Title: How far can you stretch classical mechanics?

Date: May 12, 2015 02:50 PM

URL: http://pirsa.org/15050083

Abstract: Classical and quantum theories are very different, but the gap between them may look narrow particularly if the notion of classicality is broadened. For example, if we do not impose all the classical assumptions at the same time, hidden variable theories reproduce the results of quantum mechanics. If a quantum system is restricted to Gaussian states, evolution and measurements, then classical phase space mechanics with a finite resolution fully reproduces its behavior.

We discuss two examples of such extensions. In a version of the delayed-choice experiment we allow an otherwise classical system to exhibit two types of behavior ("P" or "W"), requiring, however, objectivity: the system is at any moment either "P" or "W", but not both. It turns out that the three conditions of objectivity, determinism, and independence of hidden variables are incompatible with any theory, not only with quantum mechanics. We then consider two harmonic oscillators with a Gaussian interaction between them. If one is treated as quantum and one is described by a classical theory with a finite phase space resolution, no consistent description of this interaction is possible. The lesson is that it is hard to be a little bit quantum: it is either pointless or quantumness takes over altogether.

OUTLINE

Part 1: Gaussian QM: nearly classical, but...
Part 2: Three classical assumptions: self-contradictory

COLLABORATORS

Asher Peres, Aida Ahmadzadegan
Radu Ionicioiu, Thomas Jennewein, Robb Mann

PART 1

A story of two oscillators

Hilbert space classical mechanics
Epirestriction
Interactions
Phase space quantum mechanics

CLASSICAL MECHANICS in Hilbert space

NATURAL LANGUAGES
Classical mechanics: phase space
Quantum mechanics: Hilbert space

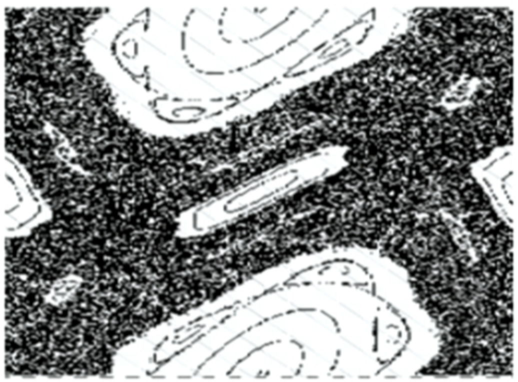
CLASSICAL MECHANICS in Hilbert space

NATURAL LANGUAGES
Classical mechanics: phase space **Quantum mechanics**: Hilbert space

Quantum mechanics in phase space

Moyal, Proc. Camb. Philos. Soc. **45**, 99 (1949)
Zachos, Fairlie, Curtright, *QM in phase space*

Classical mechanics in Hilbert space



Koopman, Proc. Natl. Acad. Sci. **17**, 315 (1931)
Von Neumann, Ann. Math **33**, 587 (1932)
Reed and Simon, vol 1
DRT, Found. Phys. **36**, 102 (2006).

CLASSICAL MECHANICS in Hilbert space

Phase space probability density ►wave function

$$\psi_c(x,k) := \sqrt{f(x,k)}$$

Liouville equation ► Schrödinger-like equation

$$\frac{\partial f(x,k)}{\partial t} = -\frac{\partial H}{\partial k}\frac{\partial f}{\partial x} + \frac{\partial H}{\partial x}\frac{\partial f}{\partial k} \quad \Rightarrow \quad \boxed{i\frac{\partial \psi_c}{\partial t} = \hat{L}\psi_c}$$

$$\boxed{\hat{L} := \frac{\partial H}{\partial k}\left(-i\frac{\partial}{\partial x}\right) - \frac{\partial H}{\partial x}\left(-i\frac{\partial}{\partial k}\right)}$$

CLASSICAL MECHANICS in Hilbert space

Phase space probability density ► wave function

$$\psi_c(x,k) := \sqrt{f(x,k)}$$

Liouville equation ► Schrödinger-like equation

$$\frac{\partial f(x,k)}{\partial t} = -\frac{\partial H}{\partial k}\frac{\partial f}{\partial x} + \frac{\partial H}{\partial x}\frac{\partial f}{\partial k} \quad \Rightarrow \quad \boxed{i\frac{\partial \psi_c}{\partial t} = \hat{L}\psi_c}$$

$$\boxed{\hat{L} := \frac{\partial H}{\partial k}\left(-i\frac{\partial}{\partial x}\right) - \frac{\partial H}{\partial x}\left(-i\frac{\partial}{\partial k}\right)}$$

Price: unobservable shift generators

$$\hat{p}_x := -i\frac{\partial}{\partial x} \qquad \hat{p}_k := -i\frac{\partial}{\partial k}$$

CLASSICAL MECHANICS in Hilbert space

- Equivalent to the usual Hamiltonian/Liouvillian dynamics
- Heisenberg picture for the operators
- Tensor products
- Entanglement
- Measurement formalism; usharp measurements
- Useful in ergodic theory

WIGNER DISTIRBUTION

LOGIC

A (quasi)probability distribution with the correct marginal

$$\langle q|\rho|q\rangle = \int W(q,p)dp\ldots$$

Evaluation

$$\rho \to \chi_\rho \to W(q,p)$$

Notation:

$$\Lambda = (x_1, k_1, \ldots, x_n, k_n)$$

$$X = (q_1, p_1, \ldots, q_n, p_n)$$

$$\hat{Z} = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_n, \hat{p}_n)$$

$$\Omega = \bigoplus_{k=1}^{n} \omega$$

$$\omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

WIGNER DISTIRBUTION

LOGIC

A (quasi)probability distribution with the correct marginal

$$\langle q|\rho|q\rangle = \int W(q,p)dp\ldots$$

Evaluation

$$\rho \rightarrow \chi_\rho \rightarrow W(q,p)$$

Notation:

$$\Lambda = (x_1, k_1, \ldots, x_n, k_n) \qquad \Omega = \bigoplus_{k=1}^{n} \omega \qquad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$X = (q_1, p_1, \ldots, q_n, p_n)$$

$$\hat{Z} = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_n, \hat{p}_n)$$

WIGNER DISTIRBUTION

$$\chi_\rho(\Lambda) = \mathrm{tr}\left(\rho \exp\left[-i\Lambda^T \Omega \hat{Z}\right]\right)$$

$$W_\rho(X) = \frac{1}{(2\pi)^{2n}} \int d^{2n}\Lambda \exp\left(i\Lambda^T \Omega X\right) \chi_\rho(\Lambda)$$

1d example

$$W(q,p) = \frac{1}{2\pi} \int dx e^{ixp} \langle q - x/2 | \rho | q + x/2 \rangle$$

WIGNER DISTIRBUTION

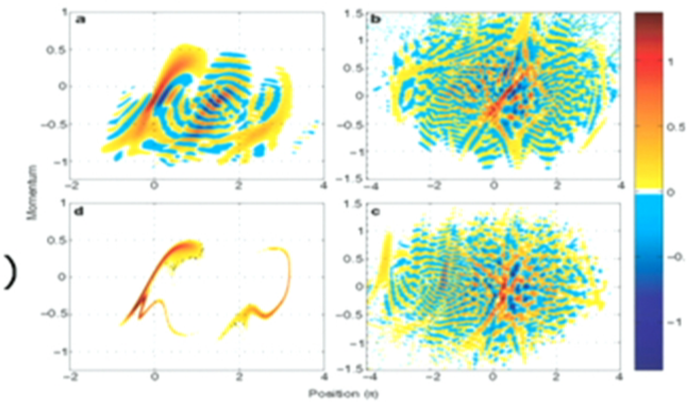
$$\chi_\rho(\Lambda) = \mathrm{tr}\left(\rho \exp\left[-i\Lambda^T \Omega \hat{Z}\right]\right)$$

$$W_\rho(X) = \frac{1}{(2\pi)^{2n}} \int d^{2n}\Lambda \exp\left(i\Lambda^T \Omega X\right) \chi_\rho(\Lambda)$$

1d example

$$W(q,p) = \frac{1}{2\pi} \int dx e^{ixp} \langle q - x/2 | \rho | q + x/2 \rangle$$

Chaotic example
Zurek, Nature **412**, 712 (2001)



GAUSSIAN STATES

Units/conventions

For each mode $l = \sqrt{\frac{\hbar}{m\omega_q}}; \quad l_p = \frac{\hbar}{l} = \sqrt{\hbar m \omega_q}, \quad q \to q/l, p \to p/l_p$

Creation/annihilation operators

$$\hat{q}_k = \frac{1}{\sqrt{2}}(\hat{a}_k + \hat{a}_k^\dagger), \quad \text{and} \quad \hat{p}_k = \frac{1}{i\sqrt{2}}(\hat{a}_k - \hat{a}_k^\dagger)$$

Hamiltonian

$$\hat{H} = \sum_k \tfrac{1}{2}\hbar\omega_k \left(\hat{p}_k^2 + \hat{q}_k^2\right) = \sum_k \hbar\omega_k \left(a_k^\dagger a_k + \tfrac{1}{2}\right)$$

