Title: Entropic Dynamics: from Entropy and Information Geometry to Quantum Mechanics

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Abstract: Our subject is Entropic Dynamics, a framework that emphasizes the deep connections between the laws of physics and information. In attempting to understand quantum theory it is quite natural to assume that it reflects laws of physics that operate at some deeper level and the goal is to discover what these underlying laws might be.

In contrast, in the entropic view no fundamental underlying dynamics is invoked. Quantum theory is an application of entropic methods of inference and the goal is to make the best possible predictions on the basis of some limited information represented by appropriate constraints. It is through the choice of microstates and of these constraints that the â€œphysicsâ€• is introduced.

In Entropic Dynamics a relational notion of entropic time is introduced as a book-keeping device to keep track of changes. We show that a non-dissipative entropic dynamics naturally leads to generic forms of Hamiltonian dynamics, and notions of information geometry naturally lead to those specific Hamiltonians (that is, those that include the correct quantum potential) that describes quantum mechanics.

Entropic Dynamics

from Entropy and Information Geometry to Quantum Mechanics

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Information Theoretic
Foundations for Physics
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Two attitudes:

(1) Physical models attempt to represent Nature.

(2) Physical models are a framework for processing information about Nature.

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probability
entropy
geometry

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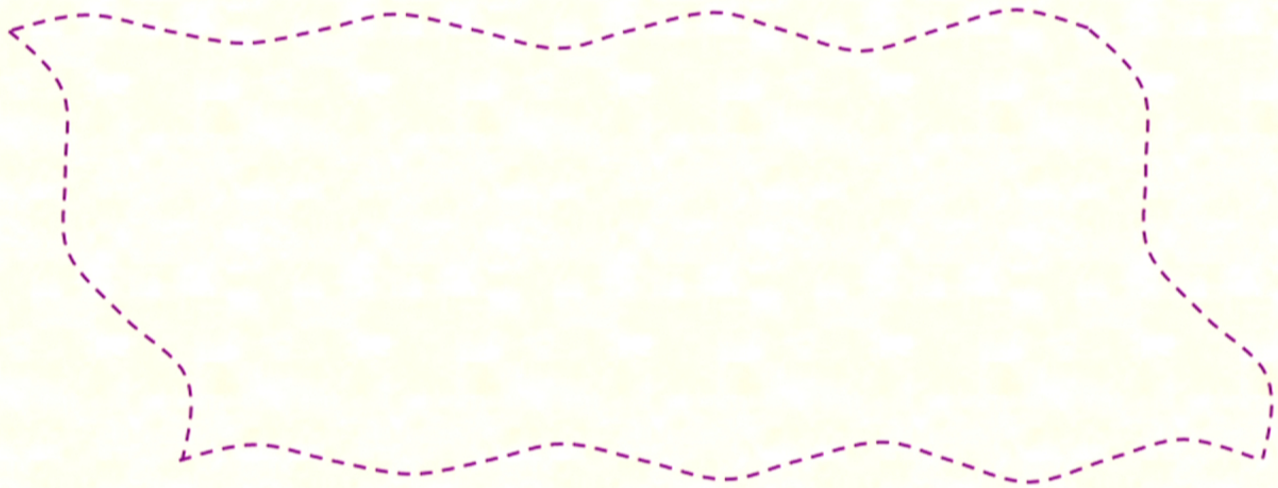
Entropic Inference:



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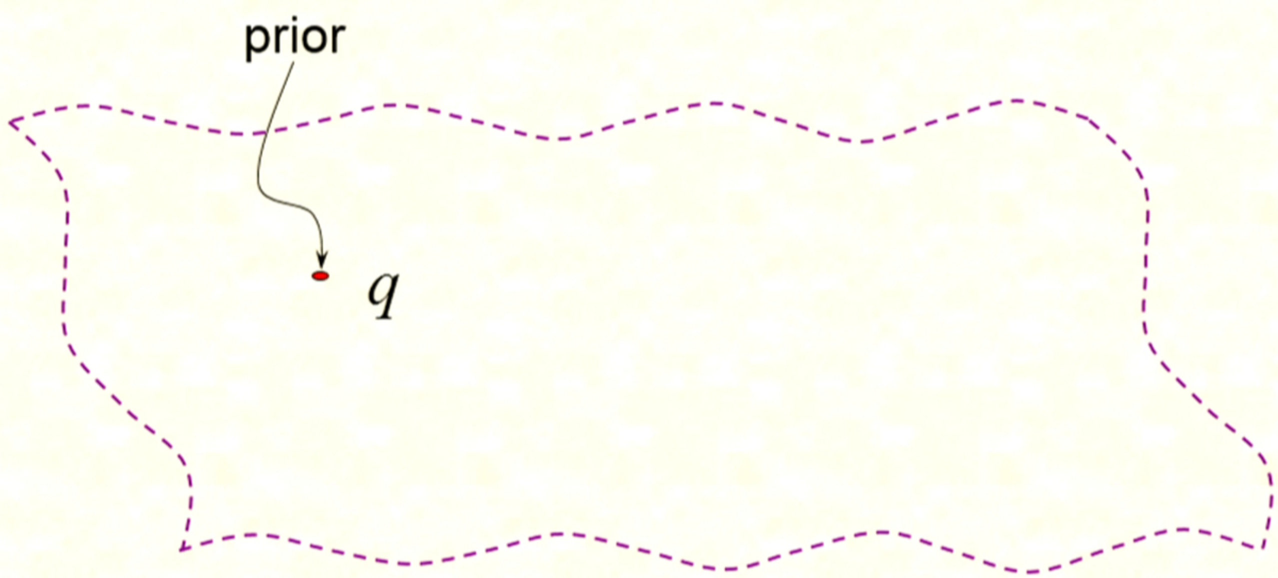

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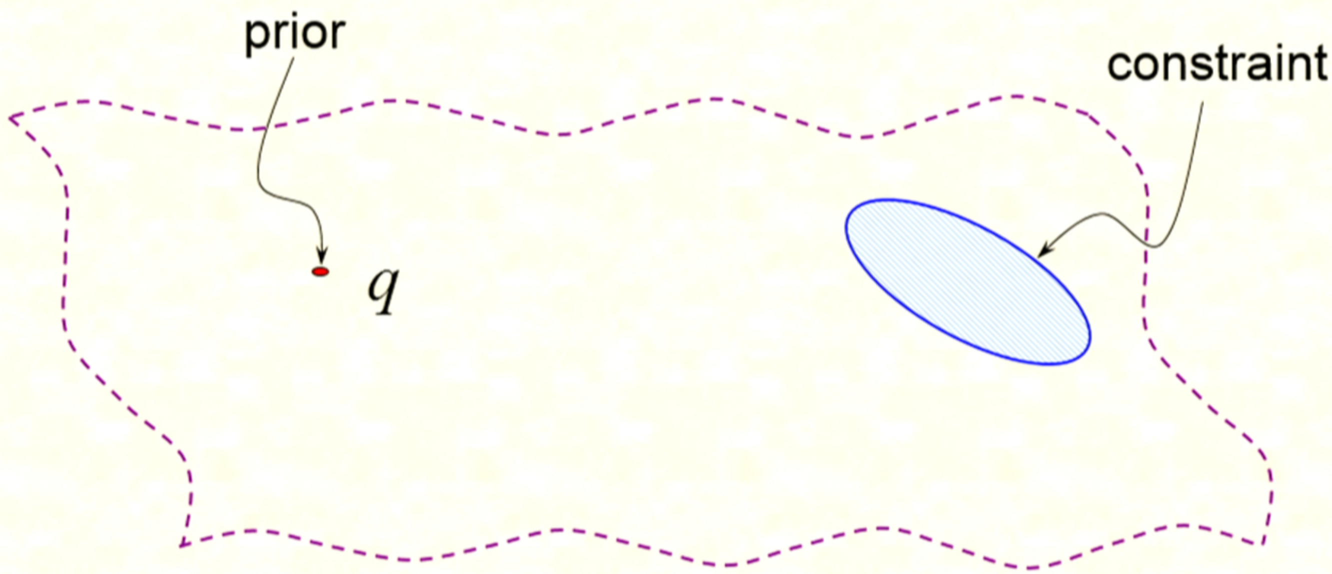

