Title: Communication without transmission

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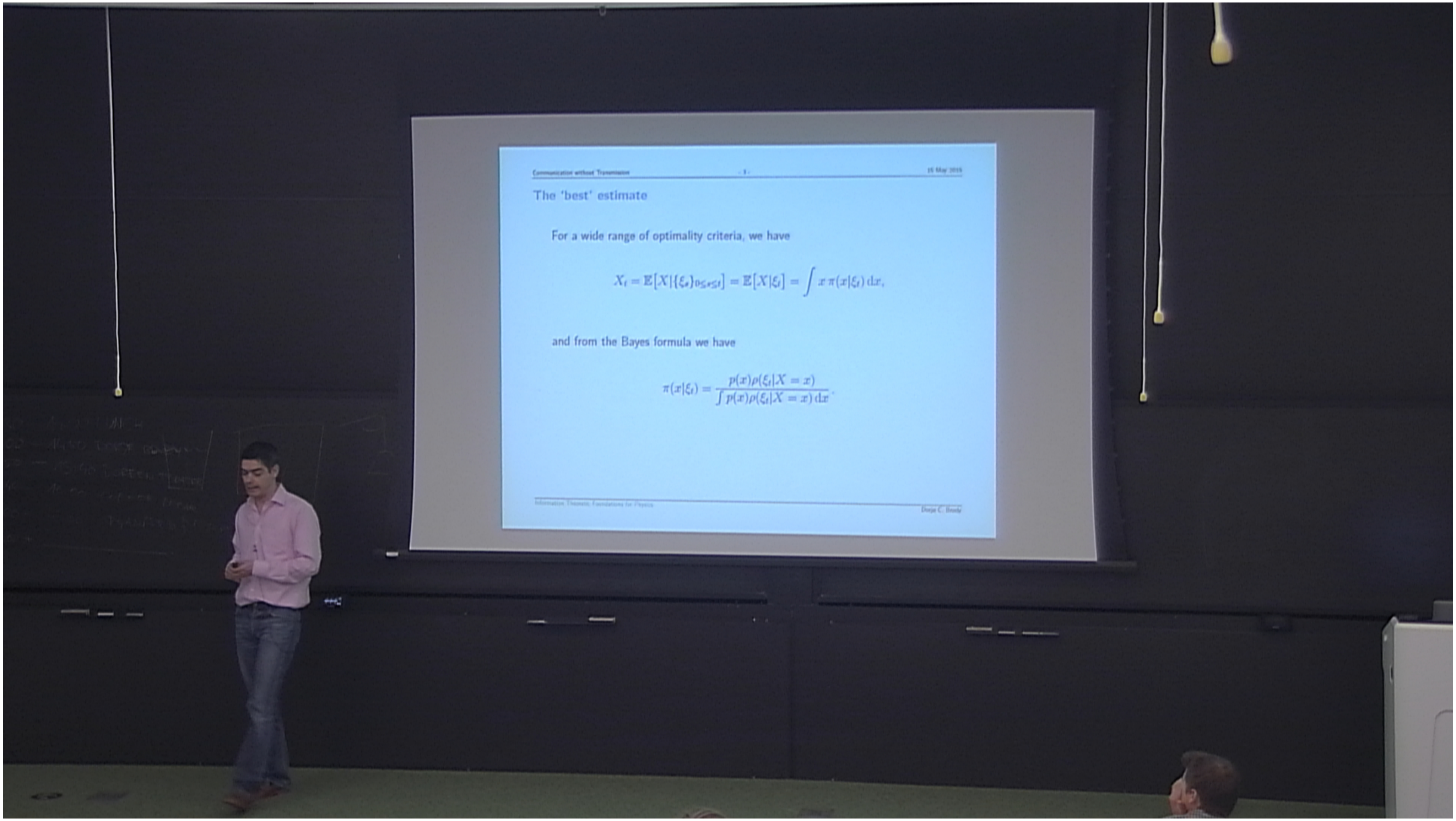
Abstract: It is sometimes envisaged that the behaviour of elementary particles can be characterised by the information content it carries, and that exchange of energy and momentum, or more generally the change of state through interactions, can likewise be characterised in terms of its information content. But exchange of information occurs only in the context of a (typically noisy) communication channel, which traditionally requires a transmitter and a receiver; whereas particles evidently are not equipped with such devices. In view of this a new concept in communication theory is put forward whereby signal processing is carried out in the absence of a transmitter; hence mathematical machineries in communication theory serves as new powerful tools for describing a wide range of observed phenomena. In the quantum context, this leads to a tentativeâ€”and perhaps speculativeâ€”idea that the dynamical evolution of the state of a quantum particle is such that the particle itself acts as if it