

Title: Nonclassicality as the failure of noncontextuality

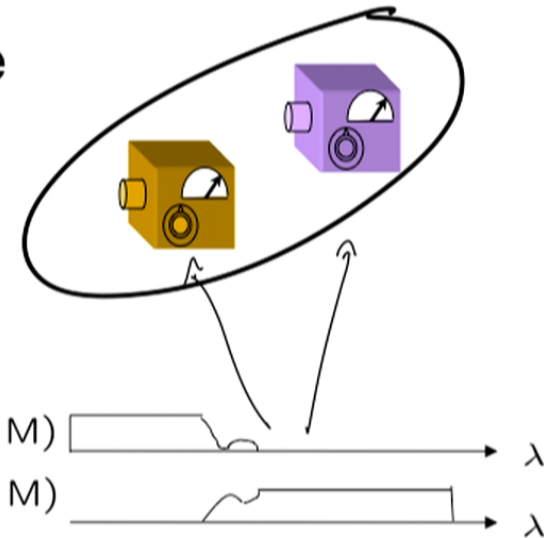
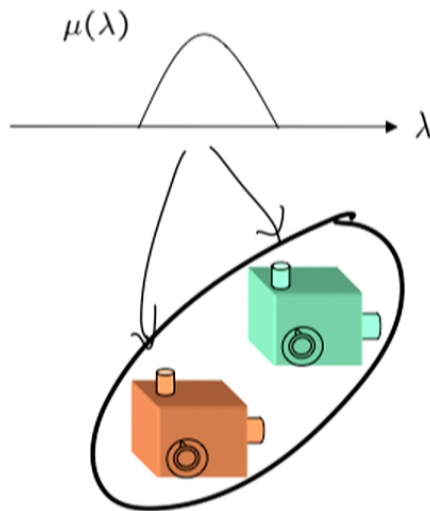
Date: May 12, 2015 11:50 AM

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Abstract: To make precise the sense in which nature fails to respect classical physics, one requires a formal notion of "classicality". Ideally, such a notion should be defined operationally, so that it can be subjected to a direct experimental test, and it should be applicable in a wide variety of experimental scenarios, so that it can cover the breadth of phenomena that are thought to defy classical understanding. Bell's notion of local causality fulfills the first criterion but not the second, because it is restricted to scenarios with two or more systems that are space-like separated. The notion of noncontextuality fulfills the second criterion, because it is applicable to any experiment (even those on a single system), but it is a long-standing question whether it can be made to fulfill the first. Previous attempts to experimentally test noncontextuality have all presumed certain idealizations that do not hold in real experiments, namely, noiseless measurements and exact operational equivalences. In this talk, I will describe how one can devise experimental tests that are free of these idealizations using an operational notion of noncontextuality that applies to both preparations and measurements. These new theoretical insights raise the bar significantly for any claim of an experimental demonstration of nonclassicality. They also provide the means of determining, for any phenomenon that is typically thought to defy classical explanation, which experimentally-testable features of that phenomenon, if any, conflict with the assumption of a noncontextual model.

Nonclassicality as the failure of noncontextuality

Robert Spekkens
Perimeter Institute



Information Theoretic Foundations of Quantum Theory
PI, May 12, 2015

What we want in a notion of nonclassicality

Subject to direct
experimental test

Applicable to a
broad range of
physical scenarios

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Failure of local causality



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Failure of noncontextuality



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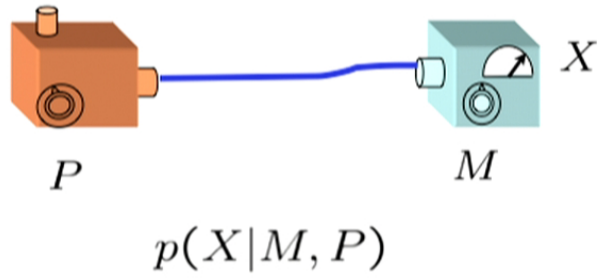
Failure of local causality



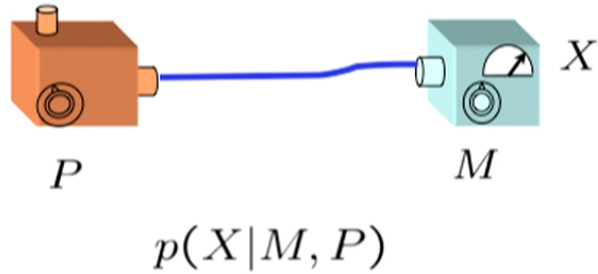
Failure of noncontextuality



Operational theory



Operational theory

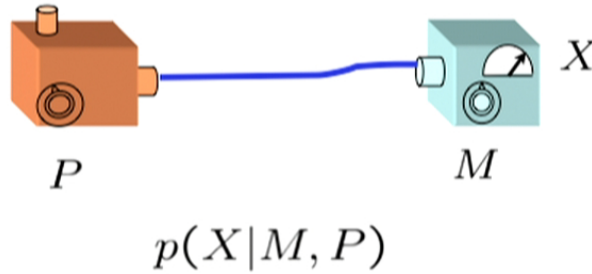


Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

λ causally mediates
between P and M

Operational theory

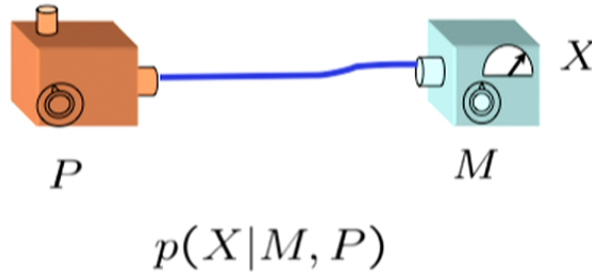


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Ontological model of an operational theory

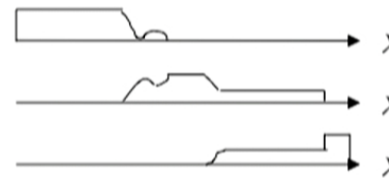
$\lambda \in \Lambda$ Ontic state space

λ causally mediates
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$$P \leftrightarrow \mu(\lambda|P)$$



$$M \leftrightarrow \xi(X|M, \lambda)$$



Operational theory



$$p(X|M, P)$$

Ontological model of an operational theory

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$$P \leftrightarrow \mu(\lambda|P)$$



