Title: Quantum mechanics from first principles

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Abstract: Quantum mechanics is derived from the principle that the universe contain as much variety as possible, in the sense of maximizing the distinctiveness of each subsystem. This is an expression of Leibniz's principles of sufficient reason and the identity of the indiscernible.

The quantum state of a microscopic system is defined to correspond to an ensemble of subsystems of the universe with identical constituents and similar preparations and environments. A new kind of interaction is posited amongst such similar subsystems which acts to increase their distinctiveness, by extremizing the variety. In the limit of large numbers of similar subsystems this interaction is shown to give rise to Bohm's quantum potential. As a result the probability distribution for the ensemble is governed by the Schroedinger equation.

The measurement problem is naturally and simply solved. Microscopic systems appear statistical because they are members of large ensembles of similar systems which interact non-locally. Macroscopic systems are unique, and are not members of any ensembles of similar systems. Consequently their collective coordinates may evolve deterministically.

This proposal could be tested by constructing quantum devices from entangled states of a modest number of quits which, by its combinatorial complexity, can be expected to have no natural copies.

Quantum mechanics and the principle of maximal variety

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PI
Information theoretic foundations of physics
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hep-th/9203041 arXiv:1205.3707 arXiv:1104.2822 and in preparation.

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Thanks to the organizers.

1. Prelude on locality
2. Heuristic principles
3. Assumptions
4. Microscopic theory of subsystems
5. Derivation of quantum mechanics
6. Deviations from quantum mechanics

We are still seeking the answer to Johnny Wheeler's question,

Why the quantum?

Two roads to the answer:

- **Interpretational approaches:** Assume the dynamics and kinematics of QM are correct and that the measurement problem and other foundational issues reflect a defect in our understanding of the theory. Seek to reformulate the theory, keeping its physical content unchallanged.

  Copenhagen, Everett, information theoretic reconstructions

- **Dynamical approaches:** Assume the measurement problem and other issues arise because QM is an incomplete description of nature. Seek to find the correct completion, which will resolve the measurement problem and have an unproblematic interpretation.

  deBroglie-Bohm, spontaneous collapse, hidden variables, Nelson, many classical interacting worlds (MIW), matrix models...

Physics is a weird combination of local and non-local.

- Local propagation of energy and information
- Non-local entanglement.

But quantum gravity suggests space is emergent. But then locality must be emergent too.

Valentini tells us that the price or reward of going out of quantum equilibrium is non-local signaling, ie locality is a feature only of quantum equilibrium. The world is awash in non-local interactions that are hidden by equilibrium. (Embrace "fine tuning")

Locality is also relative to position and motion of the observer.

Are there then defects in locality? Events that are far away in the emergent geometry of space that are causal neighbors? Is this the origin of entanglement?

The ***view of an event*** is the information reached there from the past, it contains information about the past causal neighborhood.

Events nearby in spacetime have similar views.

Reverse this:

Events that have similar views can interact, by virtue of that similarity.

Sometimes this results in their being nearby in the emergent spacetime.

But sometimes it is just because they have similar preparations, even if they are far away.

Heuristic Principles

Principle of the identity of the indiscernible (PII): *any two events or objects with isomorphic relational properties are to be identified.*

- *Global symmetries cannot be fundamental. Indeed GR has none and all the global symmetries in the standard model are accidental or broken.*
- *Relative locality: Localization is a consequence of identity, ie something is uniquely localized if it is distinguished by having a unique causal neighborhood.*

- *Hypothesis: the fundamental geometry is built from distinctiveness based on causal neighborhoods. Distance is a consequence of having disimilar causal neighborhoods.*
- *There are defects in this causal geometry. Two systems with very similar causal neighborhoods are nearby causally, even if distant in the coarse grained macroscopic metric. Hence they interact.*
- *The interactions induced between two similar systems are repulsive in that they act to increase their distinctiveness. Thus the PII is protected dynamically.*

PII forces local physics to be non-deterministic:

- *By the PII each event has a unique causal neighborhood* (arXiv:1307.6167)*:*
  - *Suppose two events A and B have isomorphic causal pasts:*

$$P(A) = P(B)$$

Then to prevent a violation of the PII their causal futures must be different

$$\Rightarrow F(A) \neq F(B)$$

Thus the same causal past implies a different causal future. Hence local physics cannot be deterministic.

