Title: Tangent field theory

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Abstract: The modern understanding of quantum field theory underlines its effective nature: it describes only those properties of a system relevant above a certain scale. A detailed understanding of the nature of the neglected information is essential for a full application of quantum information-theoretic tools to continuum theories.

I will present an operationally motivated method for deriving an effective field theory from any microscopic description of a state. The approach is based on dimensional reduction relative to a quantum distinguishability metric. It relies on a microscopic description of experimental limitations, such as a finite spatial resolution. In this picture, the emergent field observables represent cotangent vectors on the manifold of states, and are not necessarily endowed with the full semantic of standard quantum observables.

Information-theoretical foundations for quantum field theory

Cédric Bény

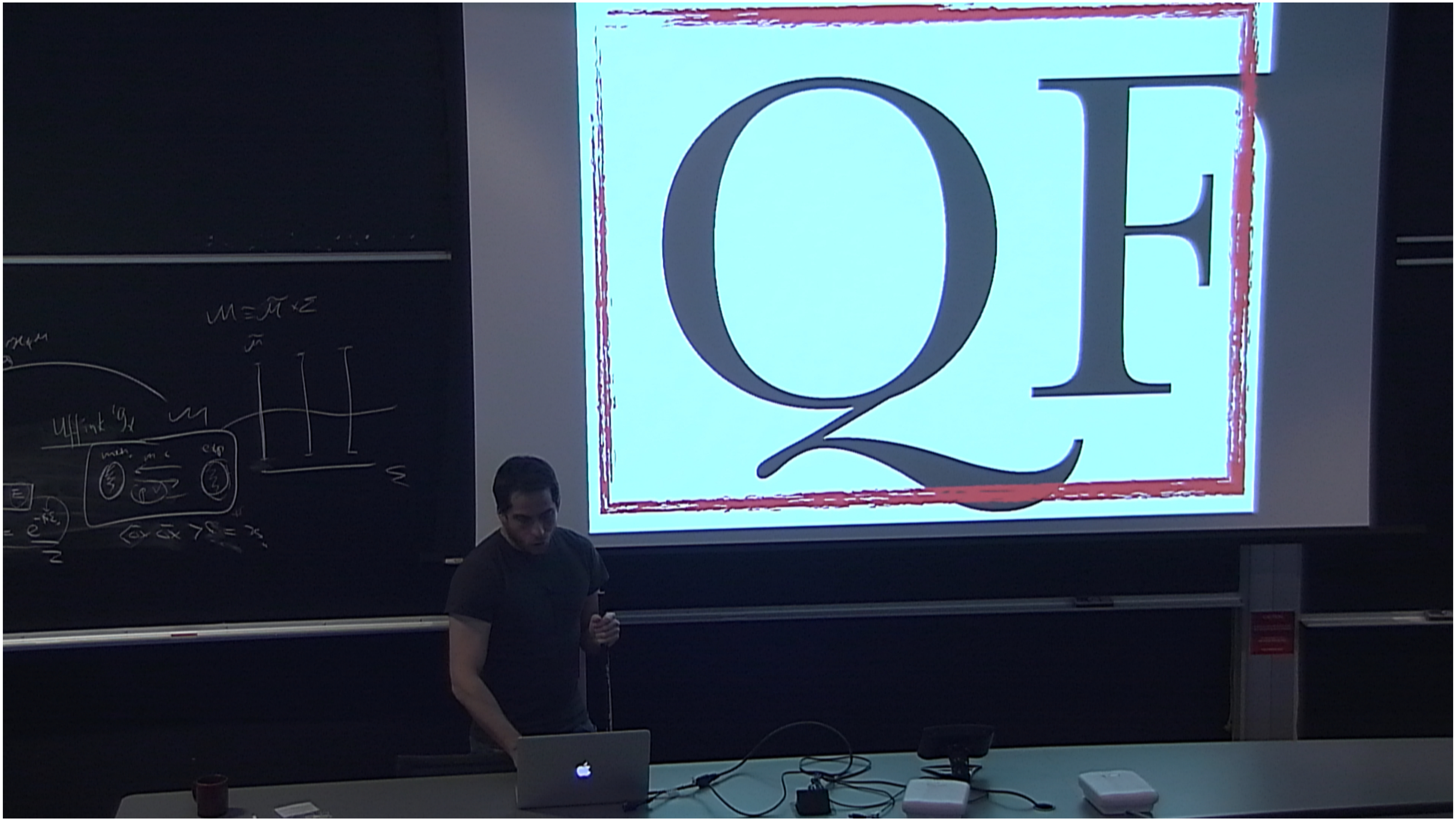
joint work with Tobias Osborne

arXiv:1402.4949

QFT

QFT

local quantum theory in the continuum


QF

$$\partial_x^2 \phi(x) + m^2 \phi(x) = 0$$

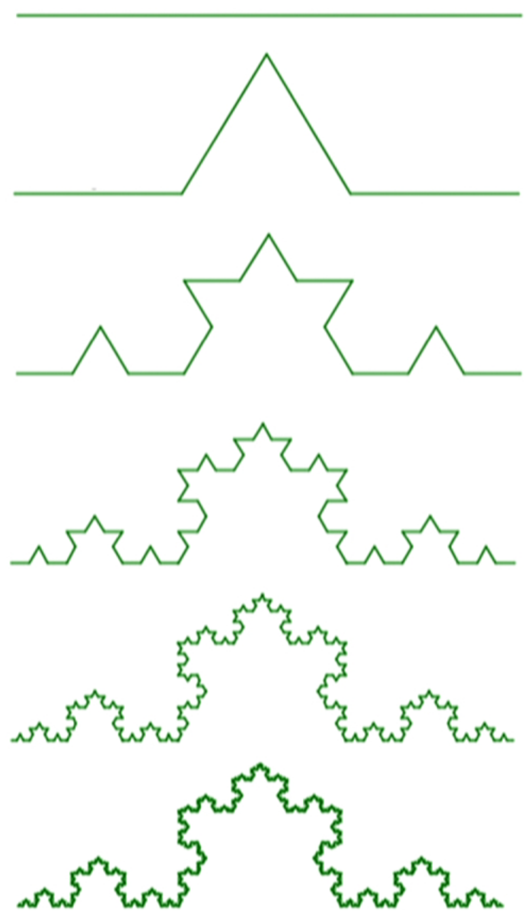
finite resolution

arbitrary

finite resolution

continuum theory

resolution → effective theory



$$\pi = \left\{3, \frac{31}{10}, \frac{314}{100}, \ldots\right\}$$