$$M \leftrightarrow \xi(X|M, \lambda)$$



$$p(X|M, P) = \int \xi(X|M, \lambda) \mu(\lambda|P) d\lambda$$

An ontological model of an operational theory is **noncontextual** if

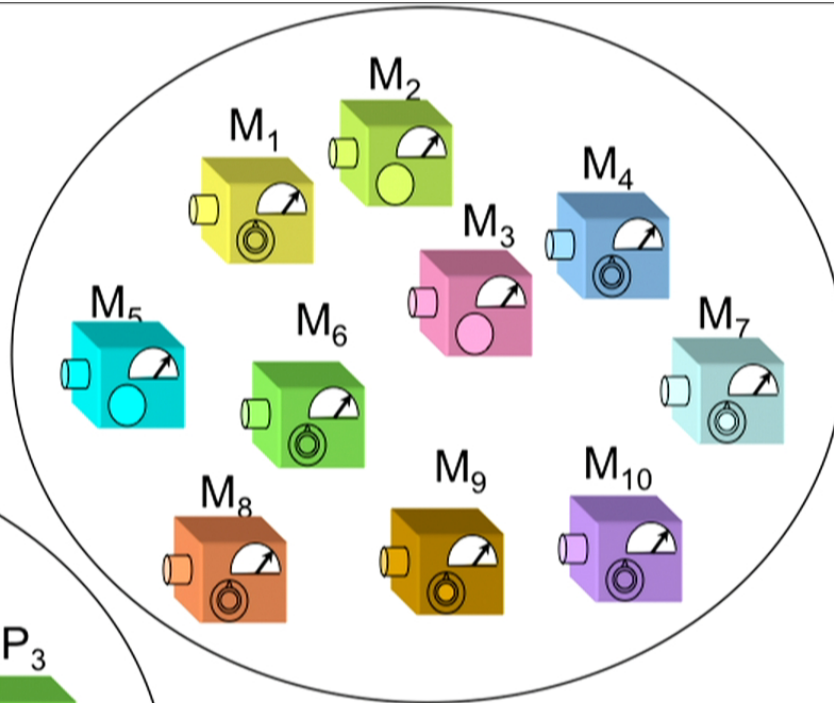
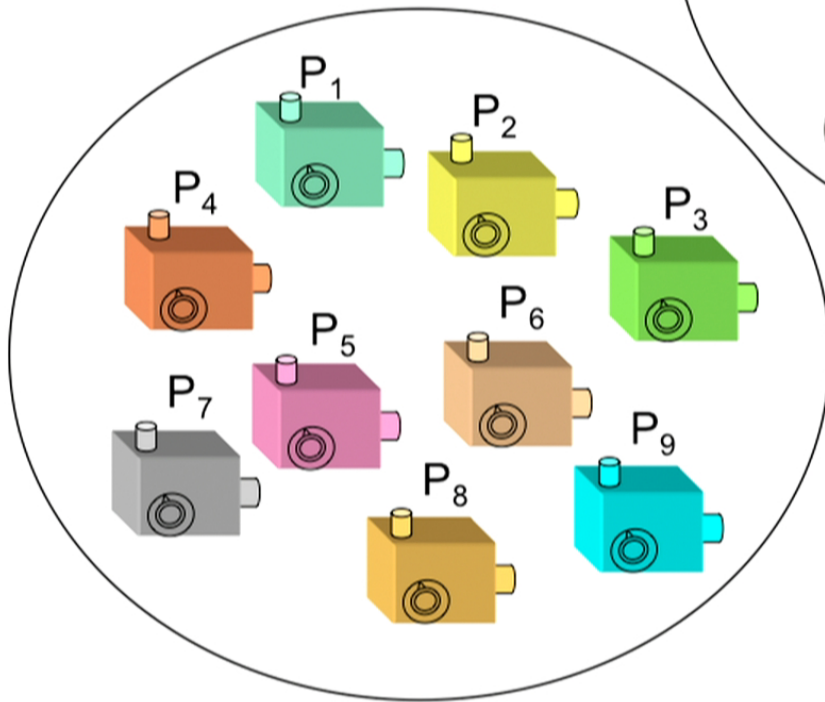
Operational equivalence of
two experimental
procedures



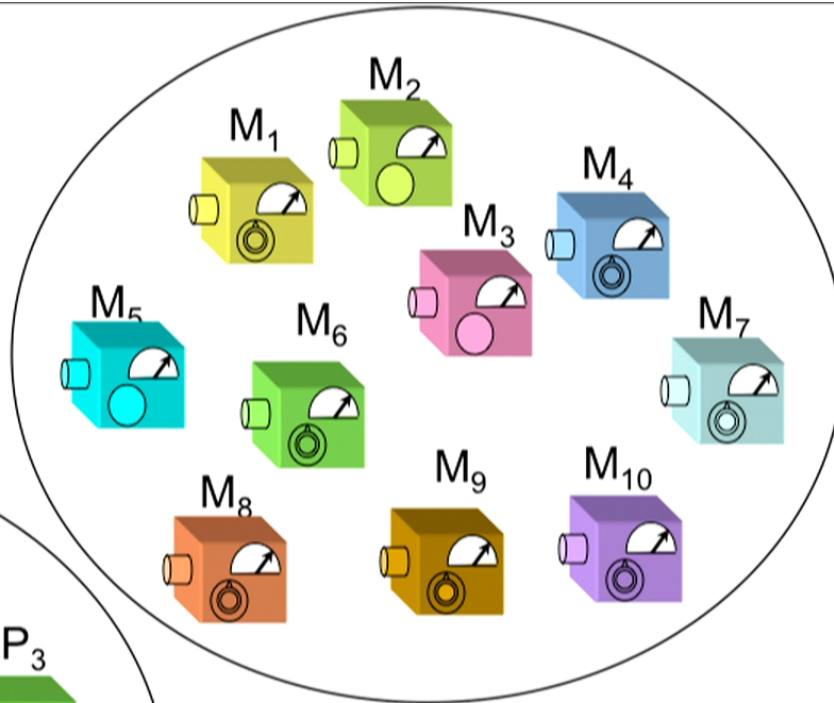
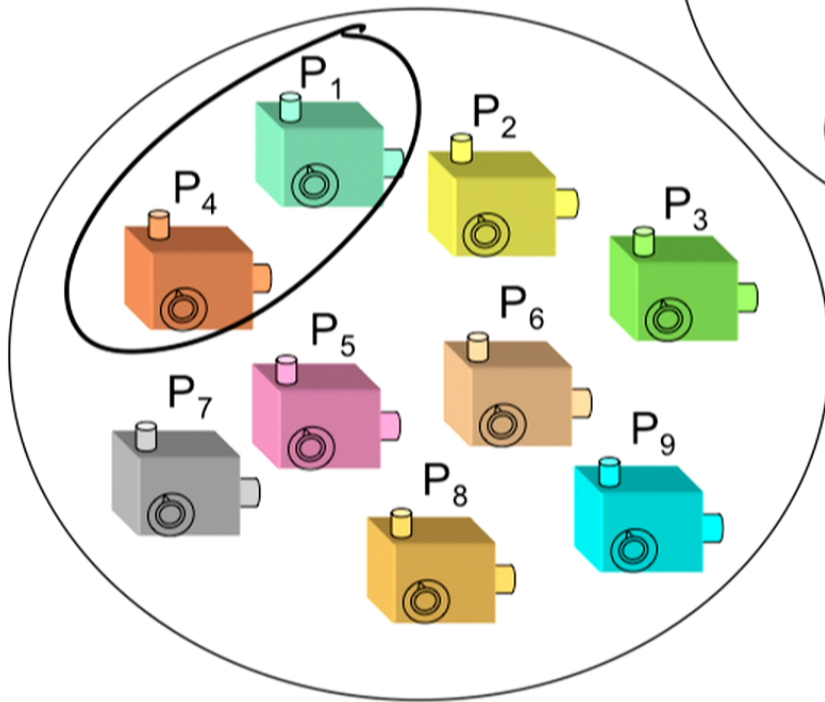
Equivalent
representations
in the ontological
model

RWS, Phys. Rev. A 71, 052108 (2005)

