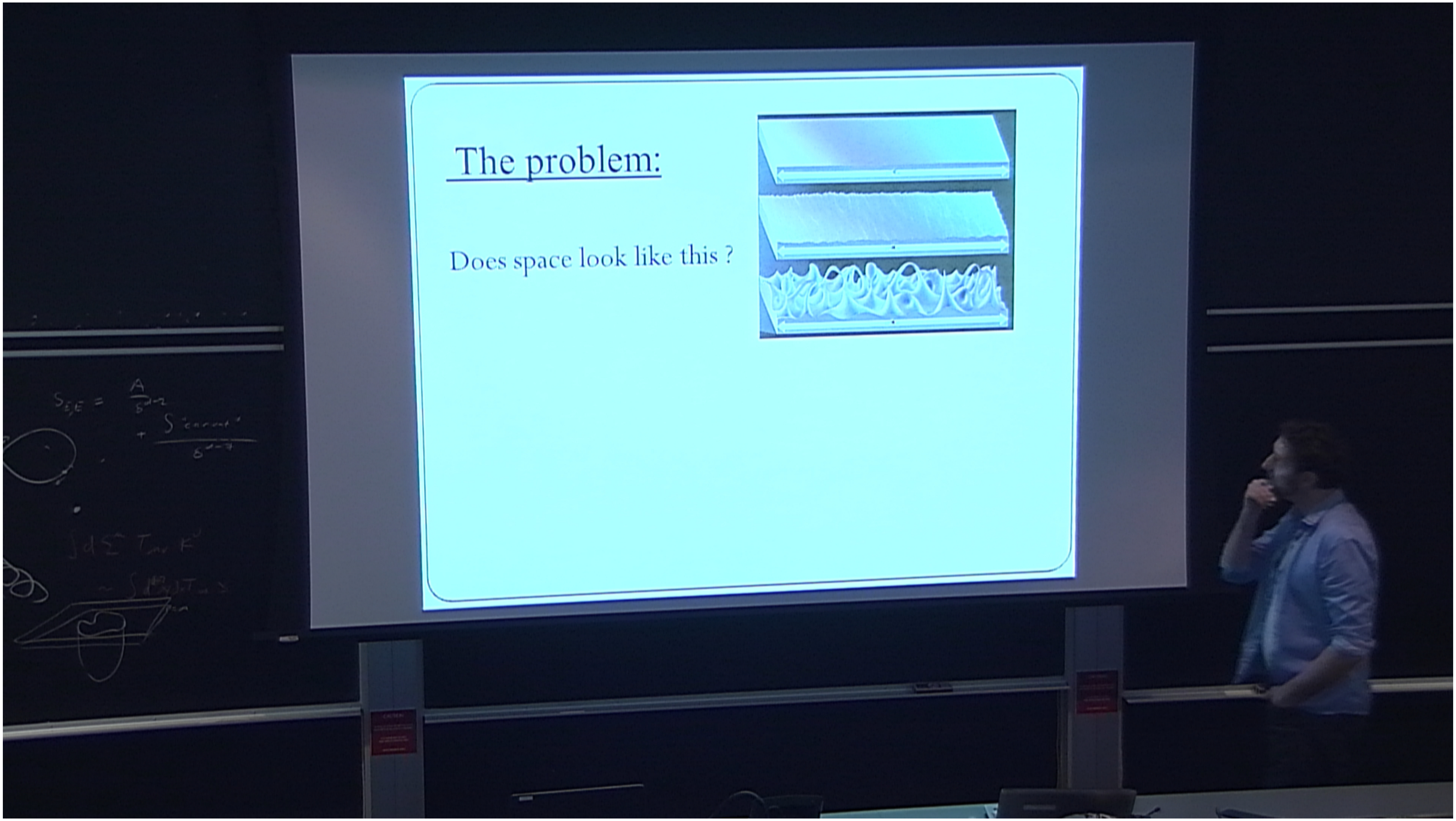
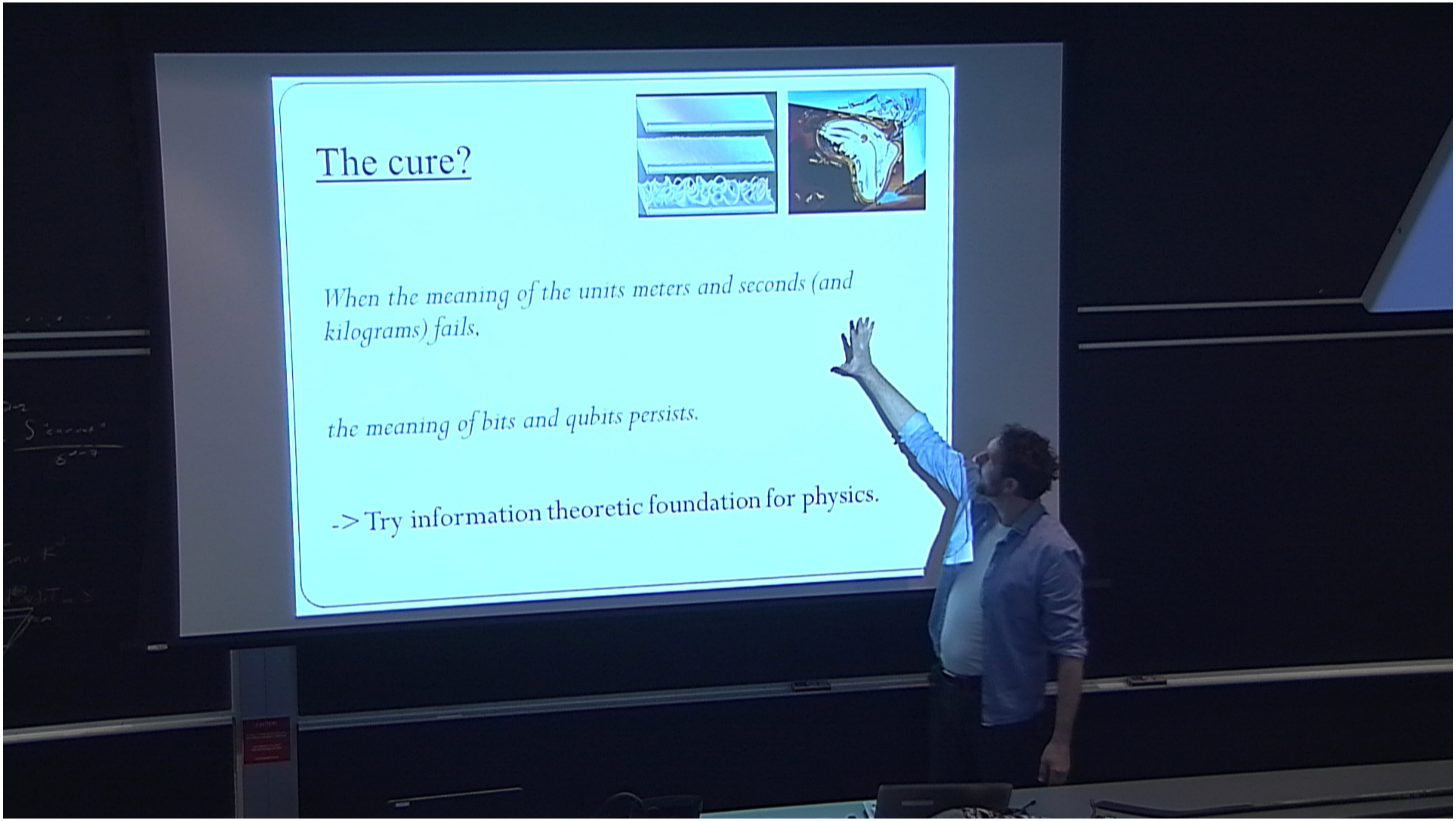
Title: What if nature is bandlimited by a Planck scale cutoff?