$x_1, x_2, \ldots \in \mathbb{Q}$ is Cauchy if

$$\forall\, \epsilon > 0 \quad \exists\, N \text{ such that } \forall\, i, j > N,\ \|x_i - x_j\| < \epsilon$$

$$\Lambda \mapsto \psi_\Lambda \quad \text{is "Cauchy" if}$$

$$\forall\, \sigma, \epsilon \quad \exists\, \Lambda_0 \;\; \text{such that} \;\; \forall\, \Lambda, \Lambda' > \Lambda_0,\; d_\sigma(\psi_\Lambda, \psi_{\Lambda'}) < \epsilon$$

experimental resolution

effectively indistinguishable

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experimental resolution

effectively indistinguishable

renormalisation

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experimental resolution

effectively indistinguishable

$$d(\rho, \rho') = \max_{0 \leq A \leq I} \mathrm{Tr}(A(\rho - \rho'))$$

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$$d(\rho, \rho') = \begin{cases} \|\rho - \rho'\|_1 \\ S(\rho' \| \rho) \\ F(\rho, \rho') \\ \dots \end{cases}$$

$$d_\sigma(\rho, \rho') := d(\mathcal{N}_\sigma(\rho), \mathcal{N}_\sigma(\rho'))$$

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information geometry

Riemannian metric on states

$$d(\rho + \epsilon X, \rho)$$

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$$d(\rho + \epsilon X, \rho) \;\; \simeq \;\; \epsilon^2 \langle X, X \rangle_\rho$$

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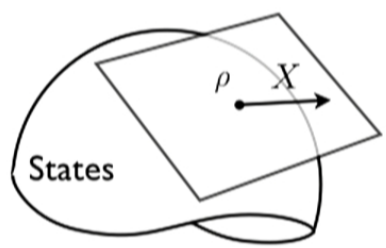
$$\rho + \epsilon X \qquad X^\dagger = X \qquad \mathrm{Tr} X = 0$$