Operational equivalence classes of preparations



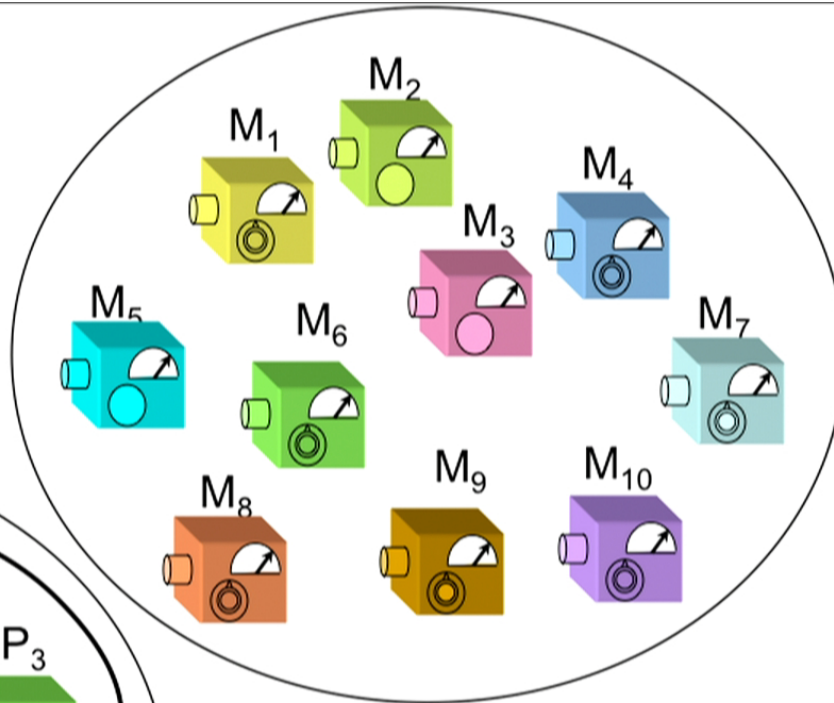
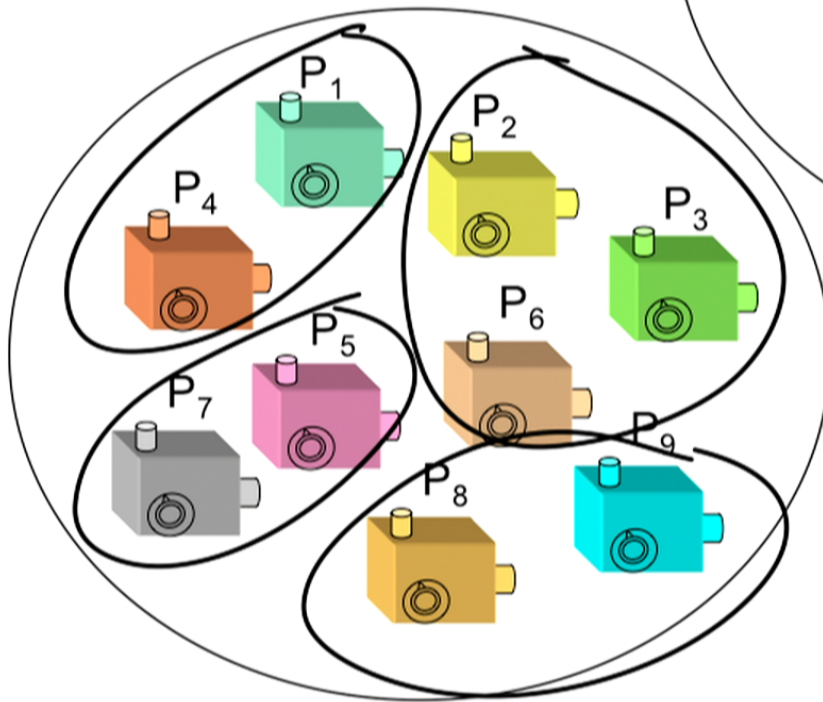
Operational equivalence classes of preparations



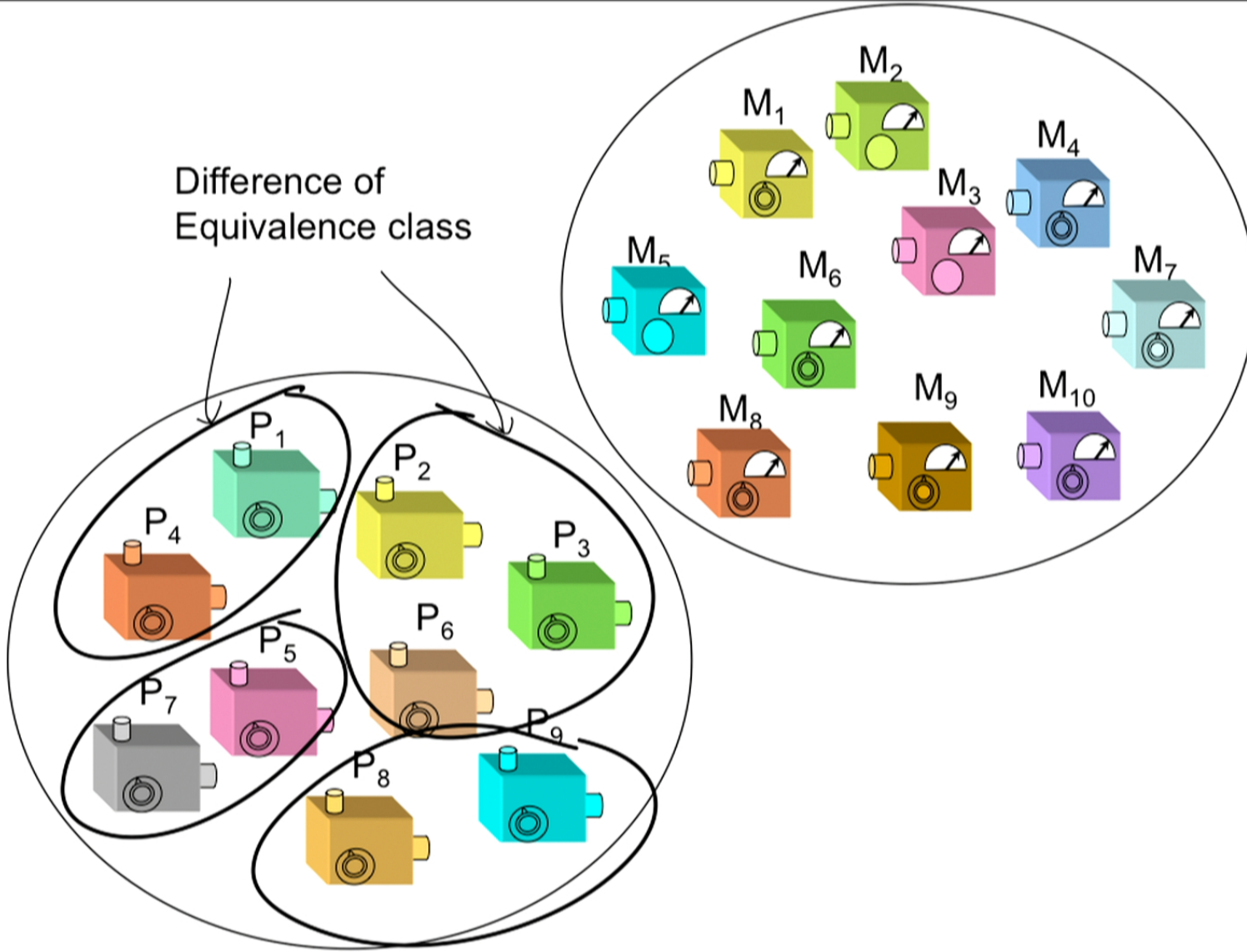
Operational equivalence
classes of preparations

