

Title: Scanning the Horizon: Information, Holography and Gravity

Date: May 13, 2015 09:50 AM

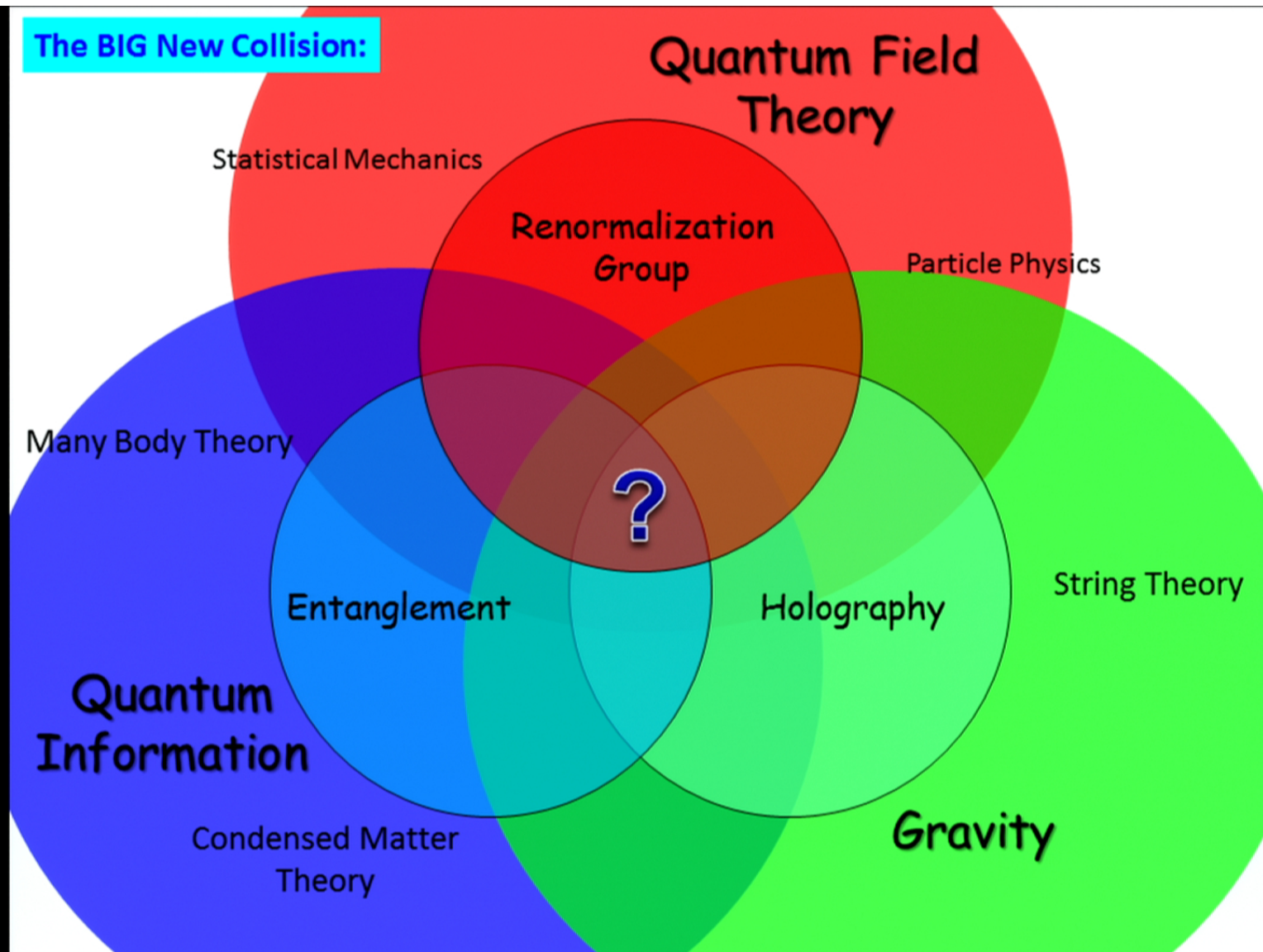
URL: <http://pirsa.org/15050086>

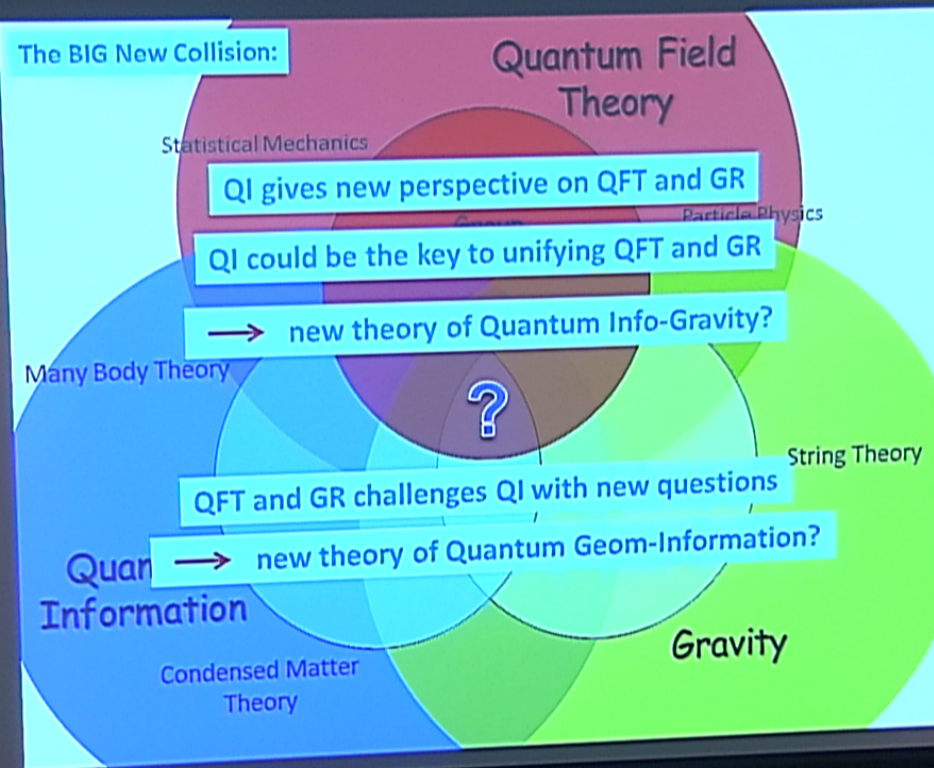
Abstract: In science, we often see new advances and insights emerging from the intersection of different ideas coming from what appeared to be disconnected research areas. The theme of my seminar will be an ongoing collision between the three topics listed in my title which has been generating interesting new insights into a variety of fields, eg, condensed matter physics, quantum field theory and quantum gravity.



Scanning the Horizon: Information, Holography & Gravity

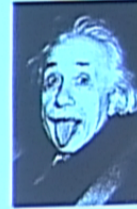
The BIG New Collision:





Einstein:

Gravity? It's all about geometry!

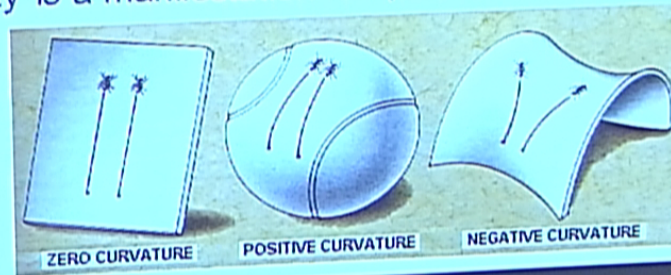


Special Relativity (1905):

Space and time are inextricably linked → Spacetime

General Relativity (1915):

Gravity is a manifestation of spacetime curvature.

