarrow$ microscopic mixing

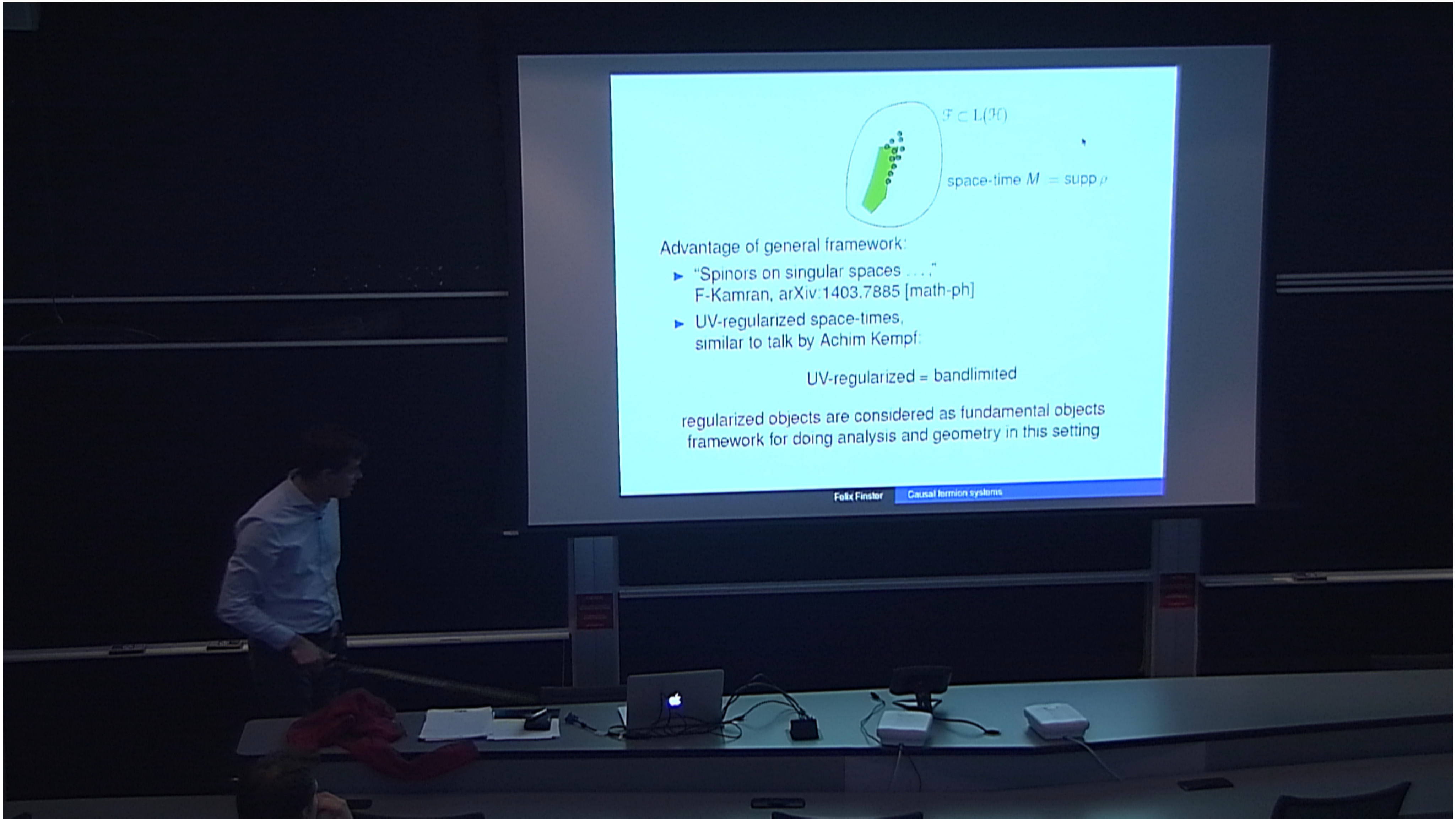
- also second-quantized bosonic field
- loop diagrams, renormalization, . . . (work in progress)

relativistic quantum field theory

arXiv:1409.2568 [math-ph], J. Math. Phys. **55** (2014) 042301

Example: curved space-time




$\mathcal{F} \subset L(\mathcal{H})$
space-time M = supp ρ
Advantage of general framework:
"Spinors on singular spaces . . . ,"
F-Kamran, arXiv:1403.7885 [math-ph]
UV-regularized space-times,
similar to talk by Achim Kempf:
UV-regularized = bandlimited
regularized objects are considered as fundamental objects
framework for doing analysis and geometry in this setting
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Information theoretic point of view

Consider the setting from perspective of information theory:
Let $(\rho, \mathcal{F}, \mathcal{H})$ be a causal fermion system,
Encodes plenty of information:

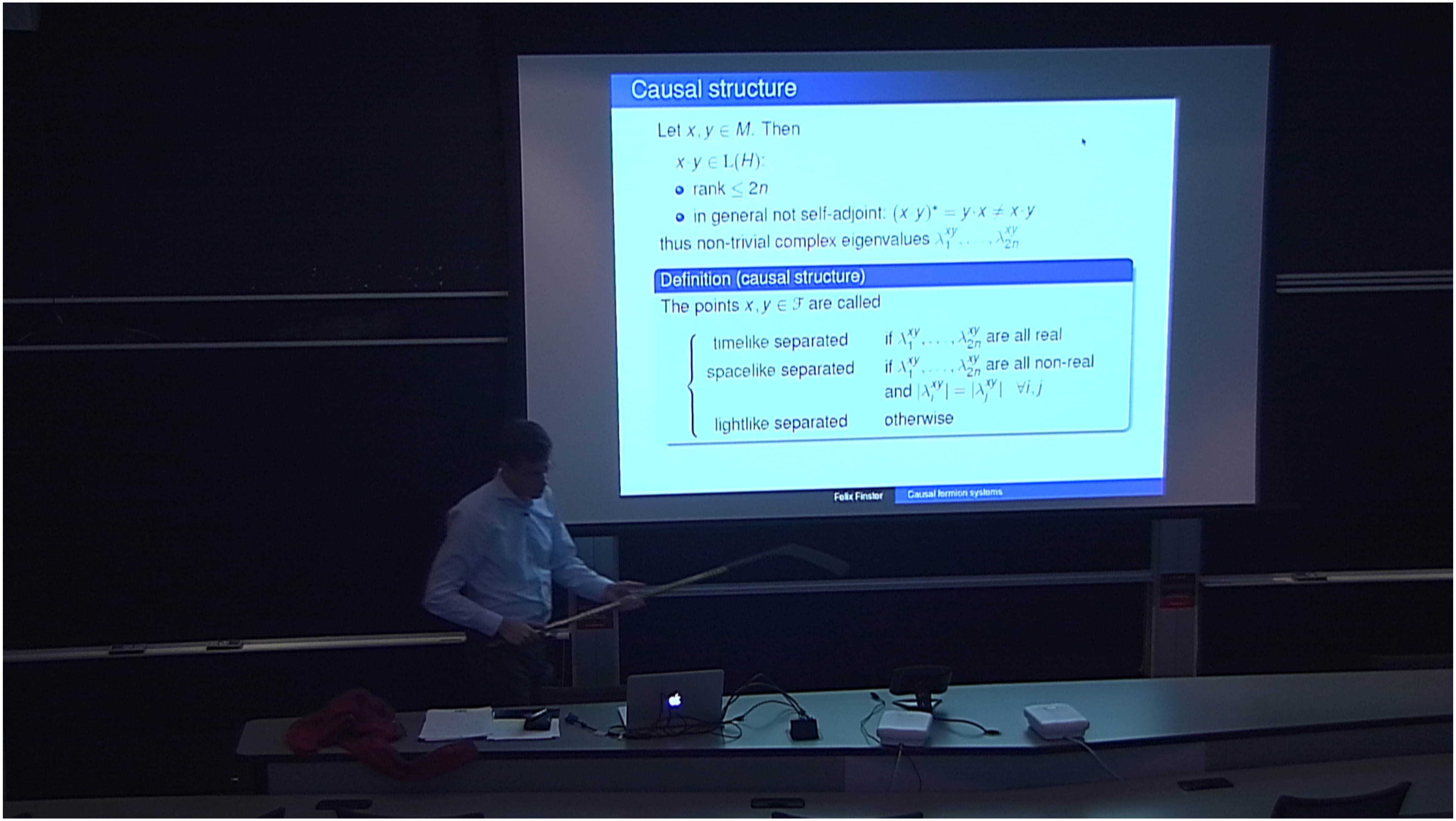
- $x \in \mathcal{F}$ has eigenvalues
- operator products xy has eigenvalues
- integrate quantities over space-time,