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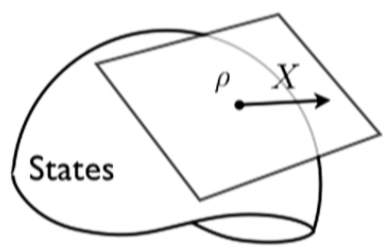

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$$d(\mathcal{N}(\rho + \epsilon X), \mathcal{N}(\rho)) \;\simeq\; \epsilon^2 \langle \mathcal{N}(X), \mathcal{N}(X) \rangle_{\mathcal{N}(\rho)}$$

$$= \epsilon^2 \|\mathcal{N}(X)\|_{\mathcal{N}(\rho)}^2$$

Information metrics

$$\|\mathcal{N}(X)\|_{\mathcal{N}(\rho)} \leq \|X\|_{\rho}$$

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$$\overset{\text{Classically}}{\Longrightarrow} \quad \langle X, Y\rangle_\rho = \mathrm{Tr}\big(X\rho^{-1}Y\big)$$

Fisher information metric

Bures metric

$$\langle X, Y \rangle_\rho = \mathrm{Tr}(X \Omega_\rho^{-1}(Y))$$

$$\Omega_\rho(A) = \frac{1}{2}(\rho A + A\rho)$$

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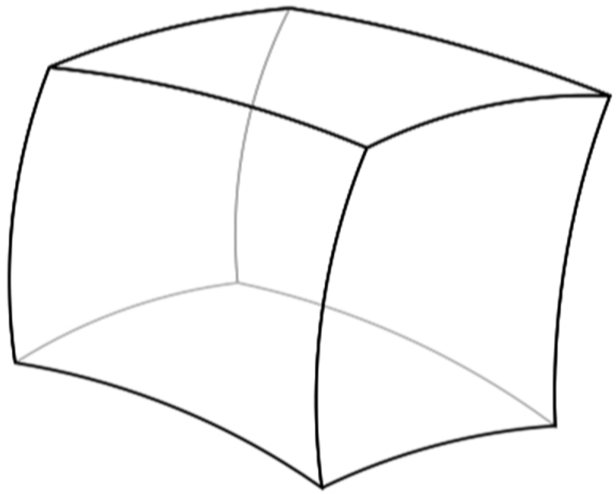
quantum Cramér-Rao bound

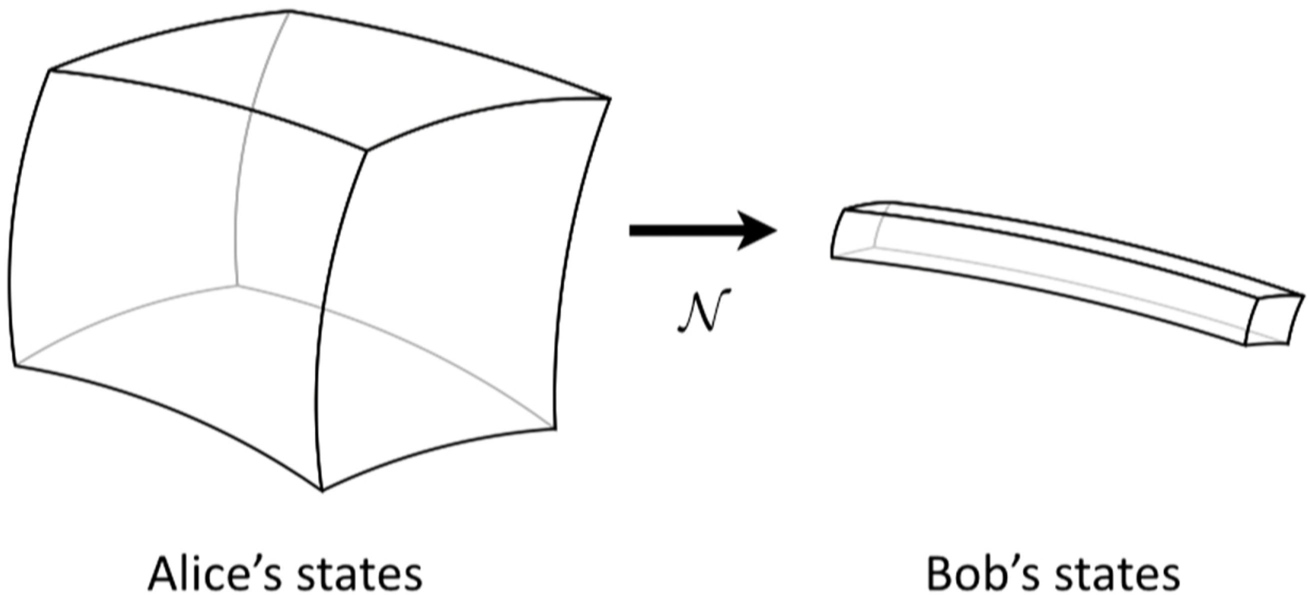
If $\quad \mathrm{Tr}(\rho_\theta A) = \theta$