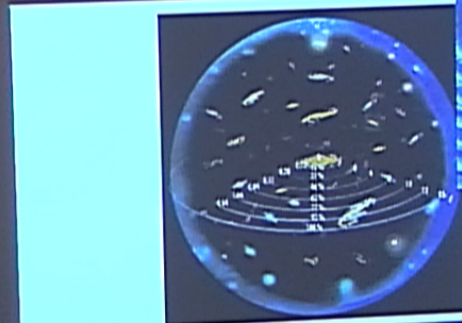
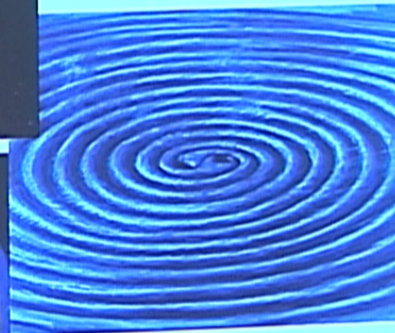


General Relativity: a rich source of new ideas



Black holes

Gravitational waves



Expanding universe

Black Hole Entropy:

- Bekenstein and Hawking: "black holes have entropy!"

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{\mathcal{A}}{4G}$$



Black Hole Entropy:

- Bekenstein and Hawking: "black holes have entropy!"

thermodynamics relativity geometry

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{A}{4G}$$

quantum gravity



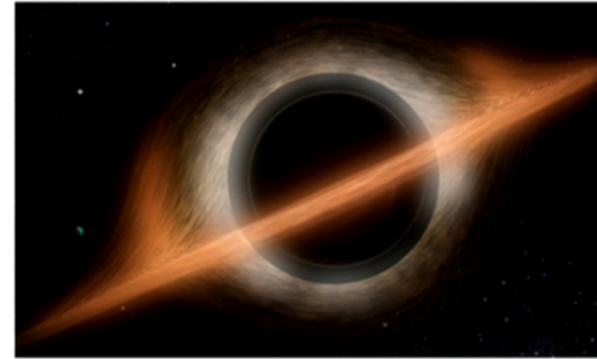
- window into the quantum theory of gravity?!!
- quantum gravity provides a fundamental scale

$$\ell_P^2 = 8\pi G \hbar / c^3$$

Black Hole Entropy:

- Bekenstein and Hawking: “black holes have entropy!”

$$S_{BH} = 2\pi \frac{\mathcal{A}}{\ell_P^2} k_B$$



- ℓ_P is really really small: $\ell_P = 10^{-35} m$
(LHC: $\sim 10^{-18} m$)

Black Hole Entropy:

- Bekenstein and Hawking: "black holes have entropy!"

$$S_{BH} = 2\pi \frac{A}{\ell_P^2} k_B$$



- ℓ_P is really really small: $\ell_P = 10^{-35} m$ (LHC: $\sim 10^{-18} m$)
- get really big entropies!!

big bag of hot gas: $S(\text{sun}) \simeq 7 \times 10^{57} k_B$

black hole: $S(1 M_\odot) \simeq 4 \times 10^{77} k_B$

Black Hole Entropy:

- Bekenstein and Hawking: "black holes have entropy!"

$$S_{BH} = 2\pi \frac{\mathcal{A}}{\ell_P^2} k_B$$

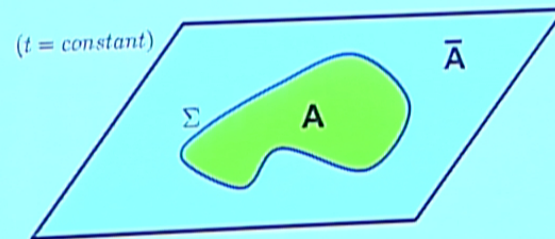


$$k_B = 1.380648810^{-23} \text{ J/K}$$

- k_B reminds us that entropy is associated with "heat"
→ black hole thermodynamics