were a "signal processor", trying to identify the stable configuration that it should settle, and adjusts its own state accordingly. It will be shown that the mathematical scheme of such a hypothesis works well for a broad class of noise structures having stationary and independent increments. (The talk will be based on work carried out in collaboration with L. P. Hughston.)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The 'best' estimate
For a wide range of optimality criteria, we have
$X_t = \mathbb{E}[X|\{\xi_s\}_{0\le s\le t}] = \mathbb{E}[X|\xi_t] = \int x\,\pi(x|\xi_t)\,\mathrm{d}x,$
and from the Bayes formula we have
$\pi(x|\xi_t) = \frac{p(x)\rho(\xi_t|X=x)}{\int p(x)\rho(\xi_t|X=x)\,\mathrm{d}x}.$
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Dorje C. Brody

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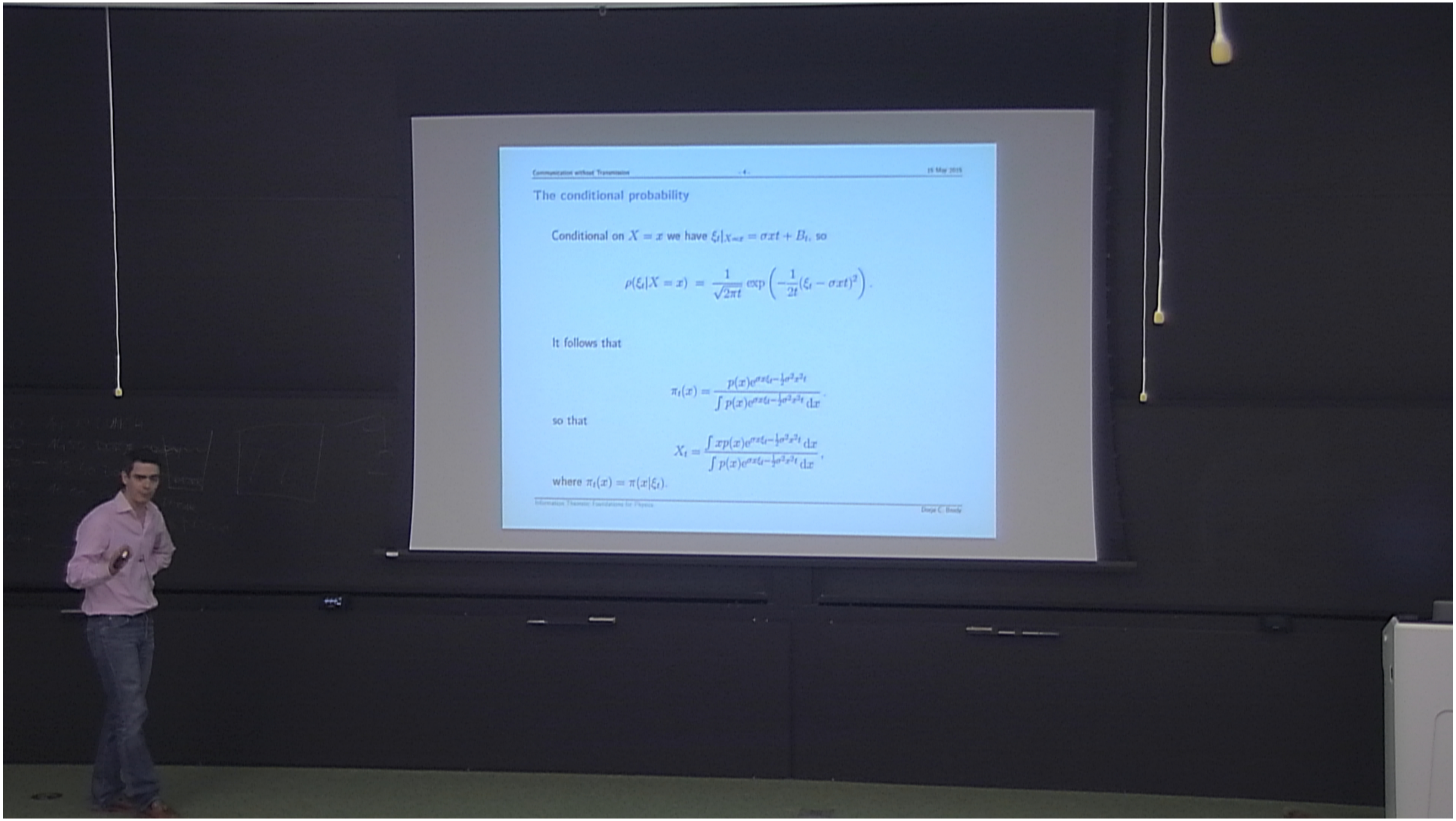

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The conditional probability
Conditional on $X = x$ we have $\xi_t|_{X=x} = \sigma x t + B_t$, so
$$\rho(\xi_t|X = x) = \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{1}{2t}(\xi_t - \sigma x t)^2\right).$$