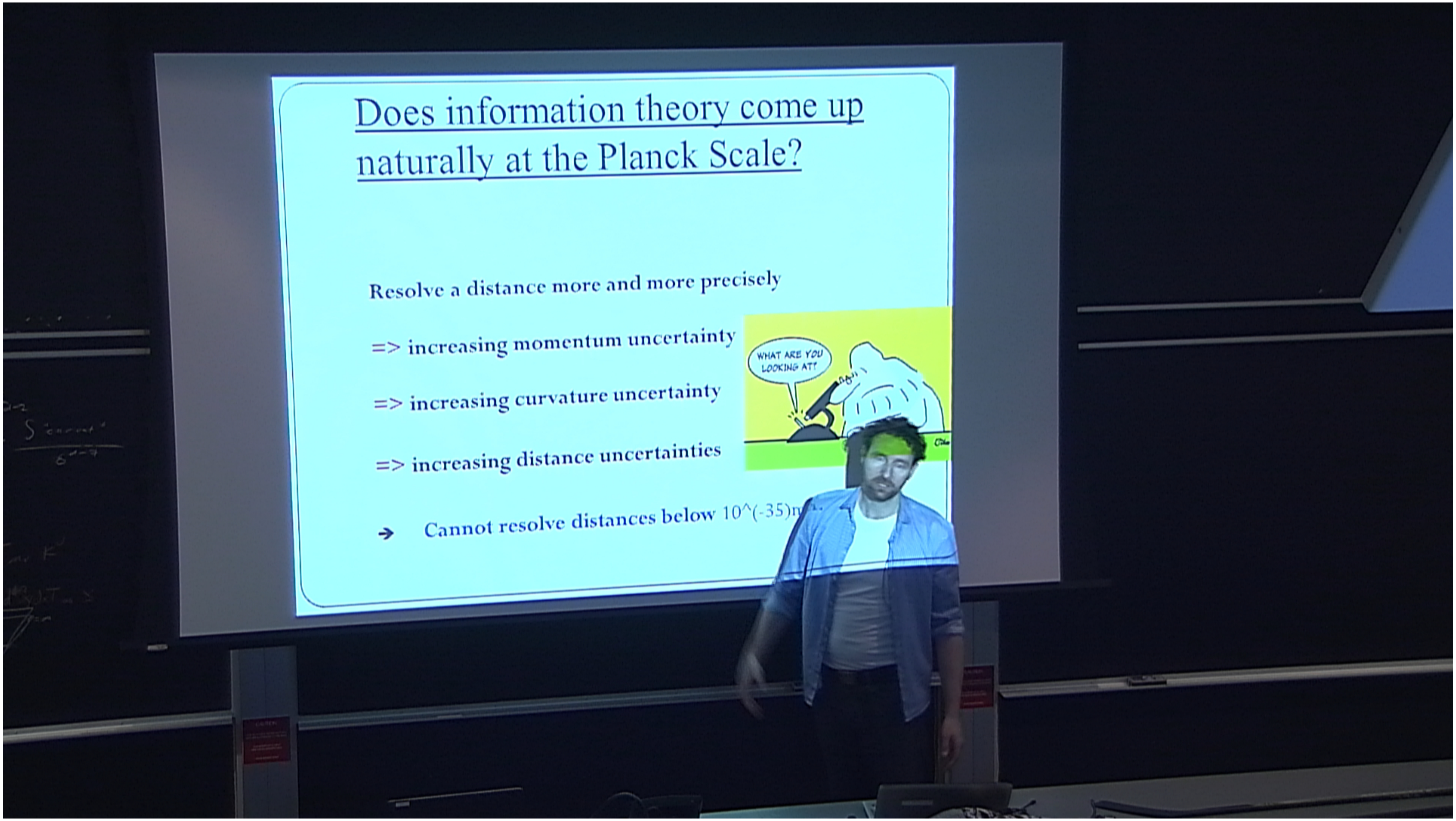
Date: May 13, 2015 11:00 AM

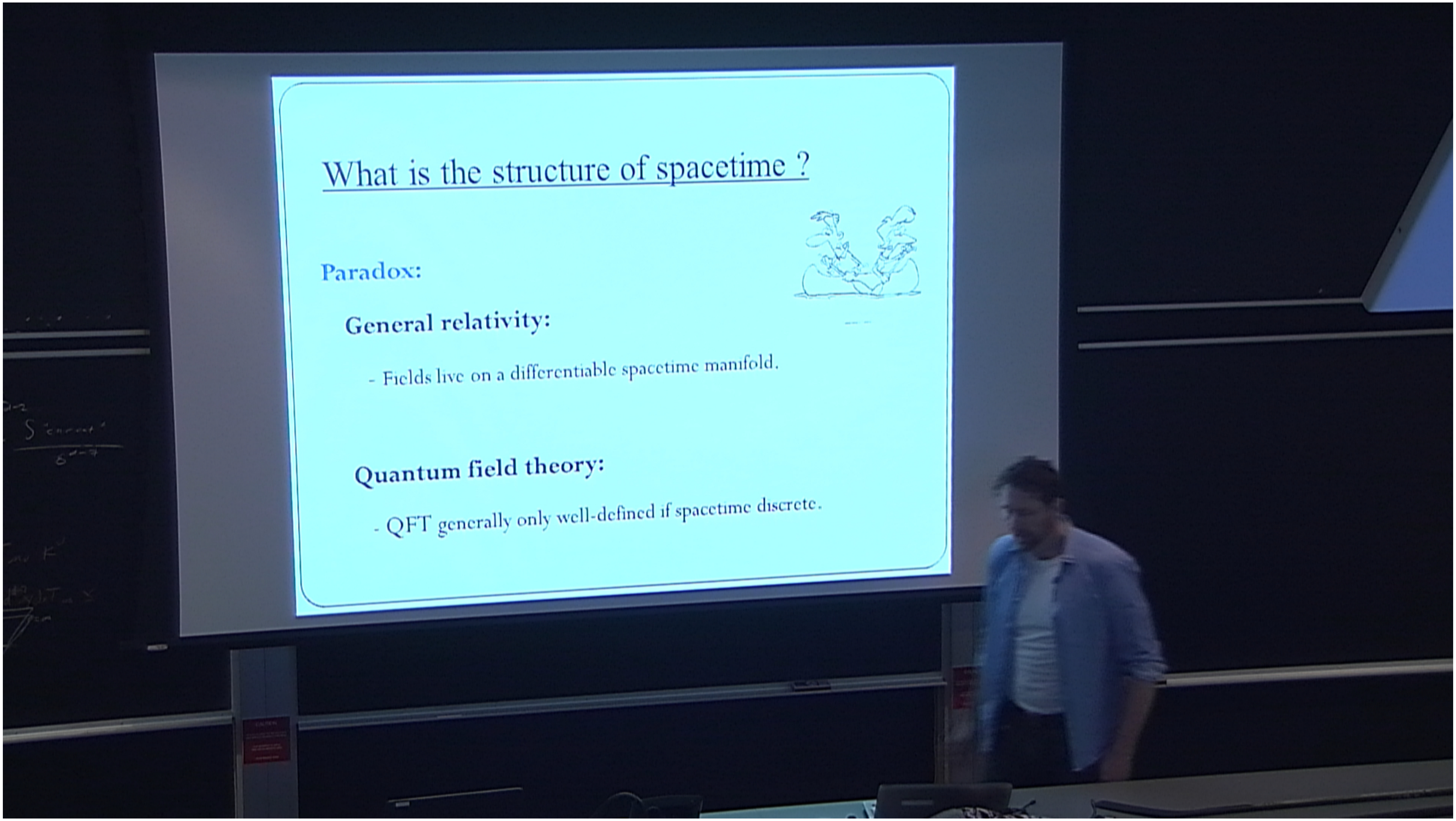
URL: http://pirsa.org/15050082

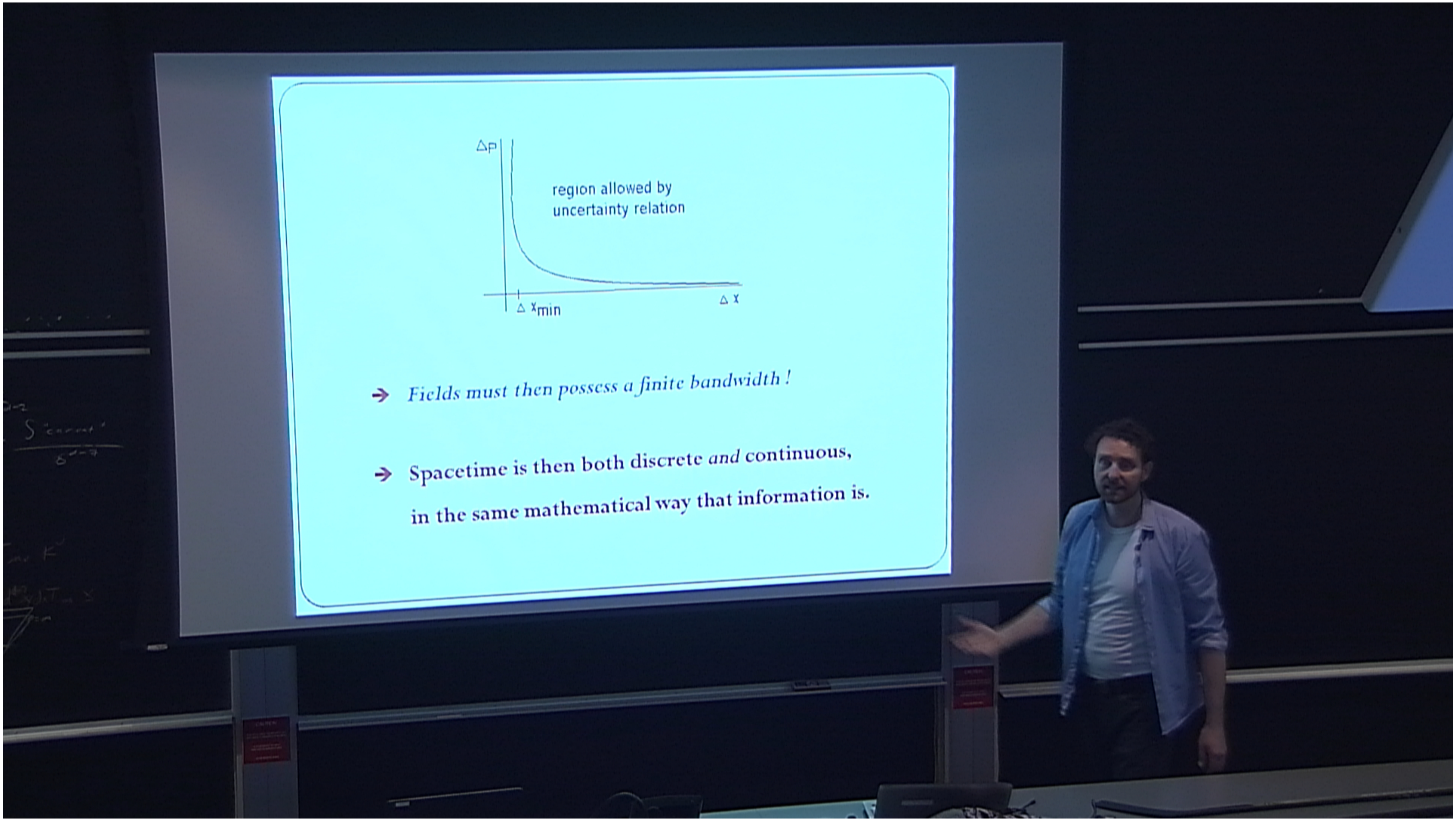
Abstract:

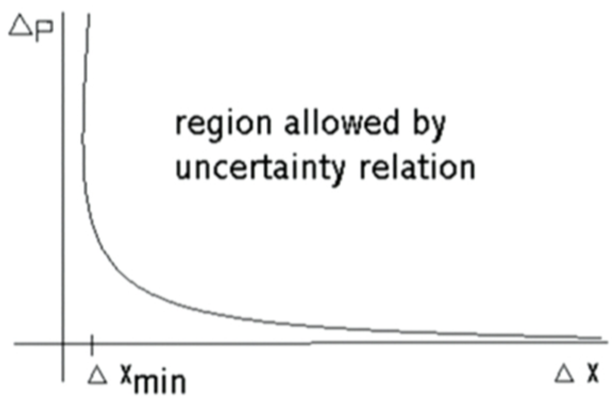

The problem:
Does space look like this ?