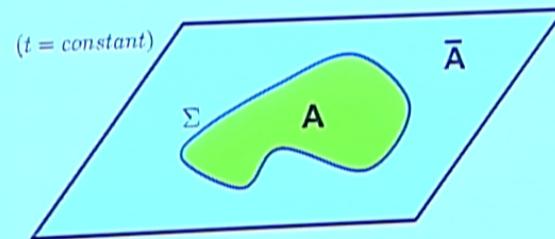
Entanglement Entropy in QFT

- entanglement entropy: general diagnostic to give a quantitative measure of entanglement using entropy to detect correlations between two subsystems
 - in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
 - integrate out degrees of freedom in "outside" region
 - remaining dof are described by a density matrix ρ_A
- calculate von Neumann entropy: $S_{EE} = -\text{Tr} [\rho_A \log \rho_A]$



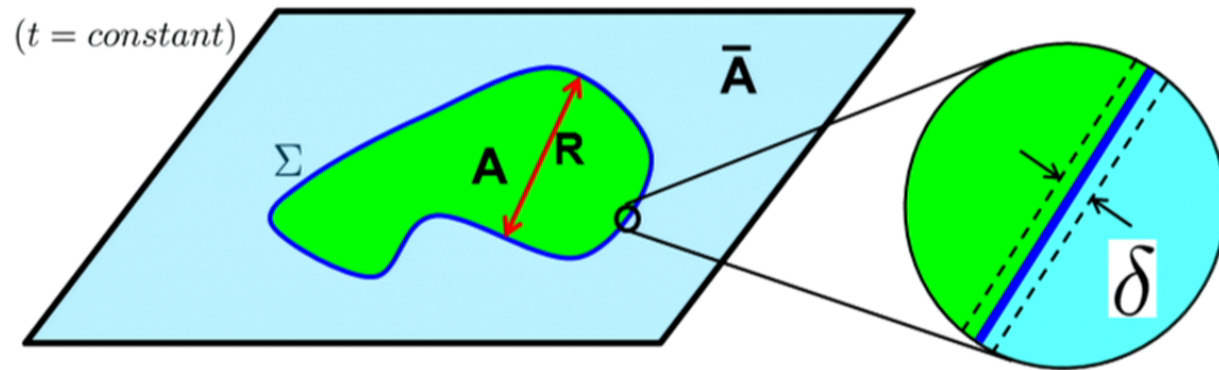
Entanglement Entropy in QFT

- entanglement entropy: general diagnostic to give a quantitative measure of entanglement using entropy to detect correlations between two subsystems
 - in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
 - integrate out degrees of freedom in "outside" region
 - remaining dof are described by a density matrix ρ_A
- calculate von Neumann entropy: $S_{EE} = -\text{Tr} [\rho_A \log \rho_A]$



Entanglement Entropy in QFT

- remaining dof are described by a density matrix ρ_A
- calculate von Neumann entropy: $S_{EE} = -\text{Tr} [\rho_A \log \rho_A]$



- result is **UV divergent!** dominated by short-distance correlations
- must regulate calculation: $\delta = \text{short-distance cut-off}$

$$S = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_2 \frac{R^{d-4}}{\delta^{d-4}} + \dots \quad d = \text{spacetime dimension}$$

→ geometric structure, eg, $S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \tilde{c}_2 \frac{f_\Sigma^{\text{"curvature"}}}{\delta^{d-4}} + \dots$

- recall Sorkin's suggestion for origin of black hole entropy:

$$S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots \quad \text{vs} \quad S_{BH} = 2\pi \frac{\mathcal{A}_{\text{horizon}}}{\ell_P^{d-2}} + \dots$$

→ "area law" suggestive of BH formula if $\delta \simeq \ell_P$
 (Sorkin '84, Bombelli, Koul, Lee & Sorkin, Srednicki, Frolov & Novikov, ...)