GAUSSIAN STATES

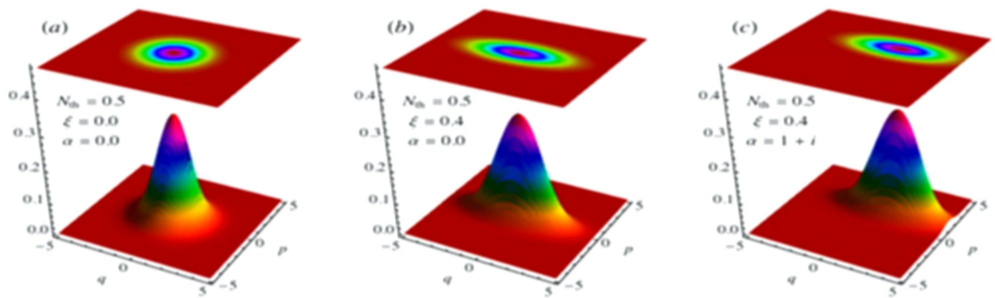
Correlation matrix

$$\sigma_{kl} := \frac{1}{2}\left\langle\left\{\hat{Z}_k, \hat{Z}_l\right\}\right\rangle - \left\langle\hat{Z}_k\right\rangle\left\langle\hat{Z}_l\right\rangle \equiv \frac{1}{2}\left\langle\left\{\Delta\hat{Z}_k, \Delta\hat{Z}_l\right\}\right\rangle$$

constraint
$\sigma + \frac{i}{2}\Omega \geq 0$

Definition

$$W(X) = \frac{\exp\left[-\frac{1}{2}\left(X - \langle\hat{Z}\rangle\right)^T \sigma^{-1}\left(X - \langle\hat{Z}\rangle\right)\right]}{\pi^n\sqrt{\det\sigma}}$$



Vaccuum
Thermal
Coherent
Squeezed

S. Olivares, Eur. Phys. J. Special Topics **203**, 3 (2012)

MACQUARADE

Gaussian interaction

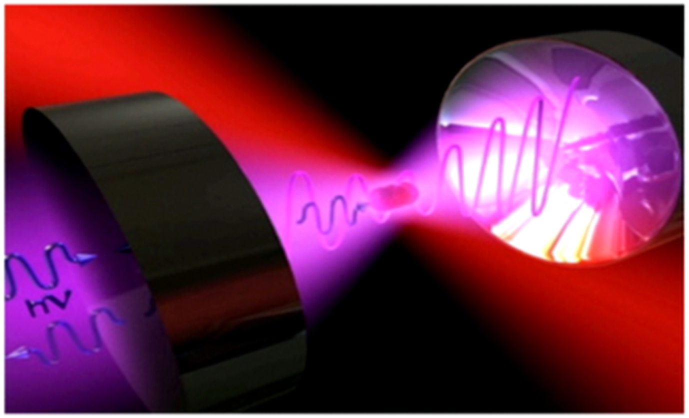
$$H = \sum_{k=1}^{n} g_k^{(1)} \, \hat{a}_k^\dagger + \sum_{k \geq l = 1}^{n} g_{kl}^{(2)} \, \hat{a}_k^\dagger \hat{a}_l + \sum_{k,l=1}^{n} g_{kl}^{(3)} \, \hat{a}_k^\dagger \hat{a}_l^\dagger + h.c.$$

Appearance

Gaussian states+Gaussian operators+Gaussian measurements are equivalent to the Koopmanian mechanics with finite phase space resolution [epirestricted]

Bartlett, Rudolph, Spekkens,
Phys. Rev. A, **86** 012103, (2012).

INTERACTION



Peres & DRT,
Phys. Rev. A **63**, 022101 (2001)
DRT, Found. Phys. **36**, 102 (2006)

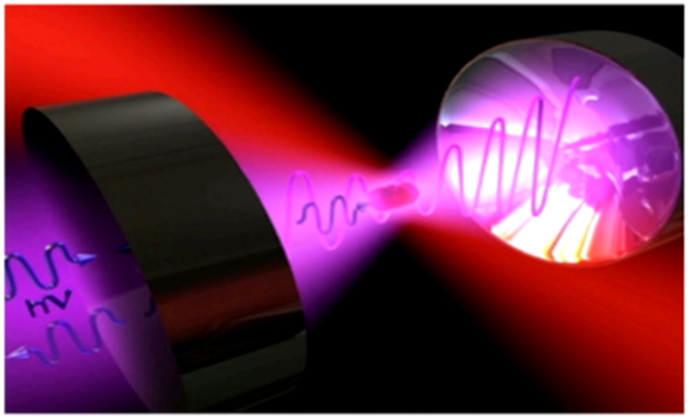
K

M

Barceló et al,
Phys Rev A **86**, 042120 (2012)

System: classical and quantum oscillators
Coupling: Gaussian

INTERACTION



Peres & DRT,
Phys. Rev. A **63**, 022101 (2001)
DRT, Found. Phys. **36**, 102 (2006)

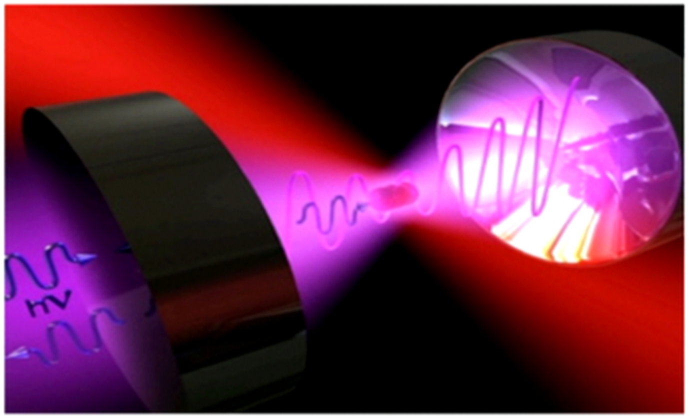
K

M

Barceló et al,
Phys Rev A **86**, 042120 (2012)

System: classical and quantum oscillators
Coupling: Gaussian

INTERACTION



Peres & DRT,
Phys. Rev. A **63**, 022101 (2001)
DRT, Found. Phys. **36**, 102 (2006)

K

System: classical and quantum oscillators
Coupling: Gaussian

Units & notation

$$\lambda := \sqrt{\frac{\kappa}{m\omega_c}}, \lambda_k := \sqrt{\kappa m \omega_c} \qquad x \to x/\lambda, \;\; k \to k/\lambda_k$$

INTERACTION

Classical oscillator

Creation and annihilation operators

$$\hat{b}_x = \tfrac{1}{\sqrt{2}}\left(\hat{x} + i\hat{p}_x\right)$$

$$\hat{b}_k = \tfrac{1}{\sqrt{2}}\left(\hat{k} + i\hat{p}_k\right)$$

INTERACTION

Classical oscillator

Creation and annihilation operators

$$\hat{b}_x = \tfrac{1}{\sqrt{2}}\left(\hat{x} + i\hat{p}_x\right)$$

$$\hat{b}_k = \tfrac{1}{\sqrt{2}}\left(\hat{k} + i\hat{p}_k\right)$$

INTERACTION

Classical oscillator

Creation and annihilation operators

$$\hat{b}_x = \tfrac{1}{\sqrt{2}}\left(\hat{x} + i\hat{p}_x\right)$$

$$\hat{b}_k = \tfrac{1}{\sqrt{2}}\left(\hat{k} + i\hat{p}_k\right)$$

Liouvillian

$$L = \omega_c(\hat{k}\hat{p}_x - \hat{x}\hat{p}_k)$$

$$= \frac{i\omega_c}{2}\left(b_k b_x^\dagger + b_x^\dagger b_k - b_k^\dagger b_x - b_x b_k^\dagger\right) = i\omega_c\left(b_x^\dagger b_k - b_k^\dagger b_x\right)$$

INTERACTION

Interaction term

$$K = H_q + L + K_i$$

Interaction term: involves both classical and quantum operators

$$\begin{aligned} K_i \;&=\; i\left(\beta^*_{0x} a b_x - \beta_{0x} b^\dagger_x a^\dagger + \beta^*_{0k} a b_k - \beta_{0k} b^\dagger_k a^\dagger\right) \\ &\quad + 2\left(\alpha_{0x} a^\dagger b_x + \alpha^*_{0x} b^\dagger_x a + \alpha_{0k} a^\dagger b_k + \alpha^*_{0k} b^\dagger_k a\right) \end{aligned}$$

EOM

$$\begin{pmatrix} \dot{q} \\ \dot{p} \\ \dot{x} \\ \dot{k} \\ \dot{p}_x \\ \dot{p}_k \end{pmatrix} = \begin{pmatrix} 0 & \omega_q & 2\alpha_x - \beta_x & 2\alpha_k - \beta_k & 0 & 0 \\ -\omega_q & 0 & 0 & 0 & 2\alpha_x + \beta_x & 2\alpha_k + \beta_k \\ -(2\alpha_x + \beta_x) & 0 & 0 & \omega_c & 0 & 0 \\ -(2\alpha_k + \beta_k) & 0 & -\omega_c & 0 & 0 & 0 \\ 0 & -2\alpha_x + \beta_x & 0 & 0 & 0 & \omega_c \\ 0 & -2\alpha_k + \beta_k & 0 & 0 & -\omega_c & 0 \end{pmatrix} \begin{pmatrix} q \\ p \\ x \\ k \\ p_x \\ p_k \end{pmatrix}$$