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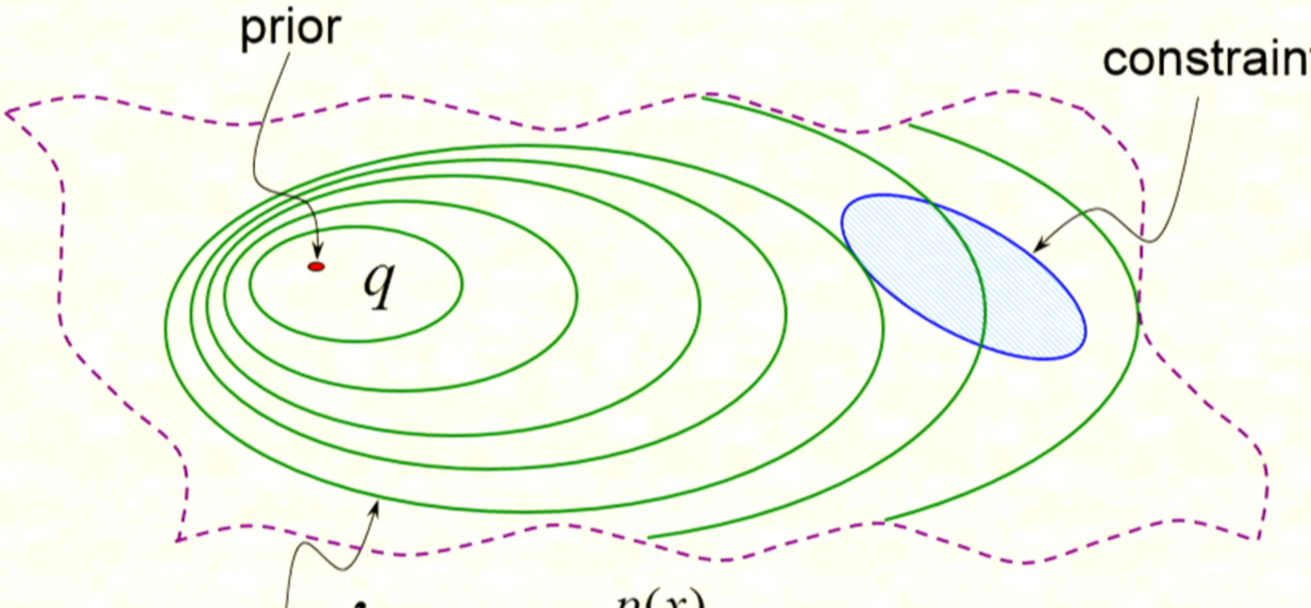

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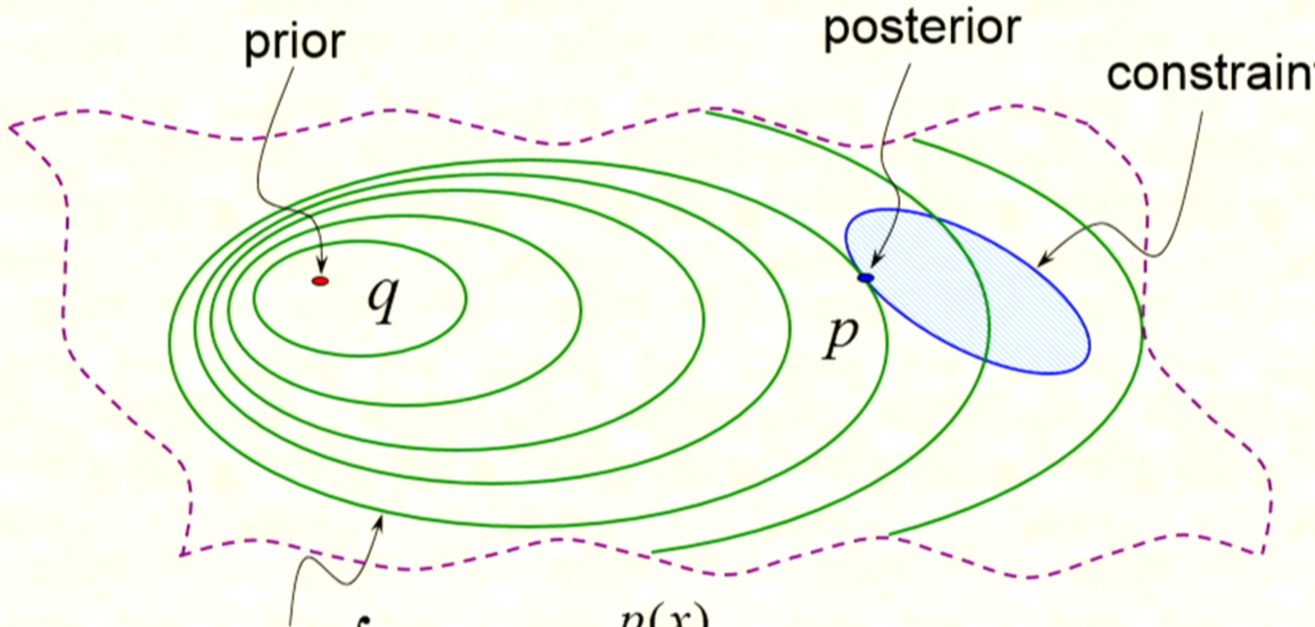