The cure?
When the meaning of the units meters and seconds (and kilograms) fails,
the meaning of bits and qubits persists.
-> Try information theoretic foundation for physics.


Does information theory come up naturally at the Planck Scale?
Resolve a distance more and more precisely
=> increasing momentum uncertainty
=> increasing curvature uncertainty
=> increasing distance uncertainties
Cannot resolve distances below 10^(-35)m
WHAT ARE YOU LOOKING AT?


What is the structure of spacetime ?
Paradox:
General relativity:
- Fields live on a differentiable spacetime manifold.
Quantum field theory:
- QFT generally only well-defined if spacetime discrete.


Δp
region allowed by
uncertainty relation
Δ x_min
Δ x
Fields must then possess a finite bandwidth !
Spacetime is then both discrete and continuous,
in the same mathematical way that information is.



➔ ***Fields must then possess a finite bandwidth !***

➔ **Spacetime is then both discrete *and* continuous,**

in the same mathematical way that information is.

Discrete = continuous, for information !

Information can be

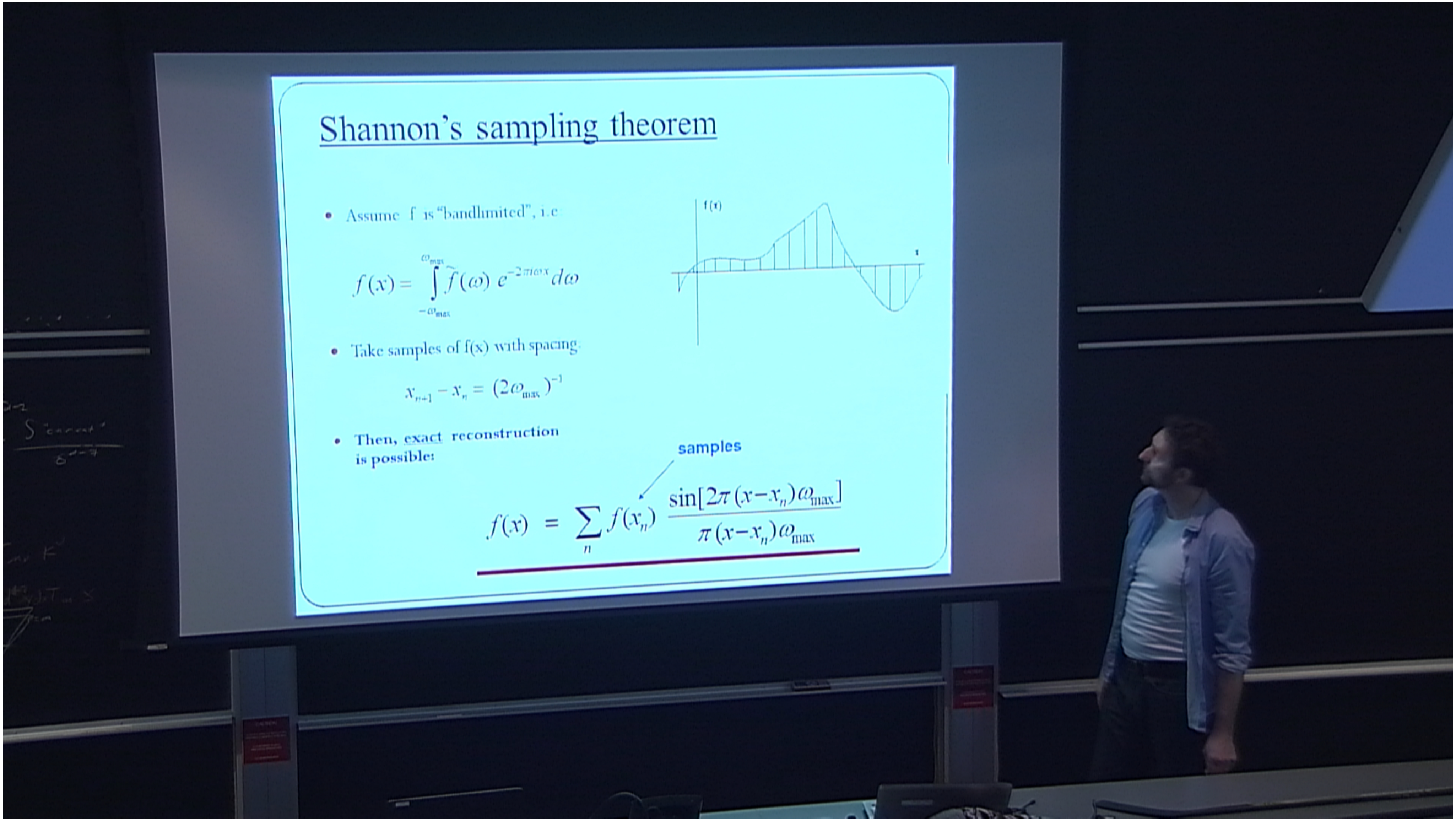
- continuous (e.g., music): 

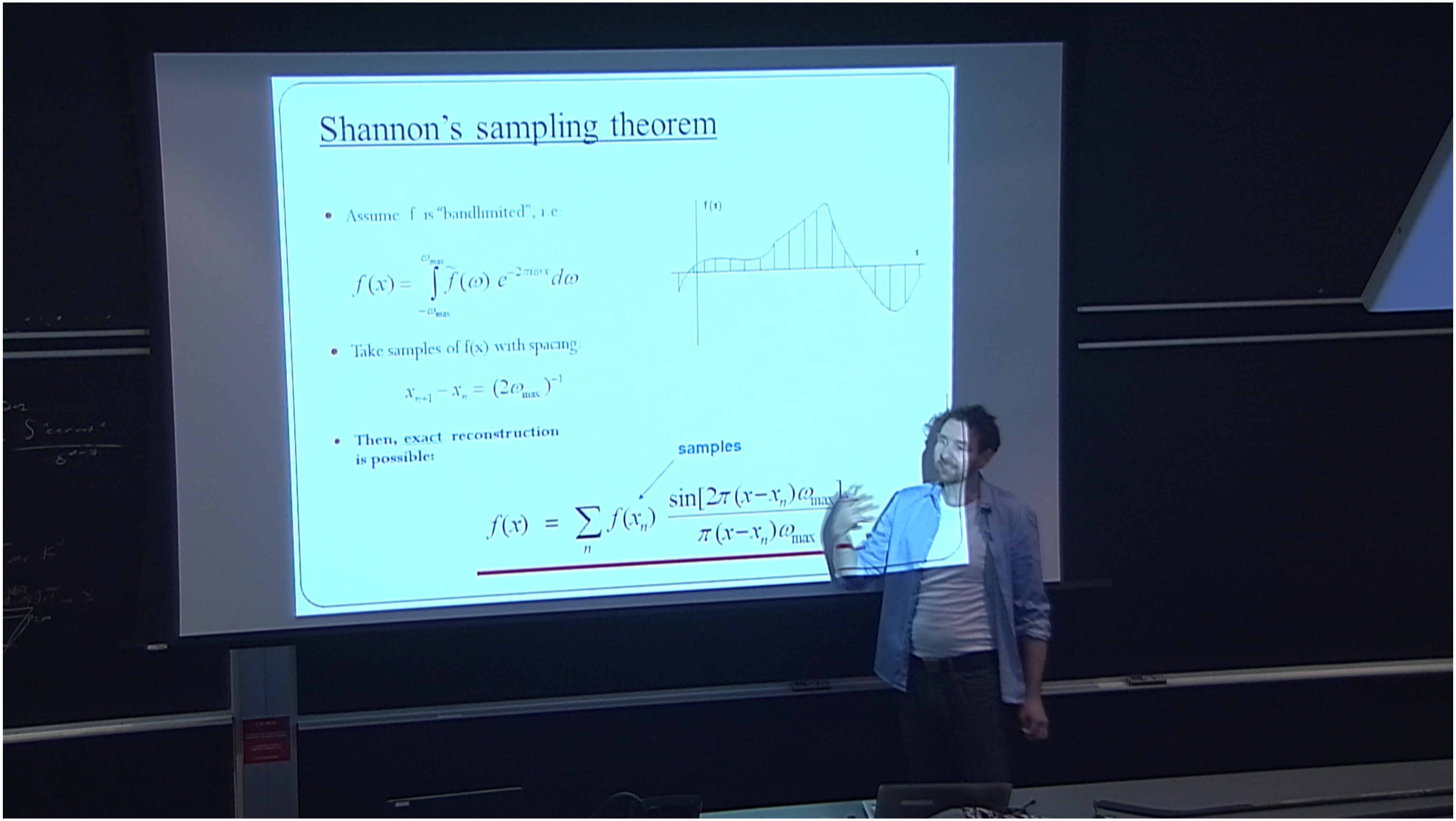
- discrete (letters, digits, etc): R 7 2 S B

Unified in 1949 by Shannon, through: **Sampling theory**

Applications ubiquitous:

- communication engineering & signal processing
- scientific data taking, e.g., in astronomy.