INTERACTION

New modes:

$$\mathbf{v}(t) = \sum_{1}^{6} C_j \mathbf{u}_j e^{\bar{\lambda}_j t}$$

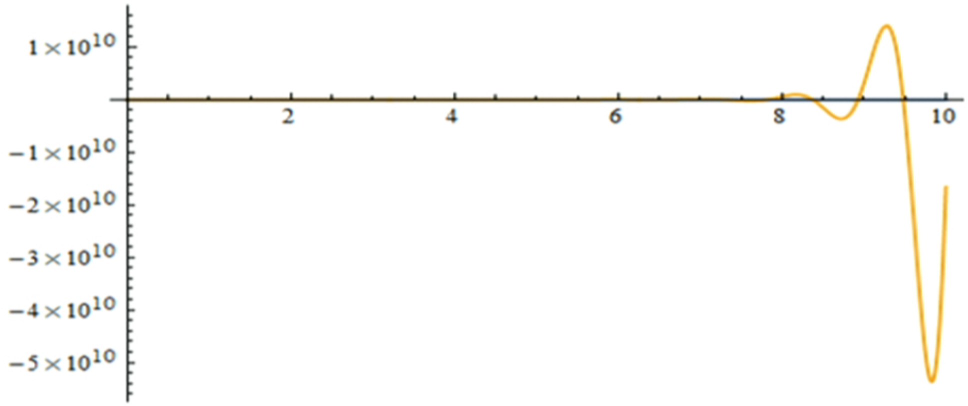
Generic situation: at least one of the coefficients has both real and imaginary parts

INTERACTION

New modes:

$$\mathbf{v}(t) = \sum_{1}^{6} C_j \mathbf{u}_j e^{\bar{\lambda}_j t}$$

Generic situation: at least one of the coefficients has both real and imaginary parts



$\langle p(t) \rangle$

$\omega_c = 4, \omega_q = 8$

INTERACTION

Avoiding unobservable operators

Possible, but:

$$\dot{q} = \omega_q p + 4\alpha_x x + 4\alpha_k k$$

$$\dot{p} = -\omega_q q$$

$$\dot{x} = \omega_c k$$

$$\dot{k} = -\omega_c x$$

No backreaction + possible resonances

INTERACTION: using phase space

Logic: Use phase space QM. Replace Moyal <>Poisson
Oscillator: Moyal is identical to Poison

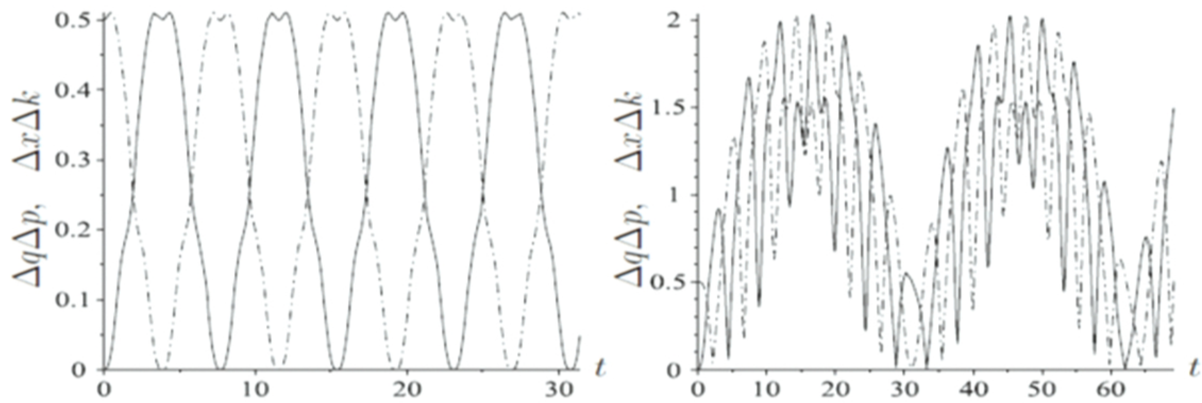


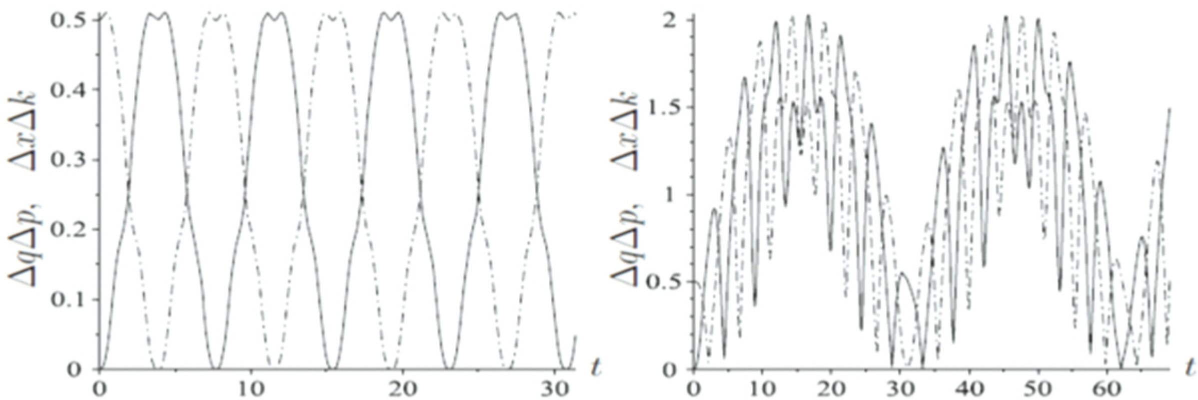
FIG. 3. In both plots, the solid line shows the time evolution of $\Delta q\, \Delta p$, and the dashed one shows $\Delta x\, \Delta k$. On the left, both frequencies are equal $\omega_q = \omega_x = 1.73$ and larger than the coupling $\sqrt{\gamma} = 1$. On the right, both frequencies are very similar to each other, with $\omega = 1.01$, $\Omega = 1$, and to the coupling $\sqrt{\gamma} = 1$. We also use $\hbar = 1$.

Quantum & classical features swap.

Barceló et al, Phys Rev A **86**, 042120 (2012)

INTERACTION: using phase space

Logic: Use phase space QM. Replace Moyal <>Poisson
Oscillator: Moyal is identical to Poison



Quantum & classical features swap.

FIG. 3. In both plots, the solid line shows the time evolution of $\Delta q\, \Delta p$, and the dashed one shows $\Delta x\, \Delta k$. On the left, both frequencies are equal $\omega_q = \omega_x = 1.73$ and larger than the coupling $\sqrt{\gamma} = 1$. On the right, both frequencies are very similar to each other, with $\omega = 1.01$, $\Omega = 1$, and to the coupling $\sqrt{\gamma} = 1$. We also use $\hbar = 1$.

Barceló et al, Phys Rev A **86**, 042120 (2012)

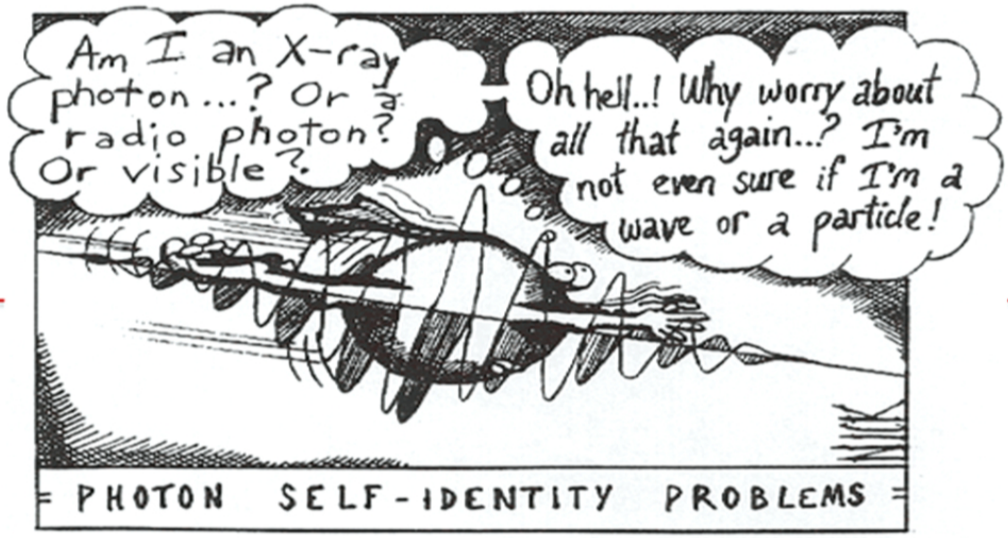
PART 2

DELAYED CHOICE & CLASSICAL PARADOXES

A bit of history
Complementarity
WDC
The use of HV
Quantum control
Classical paradoxes

- Ionicioiu and Terno, Phys. Rev. Lett. **107**, 230406 (2011).
- Céleri, Gomes, Ionicioiu, Jennewein, Mann, and. Terno, Found. Phys. **44**, 576 (2014).
- Ionicioiu, Jennewein, Mann, and Terno, Nature Comm. **5**, 3997 (2014)
- Ionicioiu, Mann, and Terno, Phys. Rev. Lett. **114**, 060405 (2015)