It follows that
$$\pi_t(x) = \frac{p(x) e^{\sigma x \xi_t - \frac{1}{2}\sigma^2 x^2 t}}{\int p(x) e^{\sigma x \xi_t - \frac{1}{2}\sigma^2 x^2 t} \, dx},$$
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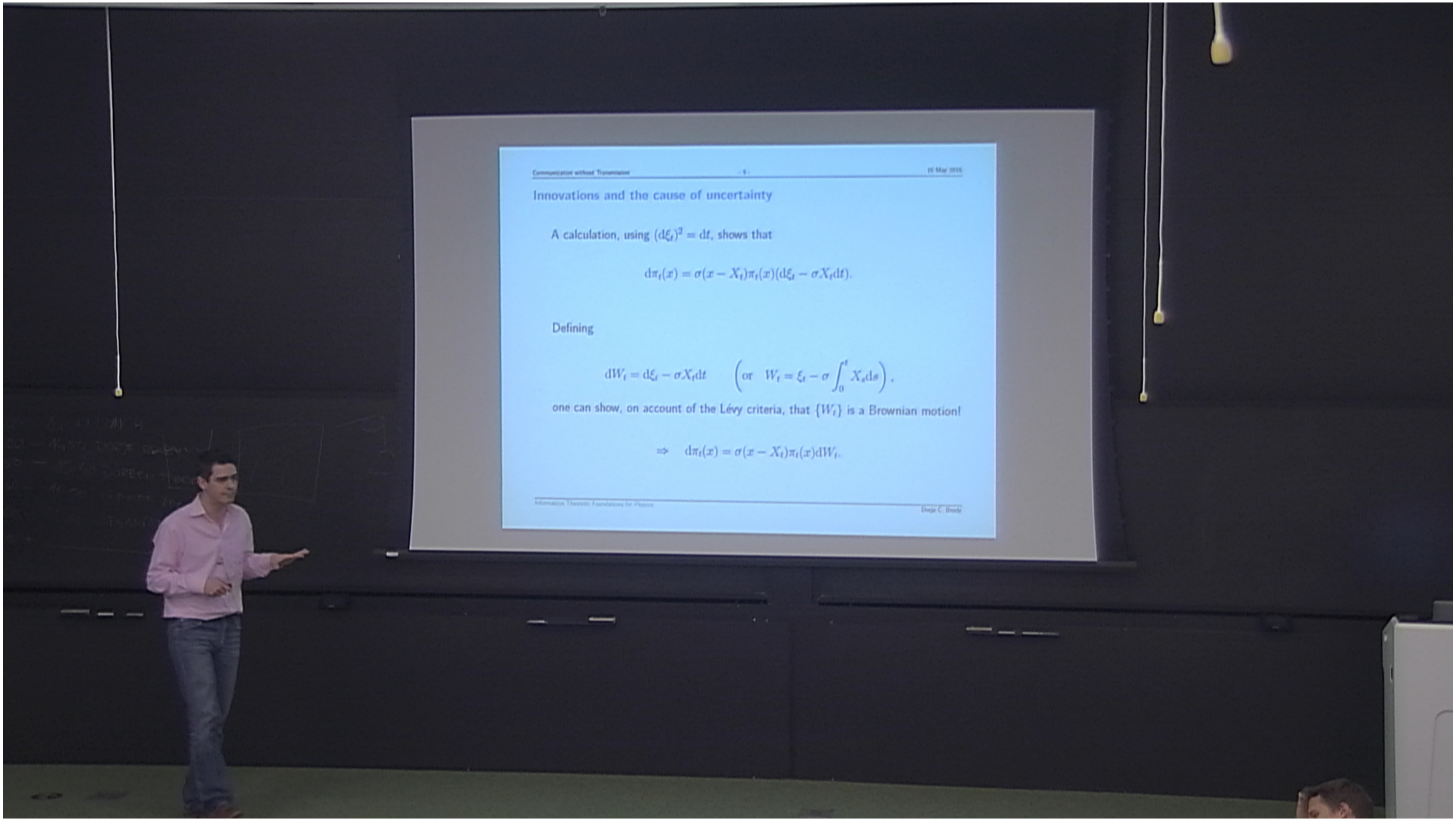
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Innovations and the cause of uncertainty
A calculation, using $(\mathrm{d}\xi_t)^2 = \mathrm{d}t$, shows that
$$\mathrm{d}\pi_t(x) = \sigma(x - X_t)\pi_t(x)(\mathrm{d}\xi_t - \sigma X_t \mathrm{d}t).$$
Defining
$$\mathrm{d}W_t = \mathrm{d}\xi_t - \sigma X_t \mathrm{d}t \qquad \left(\text{or} \quad W_t = \xi_t - \sigma \int_0^t X_s \mathrm{d}s\right),$$
one can show, on account of the Lévy criteria, that $\{W_t\}$ is a Brownian motion!
$$\Rightarrow \quad \mathrm{d}\pi_t(x) = \sigma(x - X_t)\pi_t(x)\mathrm{d}W_t.$$
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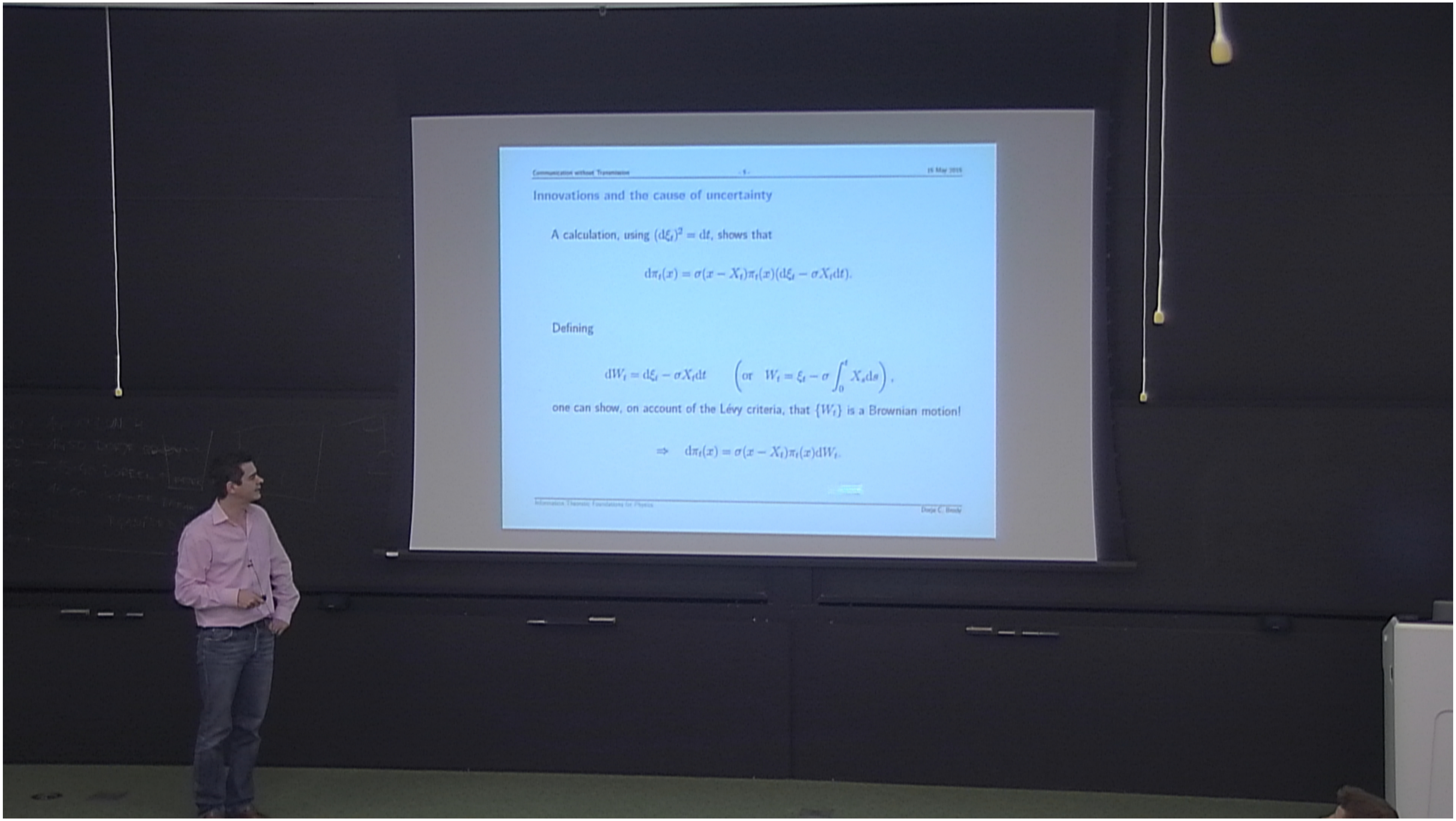
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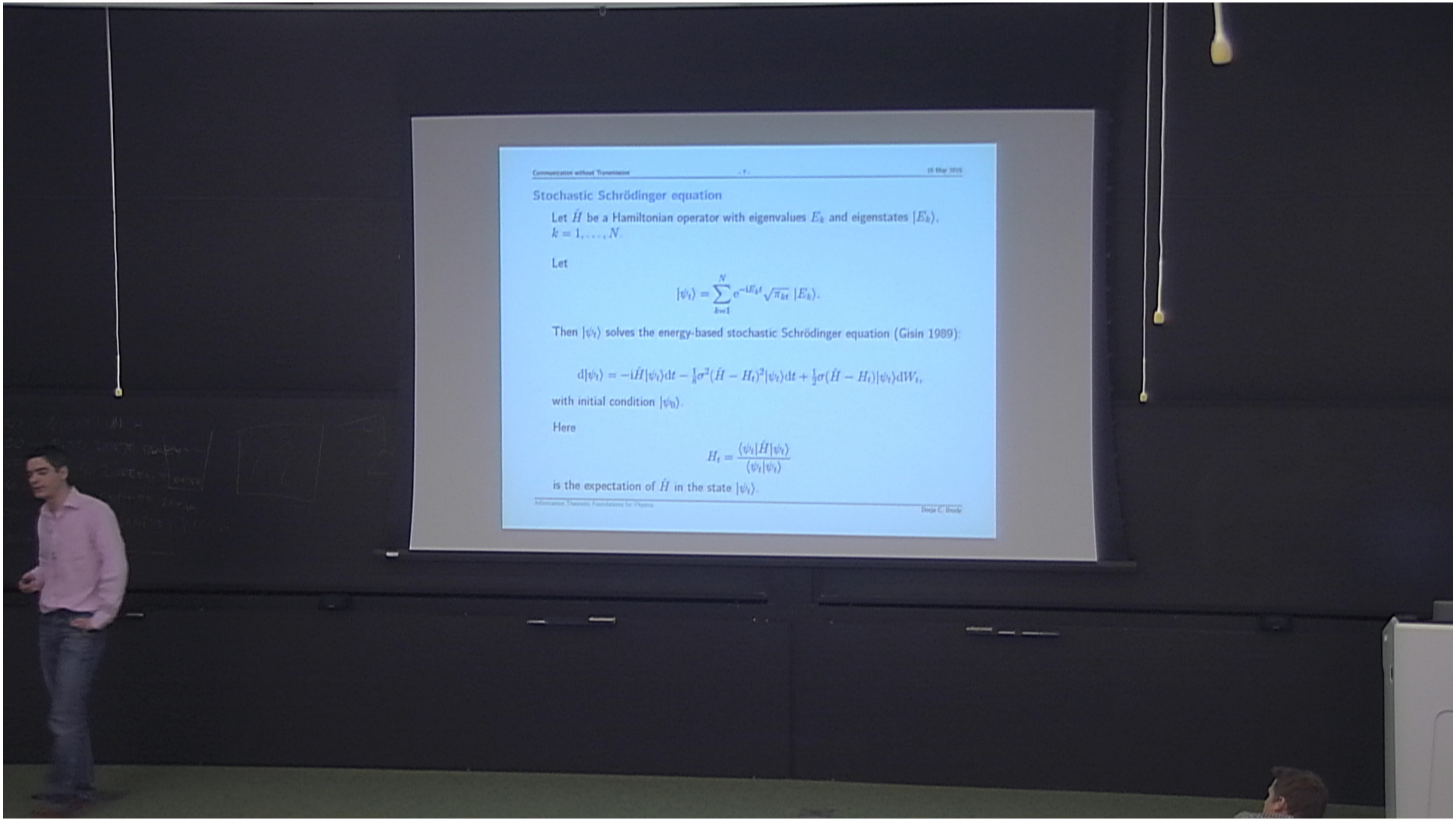
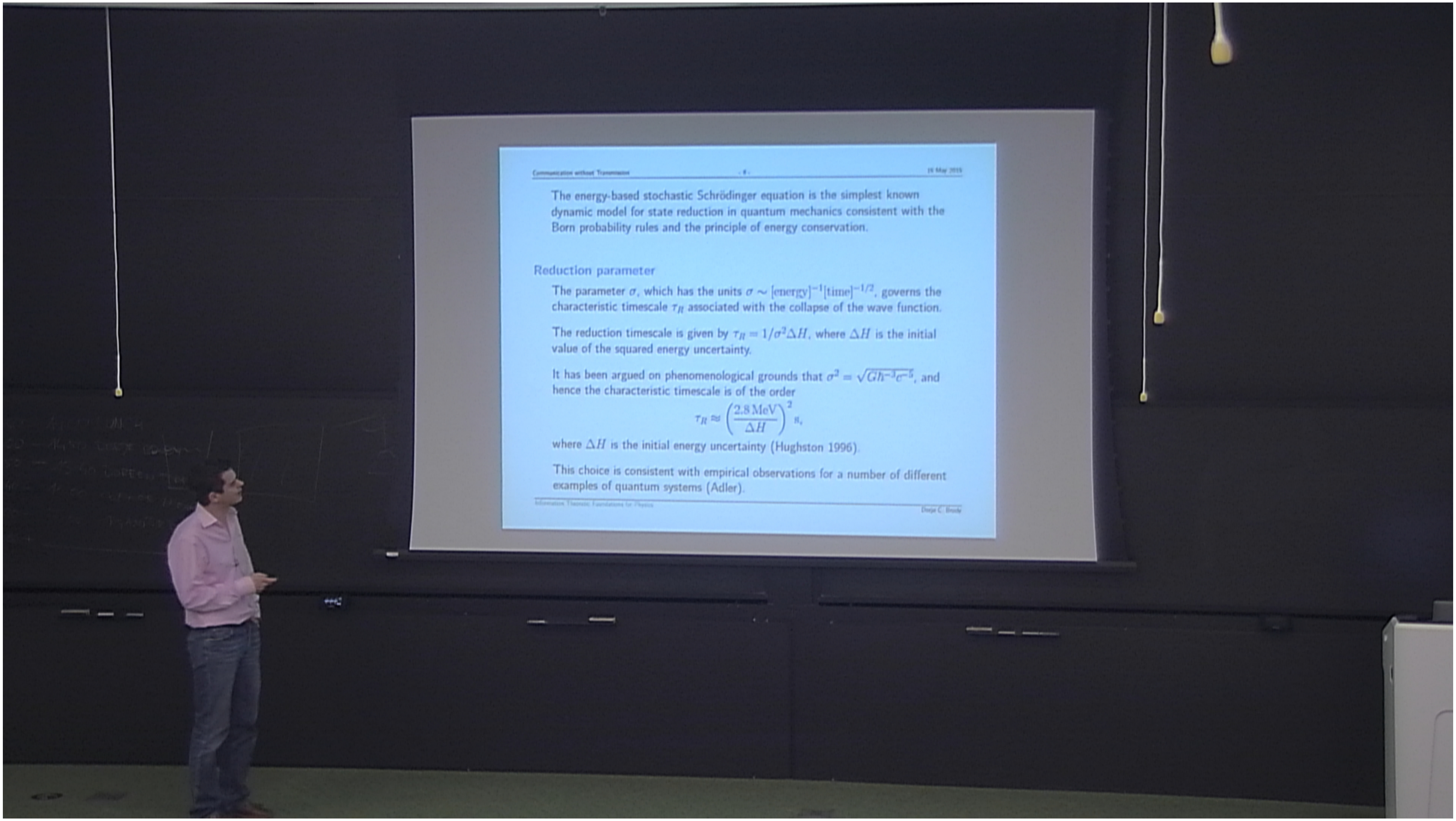
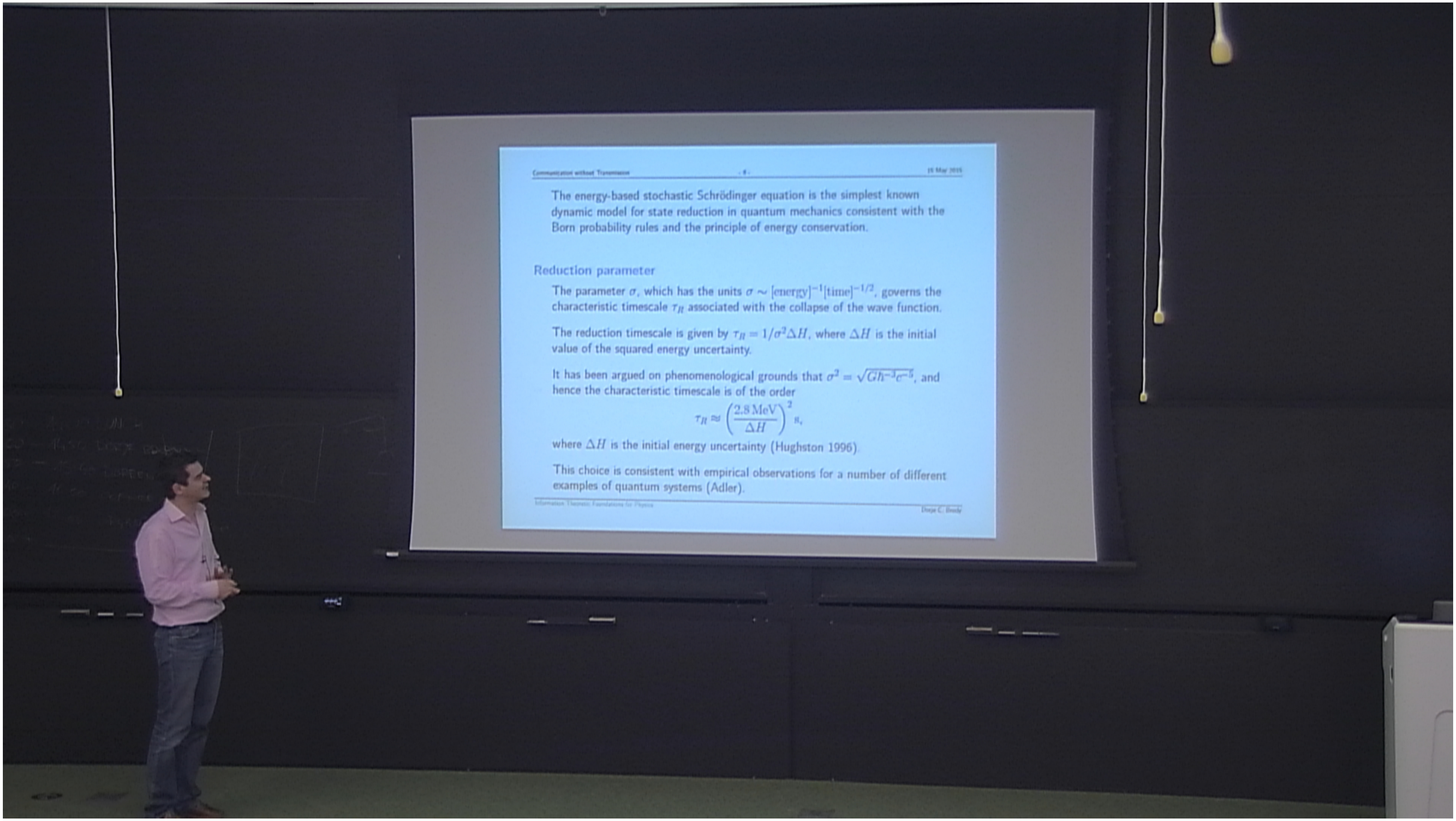


Stochastic Schrödinger equation