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Maximize $S[p,q]$ subject to the appropriate constraints.

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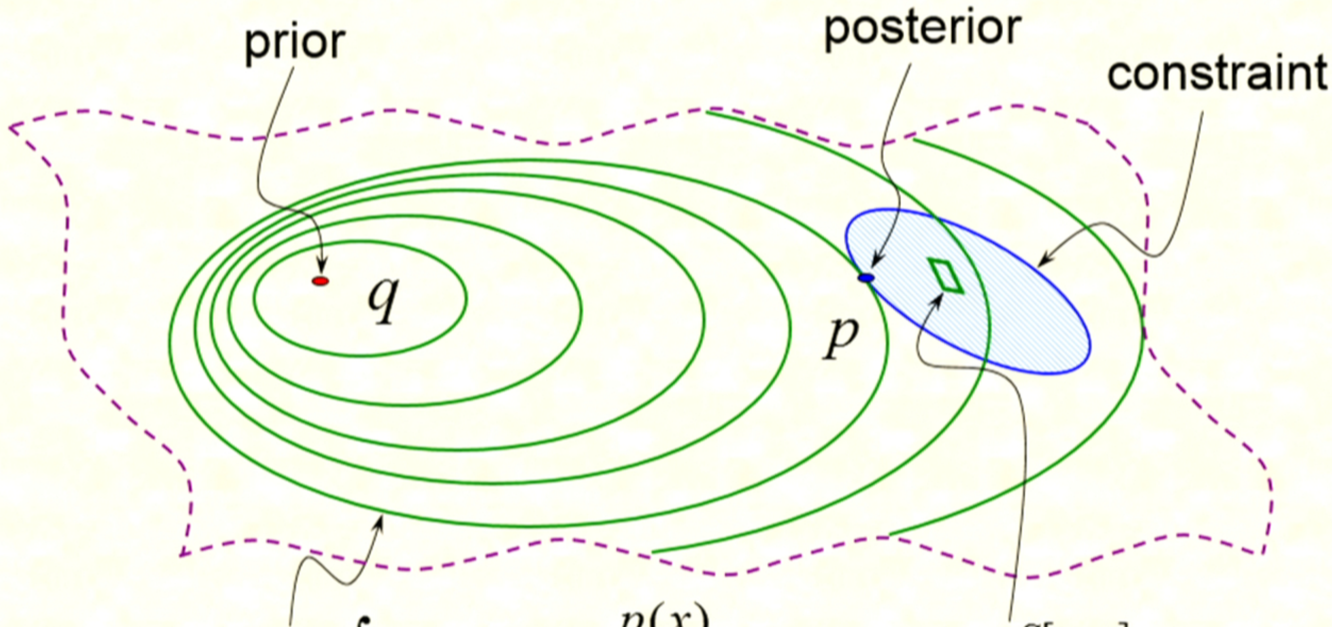

$$S[p,q] = -\int dx\, p(x) \log \frac{p(x)}{q(x)} \qquad \Pr(dV) \propto e^{S[p,q]} dV$$

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Our objective:

To derive Quantum Theory as Entropic Dynamics.