$$\mathrm{Tr}(\rho_\theta A^2) \geq \frac{1}{\| \frac{d}{d\theta} \rho_\theta \|^2_{\rho_\theta}}$$

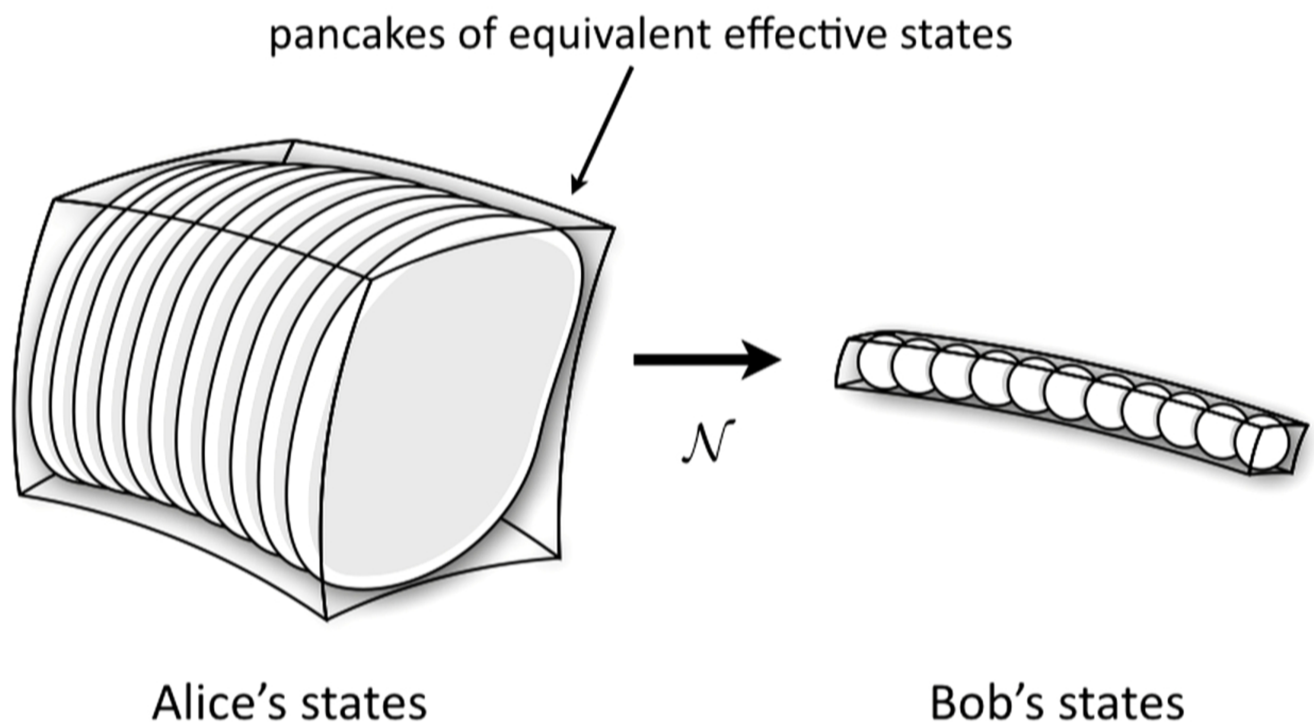
dimensionality reduction



Alice’s states



$\mathcal{N}$
Alice's states
Bob's states


pancakes of equivalent effective states
$\mathcal{N}$
Alice's states
Bob's states