$$\int_{\mathcal{F}} \cdots d\rho$$

Connection to information theory remains to be developed:

- Right now: no operational point of view
- No definitions of entropy, temperature, ...

Next: Try bring the information into a useful form.


Causal structure
Let $x, y \in M$. Then
$x \cdot y \in \mathrm{L}(H)$:
rank $\leq 2n$
in general not self-adjoint: $(x \cdot y)^* = y \cdot x \neq x \cdot y$
thus non-trivial complex eigenvalues $\lambda_1^{xy}, \ldots, \lambda_{2n}^{xy}$
Definition (causal structure)
The points $x, y \in \mathcal{F}$ are called
timelike separated if $\lambda_1^{xy}, \ldots, \lambda_{2n}^{xy}$ are all real
spacelike separated if $\lambda_1^{xy}, \ldots, \lambda_{2n}^{xy}$ are all non-real and $|\lambda_i^{xy}| = |\lambda_j^{xy}| \quad \forall i, j$
lightlike separated otherwise
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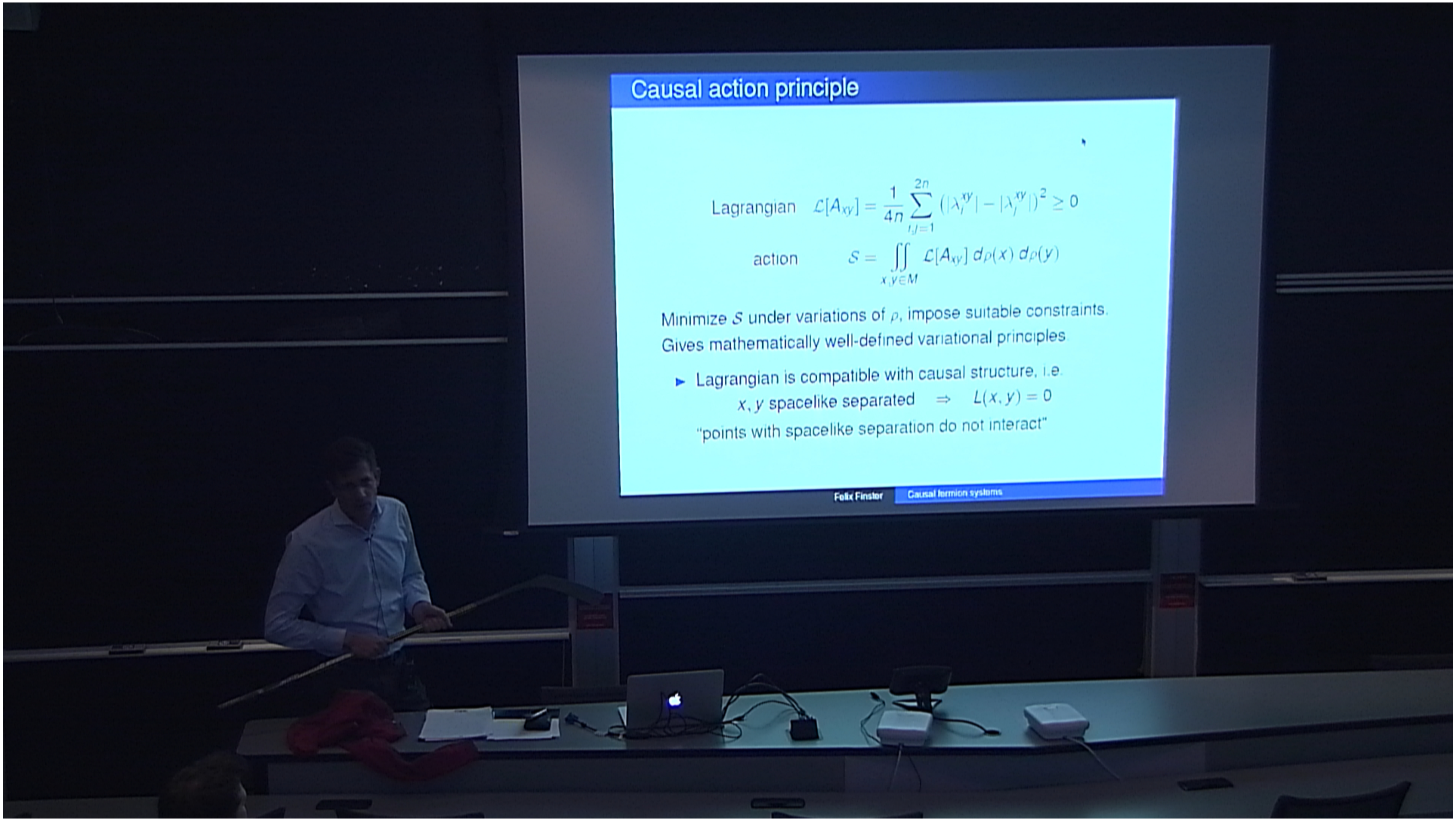