Shannon's sampling theorem
Assume f is "bandlimited", i.e.
$f(x) = \int_{-\omega_{max}}^{\omega_{max}} \tilde{f}(\omega)\, e^{-2\pi i \omega x}\, d\omega$
f(x)
x
Take samples of f(x) with spacing:
$x_{n+1} - x_n = (2\omega_{max})^{-1}$
Then, exact reconstruction is possible:
samples
$f(x) = \sum_n f(x_n) \frac{\sin[2\pi(x-x_n)\omega_{max}]}{\pi(x-x_n)\omega_{max}}$


Shannon's sampling theorem
Assume f is "bandlimited", i.e.
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Take samples of f(x) with spacing:
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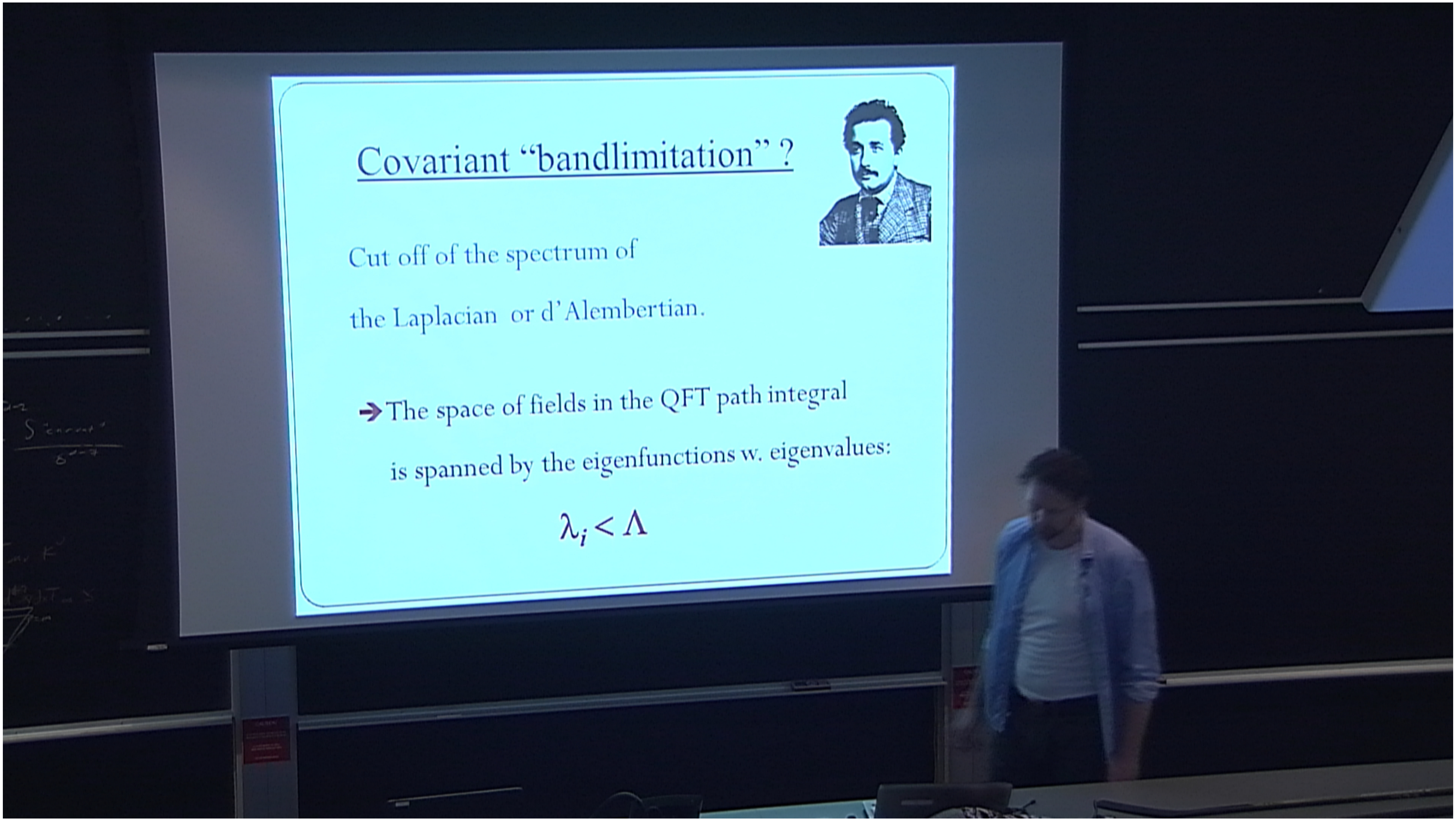
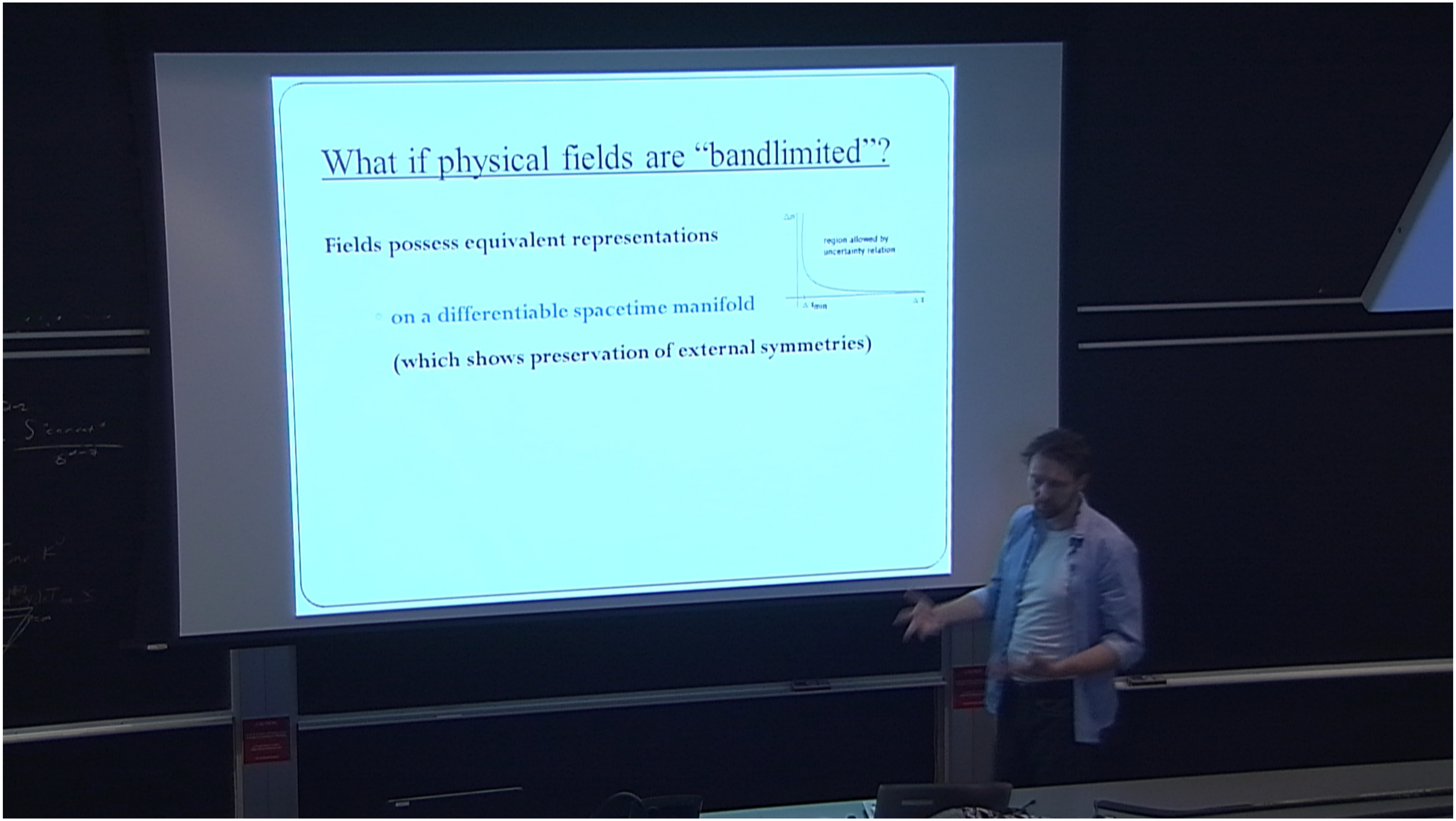
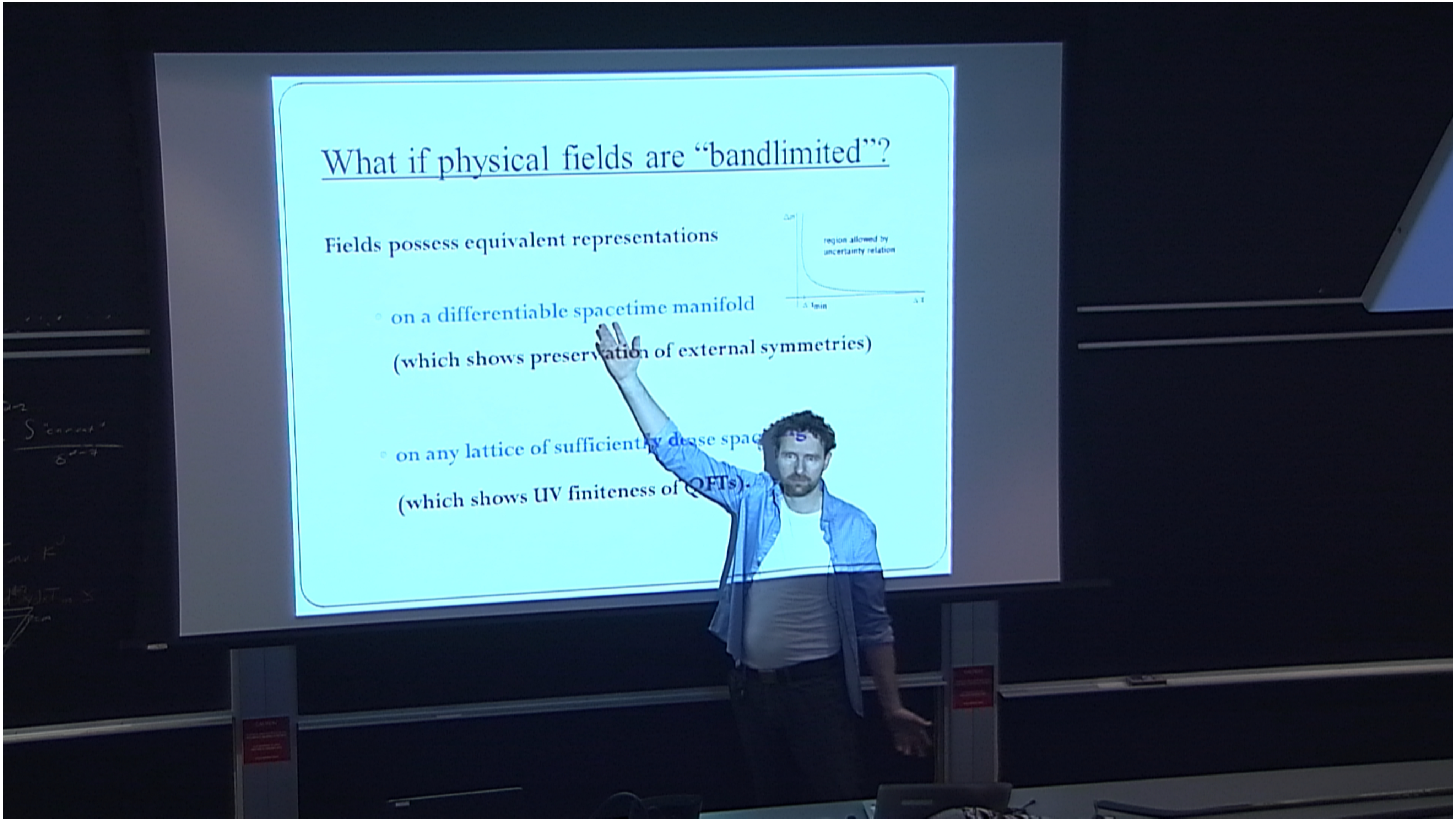