contraction ratio of direction X

$$\eta(X) = \frac{\|\mathcal{N}(X)\|_{\mathcal{N}(\rho)}}{\|X\|_\rho}$$

principal contraction directions

$$\mathcal{N}_\rho^*(\mathcal{N}(X_i)) = \eta_i^2 X_i$$

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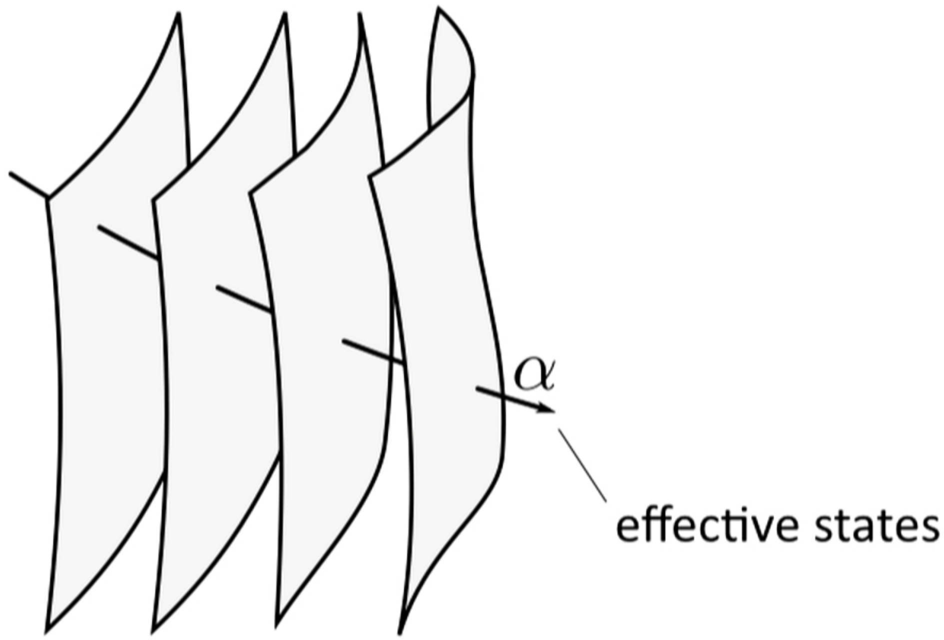
$\rho + X$ and $\rho + X'$ effectively indistinguishable if

$$\langle A_i \rangle_{\rho+X} = \langle A_i \rangle_{\rho+X'} \qquad \text{for all } i,\ \eta_i > \epsilon$$

$$A_i = \Omega_\rho^{-1}(X_i)$$

$$(\mathcal{N}_\rho^* \mathcal{N})^\dagger (A_i) = \eta_i A_i$$



α
effective states

$$\rho = e^{-H}$$

relevant observables A_i

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relevant observables A_i

pick ρ' minimising $S(\rho' \| \rho)$ on given class

QFT from spin system

$$\mathcal{N}_{\sigma,y} = \mathcal{D}_y^{\otimes N} \circ e^{\frac{1}{2}\sigma^2 \mathcal{L}}$$

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$$\mathcal{L}(\rho) = \sum_{\langle ij \rangle} \left(U_{ij} \rho U_{ij}^\dagger - \rho \right)$$

swap

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$$\langle X_A, X_B \rangle_\rho = 4 \, \mathrm{Re} \, \langle \Omega | AB | \Omega \rangle$$

If A acts on at least n sites then $\eta(X_A) \leq \mathcal{O}(y^{-n})$

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most "relevant" observables $\quad n = 1 \quad k < \frac{1}{\sigma}$

$$\phi_k = \frac{1}{\sqrt{N}} \sum_j \left[\cos(kj)(\tau_x)_j - \sin(kj)(\tau_y)_j\right]$$

$$\pi_k = \frac{1}{\sqrt{N}} \sum_j \left[\sin(kj)(\tau_x)_j + \cos(kj)(\tau_y)_j\right]$$

$$\eta_k \simeq y^{-1} e^{-\frac{1}{2}k^2\sigma^2}$$

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$$\langle\Omega|\phi_{k_1}\cdots[\phi_{k_i},\pi_{k_i}]\cdots\phi_{k_n}|\Omega\rangle = \langle\Omega|\phi_{k_1}\cdots iI\cdots\phi_{k_n}|\Omega\rangle$$