Causal action principle
Lagrangian $\mathcal{L}[A_{xy}] = \frac{1}{4n} \sum_{i,j=1}^{2n} \left(|\lambda_i^{xy}| - |\lambda_j^{xy}| \right)^2 \geq 0$
action $\mathcal{S} = \iint_{x,y \in M} \mathcal{L}[A_{xy}] \, d\rho(x) \, d\rho(y)$
Minimize $\mathcal{S}$ under variations of ρ, impose suitable constraints.
Gives mathematically well-defined variational principles
Lagrangian is compatible with causal structure, i.e.
x, y spacelike separated $\Rightarrow$ $L(x, y) = 0$
"points with spacelike separation do not interact"
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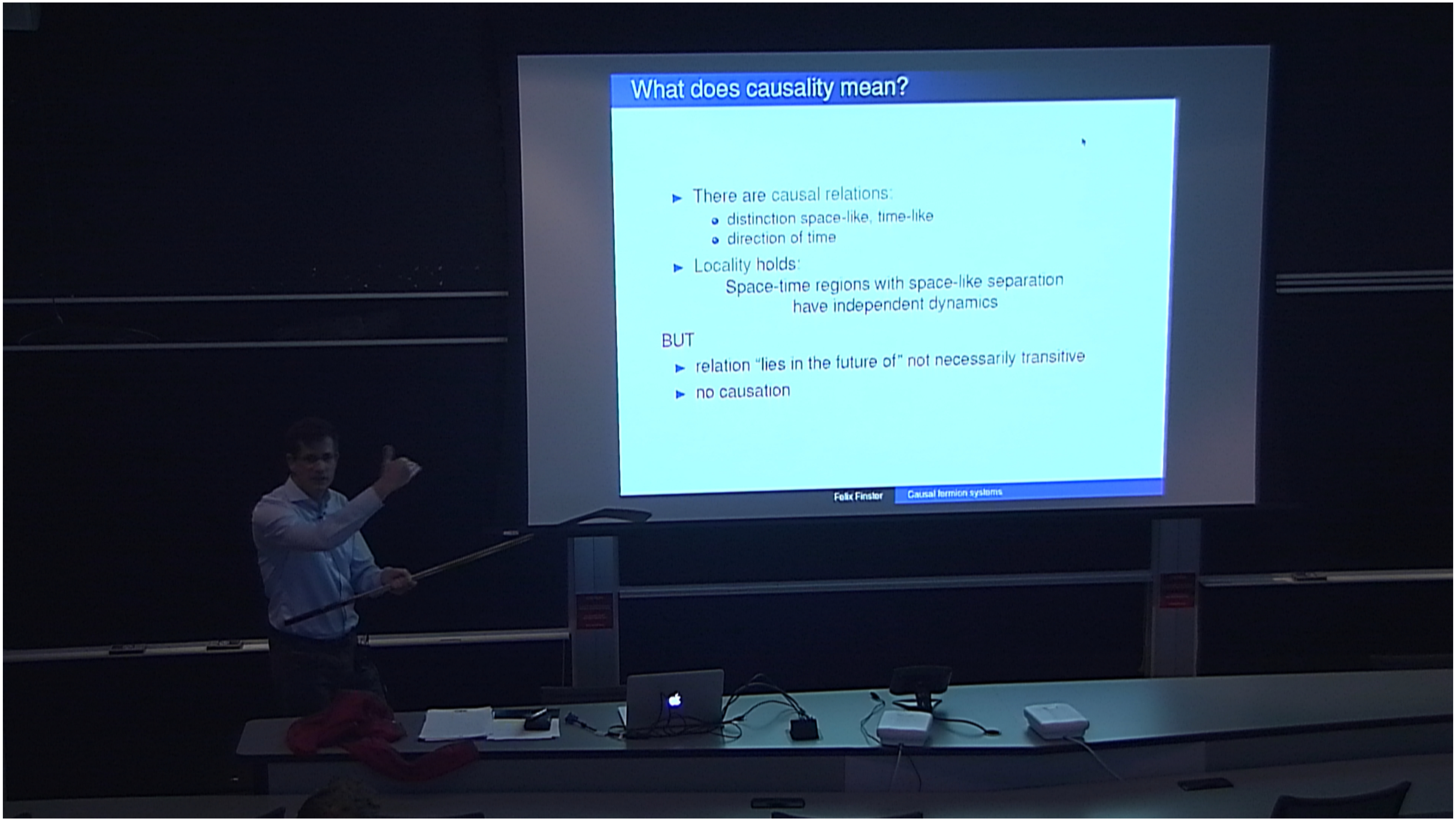
What does causality mean?
There are causal relations:
distinction space-like, time-like
direction of time
Locality holds:
Space-time regions with space-like separation
have independent dynamics
BUT
relation "lies in the future of" not necessarily transitive
no causation
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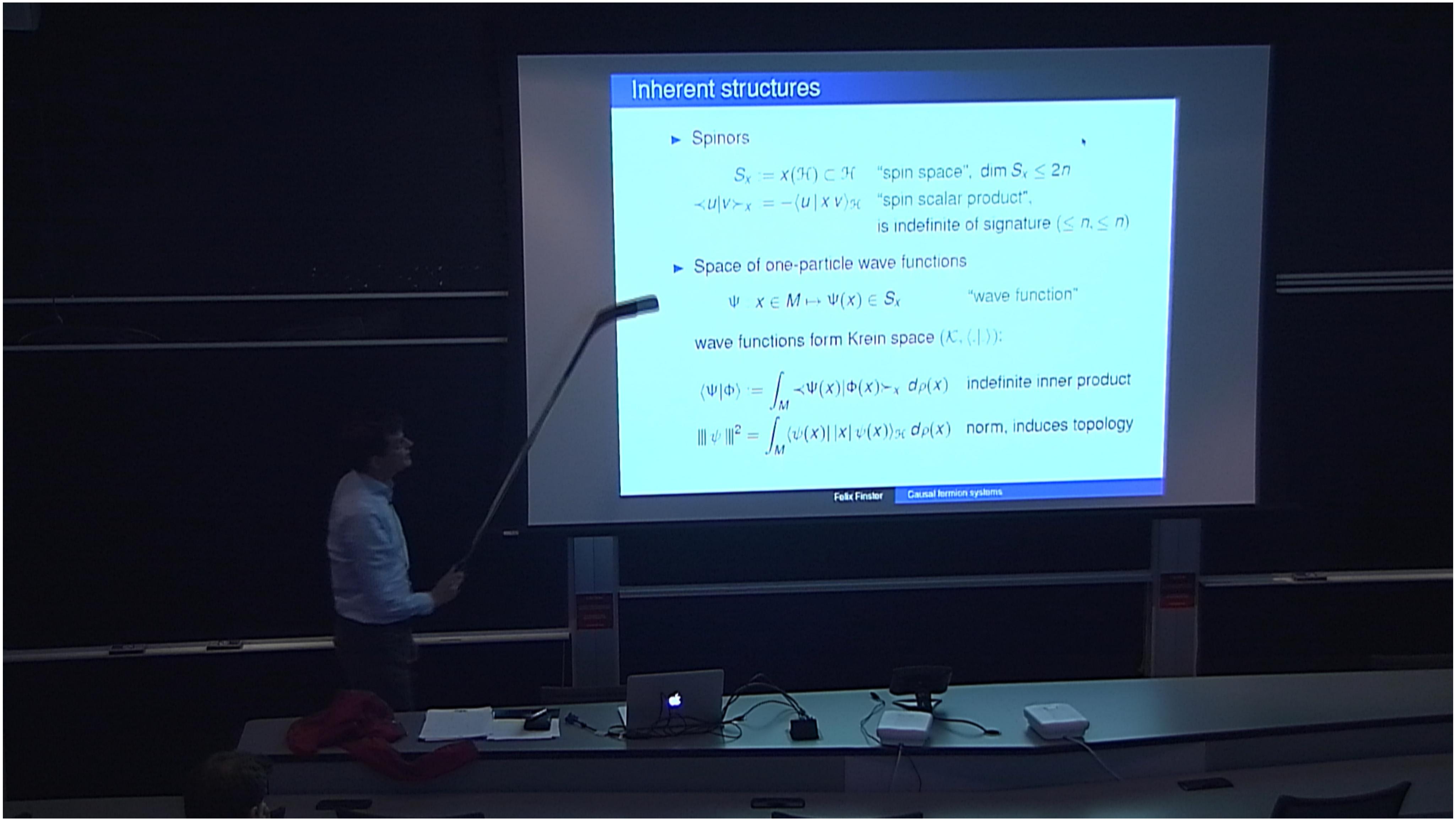
Inherent structures
Spinors
$S_x := x(\mathcal{H}) \subset \mathcal{H}$ "spin space", $\dim S_x \leq 2n$
$\prec u|v \succ_x = -\langle u \,|\, x\, v\rangle_{\mathcal{H}}$ "spin scalar product",