Let $\hat{H}$ be a Hamiltonian operator with eigenvalues E_k and eigenstates $|E_k\rangle$, $k = 1, \ldots, N$.
Let
$|\psi_t\rangle = \sum_{k=1}^{N} e^{-iE_k t} \sqrt{\pi_{kt}}\, |E_k\rangle.$
Then $|\psi_t\rangle$ solves the energy-based stochastic Schrödinger equation (Gisin 1989):
$d|\psi_t\rangle = -i\hat{H}|\psi_t\rangle dt - \frac{1}{8}\sigma^2(\hat{H} - H_t)^2|\psi_t\rangle dt + \frac{1}{2}\sigma(\hat{H} - H_t)|\psi_t\rangle dW_t,$
with initial condition $|\psi_0\rangle$.
Here
$H_t = \frac{\langle\psi_t|\hat{H}|\psi_t\rangle}{\langle\psi_t|\psi_t\rangle}$
is the expectation of $\hat{H}$ in the state $|\psi_t\rangle$.


The energy-based stochastic Schrödinger equation is the simplest known dynamic model for state reduction in quantum mechanics consistent with the Born probability rules and the principle of energy conservation.
Reduction parameter
The parameter σ, which has the units $\sigma \sim [\mathrm{energy}]^{-1}[\mathrm{time}]^{-1/2}$, governs the characteristic timescale τ_R associated with the collapse of the wave function.
The reduction timescale is given by $\tau_R = 1/\sigma^2\Delta H$, where ΔH is the initial value of the squared energy uncertainty.