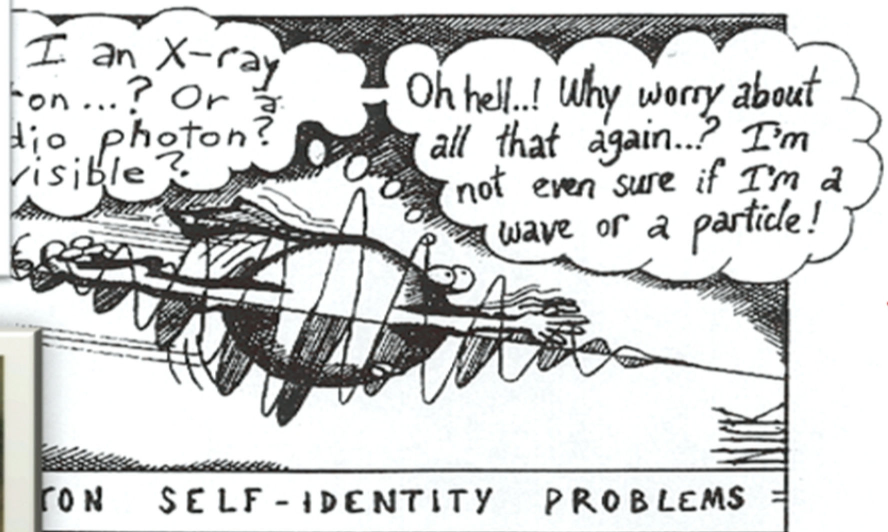
STUFF:
WAVES vs PARTICLES



Photons are particles

Photons are waves

STUFF:
WAVES vs PARTICLES



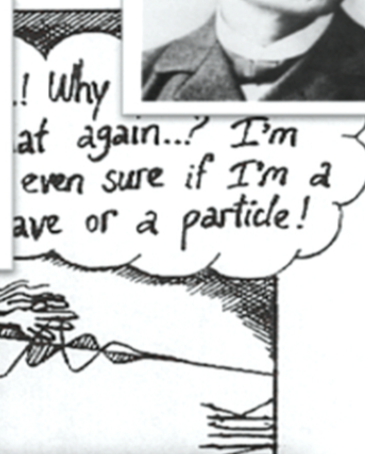
Photons are particles

Photons are waves

STUFF:

WAVES vs PARTICLES






Photons are particles

Photons are waves



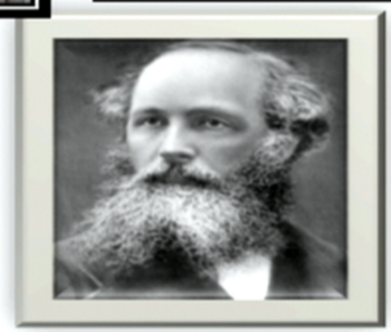

STUFF:
WAVES & PARTICLES





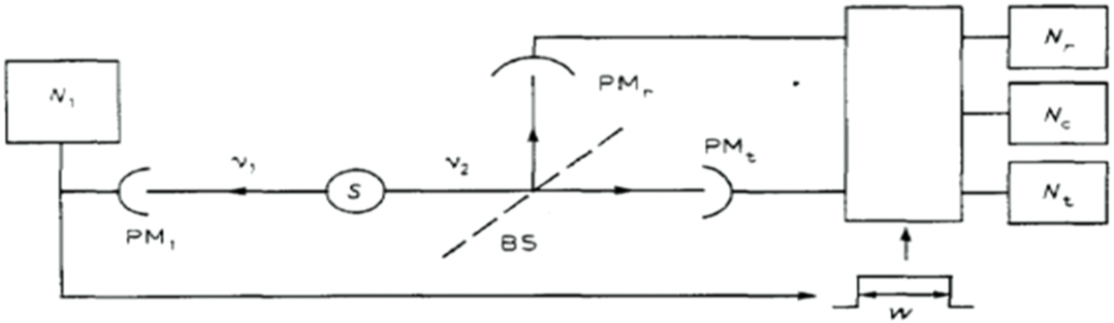




PHOTONS

at last

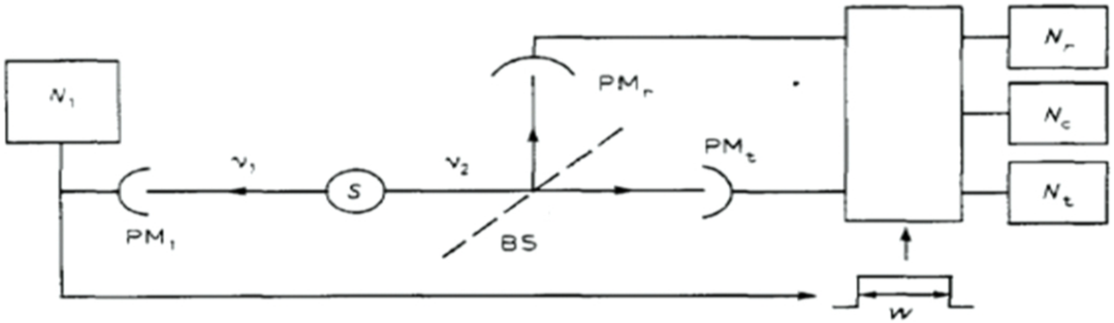


Single photons **behave as** particles

Grangier, Roger and Aspect
Europhys. Lett. **1**, 173 (1986)

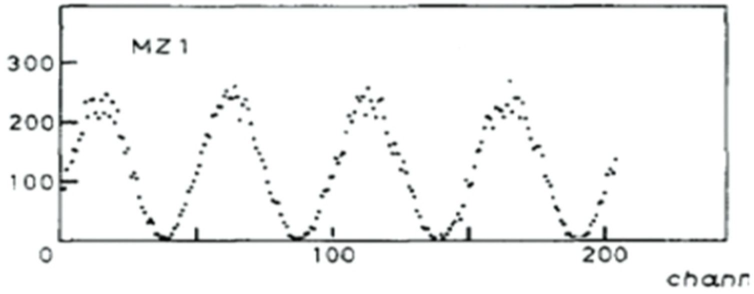
PHOTONS

at last



Grangier, Roger and Aspect
Europhys. Lett. **1**, 173 (1986)

Single photons **behave as** particles
Single photons **behave as** waves

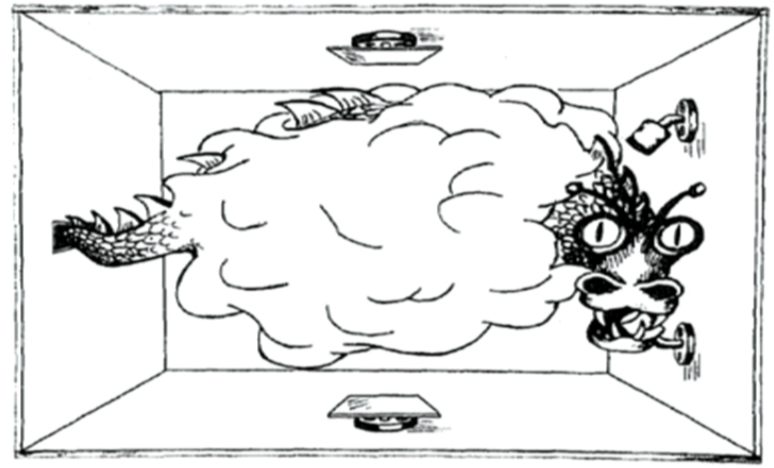


DEFINITIONS
Particles: no interference, ► single path *Waves*: interference, ► both paths

COMPLEMENTARITY:

a modern version

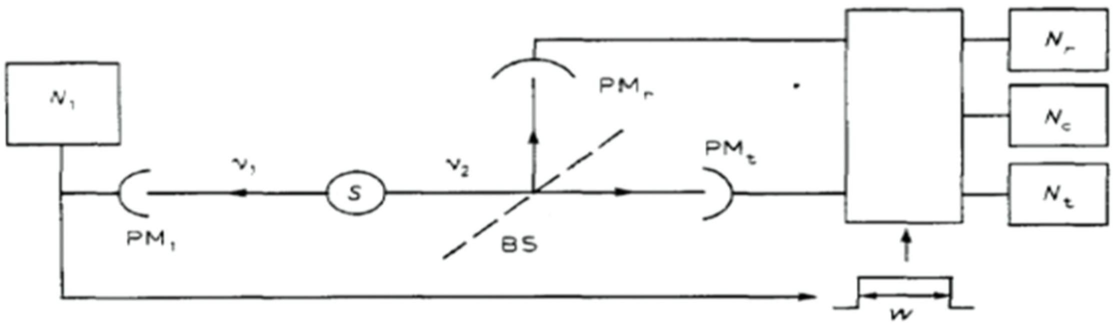
... the information provided by different experimental procedures that in principle cannot, because of the physical character of the needed apparatus, be performed simultaneously, cannot be represented by any mathematically allowed quantum state of the system. The elements of information obtainable from incompatible measurements are said to be *complementary*.