The basic hypothesis: there is a non-local interaction between similar systems which acts to increase their differences. This is the origin of quantum physics. This interaction is driven by a potential energy which measures the distinctiveness of all the pairs of similar subsystems in nature.

Assumptions

1) Quantum mechanics is necessarily a description of subsystems of the universe. It is an approximation to some other, very different theory, which might be applied to the universe as a whole.

2) *The real ensemble hypothesis:* *A quantum state refers to an ensemble of similar systems present in the universe at a given time.* By similar systems we mean systems with the same constituents, whose dynamics are subject to (within errors that can be ignored) the same Hamiltonian, and which have very similar histories and hence, in operational terms, the same preparation. arXiv:1104.2822

3) ***The basic hypothesis:*** Similar systems have a new kind of interaction with each other, just by virtue of their similarities. This interaction takes place amongst similar systems, regardless of how far apart they may be situated in space, and thus, this is how non-locality enters quantum phenomena. *These interactions prevent similar systems from becoming identical and hence protect the principle of the identity of the indiscernible.*

4) *The principle of maximal variety*.

(Julian Barbour and LS, hep-th/9203041*)*

The variety of a system of relations, V, is a measure of how easy it is to distinguish the neighborhood of every element from that of every other. This can be used as a measure of complexity, or self-organization. We also proposed this as a dynamical principle.

This principle turns the identity of the indiscernible into a dynamical principle that introduces a repulsive interaction to prevent two subsystems from having isomorphic neighborhoods.

The variety of a network, G, representing a system of relations.

- N_l (k) is the l'th neighborhood of node k.
- This is the subgraph of G including those nodes l steps from k.

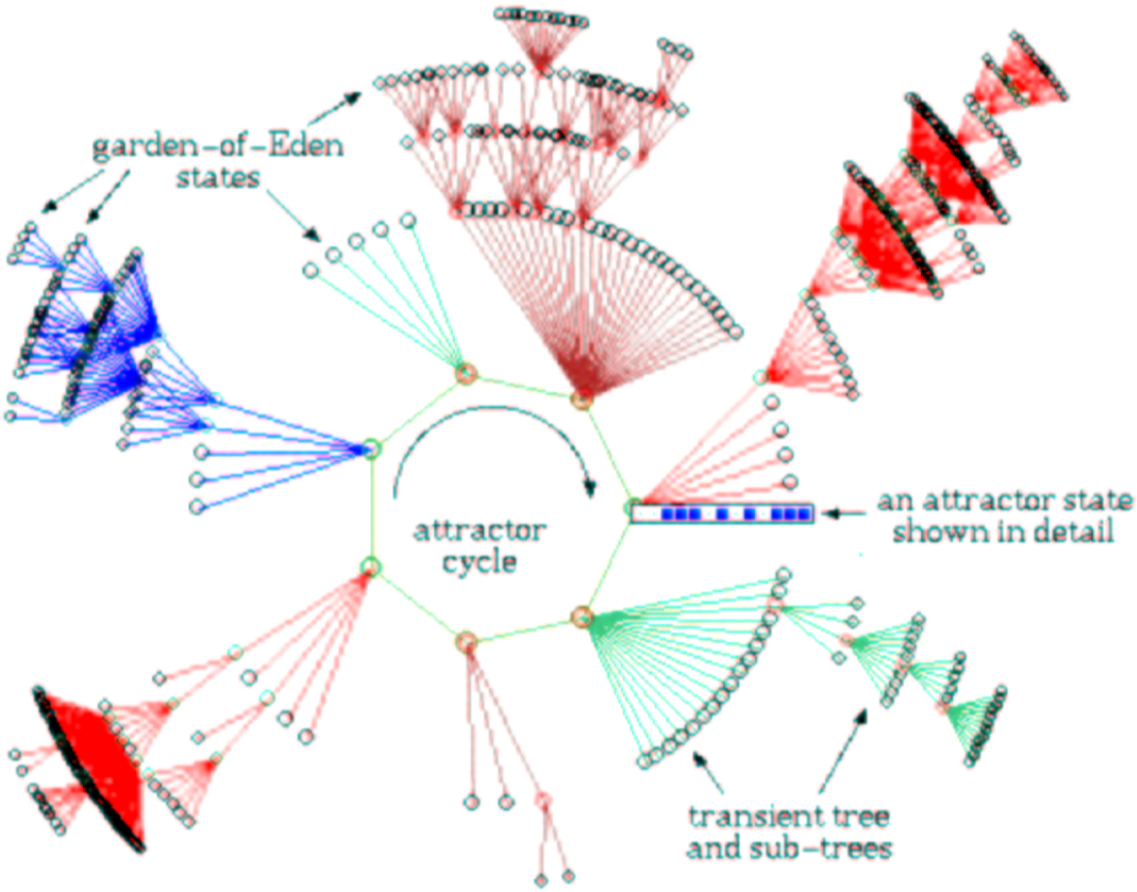
- For any pair of nodes,
- n_{kl} is the smallest n such that
- N_n (k) is not isomorphic to N_n (l).