It has been argued on phenomenological grounds that $\sigma^2 = \sqrt{G\hbar^{-3}c^{-5}}$, and hence the characteristic timescale is of the order
$$\tau_R \approx \left(\frac{2.8\,\mathrm{MeV}}{\Delta H}\right)^2 \mathrm{s},$$
where ΔH is the initial energy uncertainty (Hughston 1996).
This choice is consistent with empirical observations for a number of different examples of quantum systems (Adler).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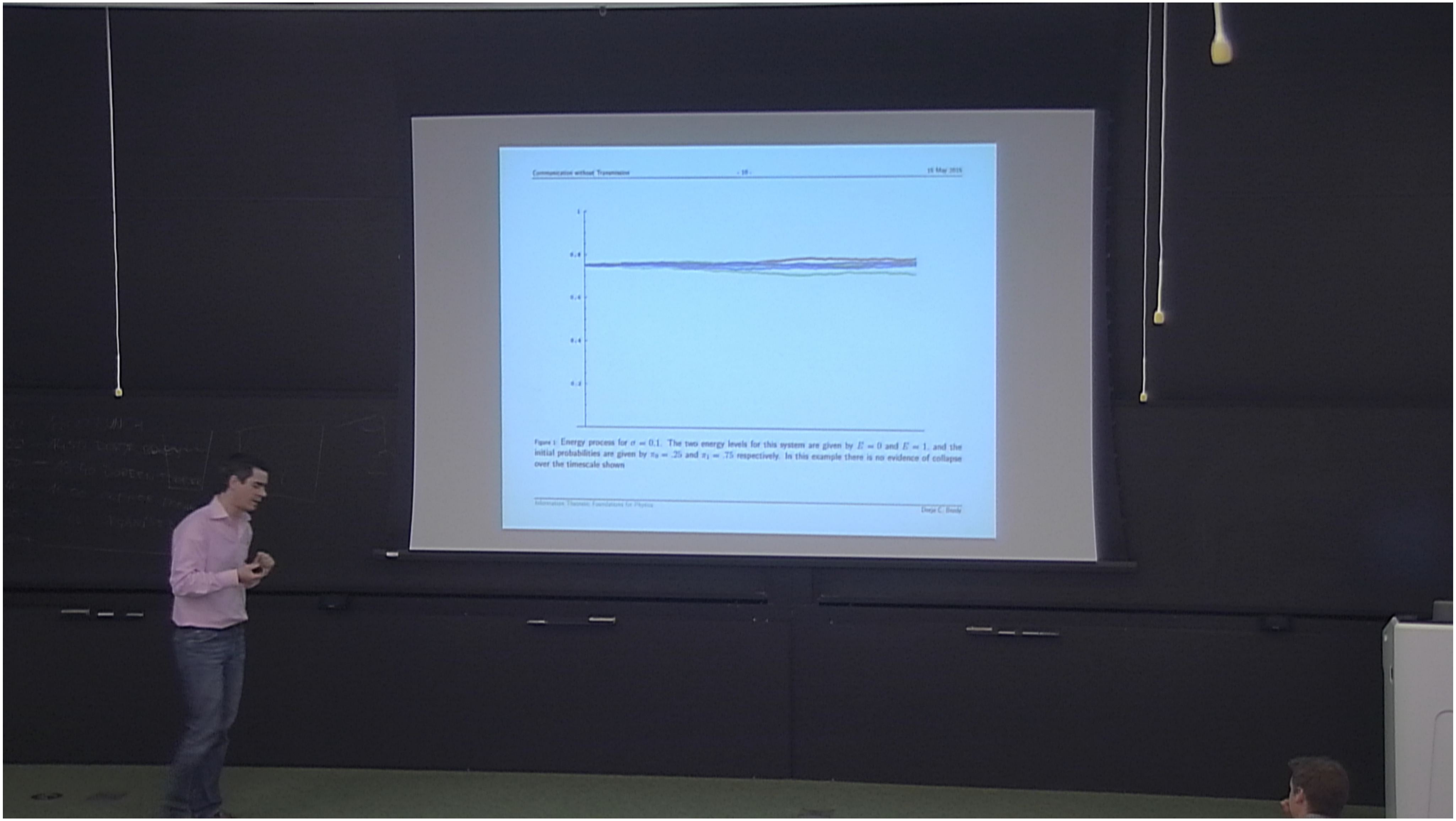
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Figure 1: Energy process for $\sigma = 0.1$. The two energy levels for this system are given by $E = 0$ and $E = 1$, and the initial probabilities are given by $\pi_0 = .25$ and $\pi_1 = .75$ respectively. In this example there is no evidence of collapse over the timescale shown
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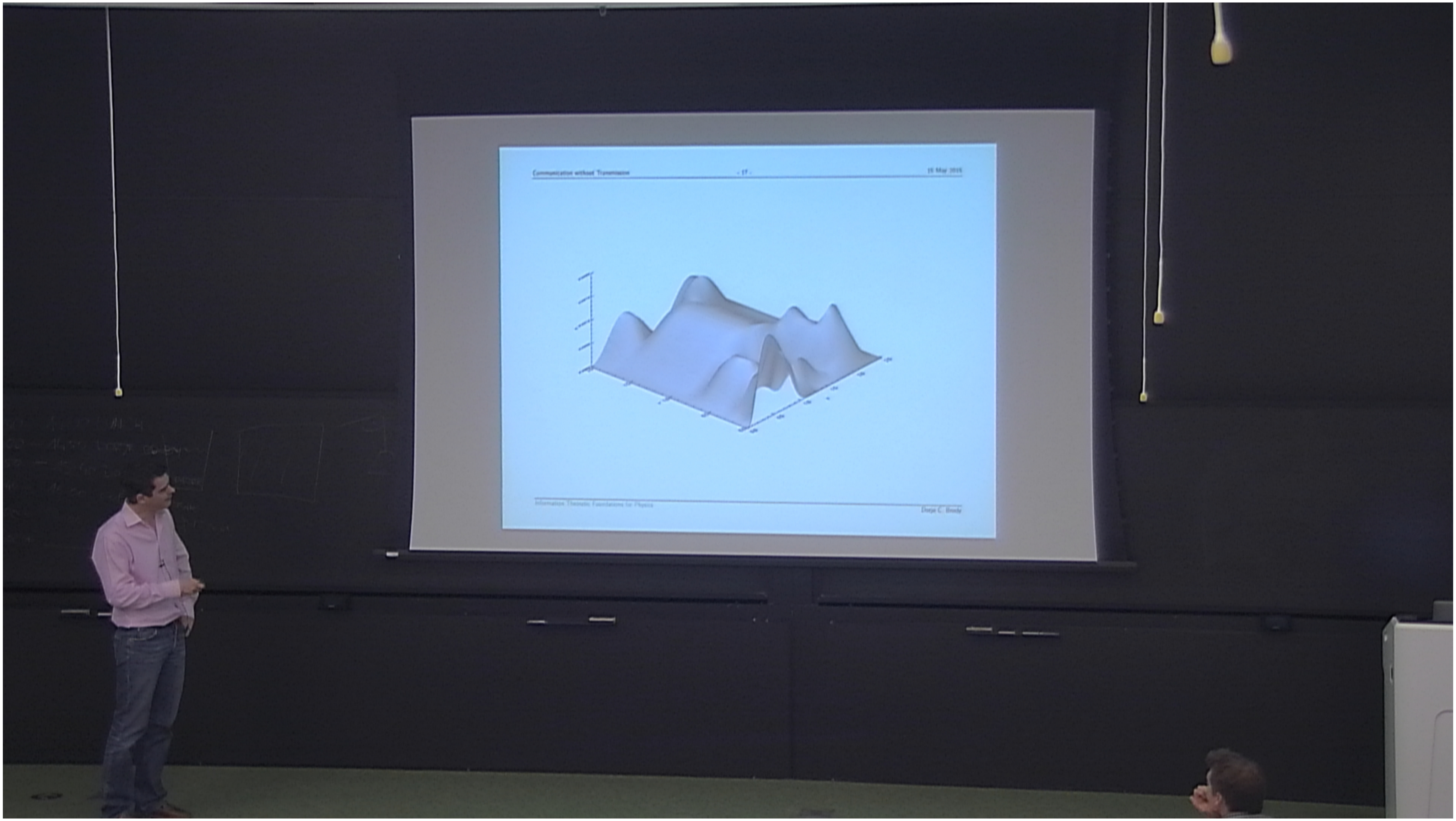
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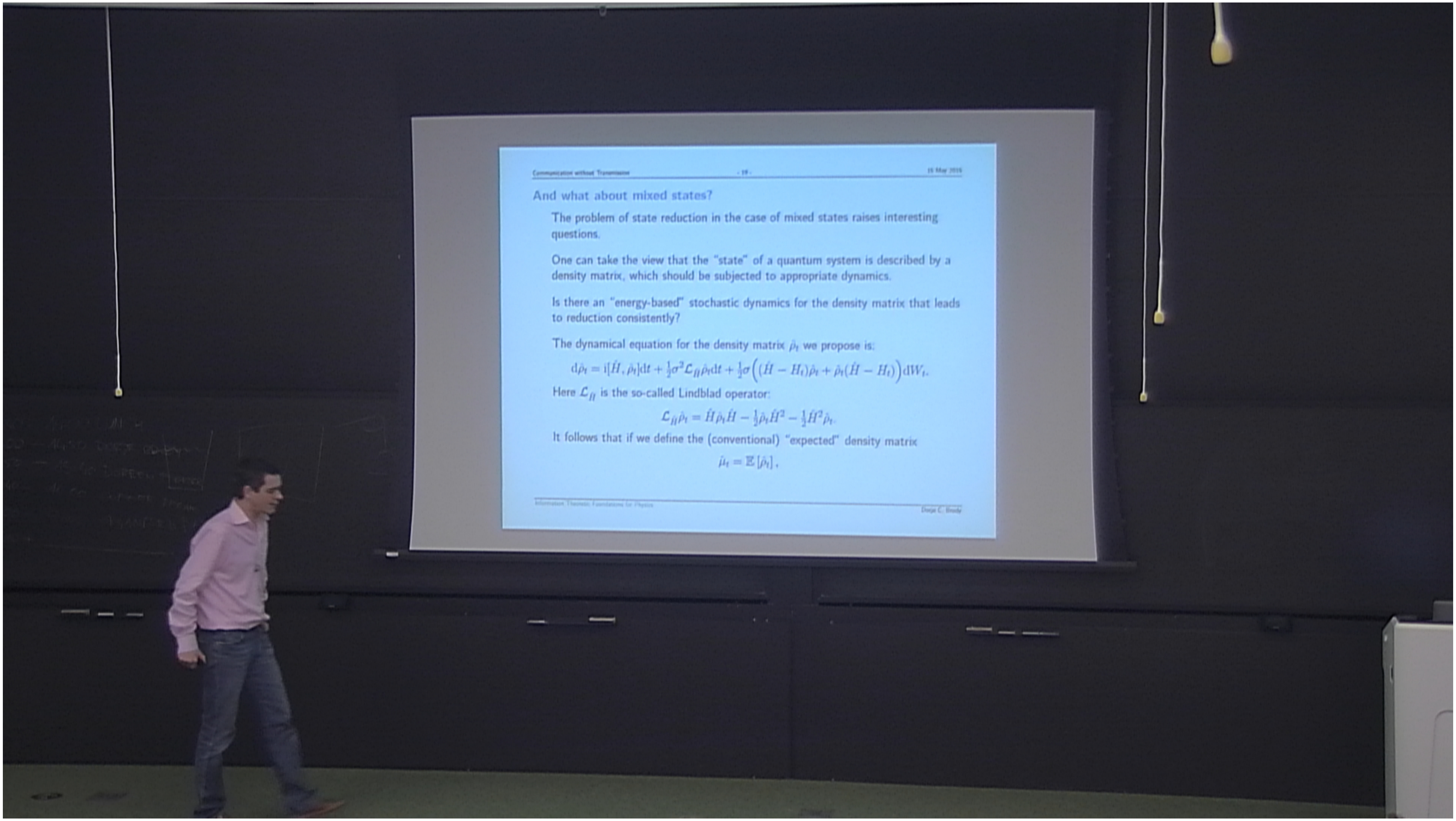
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And what about mixed states?