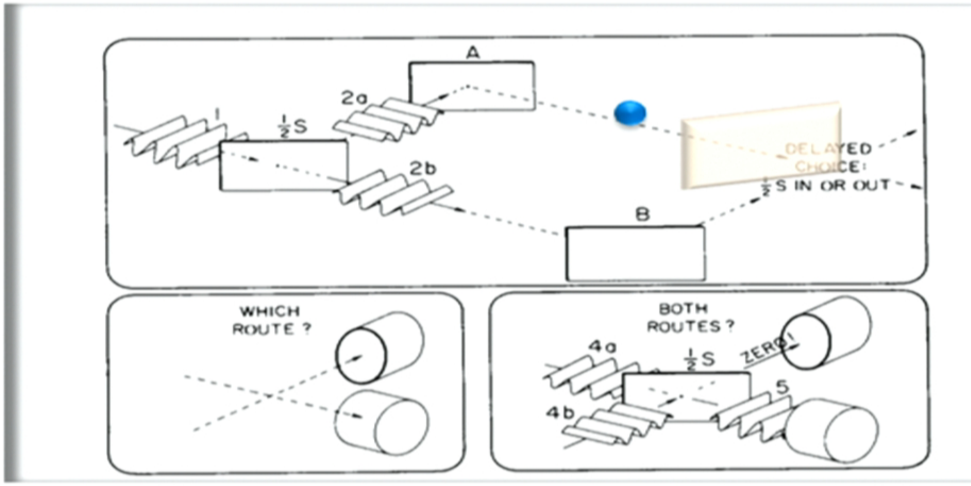
Stapp, in *Compendium of Quantum Physics*

COMPLEMENTARITY: *a conspiracy*

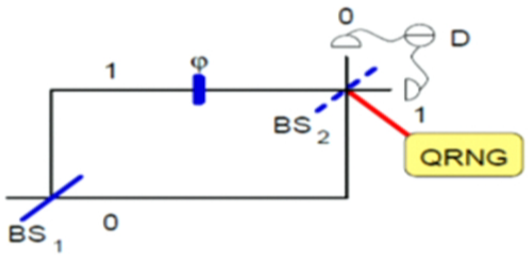
The photon could know in advance of entering the apparatus whether the latter has been set up in the "wave" configuration with BS_2 in place or the "particle" one (BS_2 removed) and adjust accordingly.



DELAYED CHOICE



Wheeler, 1978, 1984



By making the choice to close or open the MZI when the photon is already in, it is forced not to change its mind

HV THEORIES

Purpose: reproduce observed statistics and maintain classical concepts
Viewed as [likely] inadequate, but consistent world view

Counter-HV action
- ◇ consider a set-up
- ◇ make a QM prediction
- ◇ make a HV prediction
- ◇ compare
  get a contradiction
- ◇ make an experiment

HV THEORIES

Purpose: reproduce observed statistics and maintain classical concepts

Viewed as [likely] inadequate, but consistent world view

Counter-HV action

- ◇ consider a set-up
- ◇ make a QM prediction
- ◇ make a HV prediction
- ◇ compare
  - get a contradiction
- ◇ make an experiment

Counter-counter-HV action

- ❑ find a loophole
- ❑ introduce conspiratorial correlations

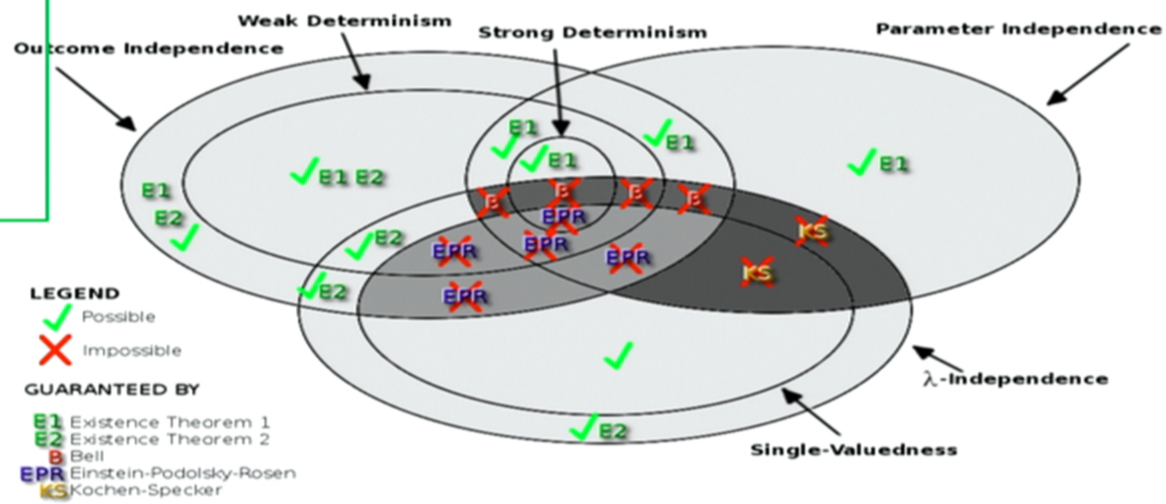
HV THEORIES

Purpose: reproduce observed statistics and maintain classical concepts
Viewed as [likely] inadequate, but consistent world view

Counter-HV action
◇ consider a set-up
◇ make a QM prediction
◇ make a HV prediction
◇ compare
get a contradiction
◇ make an experiment

Counter-counter-HV action
❑ find a loophole
❑ introduce conspiratorial correlations

Brandenburger & Yanofsky
JPA **41** 425302 (2008) ►



HV THEORIES

- **Determinism**: once hidden variables are defined, there are no residual randomness [several flavors]

$$\forall A\ \exists a:\ p(a\,|\,A,\lambda)=1$$ ◄ strong

- **Parameter independence**: the outcome of any measurement depends only on the HV and the set-up of this measurement

$$p(a\,|\,A,B,C,...,\lambda)=p(a\,|\,A,\lambda)$$

- **HV (λ-)independence**: determination of the hidden variable is independent of the choice of measurement

$$p(\lambda\,|\,A,B...)=p(\lambda\,|\,A',B'...)$$

Measurements and settings:
$A,A';B,B'$

Outcomes
$a,a';b,b'$

Brandenburger & Yanofsky, J. Phys. A **315**, 966 (2007)

HV THEORIES

- **Determinism**: once hidden variables are defined, there are no residual randomness [several flavors]

$$\forall A\ \exists a:\ p(a\,|\,A,\lambda)=1$$ ◄ strong

- **Parameter independence**: the outcome of any measurement depends only on the HV and the set-up of this measurement

$$p(a\,|\,A,B,C,\ldots,\lambda)=p(a\,|\,A,\lambda)$$

- **HV (λ-)independence**: determination of the hidden variable is independent of the choice of measurement

$$p(\lambda\,|\,A,B\ldots)=p(\lambda\,|\,A',B'\ldots)$$

Adequacy

Measurements and settings:
$A,A';B,B'$
Outcomes
$a,a';b,b'$

Brandenburger & Yanofsky, J. Phys. A **315**, 966 (2007)

Locality, contextuality, Bell inequalities,... are all derived from these three axioms

HV THEORIES

Extensions & questions

- What is the basis for assertion of wave-particle duality?
- ❑ Can we detect "it" first and decide what was it later?
- ❑ Is space-like separation necessary?
- ❑ What if the controlling devices are quantum?

HV THEORIES

Extensions & questions

- What is the basis for assertion of wave-particle duality?
- ❑ Can we detect "it" first and decide what was it later?
- ❑ Is space-like separation necessary?
- ❑ What if the controlling devices are quantum?

Conspiracy & counter-conspiracy

❑ A hidden variable λ=p,w
set at production/before splitting
❑ Reproduction of the observed data for
some $p(a,b,\lambda)$

OBJECTIVITY

a.k.a. DEFINITNESS

Photons are either particles $\lambda = p$ or waves $\lambda = w$

$$p(a \mid b = 1, \lambda = w) = (\cos^2 \tfrac{\phi}{2}, \sin^2 \tfrac{\phi}{2})$$

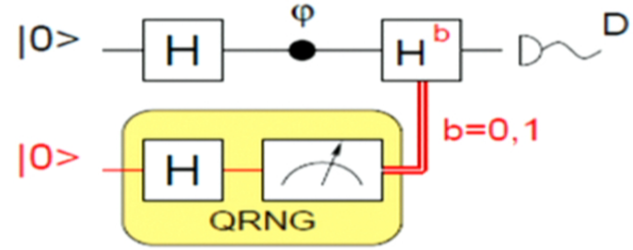
$$p(a \mid b = 0, \lambda = p) = (\tfrac{1}{2}, \tfrac{1}{2})$$

$$p(a \mid b = 0, \lambda = w) = (x, 1 - x)$$
$$p(a \mid b = 1, \lambda = p) = (y, 1 - y)$$

WDC *Logic*

data | HV theory | IC

$$q(a,b) = \sum_{\lambda} p(a\,|\,b,\lambda) \times p(\lambda\,|\,b) \times n(b)$$



Causal:

$$p(\lambda\,|\,b) = \delta_{\lambda p}\delta_{b0} + \delta_{\lambda w}\delta_{b1}$$

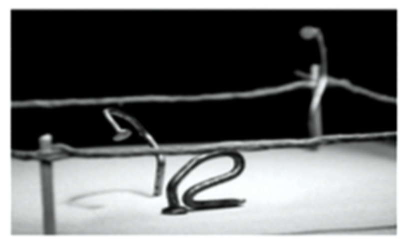
This is the target of WDC experiments. Dismissed†

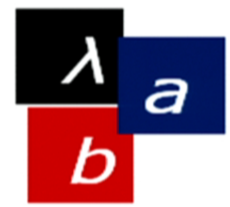
Stochastic...