is indefinite of signature $(\leq n, \leq n)$
Space of one-particle wave functions
$\psi : x \in M \mapsto \psi(x) \in S_x$ "wave function"
wave functions form Krein space $(\mathcal{K}, \langle .|.\rangle)$:
$\langle \psi|\phi\rangle := \int_M \prec \psi(x)|\phi(x)\succ_x \, d\rho(x)$ indefinite inner product
$||| \psi |||^2 = \int_M \langle \psi(x) |\, |x|\, \psi(x)\rangle_{\mathcal{H}} \, d\rho(x)$ norm, induces topology
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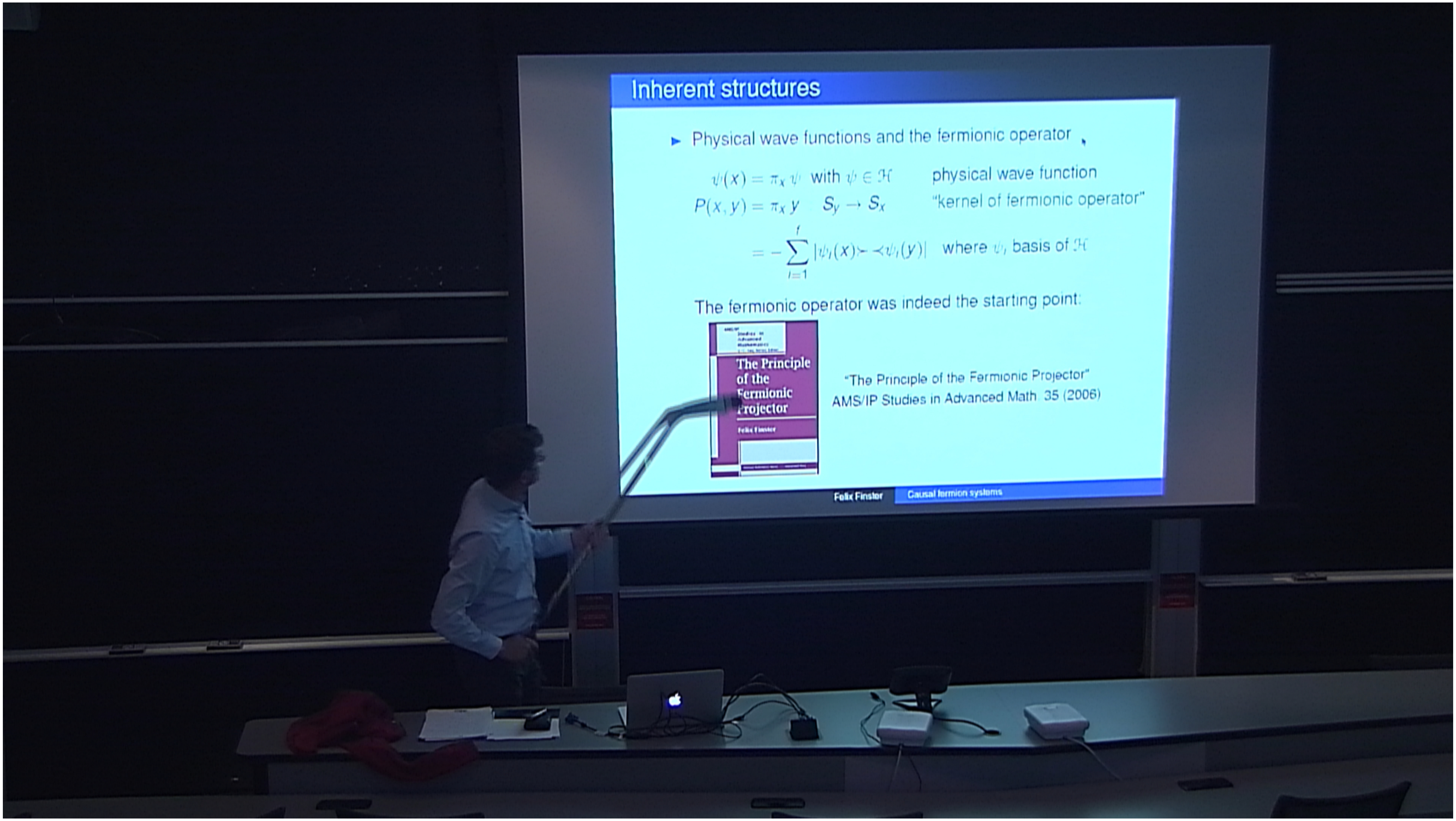
Inherent structures
Physical wave functions and the fermionic operator
$\psi(x) = \pi_x \psi$ with $\psi \in \mathcal{H}$
physical wave function
$P(x,y) = \pi_x\, y|_{S_y} : S_y \to S_x$
"kernel of fermionic operator"
$= -\sum_{i=1}^{f} |\psi_i(x) \succ \prec \psi_i(y)|$ where ψ_i basis of $\mathcal{H}$
The fermionic operator was indeed the starting point:
The Principle of the Fermionic Projector
Felix Finster
"The Principle of the Fermionic Projector"
AMS/IP Studies in Advanced Math. 35 (2006)
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Inherent structures