- The distinctiveness of the pair is

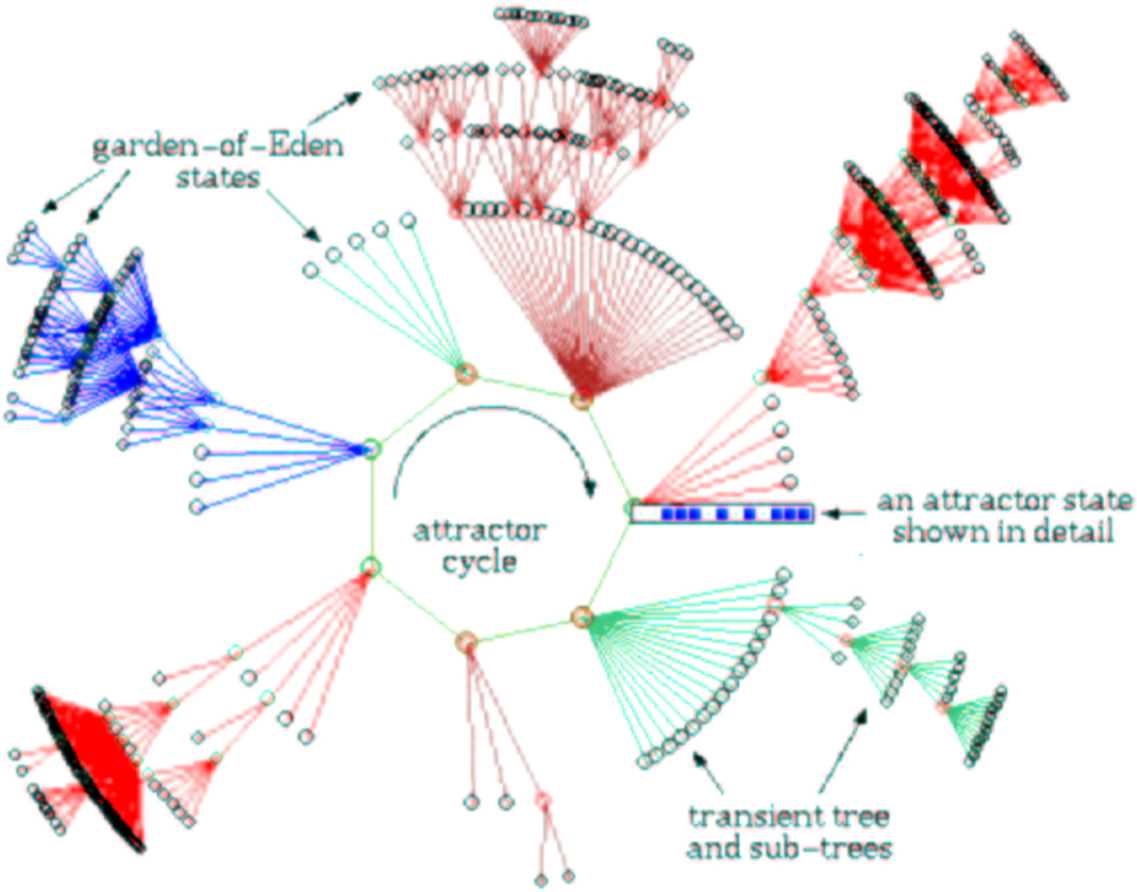
$$D(k,l) = \frac{1}{n_{kl}}$$

- The variety of G is

$$\mathcal{V} = \frac{1}{N(N-1)} \sum_{k \neq l} D(k,l) = \frac{1}{N(N-1)} \sum_{k \neq l} \frac{1}{n_{kl}}$$



High variety



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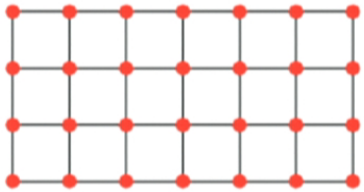

http://mathworld.wolfram.com/HexagonalGrid.html

Low variety

Configuration beables, views and variety.