→ geometric structure, eg, $S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \tilde{c}_2 \frac{f_\Sigma \text{ "curvature" }}{\delta^{d-4}} + \dots$

- recall Sorkin's suggestion for origin of black hole entropy:

$$S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots \quad \text{vs} \quad S_{BH} = 2\pi \frac{\mathcal{A}_{\text{horizon}}}{\ell_P^{d-2}} + \dots$$

→ "area law" suggestive of BH formula if $\delta \simeq \ell_P$

(Sorkin '84; Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; ...)

- problems?: leading QFT singularities are regulator dependent
- represent renormalization of gravity couplings (Susskind & Uglum; ...)
- active topic in 90's but various "technical issues" left unresolved, eg, contributions of vectors or non-minimally coupled scalars
- $S_{BH} = S_{EE}$ had revival recently with new perspectives

(Jacobson & Satz; Cooperman & Luty; Donnelly & Wall; ...)

→ geometric structure, eg, $S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \tilde{c}_2 \frac{f_\Sigma \text{ "curvature" }}{\delta^{d-4}} + \dots$

- recall Sorkin's suggestion for origin of black hole entropy:

$$S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \dots \quad \text{vs} \quad S_{BH} = 2\pi \frac{\mathcal{A}_{\text{horizon}}}{\ell_P^{d-2}} + \dots$$

→ "area law" suggestive of BH formula if $\delta \simeq \ell_P$

(Sorkin '84, Bombelli, Koul, Lee & Sorkin, Srednicki, Frolov & Novikov; ...)

- problems?: leading QFT singularities are regulator dependent
- represent renormalization of gravity couplings (Susskind & Uglum; ...)
- active topic in 90's but various "technical issues" left unresolved, eg, contributions of vectors or non-minimally coupled scalars
- $S_{BH} = S_{EE}$ had revival recently with new perspectives

(Jacobson & Satz, Cooperman & Luty, Donnelly & Wall; ...)

Black Hole Entropy:

- Bekenstein and Hawking: “black holes have entropy!”

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{\mathcal{A}}{4G}$$

microstates

geometry



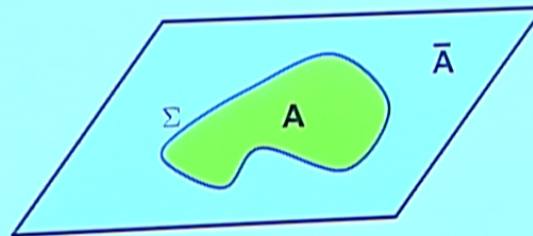
- k_B reminds us that entropy is associated with “heat”
→ black hole thermodynamics
- statistical mechanics also says: $S = -Tr [\rho_A \log \rho_A]$
→ black hole microstates
- **Sorkin: “black hole entropy is entanglement entropy”**

Proposal: Spacetime Entanglement

(Bianchi & RM)

- in a theory of quantum gravity, for any sufficiently large region A in a smooth background, consider entanglement entropy between dof describing A and $\bar{A}$; contribution describing short-range entanglement is finite and described in terms of geometry of entangling surface with leading term:

$$S_{EE} = \frac{\mathcal{A}_\Sigma}{4G_N} + \dots$$



- higher order terms similar to Wald entropy (RM, Pourhasan & Smolkin)

Proposal: Spacetime Entanglement

(Bianchi & RM)

- in a theory of quantum gravity, for any sufficiently large region A in a smooth background, consider entanglement entropy between dof describing A and $\bar{A}$; contribution describing short-range entanglement is finite and described in terms of geometry of entangling surface with leading term:

$$S_{EE} = \frac{A_{\Sigma}}{4G_N} + \dots$$

- evidence:
 1. holographic S_{EE} in AdS/CFT correspondence
 2. QFT renormalization of G_N
 3. induced gravity, eg, Randall-Sundrum 2 model
 4. Jacobson's "thermal origin" of gravity
 5. spin-foam approach to quantum gravity

AdS/CFT Correspondence:

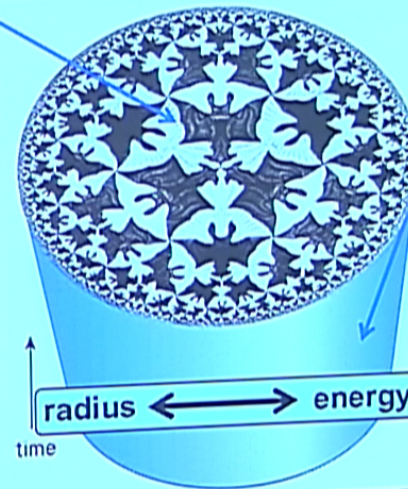
Bulk: gravity with negative Λ
in $d+1$ dimensions

Boundary: quantum field theory
without intrinsic scales
in d dimensions

anti-de Sitter
space

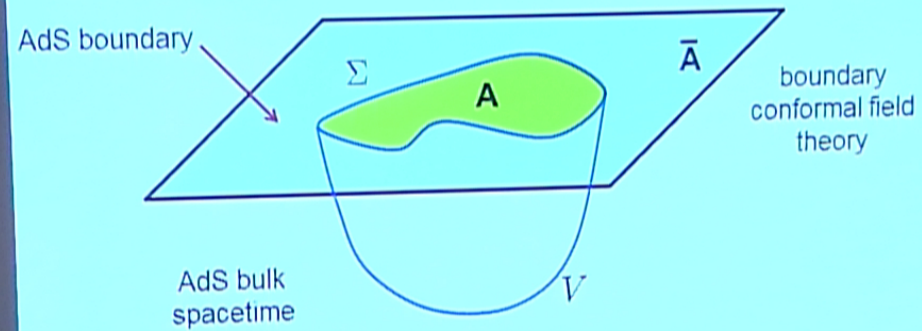
“holography”

conformal
field theory



(Ryu & Takayanagi '06)

Holographic Entanglement Entropy:

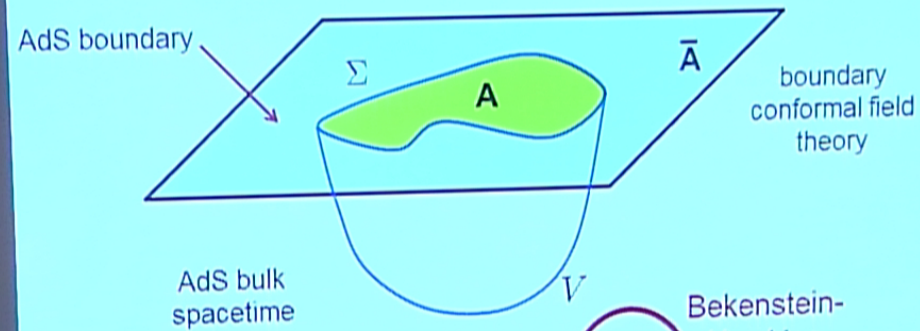


$$S(A) = \min_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

- conjecture $\longrightarrow$ many detailed consistency tests
(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, ...)
- proof $\longrightarrow$ "generalized gravitational entropy"
(Lewkowycz & Maldacena)

(Ryu & Takayanagi '06)

Holographic Entanglement Entropy:



$$S(A) = \min_{\partial V = \Sigma} \left(\frac{A_V}{4G_N} \right)$$

Bekenstein-Hawking formula

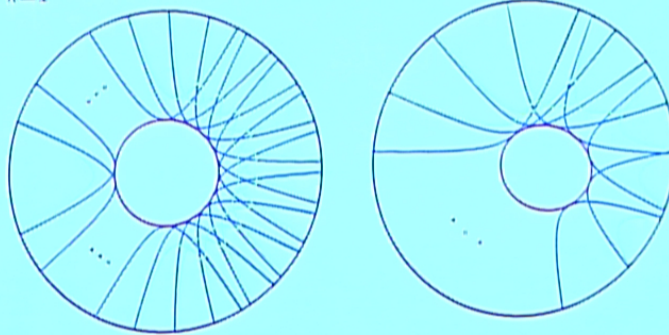
- conjecture $\longrightarrow$ many detailed consistency tests
(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, ...)
- proof $\longrightarrow$ "generalized gravitational entropy"
(Lewkowycz & Maldacena)