- Physical wave functions and the fermionic operator

$$\psi(x) = \pi_x \psi \quad \text{with } \psi \in \mathcal{H} \qquad \text{physical wave function}$$

$$P(x,y) = \pi_x y \;:\; S_y \to S_x \qquad \text{“kernel of fermionic operator”}$$

$$= -\sum_{i=1}^{f} |\psi_i(x) \succ \prec \psi_i(y)| \quad \text{where } \psi_i \text{ basis of } \mathcal{H}$$

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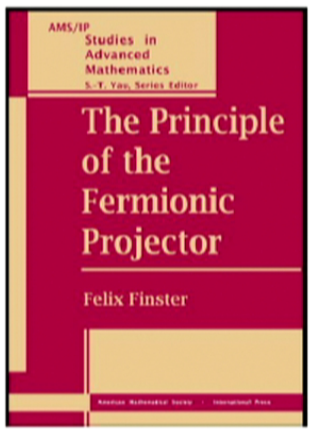

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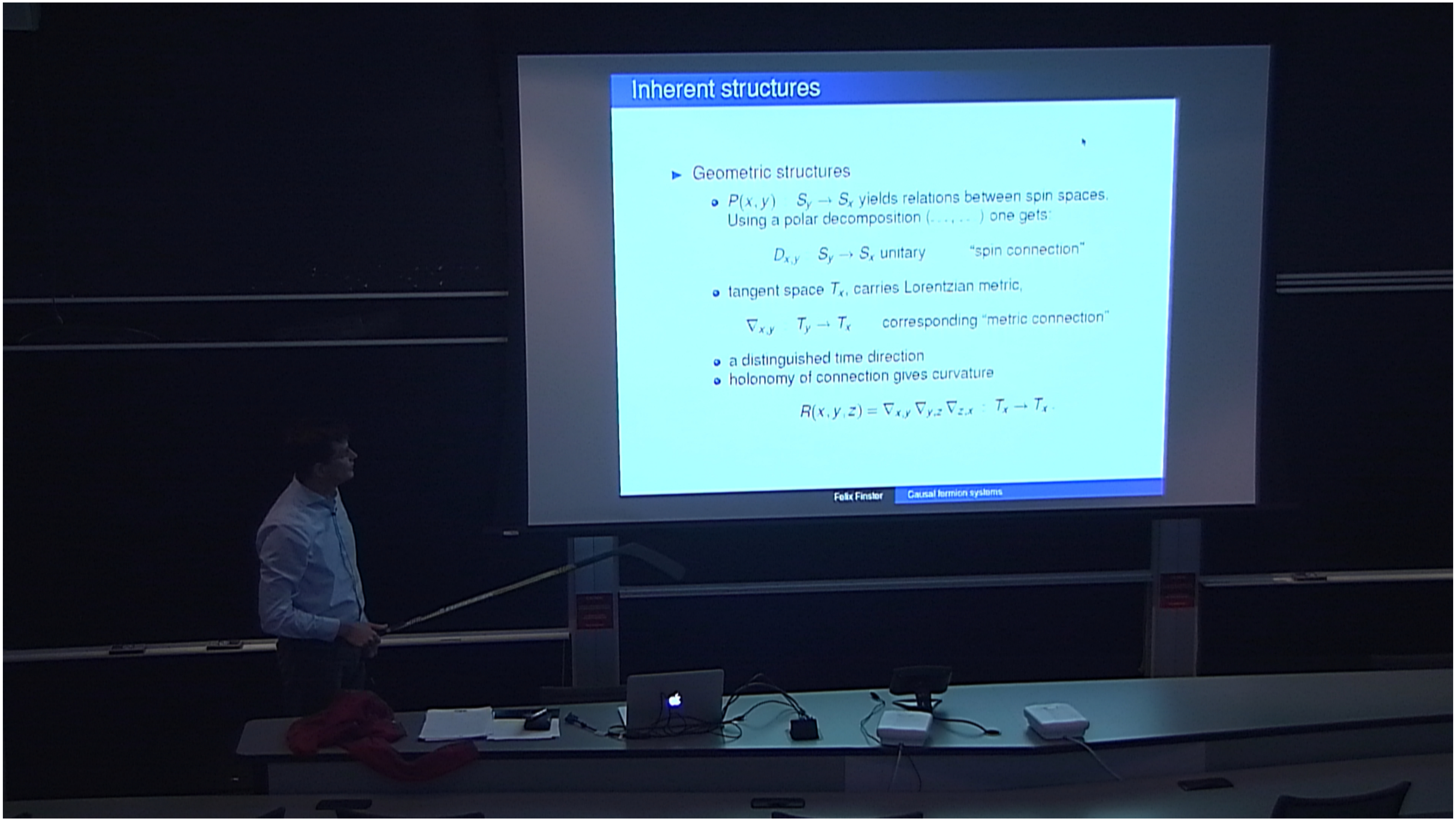
Inherent structures
Geometric structures
$P(x, y) : S_y \to S_x$ yields relations between spin spaces.
Using a polar decomposition (. . . , . . .) one gets:
$D_{x,y} : S_y \to S_x$ unitary "spin connection"
tangent space T_x, carries Lorentzian metric,
$\nabla_{x,y} : T_y \to T_x$ corresponding "metric connection"
a distinguished time direction