Our system is an ensemble consisting of N similar subsystems.

Each has a d-dimensional configuration beable x^a_k. k=1, ... N, a = 1,...,d.

We assume these live in a vector space with metric.

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Each subsystem, i, has a ***view*** of the rest of the system:

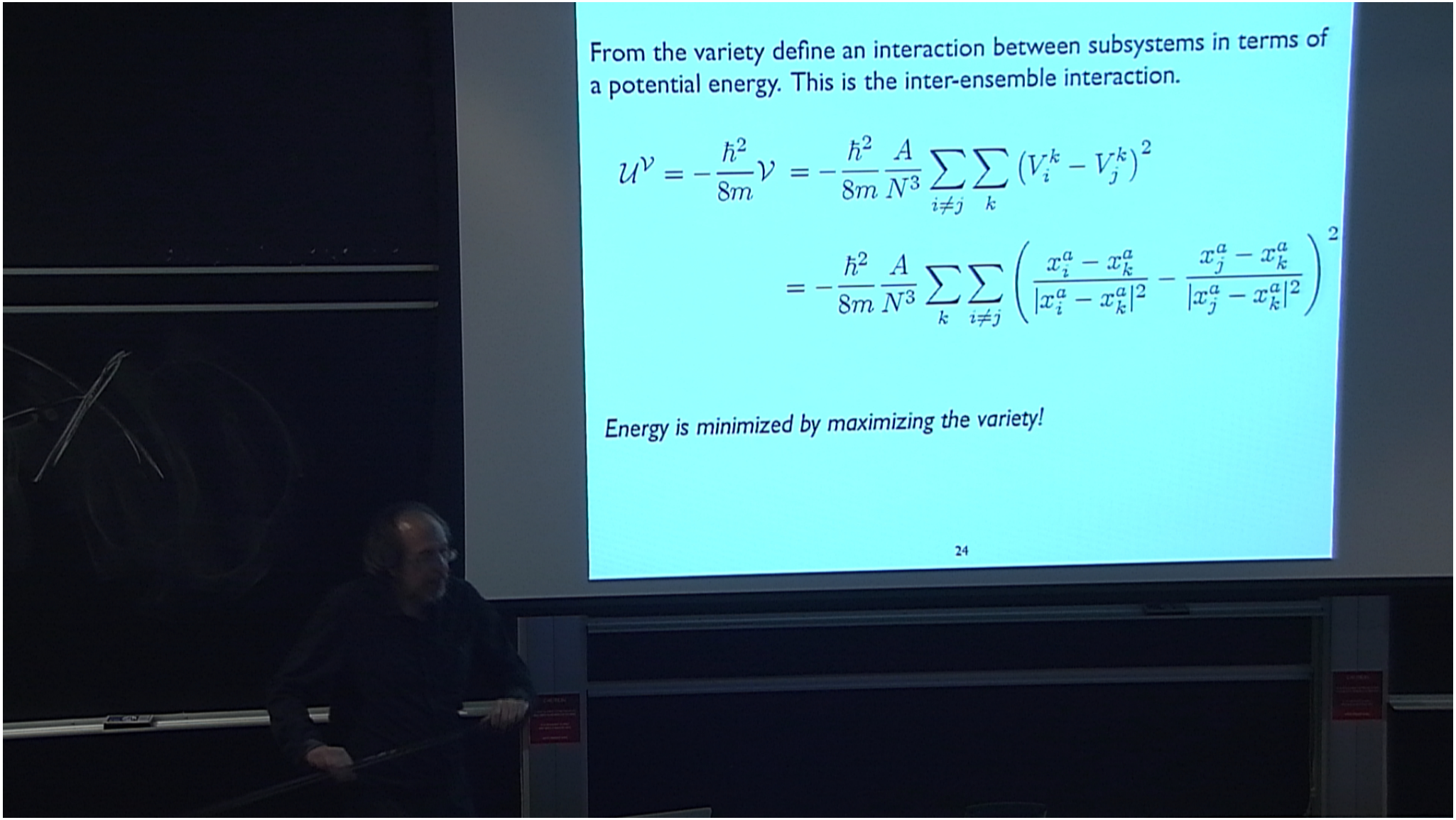
$$V_i^{ka} = \frac{x_i^a - x_k^a}{D(i,k)^2} = \frac{x_i^a - x_k^a}{|x_i^a - x_k^a|^2}$$