$$p(\lambda\,|\,b) = p(\lambda) = (\mathfrak{p}, 1-\mathfrak{p})$$

Consistency requirements
resurrect wave-particle duality † :

$$p(a\,|\,b,\lambda) = p(a\,|\,b)$$

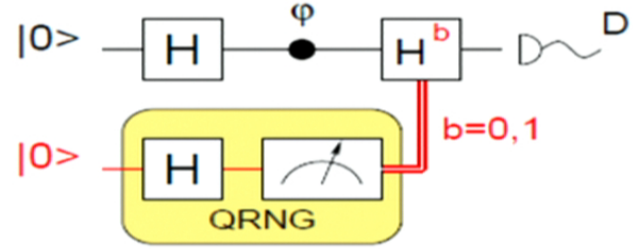




† unless "even more mind boggling" conspiracies are allowed
[e.g.: a correlation between HV of a photon & QRNG]

WDC *Logic*

$b \; \lambda \; a$



data / HV theory / IC

$$q(a,b) = \sum_{\lambda} p(a \mid b,\lambda) \times p(\lambda \mid b) \times n(b)$$

Causal:

$$p(\lambda \mid b) = \delta_{\lambda p}\delta_{b0} + \delta_{\lambda w}\delta_{b1}$$

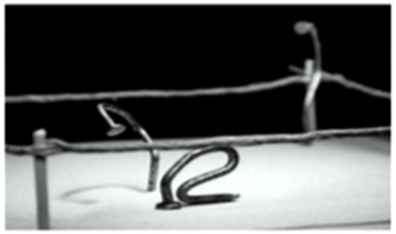
This is the target of WDC experiments. Dismissed†

Stochastic...

$$p(\lambda \mid b) = p(\lambda) = (\mathfrak{p}, 1-\mathfrak{p})$$

Consistency requirements
resurrect wave-particle duality † :

$$p(a \mid b,\lambda) = p(a \mid b)$$



$\lambda \; a \; b$

† unless "even more mind boggling" conspiracies are allowed
[e.g.: a correlation between HV of a photon & QRNG]

DELAYED CHOICE

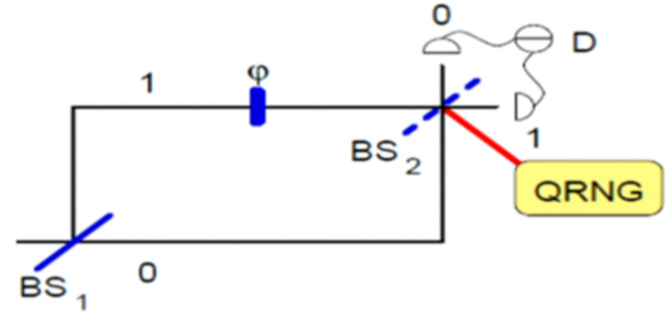
+ QRNG

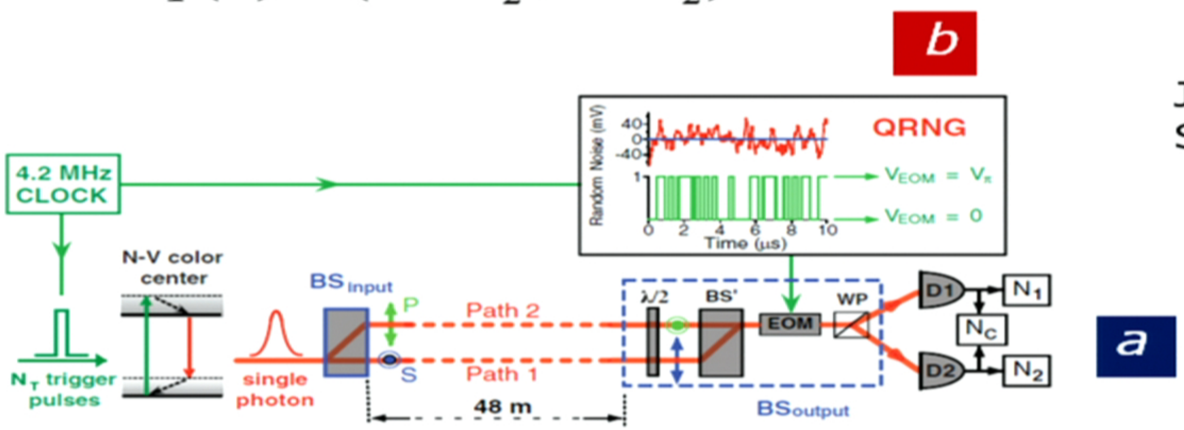
Open interferometer [particle]

$$q(a) = (\tfrac{1}{2}, \tfrac{1}{2})$$

Closed interferometer [wave]

$$q(a) = (\cos^2 \tfrac{\varphi}{2}, \sin^2 \tfrac{\varphi}{2})$$

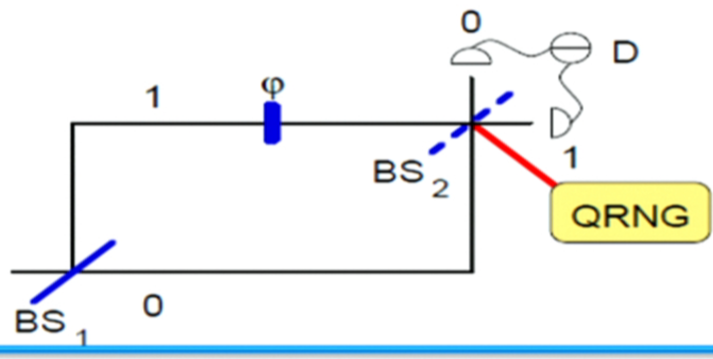


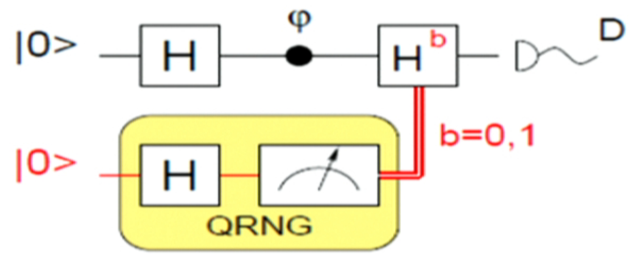


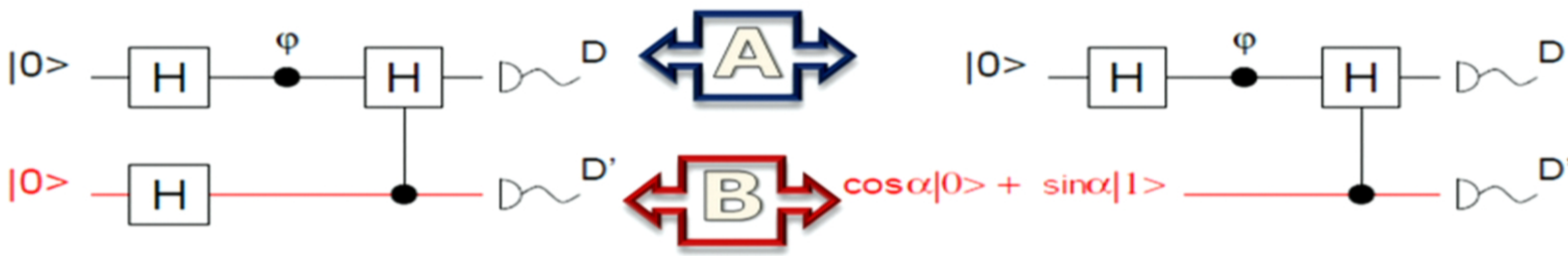
Jacques *et al.*,
Science **315**, 966 (2007)