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renormalisation group

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$$\forall\, \sigma, \epsilon \quad \exists\, \Lambda_0 \;\; \text{such that} \;\; \forall\, \Lambda > \Lambda_0,$$

$$\mathrm{Tr}(A_i^{\rho_\Lambda} \frac{d}{d\Lambda} \rho_\Lambda) = 0 \qquad \forall\, i \;\; \eta_i > \epsilon$$

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$$(\mathcal{N}^*_{\sigma,\rho_\Lambda} \mathcal{N}_\sigma)^\dagger (A_i^{\rho_\Lambda}) = \eta_i A_i^{\rho_\Lambda}$$

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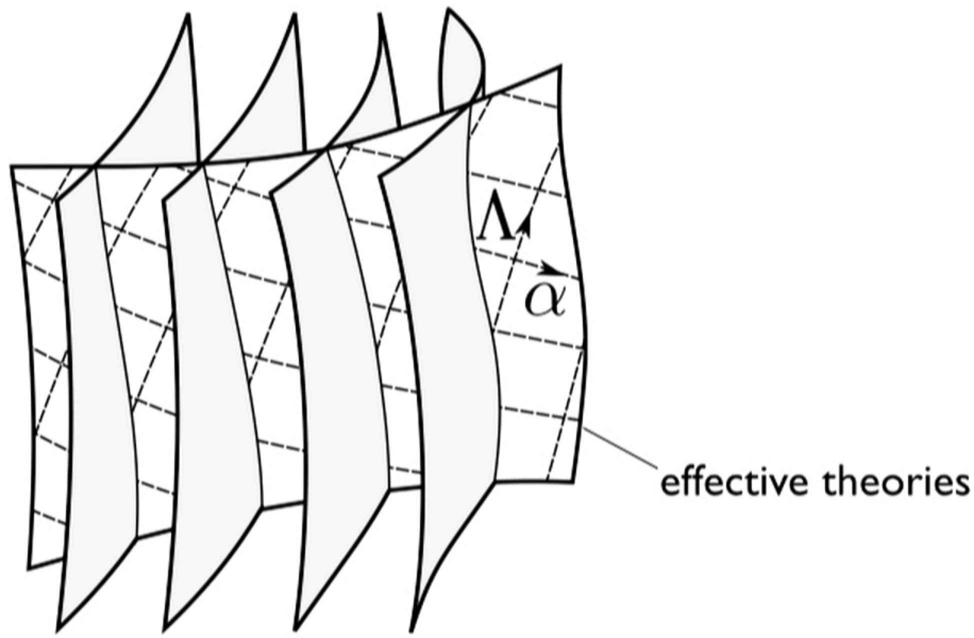
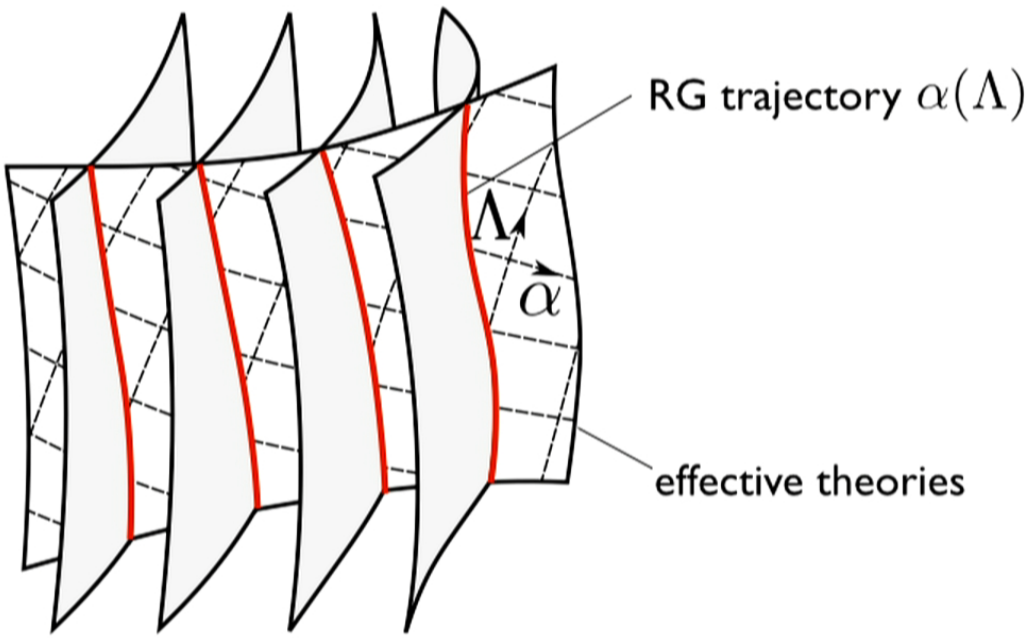
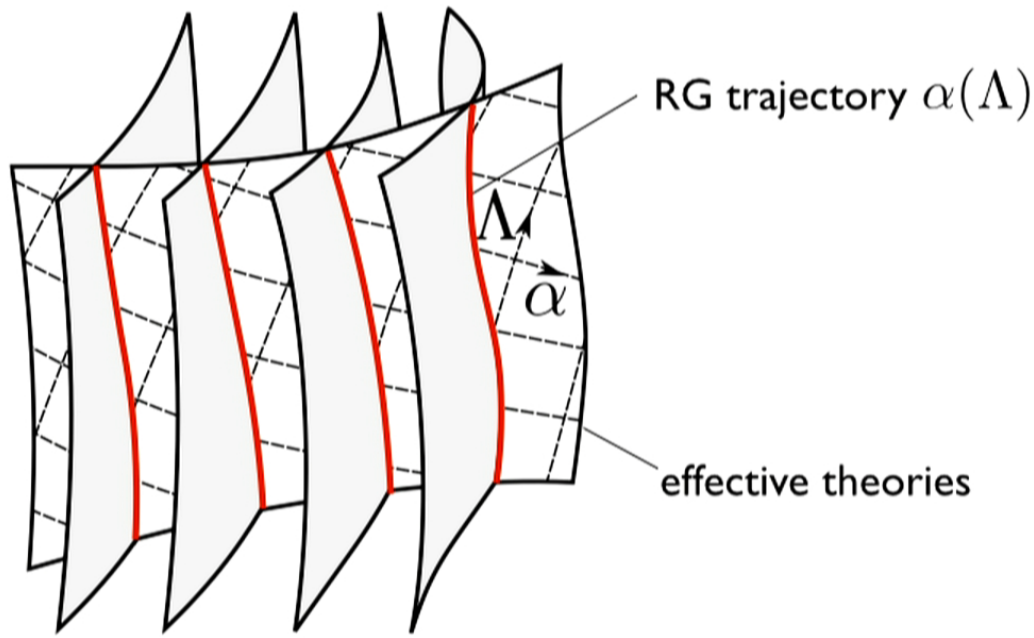