Differences between views give the ***distinctiveness*** of a pair of subsystems:

$$\mathcal{I}_{ij} = \frac{1}{N}\sum_k \left(V_i^{ka} - V_j^{ka}\right)^2$$

Sum this over all pairs to define the ***variety:***

$$\mathcal{V} = \frac{A}{N^2}\sum_{i\neq j}\mathcal{I}_{ij} = \frac{A}{N^3}\sum_{i\neq j}\sum_k \left(V_i^k - V_j^k\right)^2$$


From the variety define an interaction between subsystems in terms of a potential energy. This is the inter-ensemble interaction.
$$\mathcal{U}^{\mathcal{V}} = -\frac{\hbar^2}{8m}\mathcal{V} = -\frac{\hbar^2}{8m}\frac{A}{N^3}\sum_{i\neq j}\sum_{k}\left(V_i^k - V_j^k\right)^2$$
$$= -\frac{\hbar^2}{8m}\frac{A}{N^3}\sum_{k}\sum_{i\neq j}\left(\frac{x_i^a - x_k^a}{|x_i^a - x_k^a|^2} - \frac{x_j^a - x_k^a}{|x_j^a - x_k^a|^2}\right)^2$$
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Energy is minimized by maximizing the variety!

Phase beable:

If this were classical mechanics we would define a momenta beable p_a^k conjugate to the configuration beable x^a_k .

But in quantum mechanics the momentum density is the gradient of a phase, and this imposes restrictions on the p_a^k (Wallstrom).

So we posit that each subsystem has a **phase beable**

$$w_k = e^{\frac{\imath}{\hbar} S_k}$$

From these and the views we define a ***relational momenta:***

$$p_{ka} = -\imath \frac{1}{N} \sum_{j \neq k} V_k^{aj} \ln \left(\frac{w_j}{w_k} \right)$$

These give us the ***kinetic energy:***

$$K.E. = \mathcal{R}e \frac{Z\hbar^2}{2mN^2} \sum_{k \neq j} \frac{1}{(x_k - x_j)^2} \left[\ln \left(\frac{w_j}{w_k} \right) \right]^2$$

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The fundamental action

$$S(w,x) = \int dt \sum_k \left\{ -Z_0 \sum_{j \neq k} x_k^a \imath V_k^{ja} \frac{d}{dt} \left[\ln \left(\frac{w_j}{w_k} \right) \right] - H[x,w] \right\}$$

The Hamiltonian

$$H[x,w] = \mathcal{R}e \frac{\hbar^2}{2m} \left(Z \sum_{k \neq j} (V_k^{ja})^2 \left[\ln \left(\frac{w_j}{w_k} \right) \right]^2 - \frac{A}{4N} \sum_k \sum_{i \neq j} \left(V_i^{ka} - V_j^{ka} \right)^2 \right) + \sum_k U(x_k)$$

U(x) is a standard potential, within each subsystem.

In the large N limit this reproduces quantum dynamics

The large N limit and the continuum approximation.

The average of a function is given by a probability density ρ(x).

$$< \phi >= \frac{1}{N} \sum_k \phi(x_k) \rightarrow \int d^d z \rho(z) \phi(z)$$

For double sums we use:

$$\frac{1}{N} \sum_i \phi(x_{k+i}, x_k) \rightarrow \int_a^R d^d x \rho(z+x) \phi(z+x, z)$$

The short distance cutoff is a, reflecting the fact that for finite N nearest neighbors are not likely to come nearer to each other than

$$a(z) = \frac{1}{(N\rho(z))^{\frac{1}{d}}}$$

For the large scale cutoff, we rescale R by $N^{1/d}$:

$$R = \frac{r'}{N^{\frac{1}{d}}}$$

We hold r' fixed as we vary N.

for large R, r' is independent of ρ(z)

We next express variety in the continuum approximation.

$$\mathcal{V} = \frac{A}{N^3} \sum_k \sum_{i \neq j} \left(\frac{x_i^a - x_k^a}{|x_i^a - x_k^a|^2} - \frac{x_j^a - x_k^a}{|x_j^a - x_k^a|^2} \right)^2$$

$$\mathcal{V} = A \int d^d z \rho(z) \int_a^R d^d x \int_a^R d^d y [(\frac{x^a}{x^2} - \frac{y^a}{y^2})^2 \rho(z+x) \rho(z+y)$$