Covariant "bandlimitation" ?
Cut off of the spectrum of
the Laplacian or d'Alembertian.
➔The space of fields in the QFT path integral
is spanned by the eigenfunctions w. eigenvalues:
$\lambda_i < \Lambda$


What if physical fields are "bandlimited"?
Fields possess equivalent representations
on a differentiable spacetime manifold
(which shows preservation of external symmetries)
region allowed by
uncertainty relation


What if physical fields are “bandlimited”?
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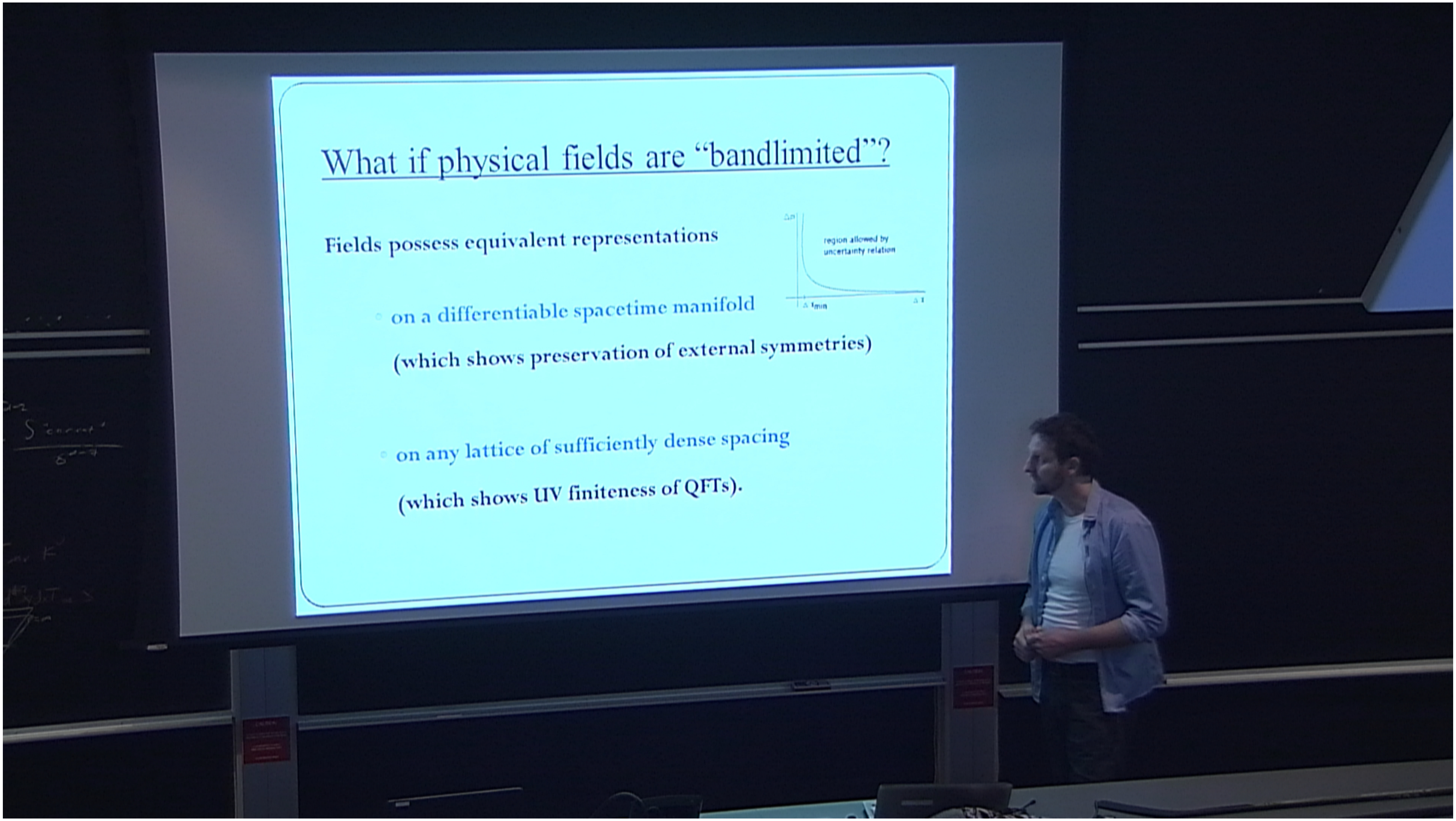
What if physical fields are "bandlimited"?
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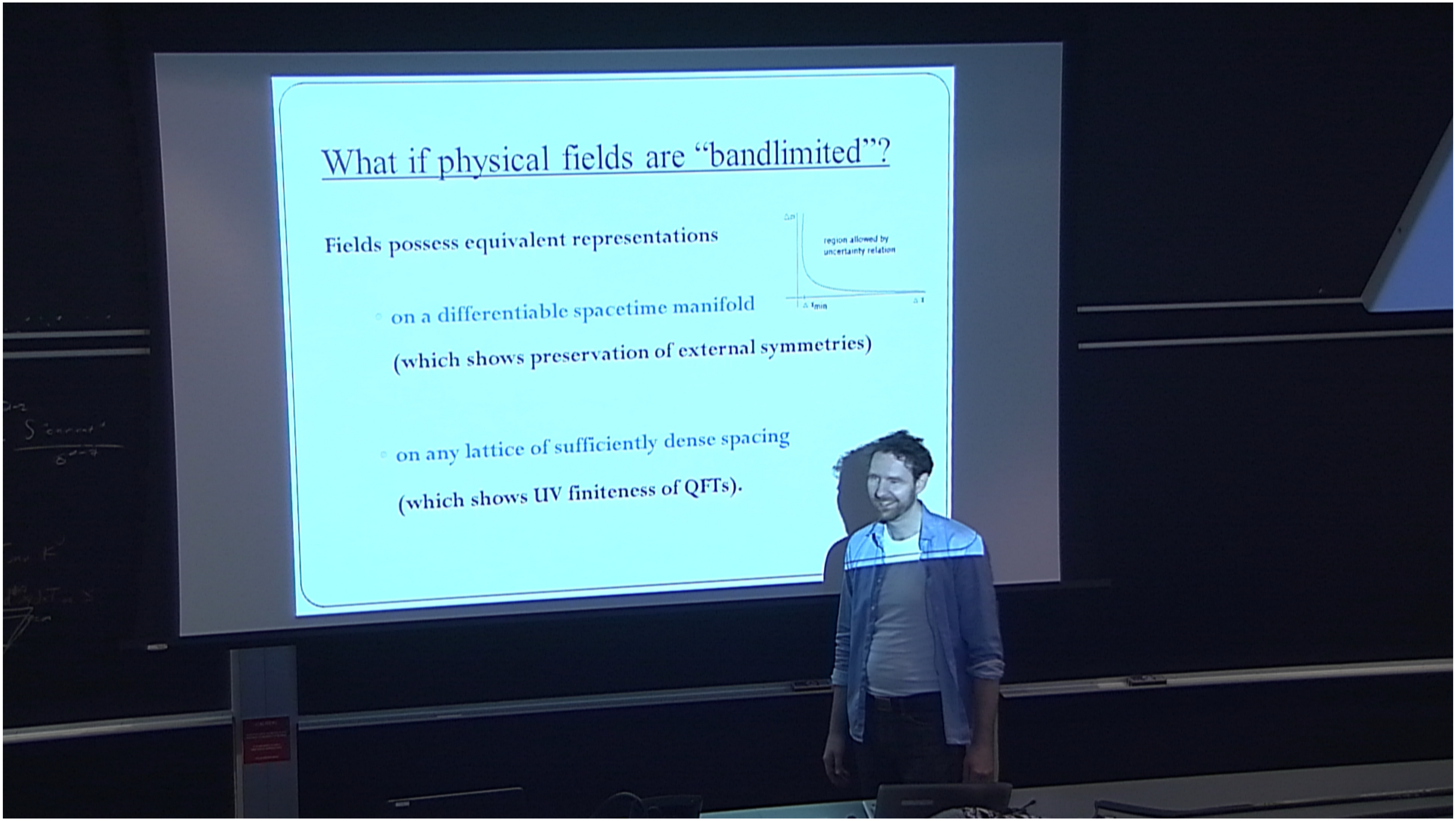
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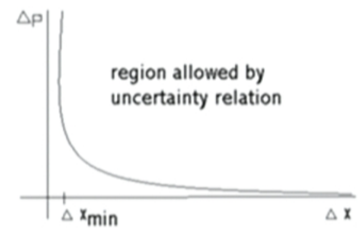
Fields possess equivalent representations