$$e(P) = e(P')$$

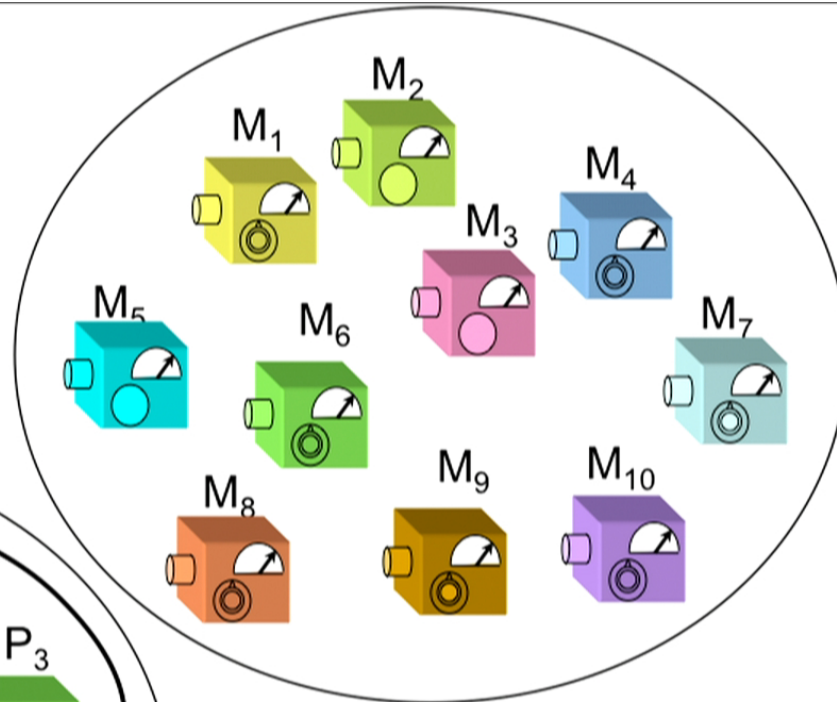
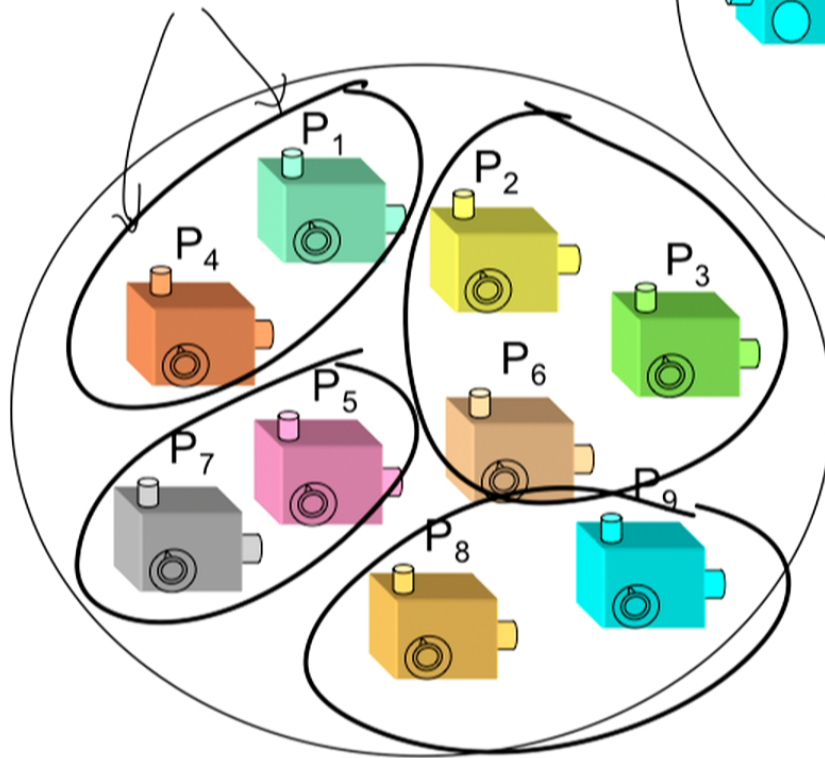
$$\forall M : p(X|P, M) = p(X|P', M)$$



Difference of
Equivalence class



Difference of context

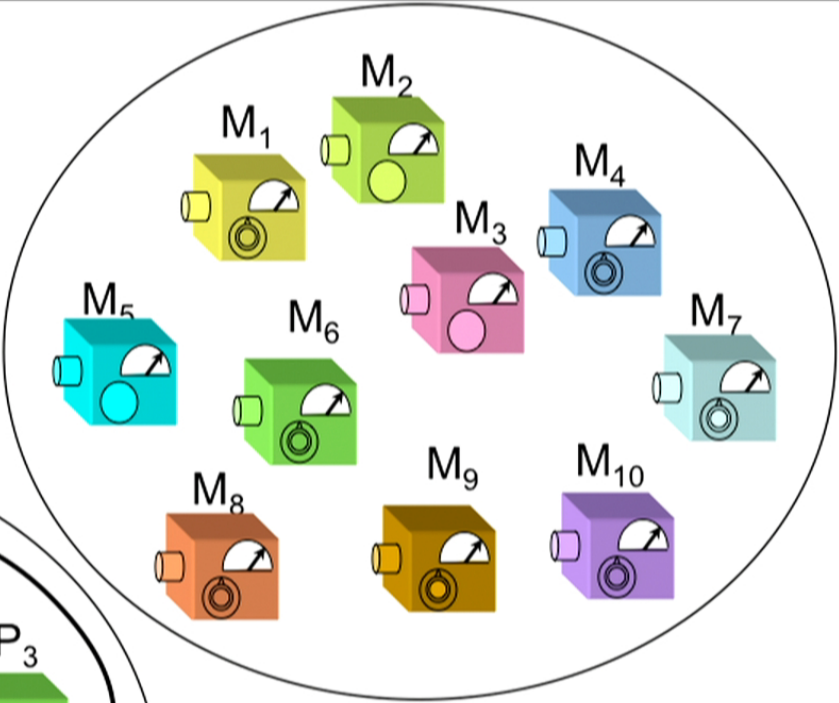
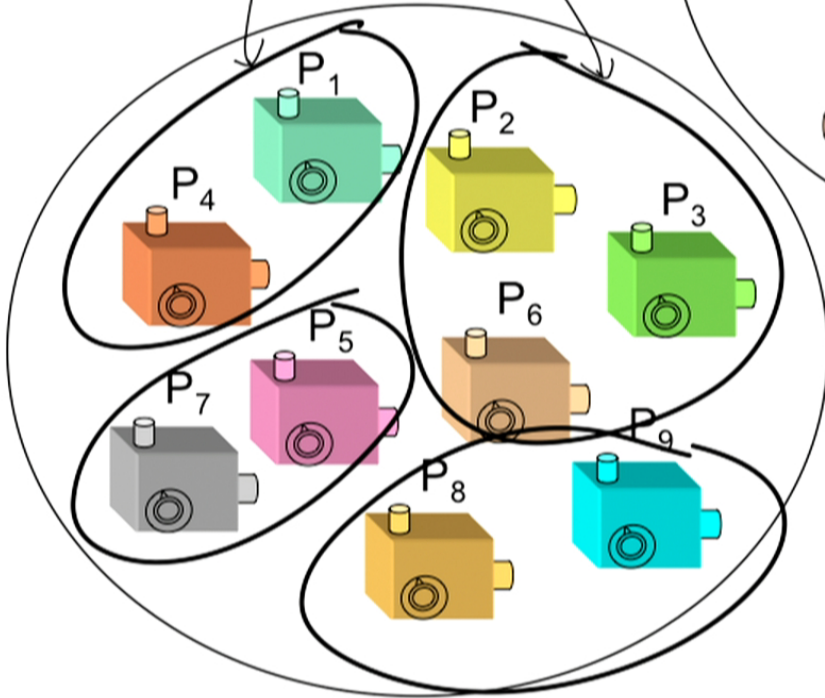