The continuum approximation to the variety is:

$$\mathcal{V} = A \int d^d z \rho(z) \int_a^R d^d x \int_a^R d^d y [(\frac{x^a}{x^2} - \frac{y^a}{y^2})^2 \rho(z+x)\rho(z+y)$$

Fix limits of integration:

$$a(z) = \frac{1}{(N\rho(z))^{\frac{1}{d}}} \qquad R = \frac{r'}{N^{\frac{1}{d}}}$$

Scale x = aα, and expand in 1/N:

$$\rho(z + a\alpha) = \rho(z) + a\alpha^a \partial_a \rho(z) + \frac{1}{2} a^2 \alpha^a \alpha^b \partial^2_{ab} \rho(z) \ldots$$

Choose normalization constant:

$$A = \frac{1}{N} \frac{d^2}{\Omega^2 (r^d - 1)^2}$$

The result is:

$$\mathcal{V} = \int d^d z \rho \left(\frac{N^{\frac{2}{d}}}{r'^2} - (\frac{1}{\rho}\partial\rho)^2 + \frac{1}{N^{\frac{2}{d}}} \frac{d}{d+2} r'^2 \frac{(\nabla^2 \rho)^2}{\rho^2} + \ldots \right)$$

The kinetic energy:

We extend the phase factors to a function of configurations. This exists because the configurations are unique.

$$w(x_k) = w_k$$

Turn the sums over configurations to integrals:

$$K.E. = \mathcal{R}e \frac{Z\hbar^2}{2mN^2} \sum_{k \neq j} \frac{1}{(x_k - x_j)^2} \left[\ln \left(\frac{w_j}{w_k} \right) \right]^2$$

$$= \frac{Z}{2m} \int d^d z \rho(z) \int_a^R d^d x \rho(x) \frac{1}{(z-x)^2} \left[S(x) - S(z) \right]^2$$

$$= \frac{1}{2m} \int d^d z \, \rho(z) \, (\partial_a S)^2 + O(\frac{1}{N^{\frac{2}{d}}})$$

$$Z = \frac{N}{(r^d - 1)}$$

For large r' and large N:

$$K.E. = \int dz\rho(z) \left[\frac{(\partial_a S)^2}{2m} + O(\frac{1}{r}) + +O(\frac{1}{N}) \right]$$

So the Hamiltonian becomes:

$$H = \int d^d z\rho(z) \left[\frac{(\partial_a S)^2}{2m} + \frac{\hbar^2}{8m}(\frac{1}{\rho}\partial_a \rho)^2 + V + O(\frac{1}{r}) + O(\frac{1}{N}) \right]$$

The last step is the symplectic structure:

$$S^0(w,x) = -Z_0 \int dt \sum_k \dot{p}_a^k x_k^a$$

The velocity of the momenta: $$\dot{p}_a^k = \frac{1}{\hbar} \sum_{j \neq k} \left[-\imath V_k^{ja} \left(\dot{S}_j - \dot{S}_k \right) \right]$$

In the continuum:

$$S^0 \to -Z_0 N \int dt \int d^d z \rho(z) z^a \dot{p}_a(z)$$

$$= \int dt \int d^d z \rho(z) \dot{S}(z)$$

$$Z_0 = \frac{dN}{\Omega(r^d - 1)}$$

$$\dot{p}_a(z) = \int_a^R d^d x \frac{x_a}{x^2} \left(\rho(z+x)\dot{S}(z+x) - \rho(z)\dot{S}(z) \right)$$

$$= \int_a^R d^d x \frac{x_a}{x^2} x^c \partial_c \left(\rho(z)\dot{S}(z) + \ldots \right)$$

$$= \frac{\Omega(r^d - 1)}{dN\rho(z)} \partial_a \left(\rho(z)\dot{S}(z) \right)$$

Putting everything together:

$$S = \int dt \int d^d z \rho(z) \left[\dot{S} + \frac{(\partial_a S)^2}{2m} + \frac{\hbar^2}{8m} (\frac{1}{\rho} \partial_a \rho)^2 + U + O(\frac{1}{r}) + O(\frac{1}{N}) \right]$$

Equations of motion:

quantum potential

$$\dot{\rho}(x^a) = \partial_a (\rho \frac{1}{m} g^{ab} \partial_b S(x^a))$$

$$-\dot{S} = \frac{1}{2m} g^{ab} (\frac{\partial S}{\partial x_{a\alpha}})(\frac{\partial S}{\partial x_{b\alpha}}) - \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + U$$

These are the real and imaginary parts of the Schroedinger equation:

$$\imath\hbar \frac{d\Psi}{dt} = \left(-\frac{\hbar^2}{2m} \nabla^2 + U \right) \Psi$$

$$\Psi(x,t) = \sqrt{\rho} e^{\frac{\imath}{\hbar} S}$$

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$\rho(x)$ is the ensemble probability distribution.