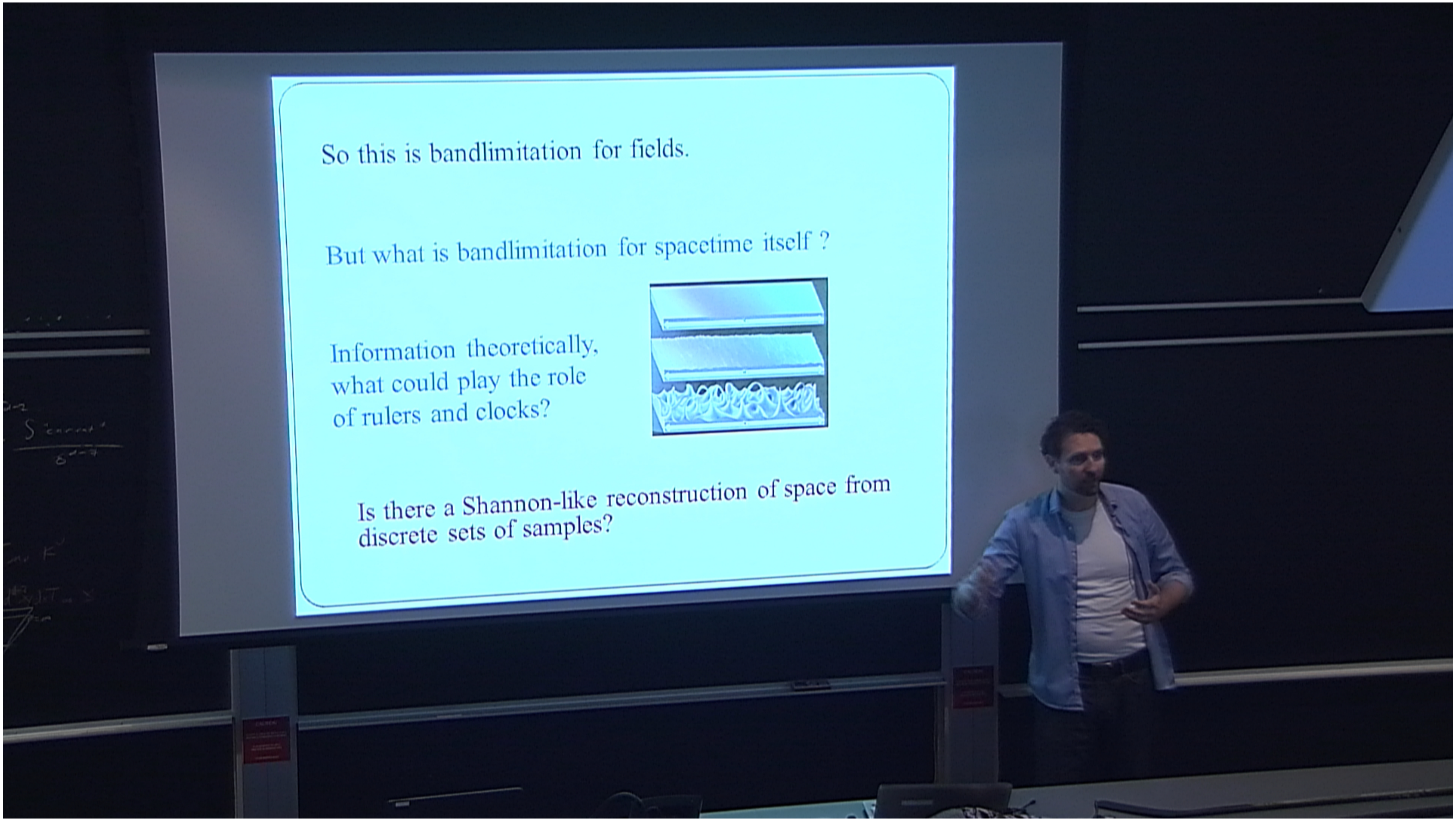


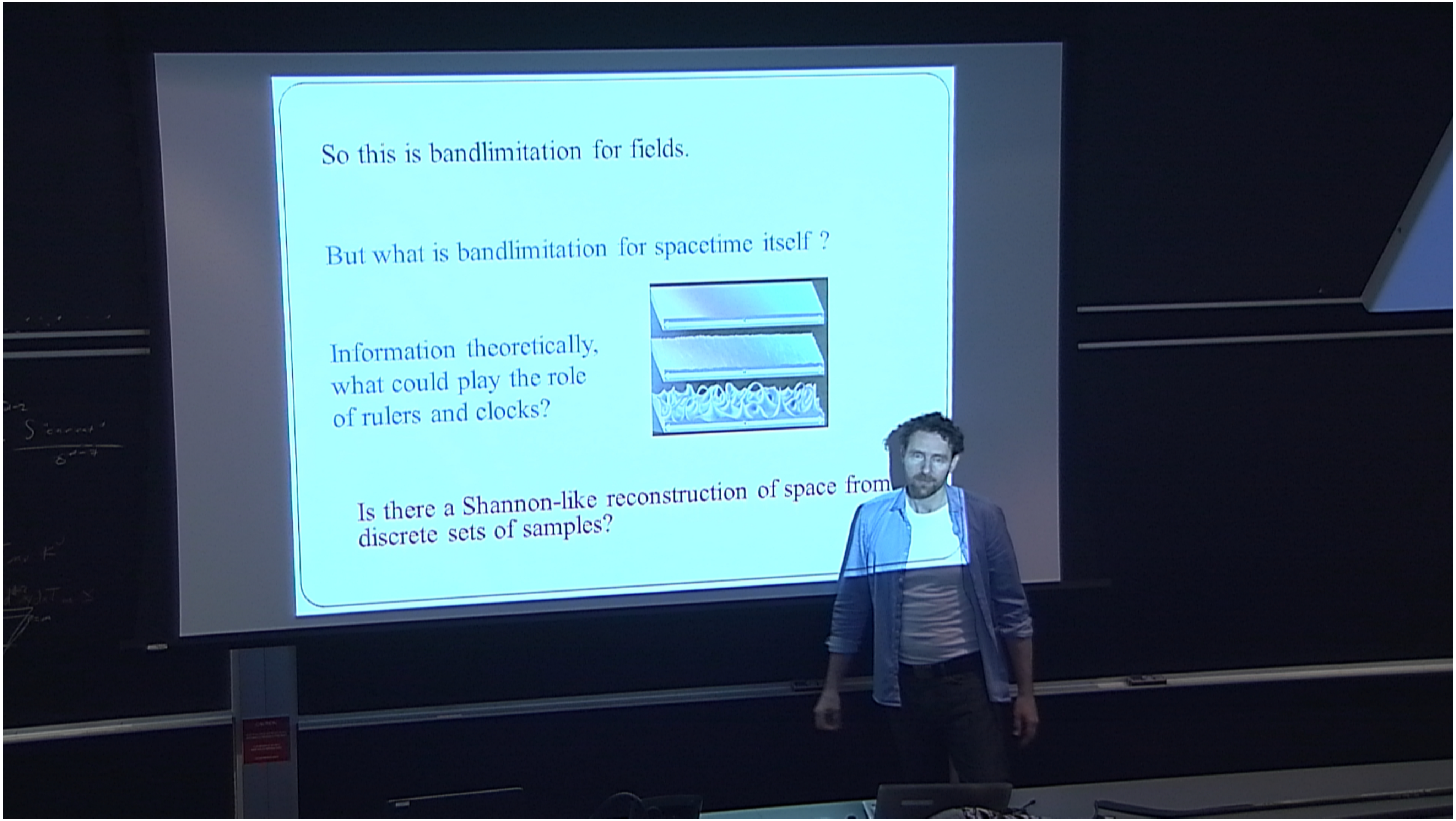
- **on a differentiable spacetime manifold**

  (which shows preservation of external symmetries)

- **on any lattice of sufficiently dense spacing**

  (which shows UV finiteness of QFTs).