holonomy of connection gives curvature
$R(x, y, z) = \nabla_{x,y} \nabla_{y,z} \nabla_{z,x} : T_x \to T_x$.
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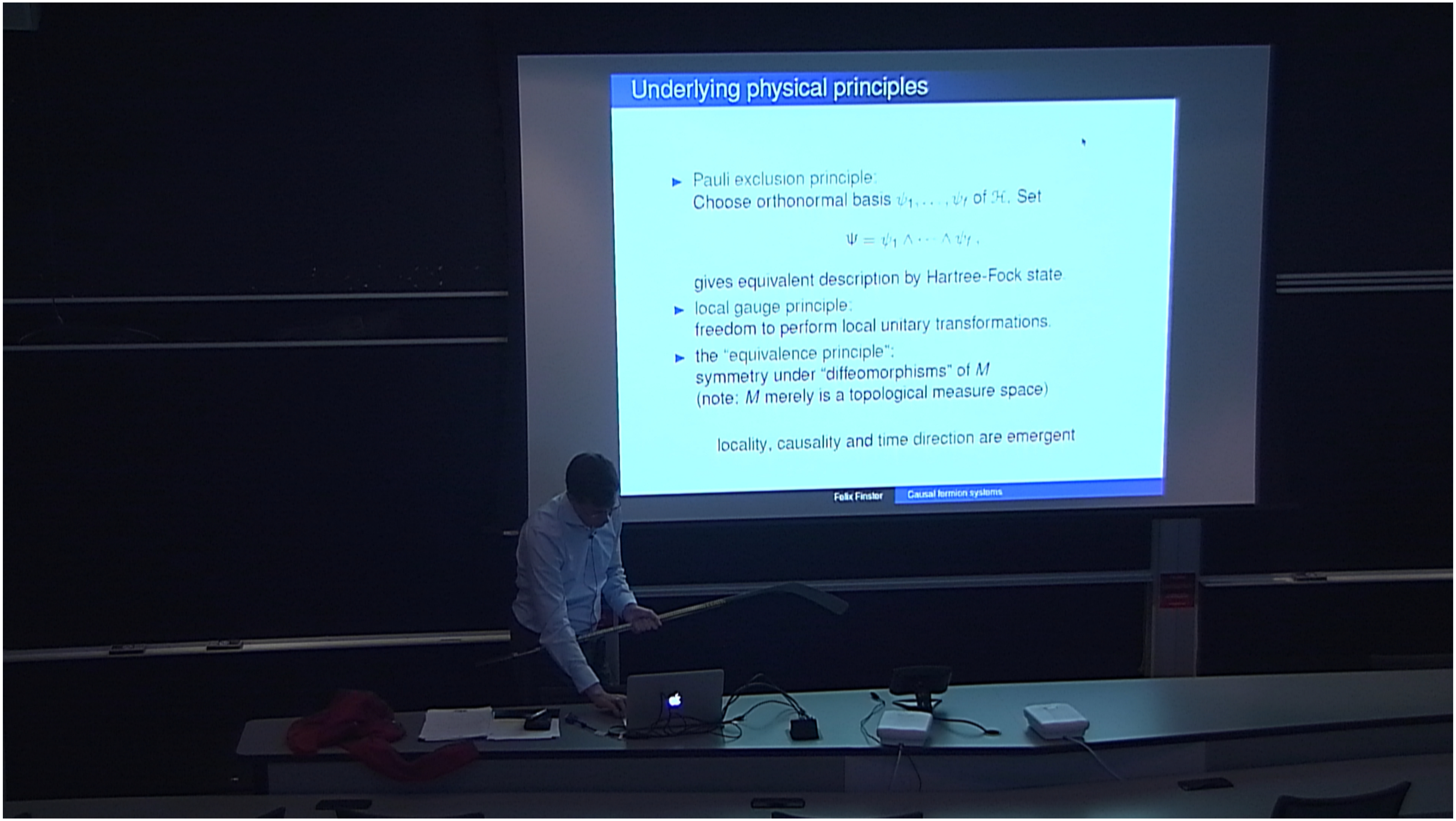
Underlying physical principles
Pauli exclusion principle:
Choose orthonormal basis $\psi_1, \ldots, \psi_f$ of $\mathcal{H}$. Set
$\Psi = \psi_1 \wedge \cdots \wedge \psi_f$,
gives equivalent description by Hartree-Fock state.
local gauge principle:
freedom to perform local unitary transformations.
the "equivalence principle":
symmetry under "diffeomorphisms" of M
(note: M merely is a topological measure space)
locality, causality and time direction are emergent
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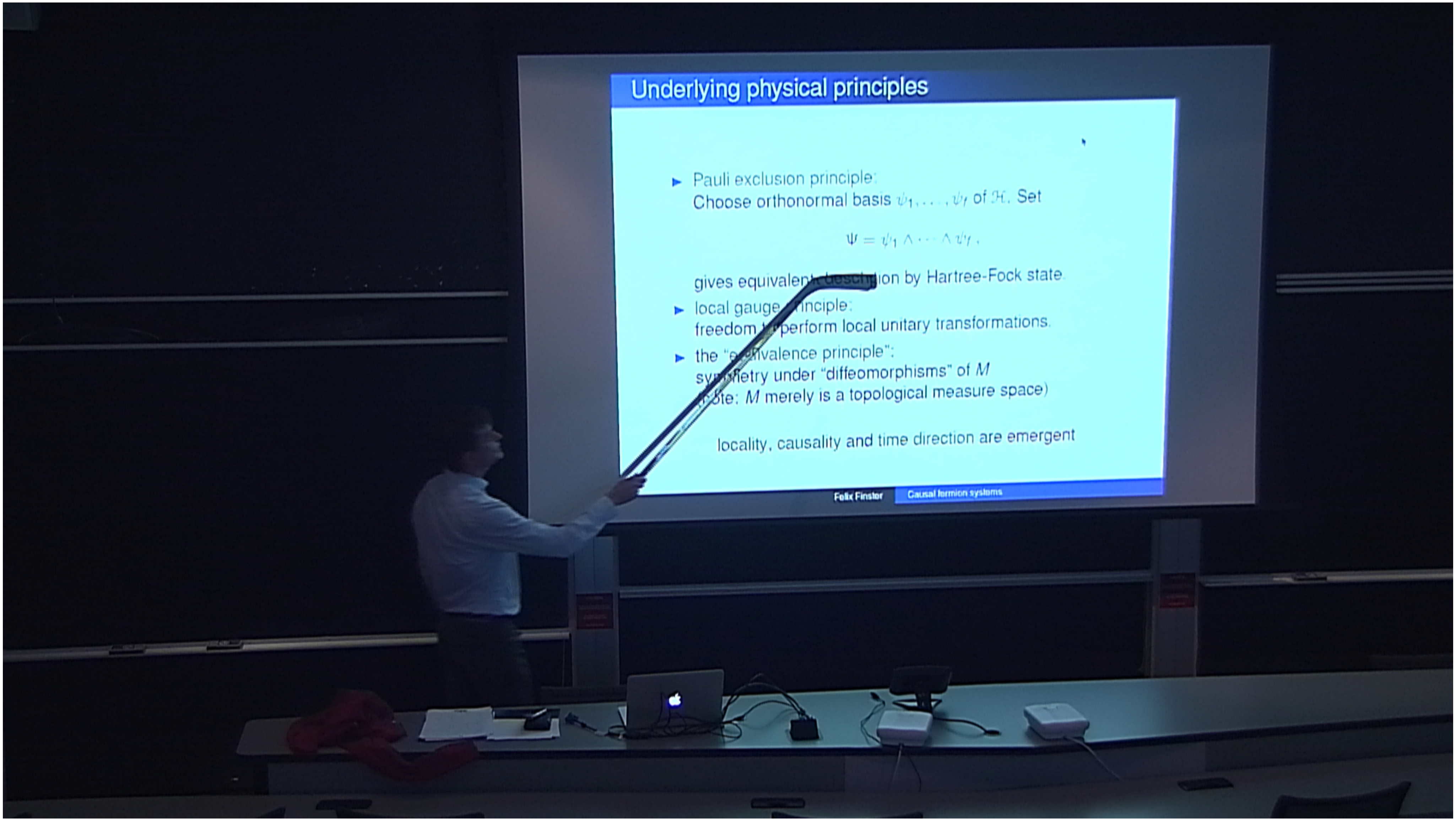
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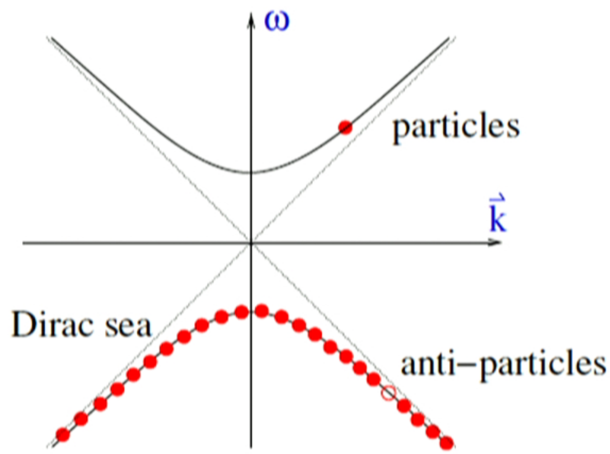
The causal action principle in the continuum limit