Spacelike separation between the source and the RNG

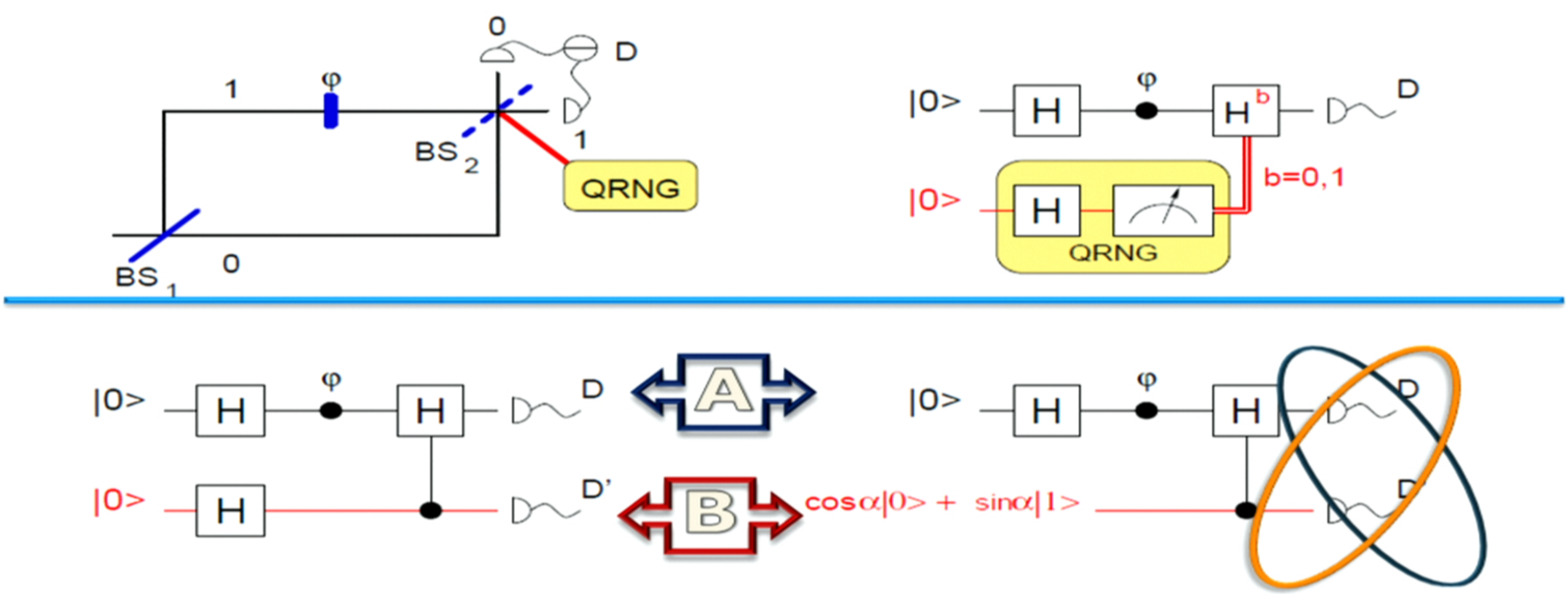
QUANTUM CONTROL







QUANTUM CONTROL



QUANTUM CONTROL

QM

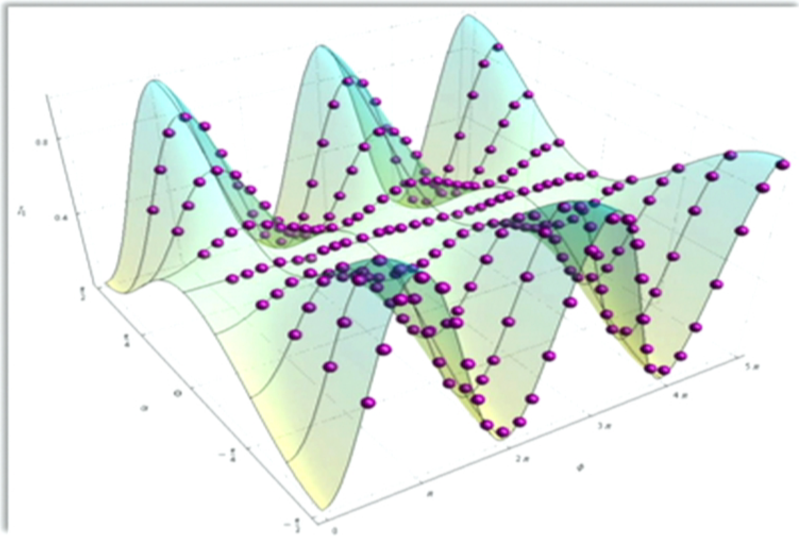
State after the gates [before the detectors]

$$|\psi_f\rangle = \cos\alpha|\psi_p\rangle|0\rangle + \sin\alpha|\psi_w\rangle|1\rangle$$

$$|\psi_p\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\phi}|1\rangle\right)$$

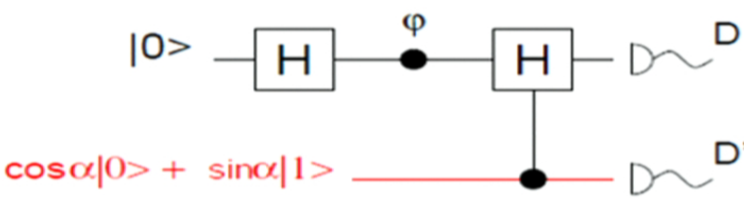
$$|\psi_w\rangle = \frac{1}{\sqrt{2}}e^{i\phi/2}\left(\cos\tfrac{\phi}{2}|0\rangle - i\sin\tfrac{\phi}{2}|1\rangle\right)$$

- Can we detect "it" first and decide what it was later
- No space-like separation
- Morphing of wave to particle stat



DC

+ Q



- We can detect "it" first and decide what was it later
- No space-like separation
- Duality restored OR HV pushed away (half-step)

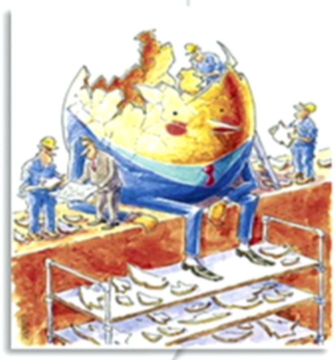
Consistency requirements resurrect wave-particle duality:

$$p(a\,|\,b,\lambda) = p(a\,|\,b)$$

$$\mathfrak{p} = 0, x = \tfrac{1}{2}$$

$$\mathfrak{p} = 1, y = \cos^2 \tfrac{\phi}{2}$$

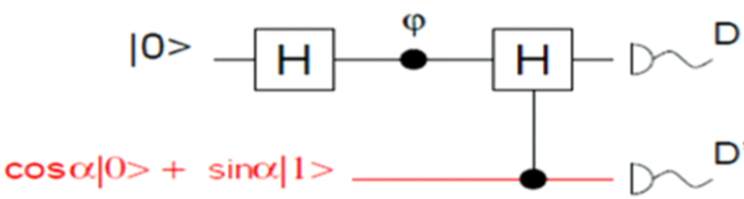
$$x = \tfrac{1}{2}, y = \cos^2 \tfrac{\phi}{2}$$



DC

+ Q

λ a b



- Can we detect "it" first and decide what was it later
- No space-like separation
- Duality restored OR HV pushed away (half-step)

Consistency requirements resurrect wave-particle duality:

$$p(a\,|\,b,\lambda) = p(a\,|\,b)$$

$$\mathfrak{p} = 0, x = \tfrac{1}{2}$$

$$\mathfrak{p} = 1, y = \cos^2\tfrac{\phi}{2}$$

$$x = \tfrac{1}{2}, y = \cos^2\tfrac{\phi}{2}$$

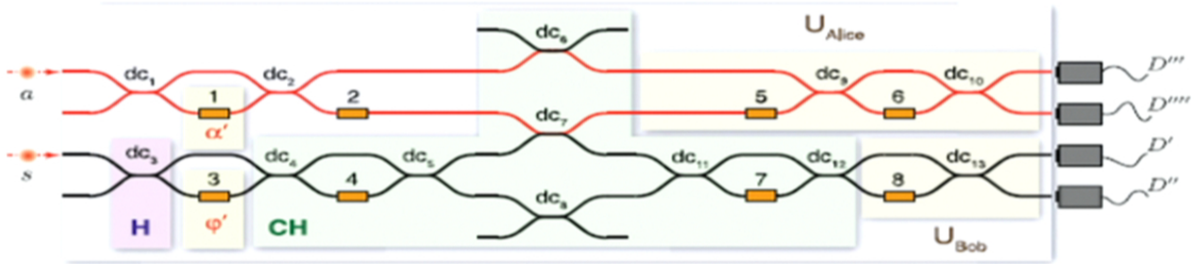
Or imply a higher level conspiracy

$$p(\lambda) = (\cos^2\alpha, \sin^2\alpha)$$

EXPERIMENTS

"exotic" systems

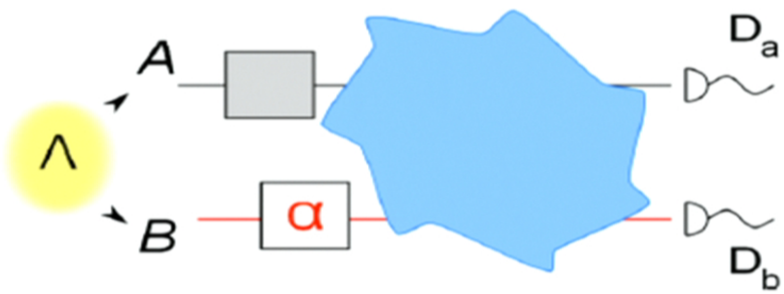
Since there is no space-like separation, it can be done on a chip



Peruzzo *et al* , Science 338, 634 (2012**).**

+ Bell test (quantumnes of the ancila)

THREE INCOMPATIBLE ASSUMPTIONS



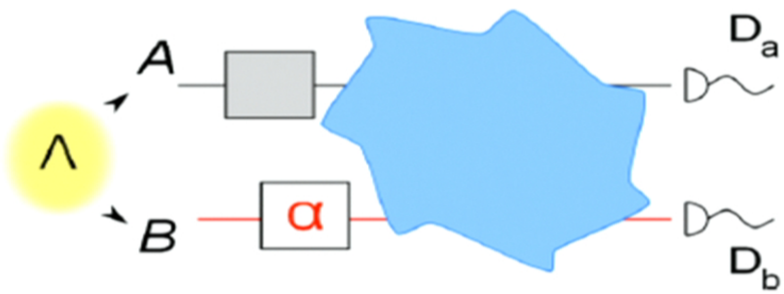
Empirical statistics

$$e(a,b) = \big(x e_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w)\big)$$

$$e(b) = (x, 1-x)$$ ◄ *controller*

two types of stats ► $\bar{e}_{\mathrm{p}}(a) = (e_p, 1-e_p)$, $\bar{e}_{\mathrm{w}}(a) = (e_w, 1-e_w)$