The problem of state reduction in the case of mixed states raises interesting questions.
One can take the view that the "state" of a quantum system is described by a density matrix, which should be subjected to appropriate dynamics.
Is there an "energy-based" stochastic dynamics for the density matrix that leads to reduction consistently?
The dynamical equation for the density matrix $\hat{\rho}_t$ we propose is:
$$\mathrm{d}\hat{\rho}_t = \mathrm{i}[\hat{H}, \hat{\rho}_t]\mathrm{d}t + \tfrac{1}{2}\sigma^2 \mathcal{L}_{\hat{H}}\hat{\rho}_t \mathrm{d}t + \tfrac{1}{2}\sigma\Big((\hat{H} - H_t)\hat{\rho}_t + \hat{\rho}_t(\hat{H} - H_t)\Big)\mathrm{d}W_t.$$
Here $\mathcal{L}_{\hat{H}}$ is the so-called Lindblad operator:
$$\mathcal{L}_{\hat{H}}\hat{\rho}_t = \hat{H}\hat{\rho}_t\hat{H} - \tfrac{1}{2}\hat{\rho}_t\hat{H}^2 - \tfrac{1}{2}\hat{H}^2\hat{\rho}_t.$$
It follows that if we define the (conventional) "expected" density matrix
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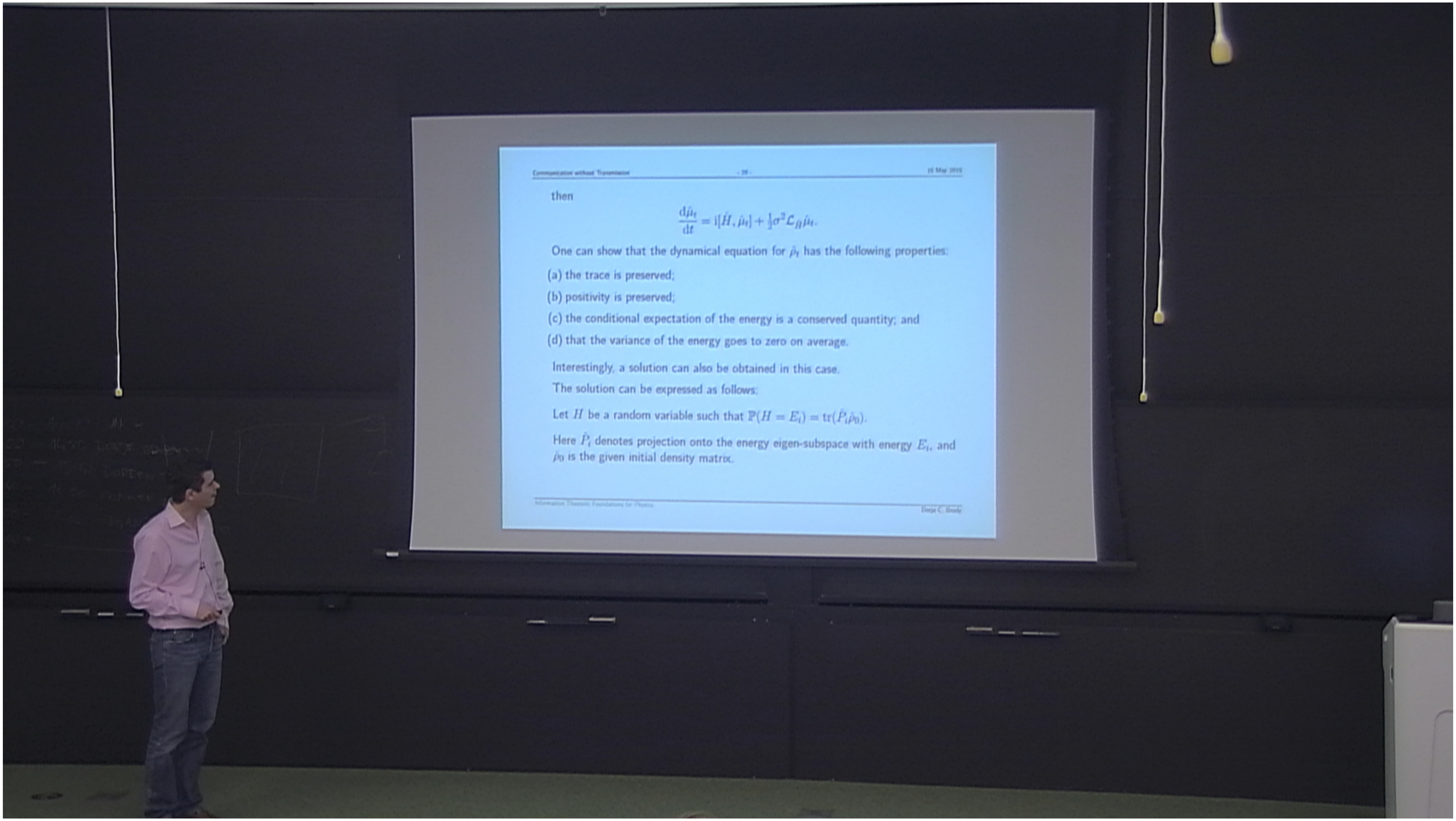
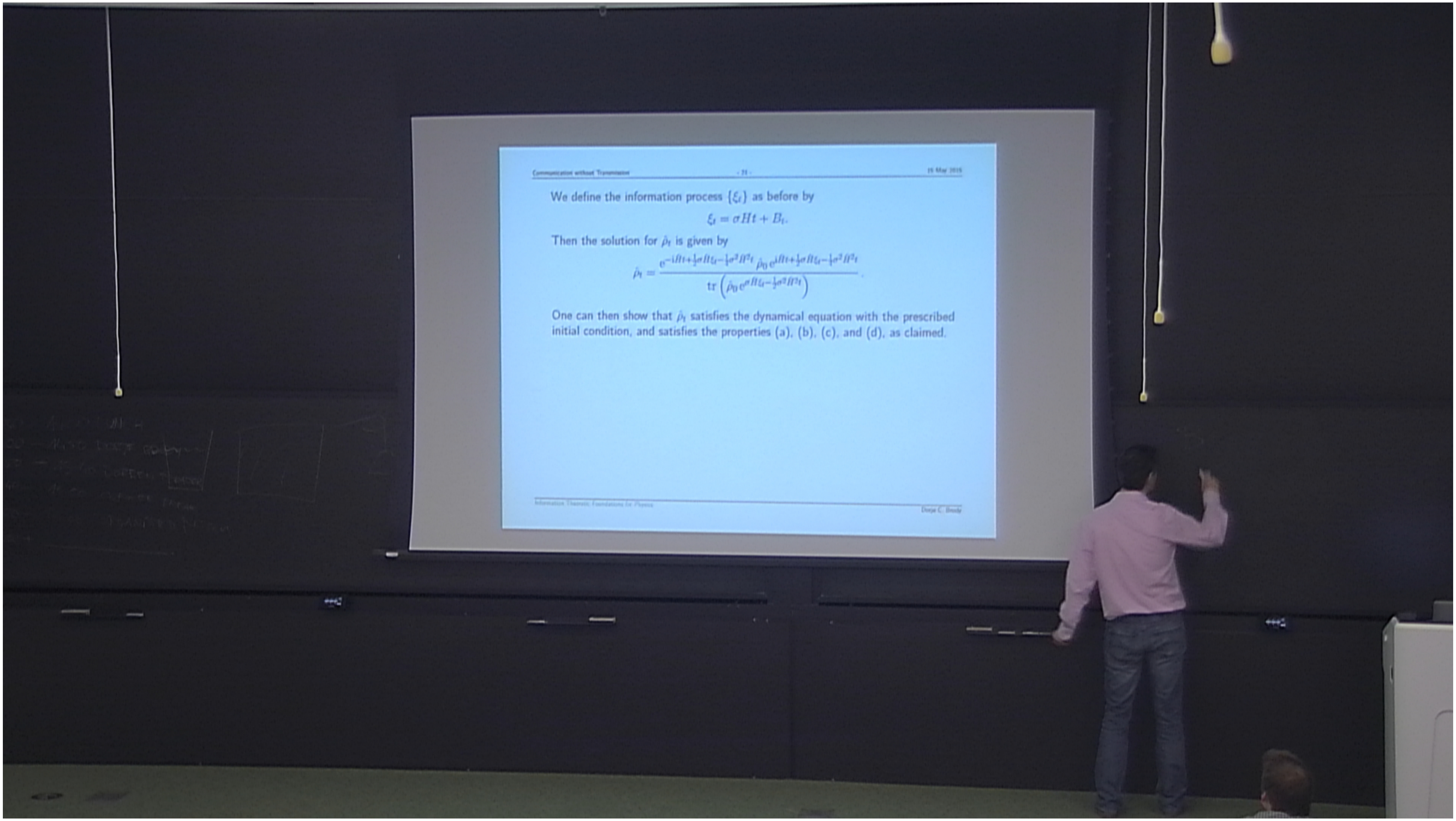

then
$$\frac{\mathrm{d}\hat{\mu}_t}{\mathrm{d}t} = \mathrm{i}[\hat{H}, \hat{\mu}_t] + \tfrac{1}{2}\sigma^2 \mathcal{L}_{\hat{H}} \hat{\mu}_t.$$
One can show that the dynamical equation for $\hat{\rho}_t$ has the following properties:
(a) the trace is preserved;
(b) positivity is preserved;
(c) the conditional expectation of the energy is a conserved quantity; and
(d) that the variance of the energy goes to zero on average.