So this is bandlimitation for fields.
But what is bandlimitation for spacetime itself ?
Information theoretically, what could play the role of rulers and clocks?
Is there a Shannon-like reconstruction of space from discrete sets of samples?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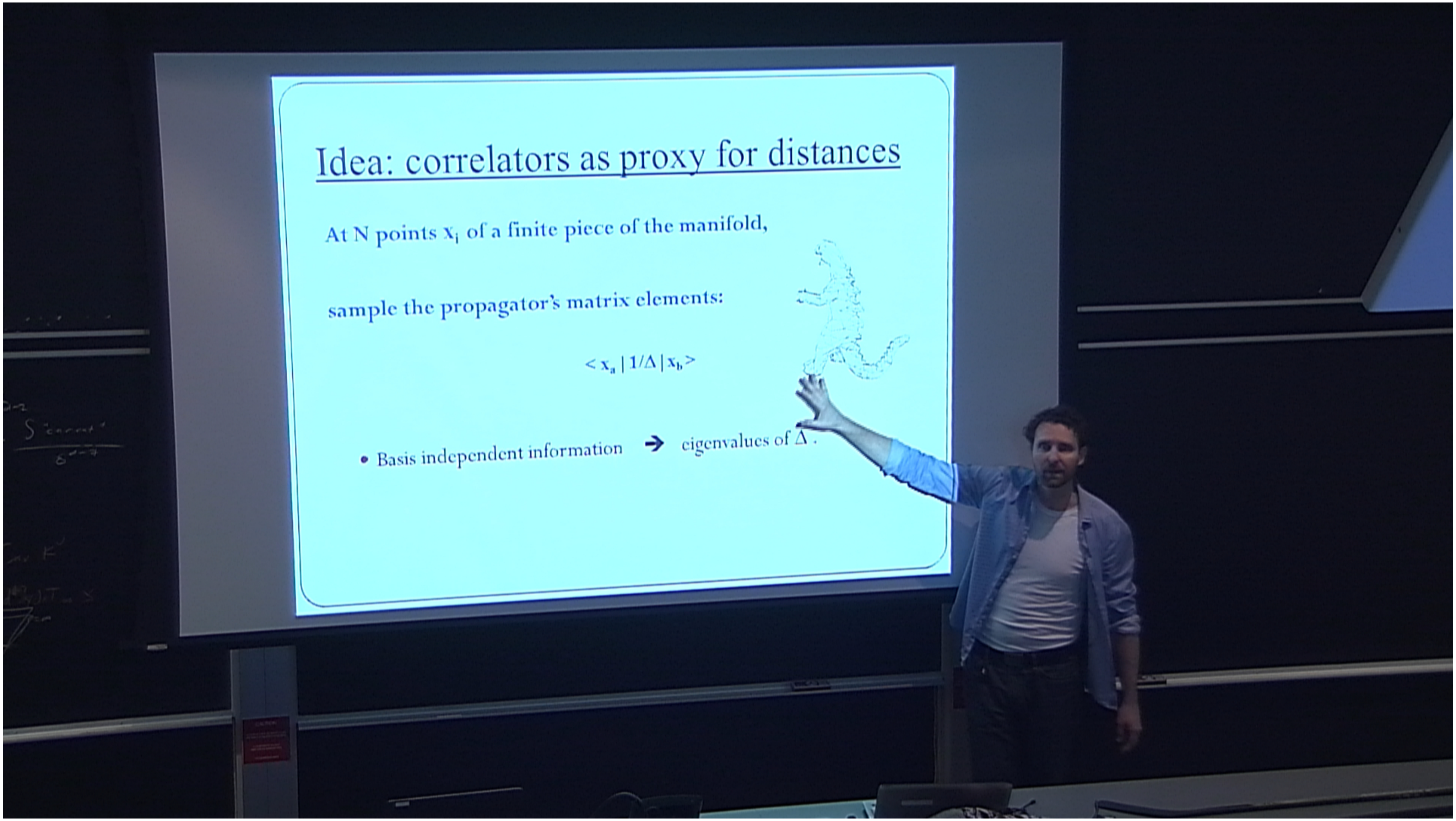
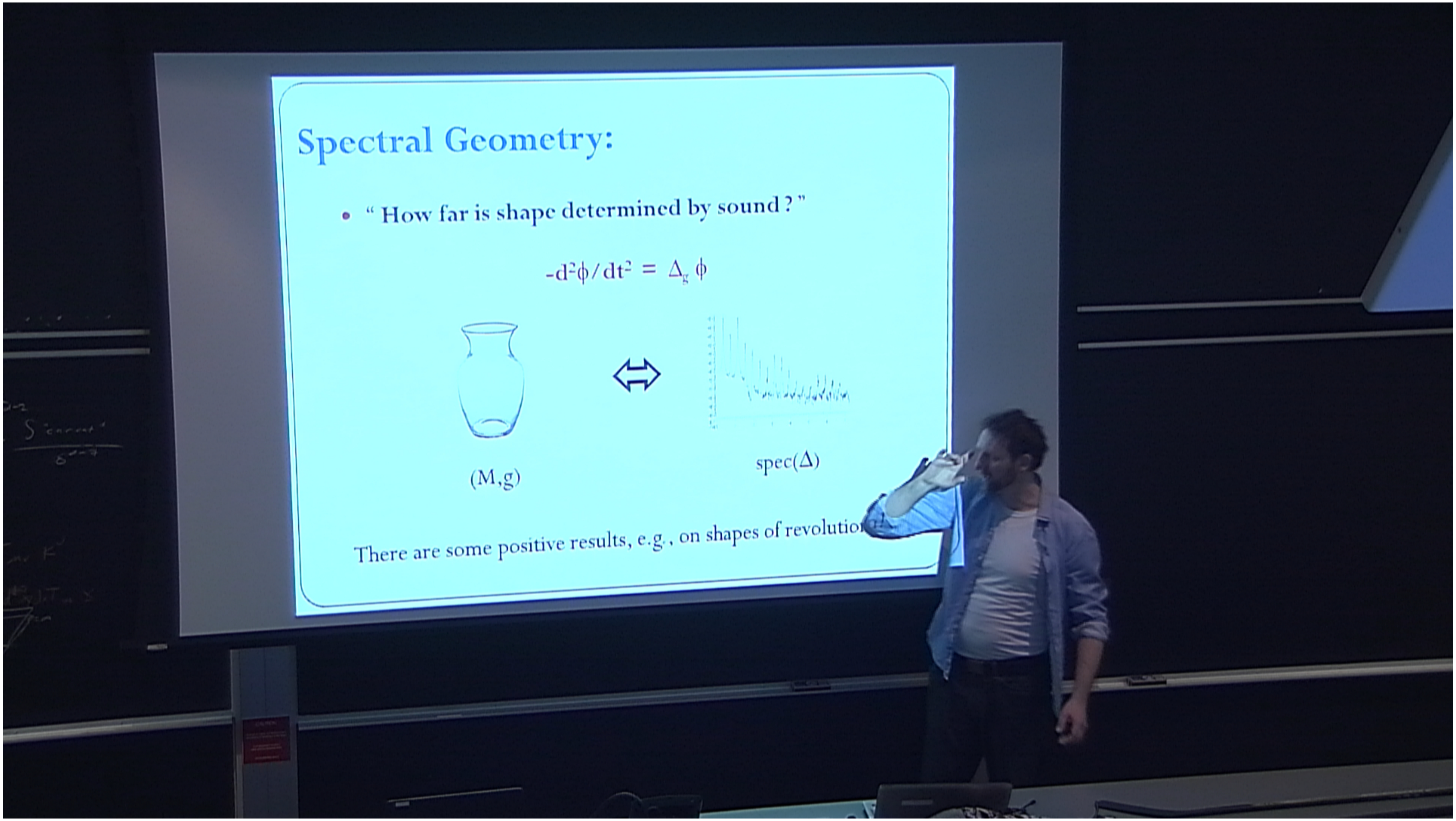
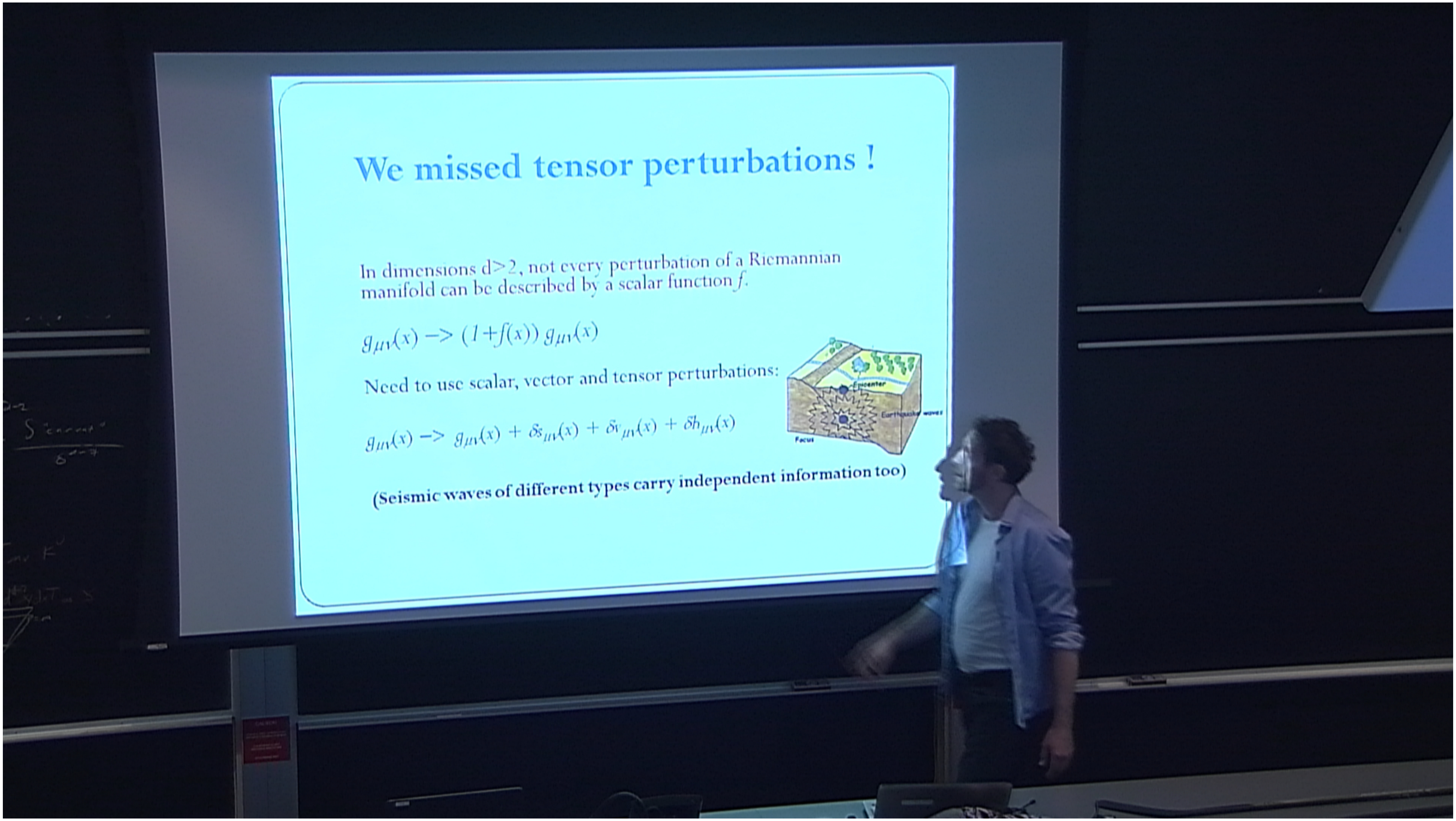
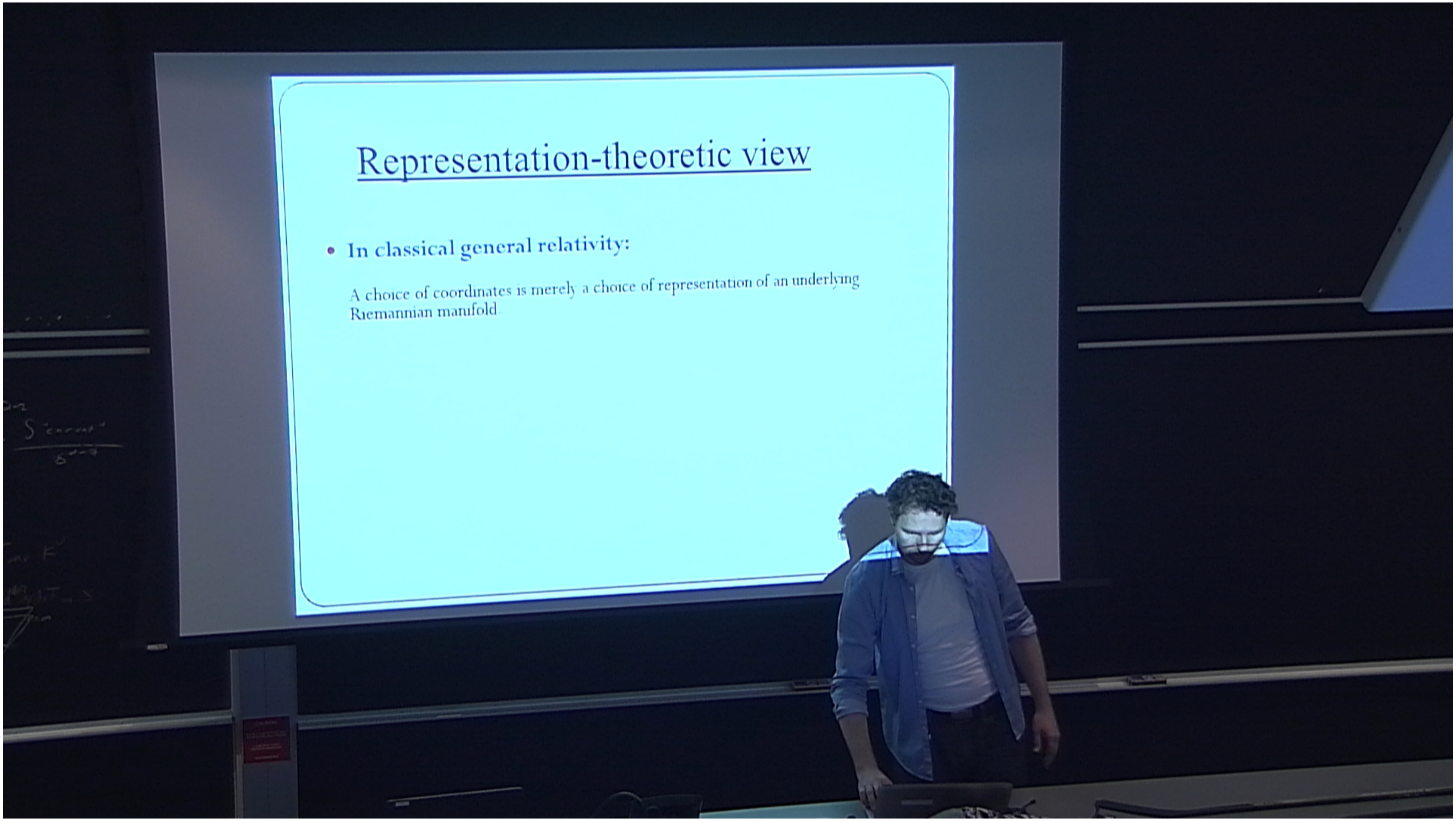