$\rho_k(x)$ is the probability distribution in the k'th subsystem.

Ergodic hypothesis:

Over time, for all k, $\rho_k(x) \rightarrow \rho(x)$

Quantum statistics: ρ(x)

Let us have a system of M identical particles with configurations x^a_{Ik} ***I=1,...M,*** ***a =1,...d,*** ***k=1,...n*** and phases w_k

PII: there is no effect of switching the coordinates of two identical particles, so we require

$$\rho(x^a_{k1}, x^a_{k2}, \ldots, t) = \rho(x^a_{k2}, x^a_{k1}, \ldots, t)$$
$$\dot{\rho}(x^a_{k1}, x^a_{k2}, \ldots, t) = \dot{\rho}(x^a_{k2}, x^a_{k1}, \ldots, t)$$

We plug these into the equations of motion to deduce

$$S(x^a_{k1}, x^a_{k2}, \ldots, t) = S(x^a_{k2}, x^a_{k1}, \ldots, t) + \phi$$

Doing this twice, and recalling that S is defined up to 2πn, we find

$$w(x_{k1}, x_{k2}) = e^{\frac{i}{\hbar} S(x_{k1}, x_{k2})} \rightarrow w(x_{k2}, x_{k1}) = \pm w(x_{k1}, x_{k2})$$

ie bosons and fermions.

The solution to the measurement problem:

Microscopic systems are quantum because they have large numbers of near copies in the universe, hence the variety interaction works on them and they find themselves members of large ensembles of similar systems.

Macroscopic systems are unique. They have no copies and are not parts of any ensembles. Hence they are not subject to quantum uncertainty. Their collective coordinates obey the fundamental deterministic dynamics.

Experimental tests?

Non-linear corrections to the Schroedinger equation:

The leading correction to the variety potential is:

$$\mathcal{U}^{\Delta\mathcal{V}} = -\frac{\hbar^2}{8m}\Delta\mathcal{V} = -\frac{1}{N^{\frac{2}{d}}}\frac{\hbar^2 r'^2}{8m}\int d^d z\rho\ (\frac{1}{\rho}\nabla^2\rho)^2$$

This gives a correction to the quantum potential

$$-\dot{S} = \frac{1}{2m}g^{ab}(\frac{\partial S}{\partial x_{a\alpha}})(\frac{\partial S}{\partial x_{b\alpha}}) + U - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + \Delta U^Q(\rho)$$

Which adds a non-linear term to the Schroedinger equation

$$\imath\hbar\frac{d\Psi}{dt} = \left(-\frac{\hbar^2}{2m}\nabla^2 + U + \Delta U^Q(\bar{\Psi}\Psi)\right)\Psi$$

This gives a correction to energy eigenvalues

$$\Delta E = \int d^d z\bar{\Psi}\Delta U^Q(\bar{\Psi}\Psi)\Psi$$

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The leading correction is

$$\Delta U^Q = \frac{r'^2}{N^{\frac{2}{d}}} \frac{d}{d+2} \frac{\hbar^2}{2m} \left[\frac{\nabla^4 \rho}{\rho} - 2\frac{(\nabla^2\rho)^2}{\rho^2} - 2\frac{(\nabla^a\rho)(\nabla_a\nabla^2\rho)}{\rho^2} \right]$$

This gives an order of magnitude correction to atomic energy levels.

$$\boxed{\frac{\Delta E}{E} \approx \frac{1}{N^{\frac{2}{d}}} \frac{r'^2}{a_0^2}}$$

Can we use rare states where N is small and d is large?

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More possible experimental tests

- Could we construct mesoscopic systems that by their combinatorial complexity are unique, and so have no natural copies? Could we see that they do not obey quantum mechanics but instead obey the fundamental theory? This might be done with a modest number of entangled q-bits.

- If we can do this, can we construct systems with small N=2,3,4,... and study the variety potential in detail?

Thank you