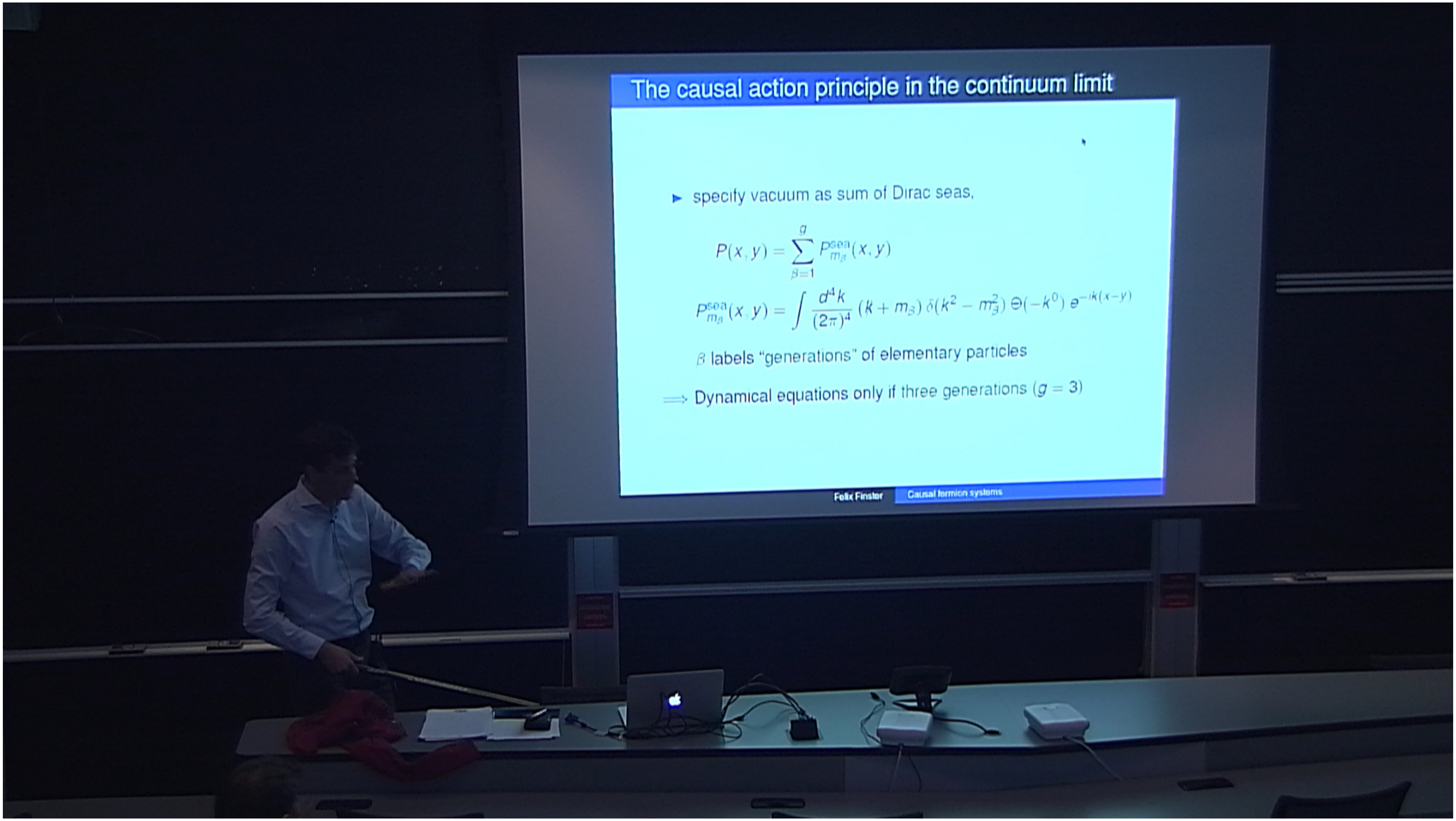
Specify vacuum:

- Choose $\mathcal{H}$ as the space of all negative-energy solutions, hence "Dirac sea"



Fixes length scale ("Compton length")

- Introduce ultraviolet regularization, "quantum geometry"
  Fixes length scale ("Planck length")