“Holography”:

(Balasubramanian, Chowdhury, Czech, de Boer & Heller;
RM, Rao & Sugishita, Czech Dong & Sully; Headrick, RM & Wein)

- differential entropy is an observable in the boundary CFT that measures the BH entropy of closed surfaces in bulk geometry

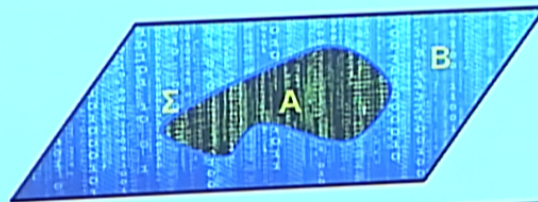
$$\frac{\mathcal{A}}{4G_N} = \sum_{k=1}^{\infty} [S(I_k) - S(I_k \cap I_{k+1})] = - \oint d\lambda \left. \frac{\partial S(\gamma_L(\lambda'), \gamma_R(\lambda))}{\partial \lambda'} \right|_{\lambda'=\lambda}$$



→ interpreted as QI task: “constrained state swapping”
(Czech, Hayden, Lashkari & Swingle)

Spacetime Geometry = Entanglement

- Bekenstein-Hawking formula: spacetime geometry encodes S_{BH}
- black hole entropy is entanglement entropy (Sorkin,)
- use BH formula for holographic entanglement entropy (Ryu & Takayanagi,)
- connectivity of spacetime requires entanglement (van Raamsdonk)
- spacetime entanglement conjecture (Bianchi & RM)
- AdS spacetime as a tensor network (MERA) (Swingle, Vidal,)
- "ER = EPR" conjecture (Maldacena & Susskind)
- hole-ographic spacetime (Balasubramanian, Chowdhury, Czech, de Boer & Heller; RM, Rao & Sugishita, Czech Dong & Sully;)



Gravitation from Entanglement:

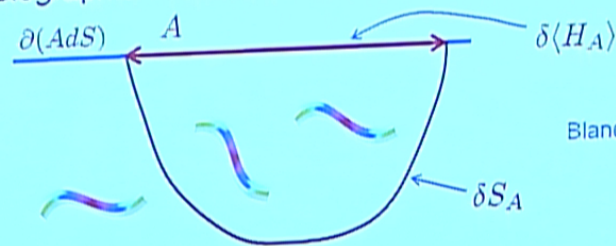
- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
 $= \exp(-H_A)$ $\leftarrow$ modular Hamiltonian
 $\longrightarrow$ "1st law" of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$

(Casini, Huerta & RM;
Blanco, Casini, Hung & RM)

Gravitation from Entanglement:

- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
 $= \exp(-H_A)$ ← modular Hamiltonian
 → “1st law” of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$
- generally H_A is “nonlocal mess” but H_A has simple form for CFT and spherical entangling surface:

$$H_A = 2\pi \int_{|x| < R} d^{d-1}x \frac{R^2 - r^2}{2R} T_{00}(\vec{x})$$
- holographic realization:

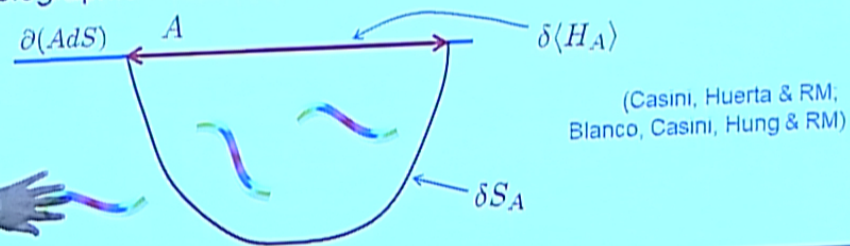


(Casini, Huerta & RM;
Blanco, Casini, Hung & RM)