Interestingly, a solution can also be obtained in this case.
The solution can be expressed as follows:
Let H be a random variable such that $\mathbb{P}(H = E_i) = \mathrm{tr}(\hat{P}_i \hat{\rho}_0)$.
Here $\hat{P}_i$ denotes projection onto the energy eigen-subspace with energy E_i, and $\hat{\rho}_0$ is the given initial density matrix.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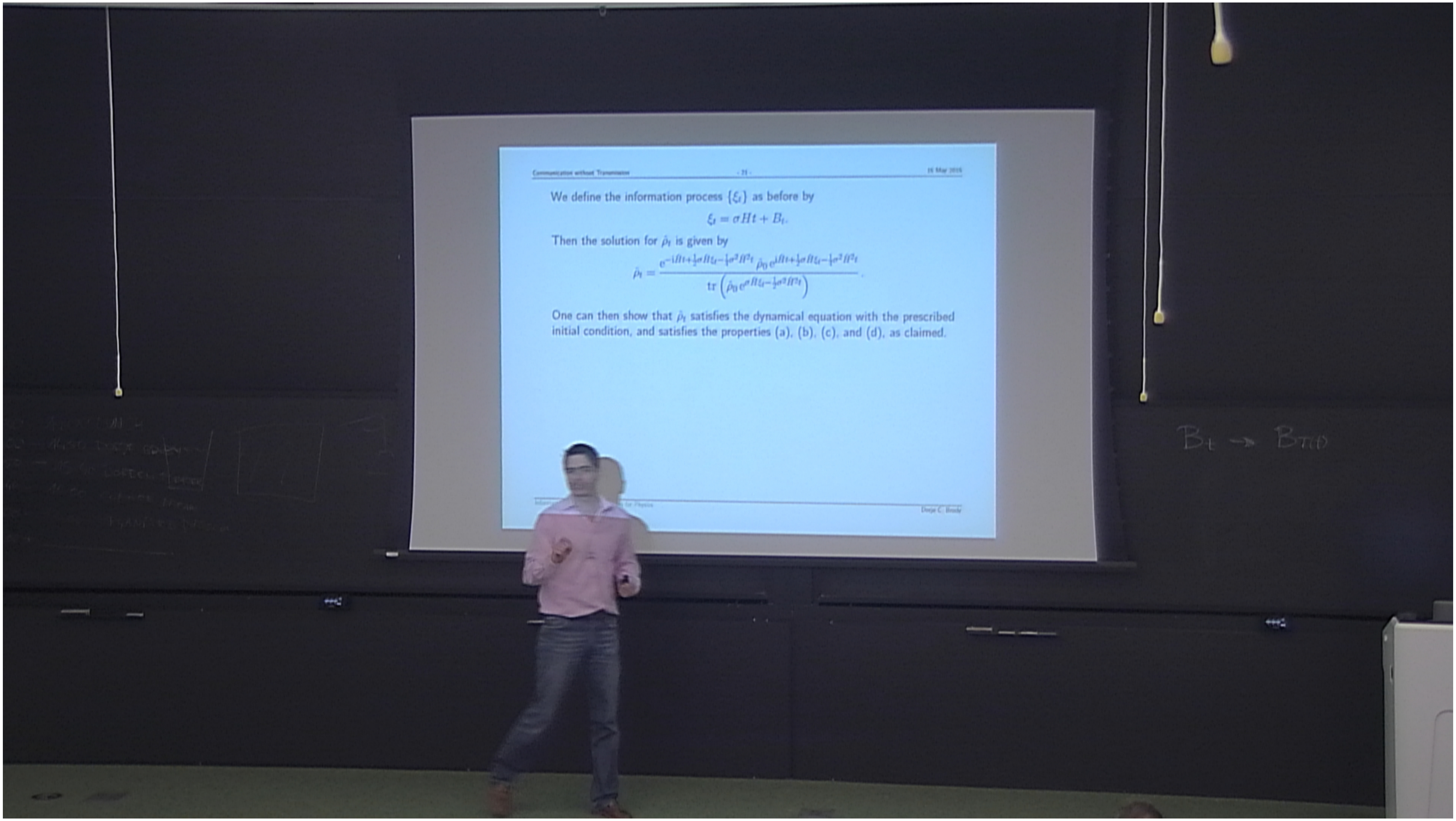
One can then show that $\hat{\rho}_t$ satisfies the dynamical equation with the prescribed initial condition, and satisfies the properties (a), (b), (c), and (d), as claimed.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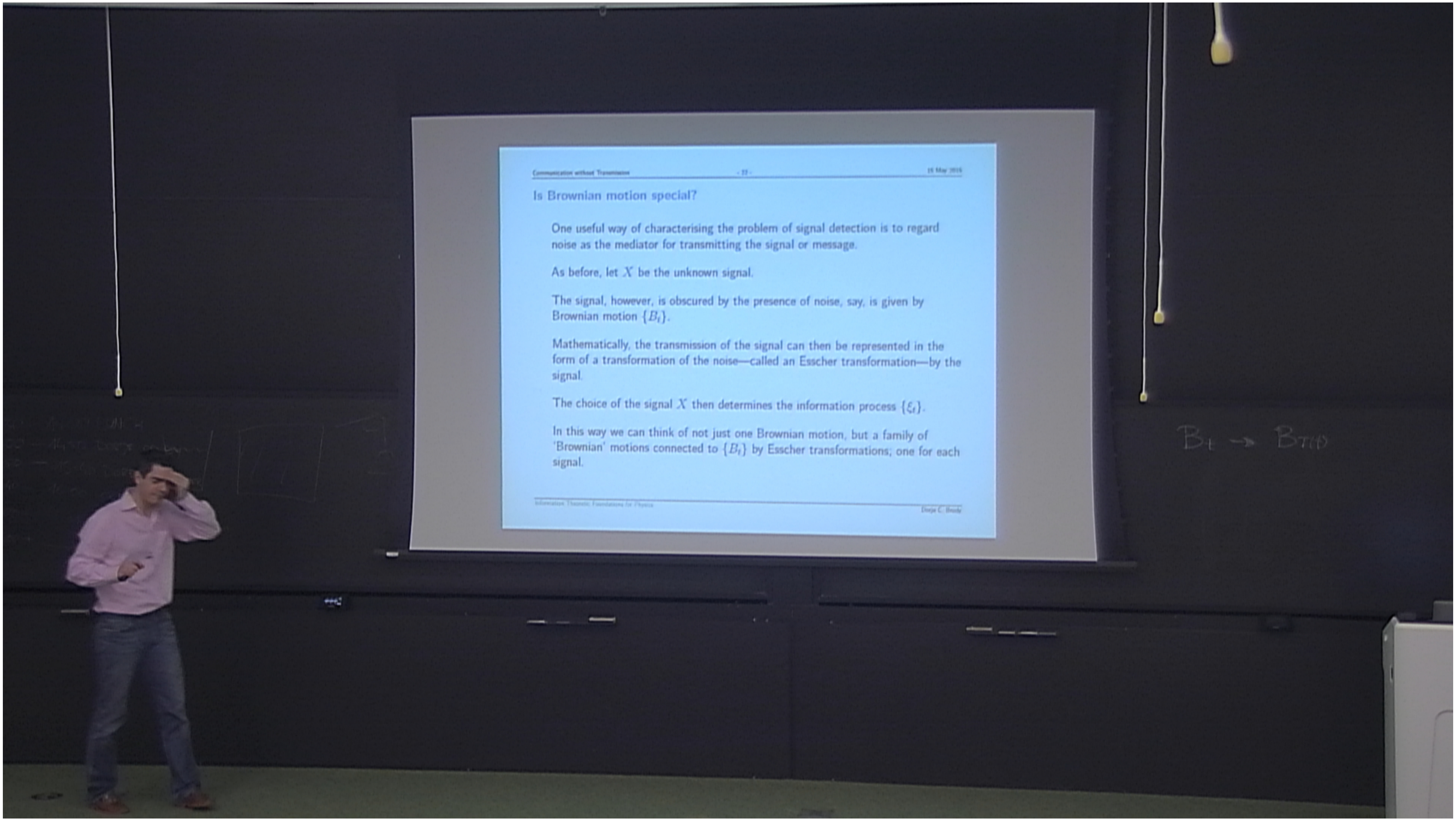
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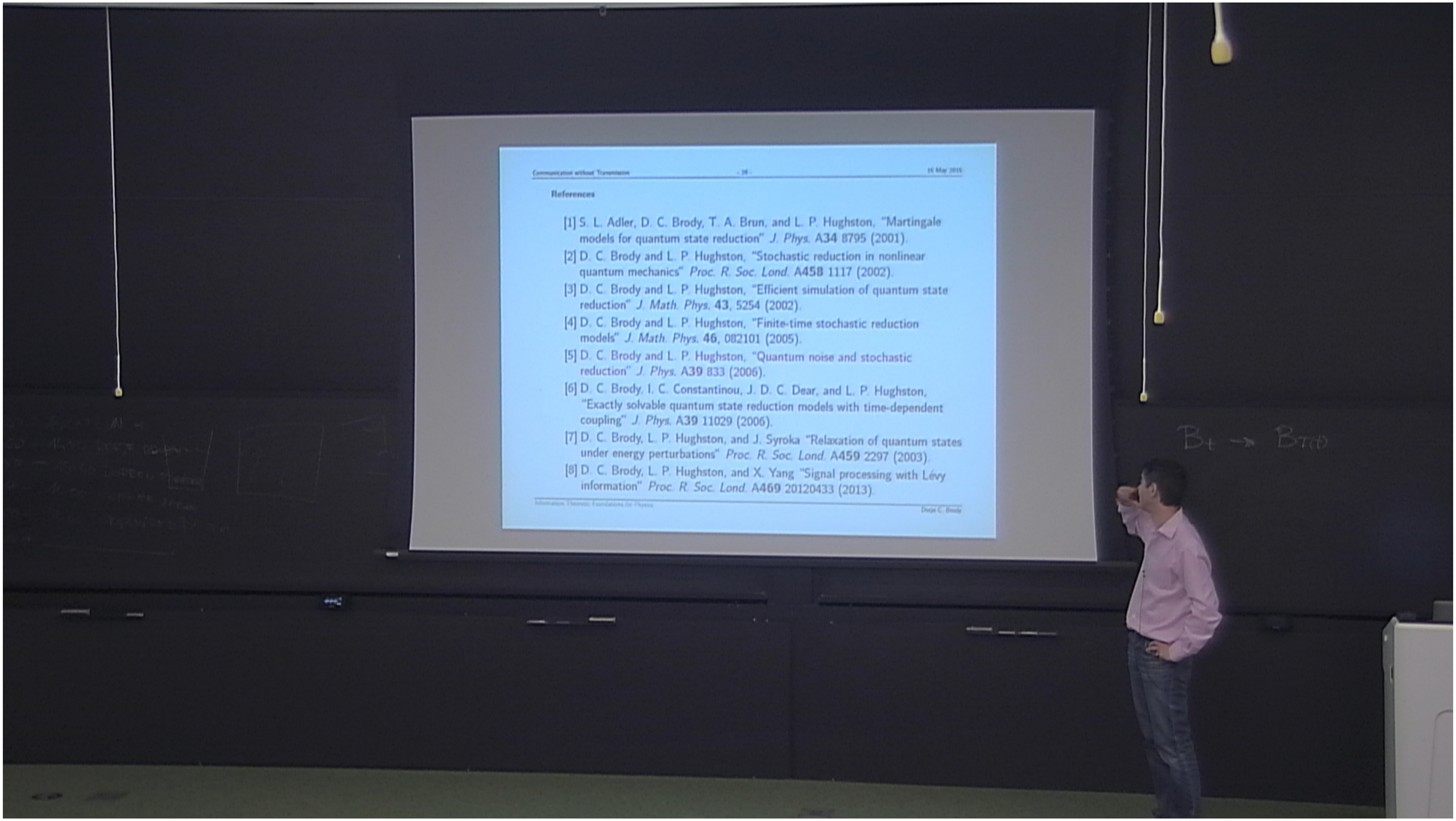
Is Brownian motion special?