Λ
$\vec{\alpha}$
effective theories


RG trajectory $\alpha(\Lambda)$
Λ
$\vec{\alpha}$
effective theories


RG trajectory $\alpha(\Lambda)$
Λ
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effective theories

perturbative QFT

$$\mathcal{N}^\dagger(e^{i\Phi(f)}) = e^{-\frac{1}{2}(f,Yf)}e^{i\Phi(Xf)}$$

$$X(f)(\phi,\pi) = f(N_\sigma \star \phi, N_\sigma \star \pi)$$

$$Y(f)(\phi,\pi) = f(y_\phi^2\,\phi, y_\pi^2\,\pi)$$

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spatial imprecision

$$Y(f)(\phi,\pi) = f(y_\phi^2\phi, y_\pi^2\pi)$$

field value imprecisions

$\phi_k,\ \pi_k$ are principal with relevance

$$\eta_k^{\phi} \simeq \frac{1}{\frac{\beta\omega_k}{2}\coth\frac{\beta\omega_k}{2} + \beta\omega_k^2 y_\phi^2\, e^{k^2\sigma^2}}$$

$$\eta_k^{\pi} \simeq \frac{1}{\frac{\beta\omega_k}{2}\coth\frac{\beta\omega_k}{2} + \beta y_\pi^2\, e^{k^2\sigma^2}}$$

$$y_\phi y_\pi \gg 1$$

renormalisation group equations

$$\forall\, \sigma \quad \exists\, \Lambda_0 \;\; \text{such that} \;\; \forall\, \Lambda > \Lambda_0,$$

$$\frac{d}{d\Lambda}\langle \phi_{k_1} \cdots \phi_{k_n} \rangle_\Lambda = 0 \qquad \forall \;\; n, \; \Sigma_i k_i^2 < \frac{1}{\sigma^2}$$

that’s all for now