The causal action principle in the continuum limit
specify vacuum as sum of Dirac seas,
$P(x,y) = \sum_{\beta=1}^{g} P^{\text{sea}}_{m_\beta}(x,y)$
$P^{\text{sea}}_{m_\beta}(x,y) = \int \frac{d^4k}{(2\pi)^4} (\not{k} + m_\beta)\, \delta(k^2 - m_\beta^2)\, \Theta(-k^0)\, e^{-ik(x-y)}$
β labels "generations" of elementary particles
$\Longrightarrow$ Dynamical equations only if three generations ($g = 3$)
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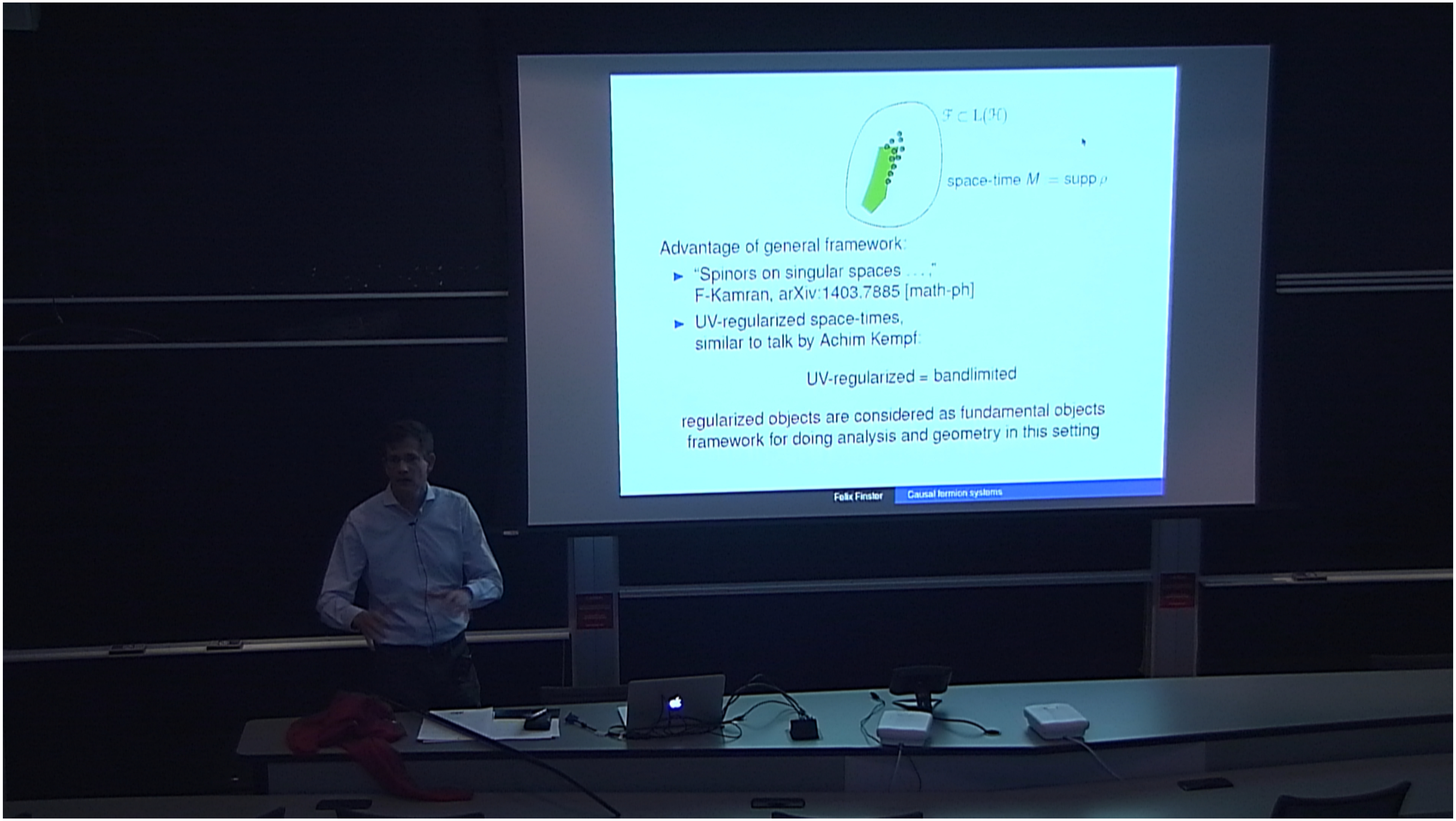
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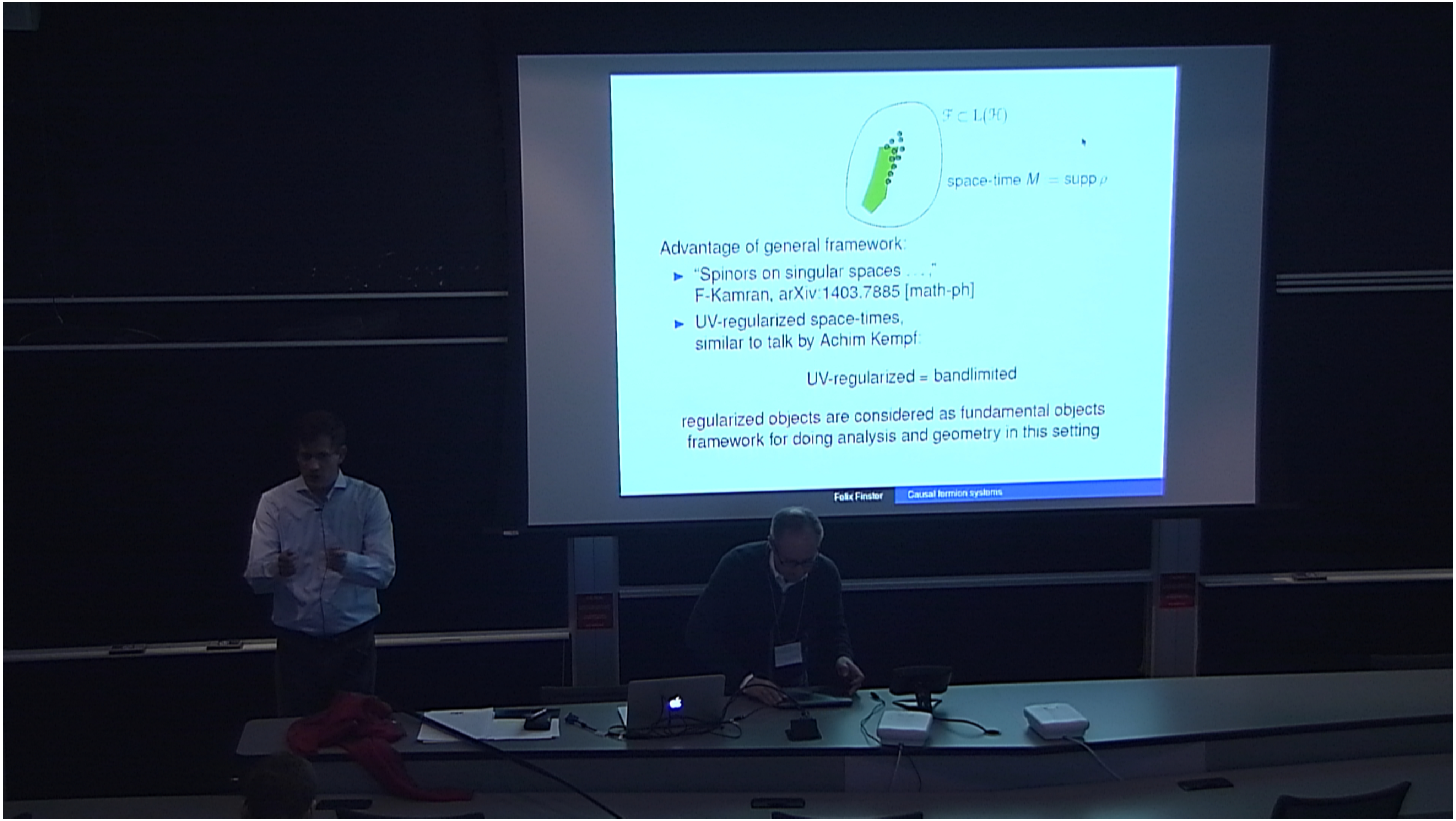
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