One useful way of characterising the problem of signal detection is to regard noise as the mediator for transmitting the signal or message.
As before, let X be the unknown signal.
The signal, however, is obscured by the presence of noise, say, is given by Brownian motion $\{B_t\}$.
Mathematically, the transmission of the signal can then be represented in the form of a transformation of the noise—called an Esscher transformation—by the signal.
The choice of the signal X then determines the information process $\{\xi_t\}$.
In this way we can think of not just one Brownian motion, but a family of 'Brownian' motions connected to $\{B_t\}$ by Esscher transformations; one for each signal.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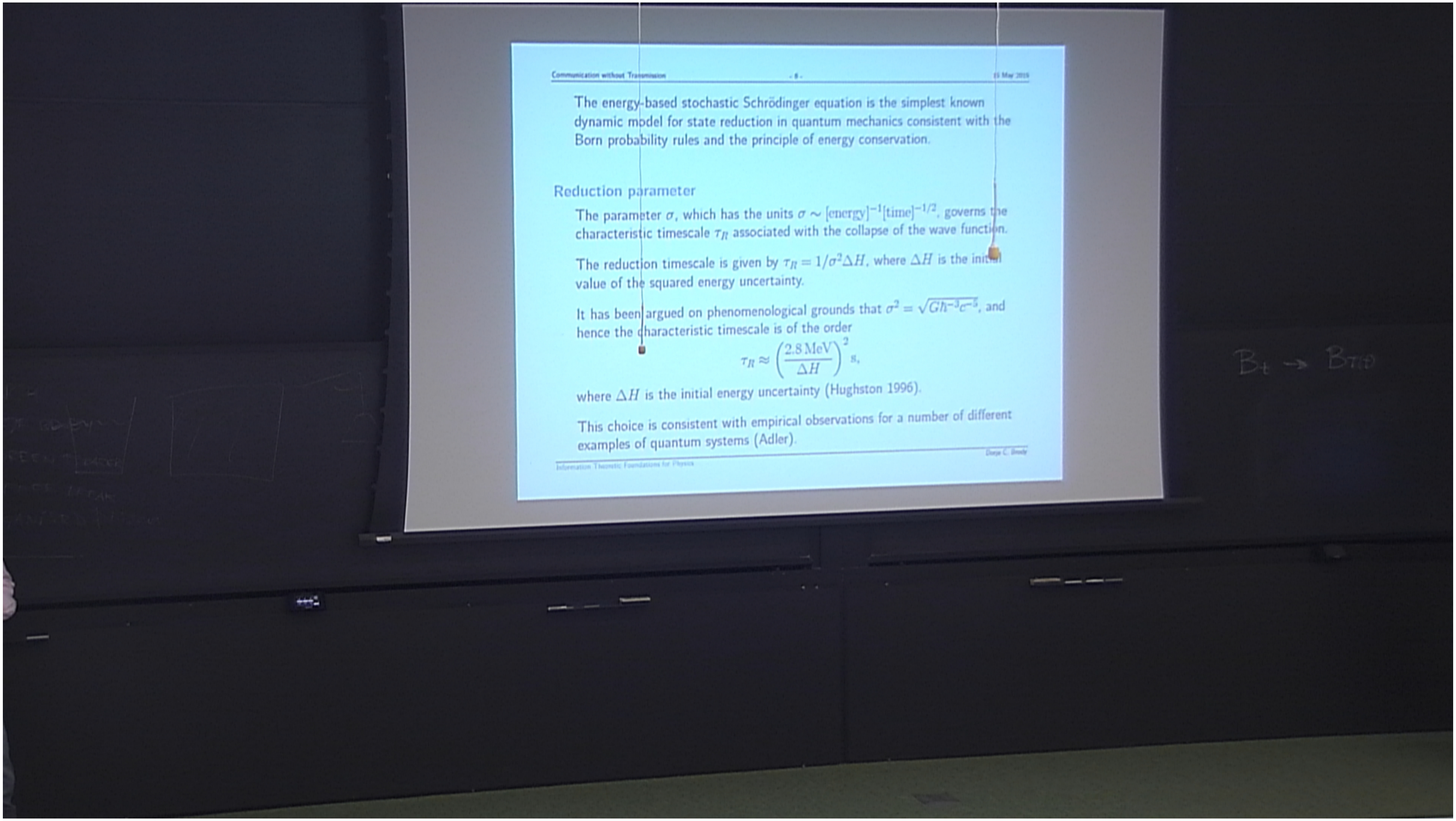

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