Example from quantum theory

Different density op's

ρ

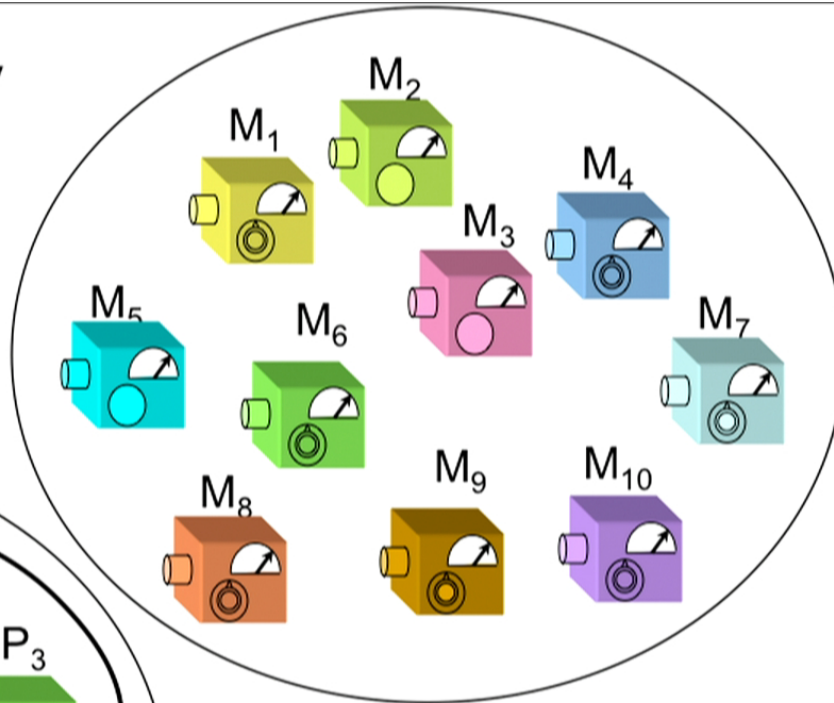
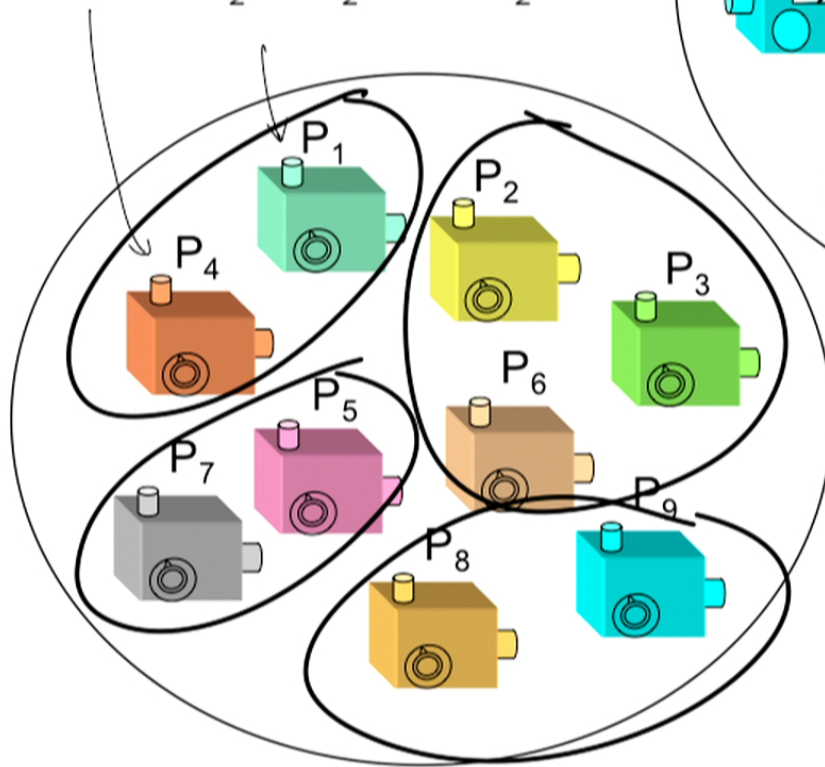
ρ'



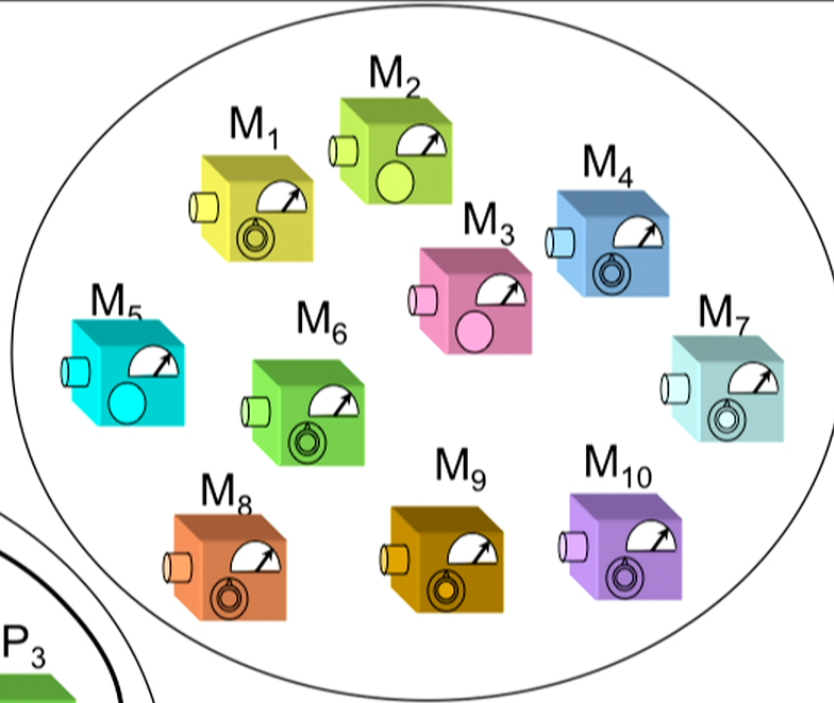
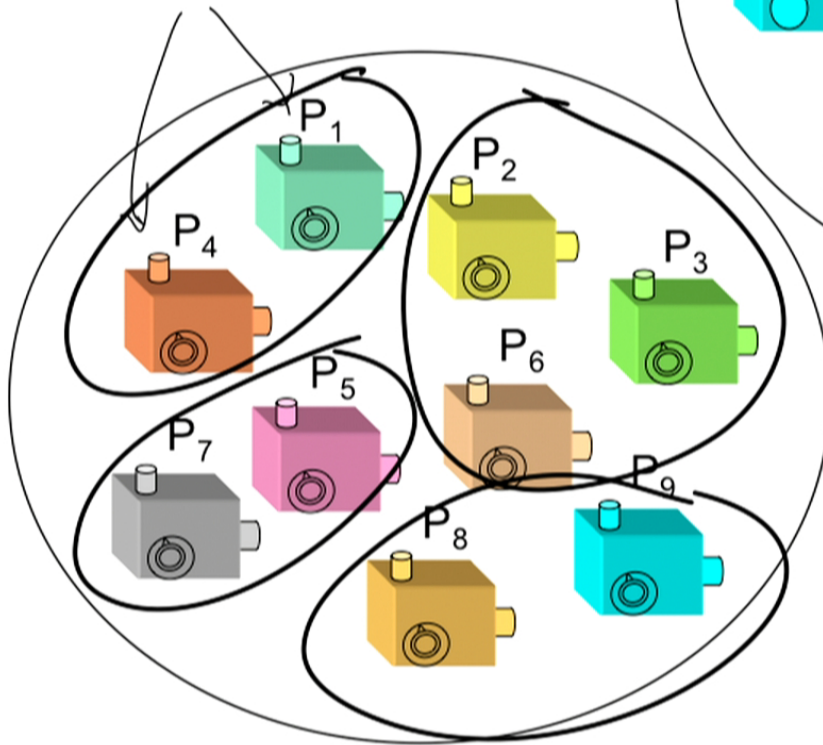
Example from quantum theory