The goal is not to discover the Schrödinger equation.

The goal is to identify the subject matter
and the information

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The goal is to predict the positions of particles x.

(Or fields, or any other configurational variables).

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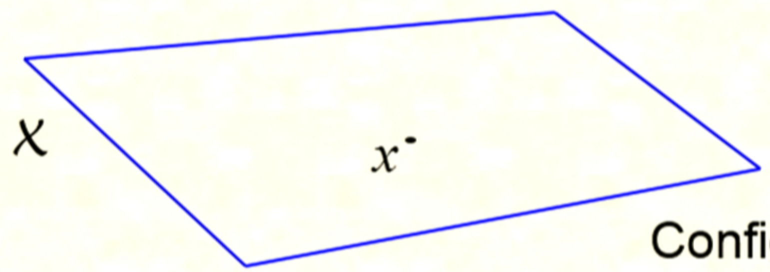

Configuration space
(→ non-locality)

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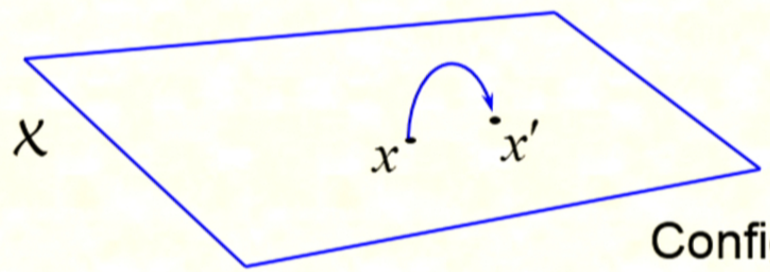

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First goal: find $P(x' \mid x)$

Entropic Dynamics

Maximize $$S[P,Q] = -\int dx'\, P(x' \mid x) \log \frac{P(x' \mid x)}{Q(x' \mid x)}$$

uniform

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Constraints:

1) Motion is continuous. Impose short steps:

$$\langle \Delta x_n^a \Delta x_n^b \rangle \delta_{ab} = \kappa_n \qquad n = 1 \dots N$$

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The result:

$$P(x' \mid x) = \frac{1}{\zeta} \exp[-\frac{1}{2}\sum_n \alpha_n \delta_{ab} \Delta x_n^a \Delta x_n^b + \alpha' \Delta x^A \partial_A \phi]$$

Displacement: $\Delta x^A = \langle \Delta x^A \rangle + \Delta w^A$

Expected drift : $\langle \Delta x_n^a \rangle = \frac{\alpha'}{\alpha_n} \delta^{ab} \frac{\partial \phi}{\partial x_n^b}$

Fluctuations: $\langle \Delta w_n^a \rangle = 0$

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In ED time is a book-keeping device introduced to keep track of the accumulation of many small changes.

Three ingredients:

(1) Introduce the notion of an instant.

(2) Instants are ordered.

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(3) Introduce duration: the interval between instants.

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"Entropic" Time

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$$\rho(x',t') = \int dx\, P(x' \mid x)\rho(x,t)$$

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(3) Duration: the interval between instants

The foundation of any notion of time is dynamics.

For large α_n the dynamics is all in the fluctuations:

$$\langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n} \delta^{ab}$$

Define duration so that motion looks simple:

$$\alpha_n(x,t) = \frac{C_n}{\Delta t}$$

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Entropic dynamics:

$$\rho(x',t') = \int dx\, P(x' \mid x)\rho(x,t)$$

Fokker-Planck equation: $\partial_t \rho = -\partial_A(\rho v^A)$

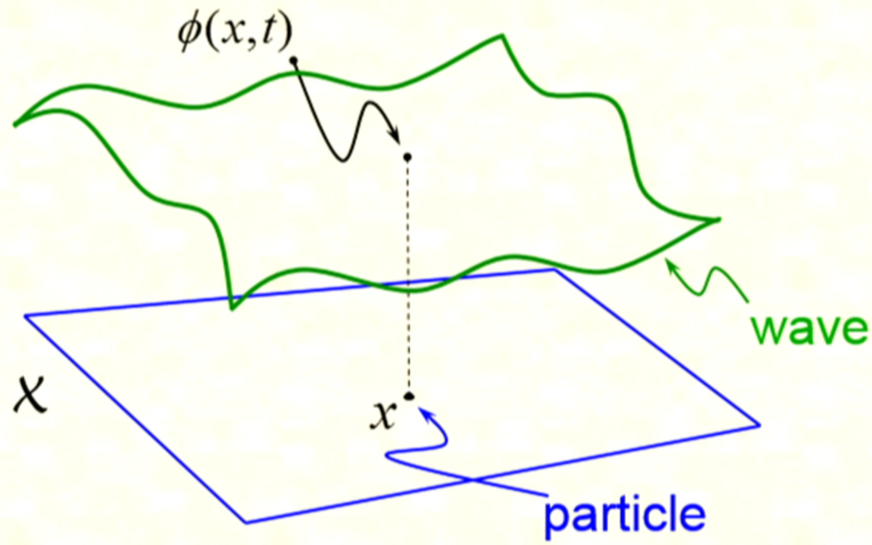
$$m_{AB} v^B = \partial_A \Phi$$

$$m_{AB} = m_n \delta_{ab}$$

$$\frac{\Phi}{\hbar} = \phi - \log \rho^{1/2}(x,t)$$

Problem: this is standard diffusion, not QM!

Solution: allow ϕ to be dynamic.



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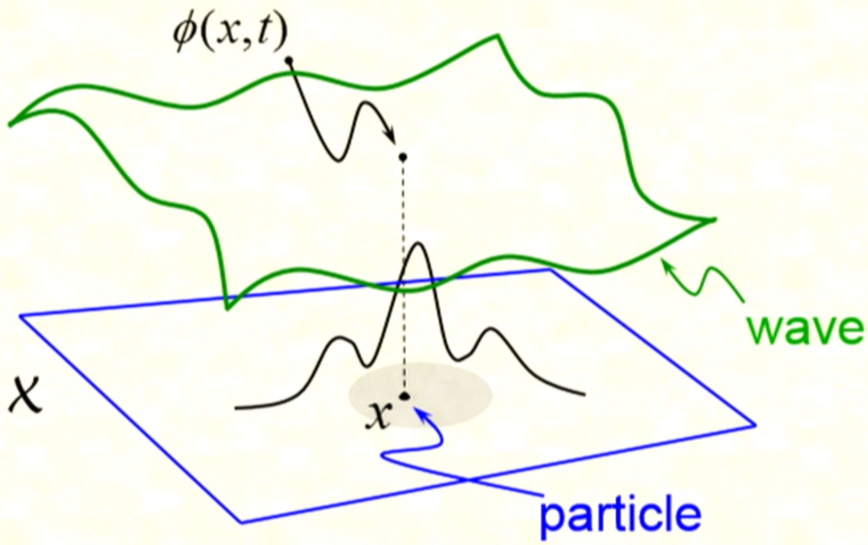

Non-dissipative diffusion (E. Nelson 1979)

Non-dissipative dynamics?

Define $\widetilde{H}[\rho,\Phi]$ so that

$$\partial_t \rho = \frac{\delta \widetilde{H}}{\delta \Phi} = -m^{AB}\partial_A\left(\rho\,\partial_B \Phi\right)$$

$$\widetilde{H}[\rho,\Phi] = \int d^{3N}x\,\rho\,\frac{1}{2}m^{AB}\partial_A\Phi\partial_B\Phi + F[\rho]$$

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Impose "energy" conservation

$$\frac{d\widetilde{H}}{dt} = \int dx \left[\frac{\delta \widetilde{H}}{\delta \Phi} \partial_t \Phi + \frac{\delta \widetilde{H}}{\delta \rho} \partial_t \rho \right] = 0$$

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$$\Rightarrow \quad \partial_t \rho = \frac{\delta\widetilde{H}}{\delta\Phi} \quad \text{and} \quad \partial_t \Phi = -\frac{\delta\widetilde{H}}{\delta\rho}$$

Conclusion:

1) Hamilton's equations

$$\partial_t \rho = \frac{\delta \widetilde{H}}{\delta \Phi} \quad \text{and} \quad \partial_t \Phi = -\frac{\delta \widetilde{H}}{\delta \rho}$$

2) Poisson brackets, symplectic structure,...