Gravitation from Entanglement:

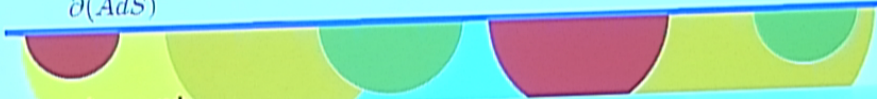
- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
 $= \exp(-H_A)$ ← modular Hamiltonian
 → “1st law” of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$
- generally H_A is “nonlocal mess” but H_A has simple form for CFT and spherical entangling surface:

$$H_A = 2\pi \int_{|x| < R} d^{d-1}x \frac{R^2 - r^2}{2R} T_{00}(\vec{x})$$
- holographic realization:



Gravitation from Entanglement in AdS/CFT:

- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
 $\longrightarrow$ "1st law" of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$
(Blanco, Casini, Hung & RM)
- holographic realization:
 $\partial(AdS)$


~~entanglement spacetime~~ provides both the stage for physical phenomena
 and the agent which manifests gravitational dynamics

- apply 1st law for spheres of all sizes, positions and in all frames:

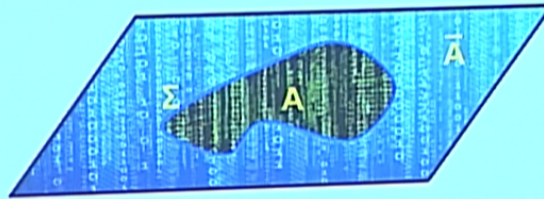
1st law of S_{EE} $\longleftrightarrow$ bulk geometry satisfies
 linearized Einstein eq's

(Lashkari, McDermott & Van Raamsdonk; Swingle & Van Raamsdonk;
 Faulkner, Guica, Hartman, RM & Van Raamsdonk)

Geometry + Gravity
= Entanglement

Conclusions:

- quantum information (& entanglement) provide new perspectives for quantum fields and quantum gravity
- QFT and QG challenge quantum information with new questions
- QI could be key to unification of quantum theory and gravity



- S_{BH} provides connection between entanglement and geometry
- spacetime entanglement: S_{BH} applies for generic large regions
- entanglement goes beyond geometry; knows gravity dynamics

Still lots to explore!

Conclusions:

- quantum information (& entanglement) provide new perspectives for quantum fields and quantum gravity
- QFT and QG challenge quantum information with new questions
- QI could be key to unification of quantum theory and gravity

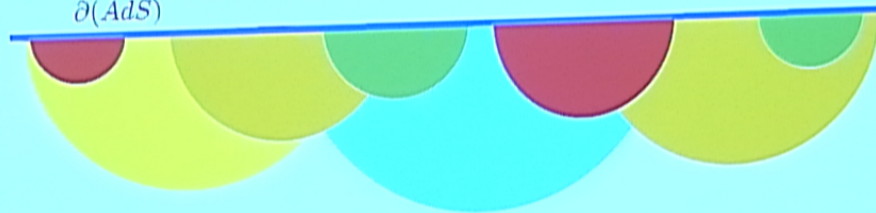


- S_{BH} provides connection between entanglement and geometry
- spacetime entanglement: S_{BH} applies for generic large regions
- entanglement goes beyond geometry; knows gravity dynamics

Still lots to explore!

Gravitation from Entanglement in AdS/CFT:

- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
→ “1st law” of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$
(Blanco, Casini, Hung & RM)
- holographic realization:
 $\partial(AdS)$



- apply 1st law for spheres of all sizes, positions and in all frames:
1st law of S_{EE} $\longleftrightarrow$ **bulk geometry satisfies linearized Einstein eq's**

(Lashkari, McDermott & Van Raamsdonk; Swingle & Van Raamsdonk;
Faulkner, Guica, Hartman, RM & Van Raamsdonk)