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

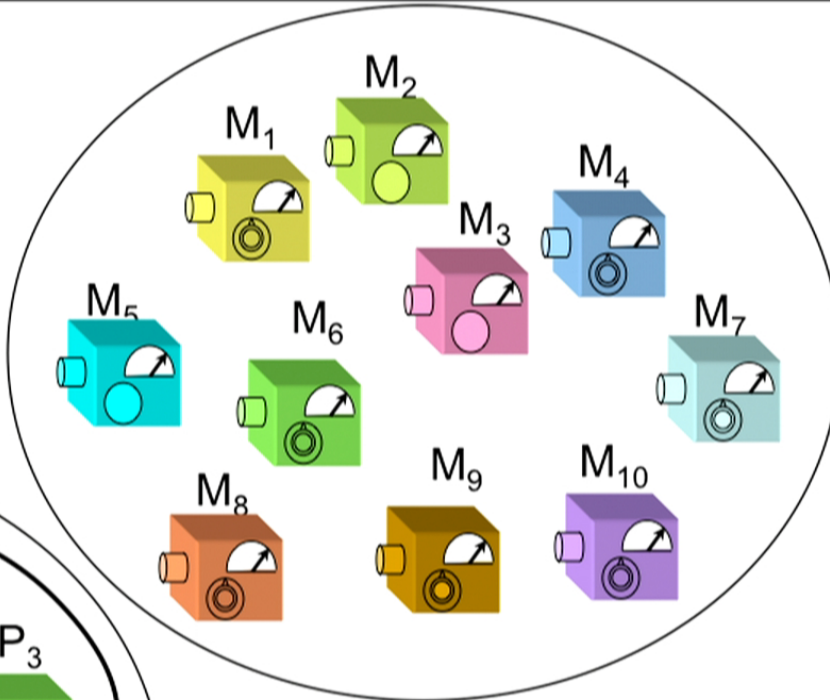
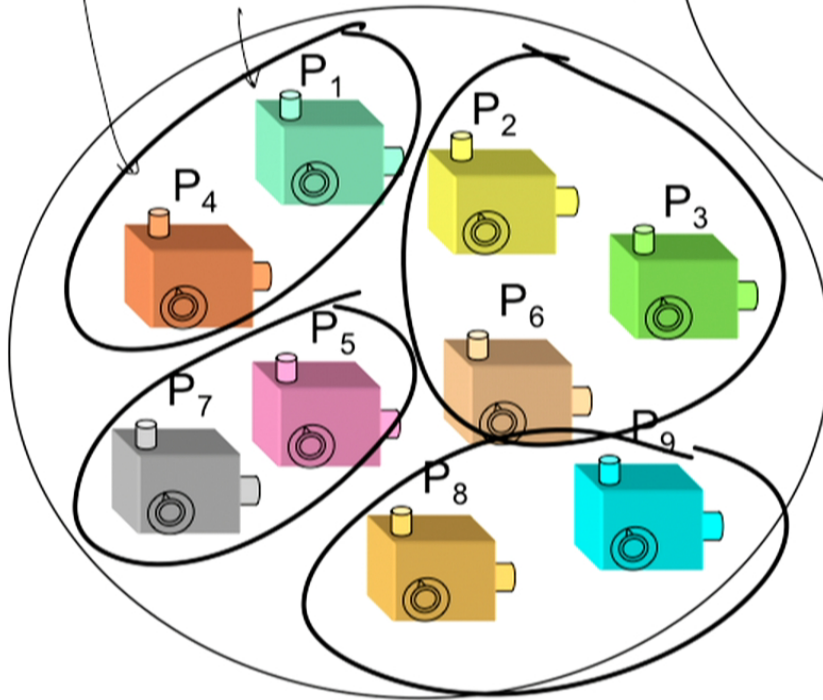
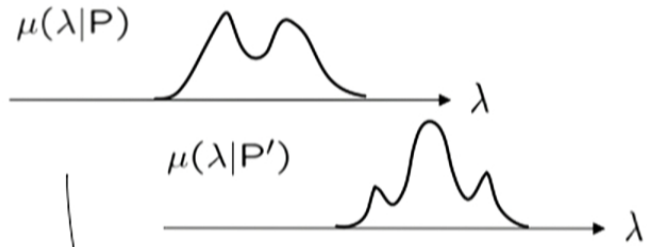
$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