THREE INCOMPATIBLE ASSUMPTIONS



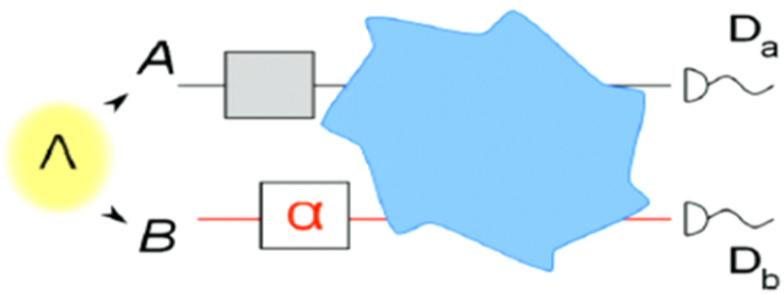
Empirical statistics

$$e(a,b) = \big(xe_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w)\big)$$

$$e(b) = (x, 1-x)$$ ◄ *controller*

two types of stats ► $\bar{e}_{\mathrm{p}}(a) = (e_p, 1-e_p)$, $\bar{e}_{\mathrm{w}}(a) = (e_w, 1-e_w)$

THREE INCOMPATIBLE ASSUMPTIONS



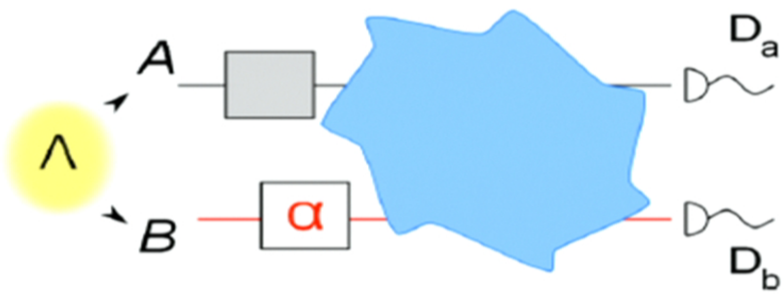
Empirical statistics

$$e(a,b) = \big(xe_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w)\big)$$

$e(b) = (x, 1-x)$ ◄ *controller*

two types of stats ► $\bar{e}_\mathrm{p}(a) = (e_p, 1-e_p)$, $\bar{e}_\mathrm{w}(a) = (e_w, 1-e_w)$

THREE INCOMPATIBLE ASSUMPTIONS



(*) Adequacy

Empirical statistics

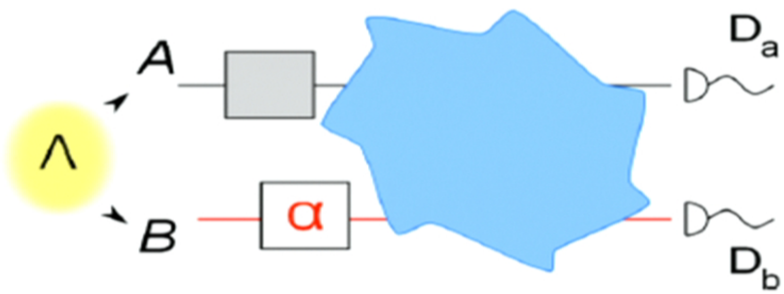
$$e(a,b) = \big(xe_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w)\big)$$

$e(b) = (x, 1-x)$ ◄ *controller*

two types of stats ► $\bar{e}_{\mathrm{p}}(a) = (e_p, 1-e_p), \qquad \bar{e}_{\mathrm{w}}(a) = (e_w, 1-e_w)$

$$e(a,b) = p(a,b) = \sum_{\Lambda} p(a,b,\Lambda) = \sum_{\Lambda} p(a,b|\Lambda)\, p(\Lambda)$$

THREE INCOMPATIBLE ASSUMPTIONS



(*) Adequacy

Empirical statistics

$$e(a,b) = \big(xe_p, (1-x)e_w, x(1-e_p), (1-x)(1-e_w)\big)$$

$$e(b) = (x, 1-x)$$ ◄ *controller*

two types of stats ► $\bar{e}_{\mathrm{p}}(a) = (e_p, 1-e_p), \qquad \bar{e}_{\mathrm{w}}(a) = (e_w, 1-e_w)$

$$e(a,b) = p(a,b) = \sum_{\Lambda} p(a,b,\Lambda) = \sum_{\Lambda} p(a,b|\Lambda)\, p(\Lambda)$$

THREE INCOMPATIBLE ASSUMPTIONS

The system is definitely s one or another

$$p(a|b=1,\lambda=\mathsf{w}) = \bar{e}_{\mathsf{w}}(a)$$

$$p(a|b=0,\lambda=\mathsf{p}) = \bar{e}_{\mathsf{p}}(a)$$

(i) Objectivity

$$\lambda = \lambda(\Lambda)$$

THREE INCOMPATIBLE ASSUMPTIONS

The system is definitely s one or another

$$p(a|b=1,\lambda=\mathsf{w}) = \bar{e}_{\mathsf{w}}(a)$$

$$p(a|b=0,\lambda=\mathsf{p}) = \bar{e}_{\mathsf{p}}(a)$$

(i) Objectivity

$$\lambda = \lambda(\Lambda)$$

HV theory is (weakly) deterministic

$$p(a,b|\Lambda) = \chi_{ab}(\Lambda)$$

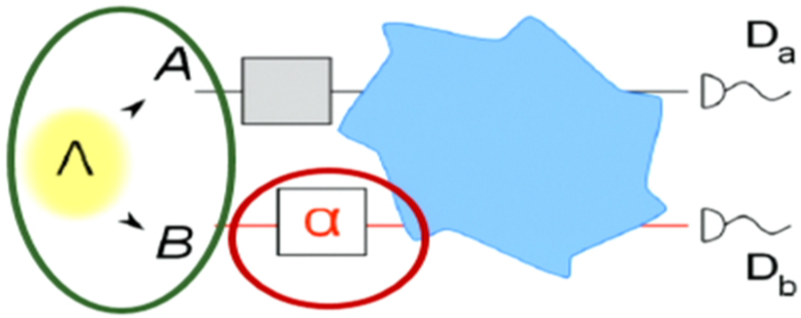
(ii) Determinism

00	01
10	11

Boundaries of the regions depend on the settings

THREE INCOMPATIBLE ASSUMPTIONS

Is λ-independent



(iii) Independence

$p(\Lambda)$ is independent of the settings

IMT, Phys. Rev. Lett. **114**, 060405 (2015)

Adequacy | three assumptions

Objectivity: two types of statistics e_p, e_w
Determinism: HV determines all the outcomes
Independence: single HV that is not influenced by the settings

THREE INCOMPATIBLE ASSUMPTIONS

LOGIC

Stage 1: find a unique non-trivial solution to (i)-(iii)

Ignoring how it arises from Λ

THREE INCOMPATIBLE ASSUMPTIONS

LOGIC

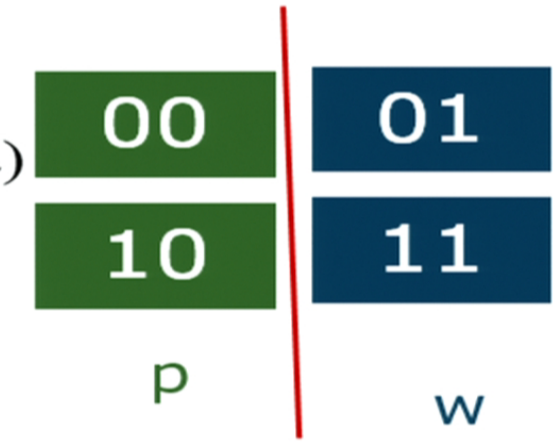
Stage 1: find a unique non-trivial solution to (i)-(iii)

Ignoring how it arises from Λ

Exists, but

$$p_s(\lambda \mid b) = \delta_{\lambda p}\delta_{b0} + \delta_{\lambda w}\delta_{b1} = p_s(b \mid \lambda)$$

$$p_s(a, b, \lambda) = e(a, b) p_s(b|\lambda)$$



Stage 2: derive a contradiction

By checking how the boundaries shift (long) OR

$$p_s(\lambda) = [x(\alpha), 1 - x(\alpha)] \equiv p[\lambda(\Lambda)]$$

THREE INCOMPATIBLE ASSUMPTIONS

Adequacy | three assumptions
Objectivity: two types of statistics e_p, e_w
Determinism: HV determines all the outcomes
Independence: single HV that is not influenced by the settings

THREE INCOMPATIBLE ASSUMPTIONS

Adequacy | three assumptions
Objectivity: two types of statistics e_p, e_w
Determinism: HV determines all the outcomes
Independence: single HV that is not influenced by the settings

CONSISTENT =QM

SUMMARY

- ❑ Interactions are complicated
- ❑ Quantum control: practical & conceptual features
- ❑ Don’t (always) blame quantum mechanics

Thanks to

Roger Colbeck
Bert Englert
Mile Gu
Gerard Milburn
Alberto Peruzzo
Valerio Scarani