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$\Rightarrow$ indeterministic "classical" mechanics

Choosing $F[\rho]$: information geometry

$$g_{AB}(\theta) = \int dx \, p(x \mid \theta) \frac{\partial \log p(x \mid \theta)}{\partial \theta^A} \frac{\partial \log p(x \mid \theta)}{\partial \theta^B}$$

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Two tensors:

$P(x' \mid x)$

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$$P(x' \mid x) \quad \Rightarrow \quad g_{AB}(x) \propto m_{AB}$$

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Combine ρ and Φ into $\Psi = \rho^{1/2} e^{i\Phi/\hbar}$

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$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2} m^{AB} \partial_A \partial_B \Psi + \frac{\hbar^2}{2} m^{AB} \frac{\partial_A \partial_B |\Psi|}{|\Psi|} \Psi + \frac{\delta F}{\delta \rho} \Psi$$

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to get QM: $i\hbar\partial_t \Psi = -\frac{\hbar^2}{2} m^{AB} \partial_A \partial_B \Psi + V\Psi$

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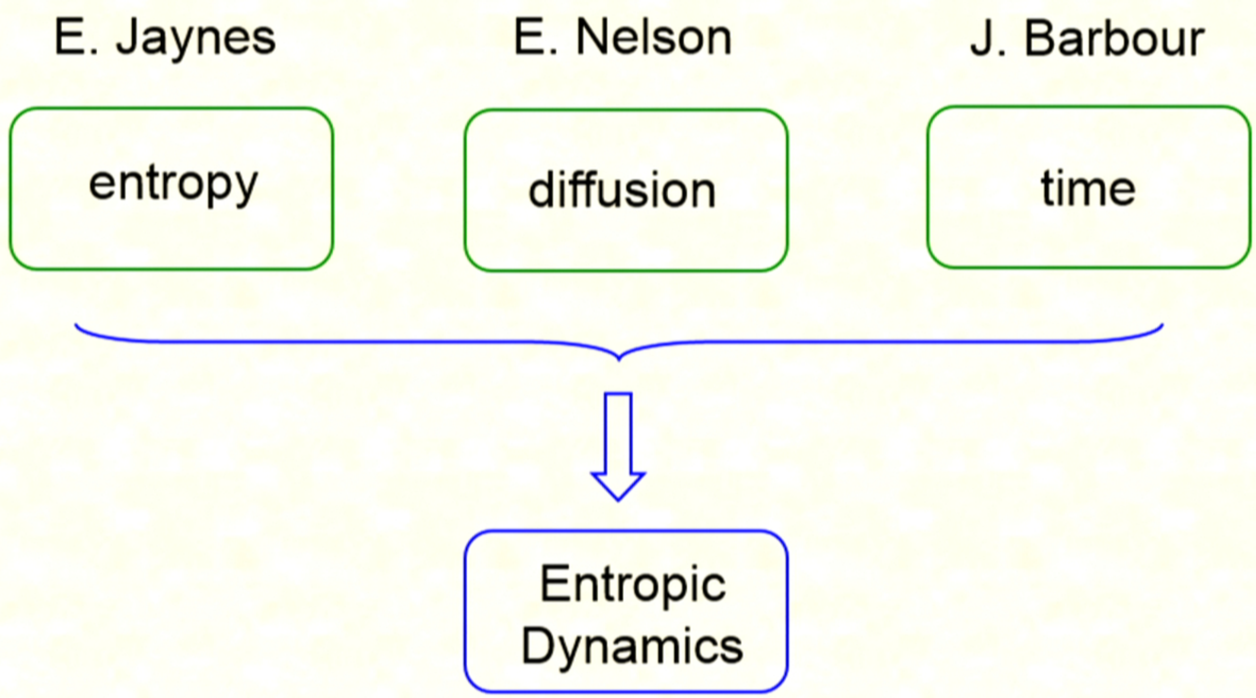

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