Idea: correlators as proxy for distances
At N points x_i of a finite piece of the manifold,
sample the propagator's matrix elements:
$< x_a \mid 1/\Delta \mid x_b >$
• Basis independent information → eigenvalues of Δ .

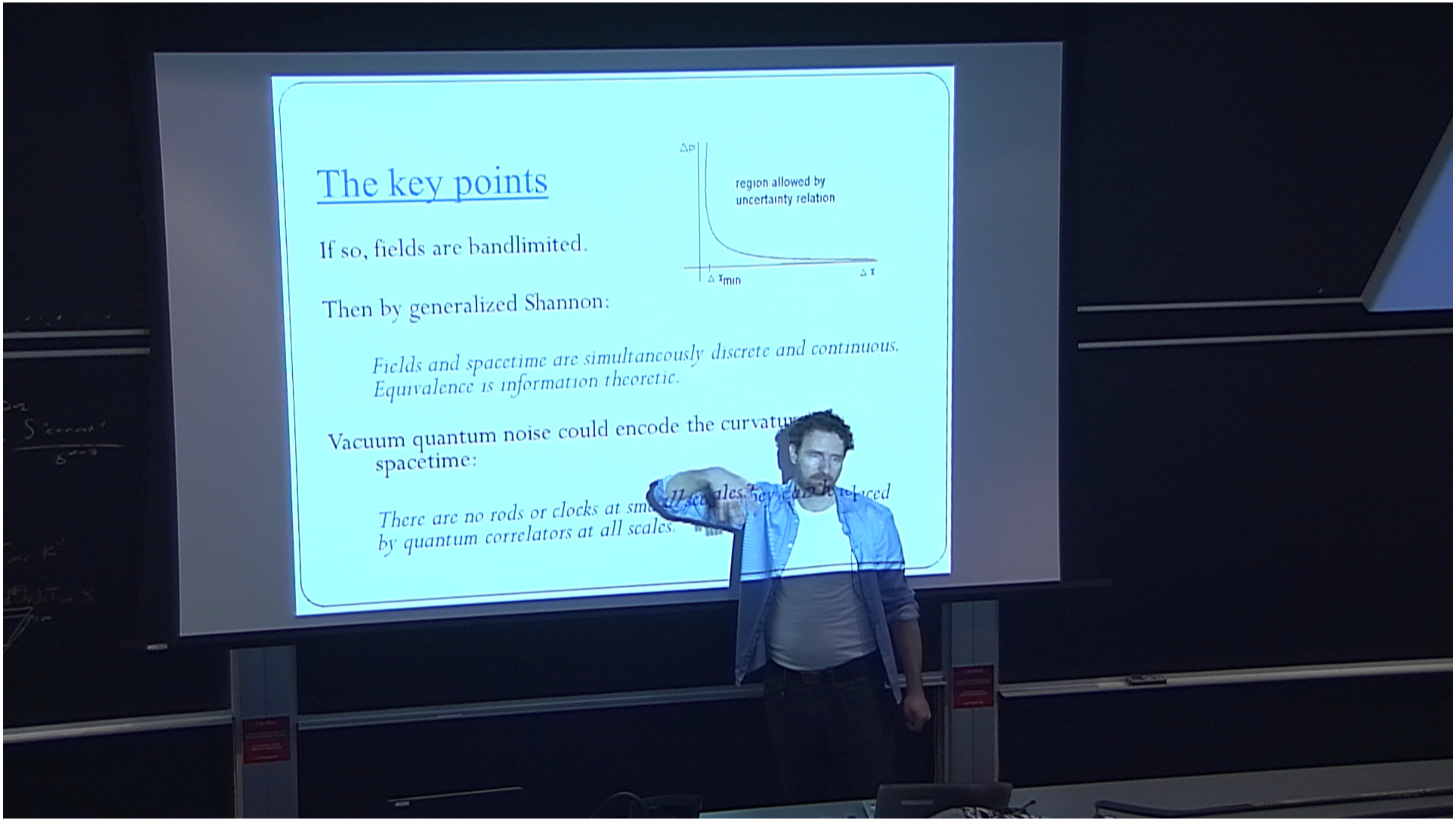

Spectral Geometry:
• “ How far is shape determined by sound ? ”
-d²φ/dt² = Δg φ
(M,g)
spec(Δ)
There are some positive results, e.g., on shapes of revolutio


Prospect:
Can one hear a spacetime's curvature in its quantum noise?
Gravitational Lens in Abell 2218
HST · WFPC2
Deep link between gravity and quantum theory?


Counting degrees of freedom
# of eigenvalues = # of eigenfunctions
$\{f_n\}$ and $\{\mu_n\}$ both now have N coefficients.
$\{S_{nm}\}$ is a generic square NxN matrix,
which should generically be invertible.
Let's carry it out explicitly and iterate!


We missed tensor perturbations !
In dimensions d>2, not every perturbation of a Riemannian manifold can be described by a scalar function f.
$g_{\mu\nu}(x) \rightarrow (1+f(x))\, g_{\mu\nu}(x)$
Need to use scalar, vector and tensor perturbations:
$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \delta s_{\mu\nu}(x) + \delta v_{\mu\nu}(x) + \delta h_{\mu\nu}(x)$
(Seismic waves of different types carry independent information too)
Epicenter
Focus


Representation-theoretic view
In classical general relativity:
A choice of coordinates is merely a choice of representation of an underlying Riemannian manifold.


The key points
If so, fields are bandlimited.
Then by generalized Shannon:
Fields and spacetime are simultaneously discrete and continuous.
Equivalence is information theoretic.
Vacuum quantum noise could encode the curvatu
spacetime:
There are no rods or clocks at sm
by quantum correlators at all scales.
region allowed by
uncertainty relation
Δp
Δx
Δ x_min