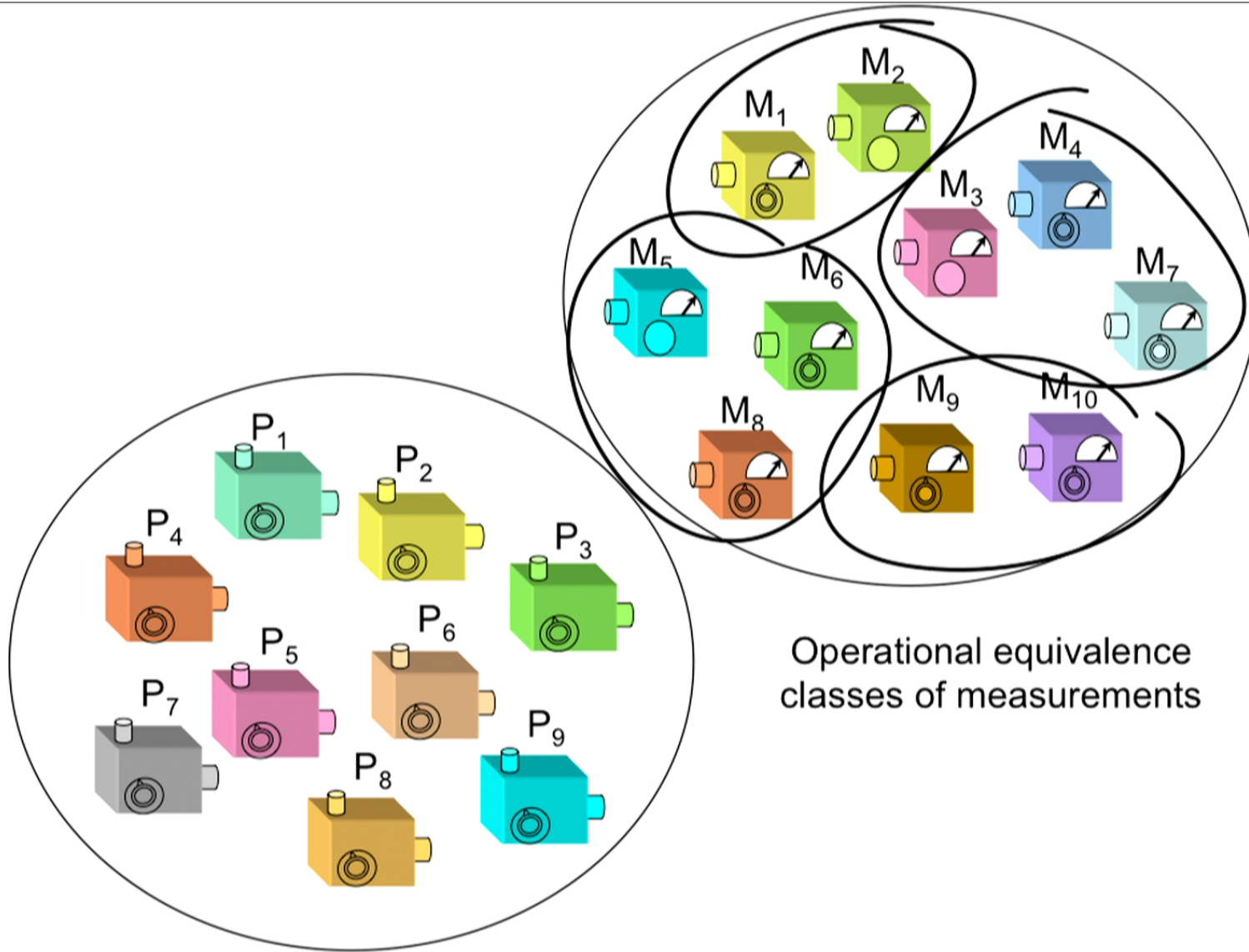


Preparation noncontextual model

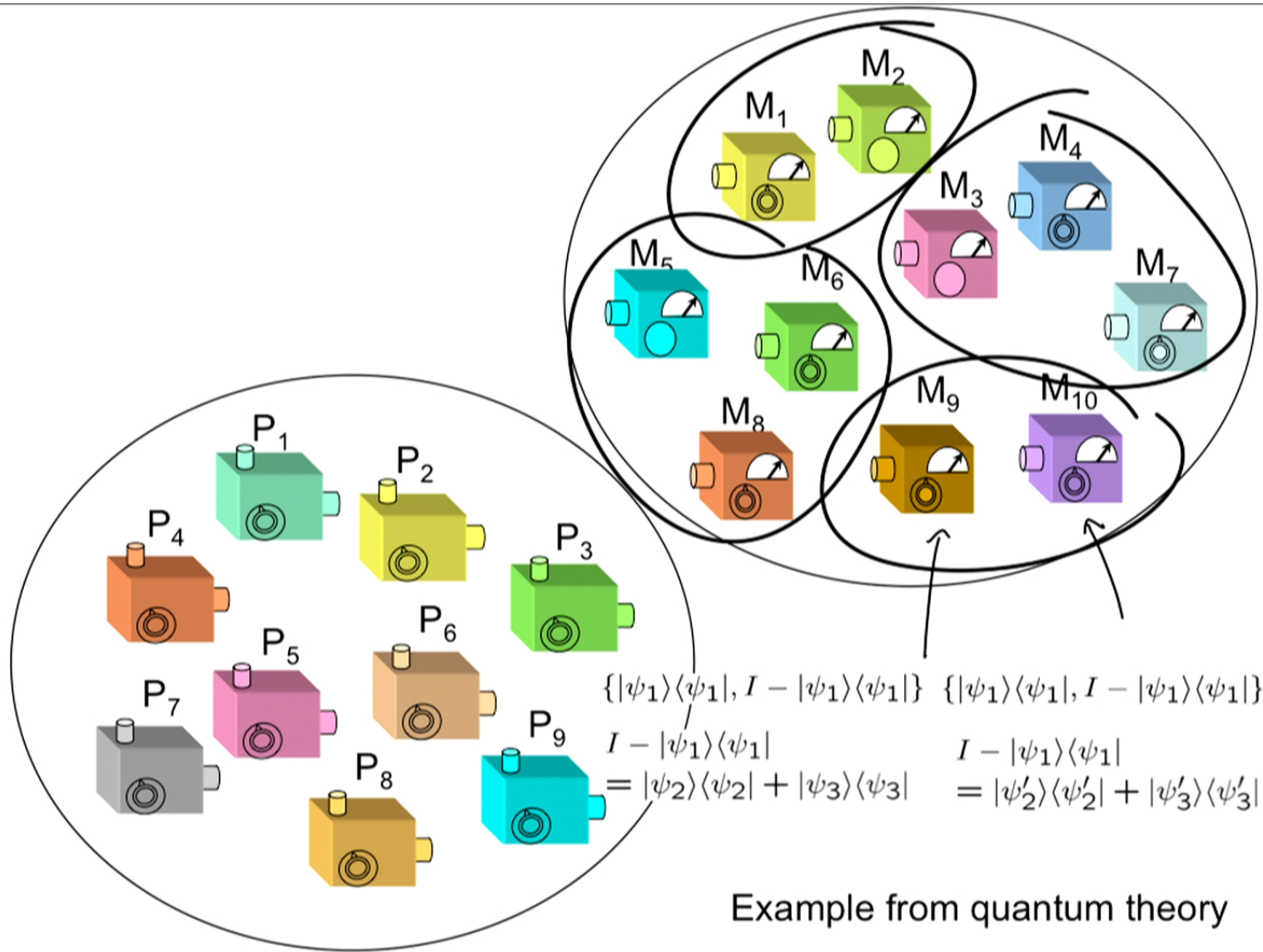


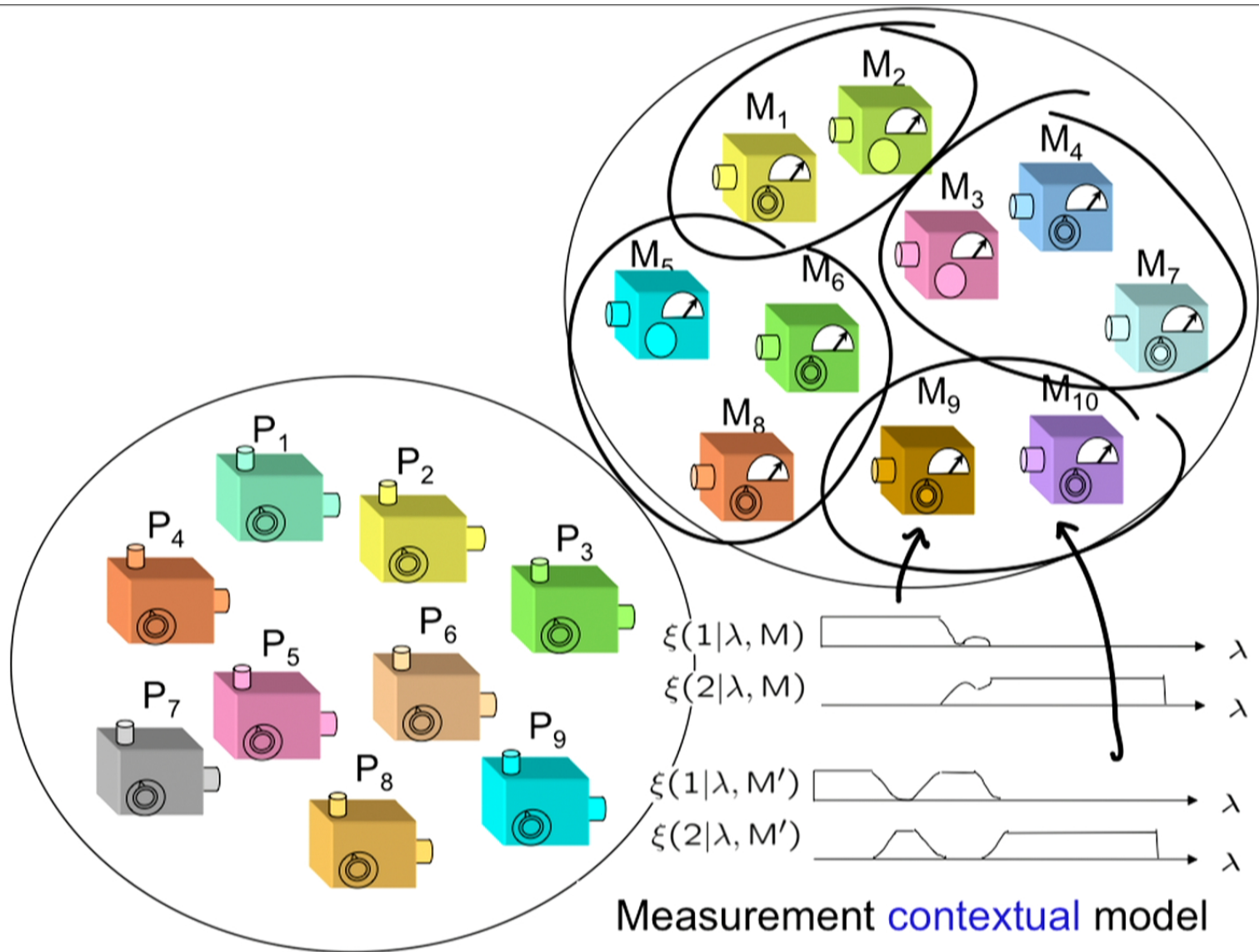
Preparation contextual model





Operational equivalence classes of measurements





Measurement contextual model

$$\begin{array}{ccc}
 e(P) = e(P') & \text{Preparation} & \\
 \forall M : p(X|P, M) = p(X|P', M) & \text{noncontextuality} & \\
 & \longrightarrow & \mu(\lambda|P) = \mu(\lambda|P')
 \end{array}$$

$$\begin{array}{ccc}
 e(M) = e(M') & \text{Measurement} & \\
 \forall P : p(X|P, M) = p(X|P, M') & \text{noncontextuality} & \\
 & \longrightarrow & \xi(X|\lambda, M) = \xi(X|\lambda, M')
 \end{array}$$



Noncontextuality is an analogue of Leibniz's principle of the identity of indiscernibles:

The ontological identity of operational indiscernibles

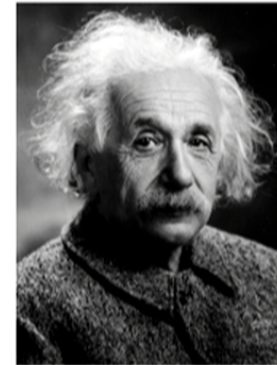


Noncontextuality is an analogue of Leibniz's principle of the identity of indiscernibles:

The ontological identity of operational indiscernibles

Noncontextuality is an analogue of Einstein's strong equivalence principle

An information-theoretic equivalence principle



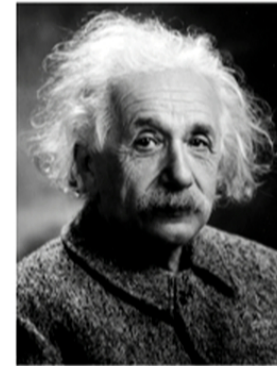


Noncontextuality is an analogue of Leibniz's principle of the identity of indiscernibles:

The ontological identity of operational indiscernibles

Noncontextuality is an analogue of Einstein's strong equivalence principle

An information-theoretic equivalence principle



Noncontextuality is an analogue of the principle of no fine-tuning used in causal inference

To achieve context-independence at the operational level while having context-dependence at the ontological level requires fine-tuning

Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noise?

Obstacle #2: How to contend with inexactness of operational equivalences?

Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noise?

Obstacle #2: How to contend with inexactness of operational equivalences?

Joint work with:

Ravi Kunjwal, Matt Pusey (theory)

Mike Mazurek, Kevin Resch (experiment)

Obstacle #1: How to contend with noise?

Previous no-go results

measurement noncontextuality

and

outcome determinism
for projective measurements



contradiction

Obstacle #1: How to contend with noise?

Previous no-go results

measurement noncontextuality

and

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contradiction

It turns out that

preparation noncontextuality

and

Facts about projective
measurements



outcome determinism for
projective measurements

Obstacle #1: How to contend with noise?

Previous no-go results

measurement noncontextuality

and

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contradiction

It turns out that

preparation noncontextuality

and

Facts about projective
measurements



outcome determinism for
projective measurements

And therefore:

universal noncontextuality

and

Facts about projective
measurements



contradiction

Obstacle #1: How to contend with noise?

Recast as:

universal noncontextuality

and

Certain operational
equivalences

Perfect correlations



contradiction

And then as:

universal noncontextuality

and

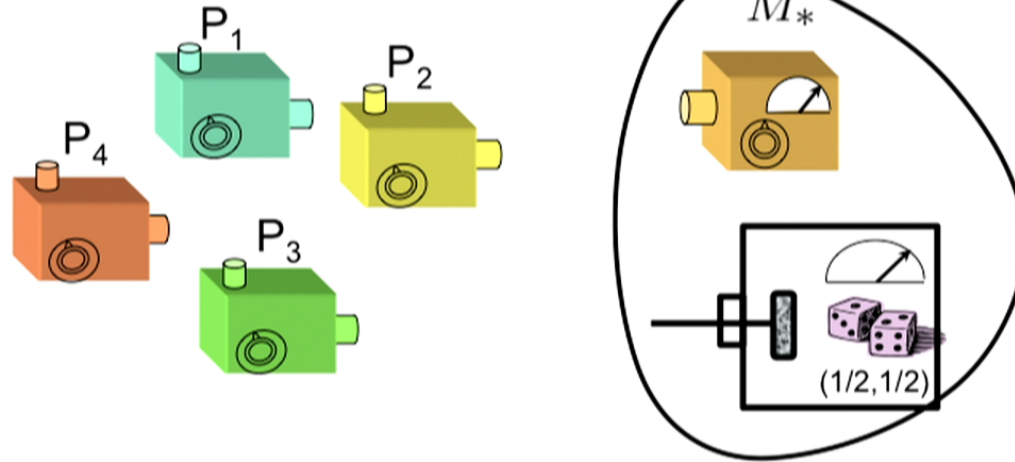
Certain operational
equivalences

**Degree of correlation above
some bound**



contradiction

A noncontextuality inequality robust to experimental noise

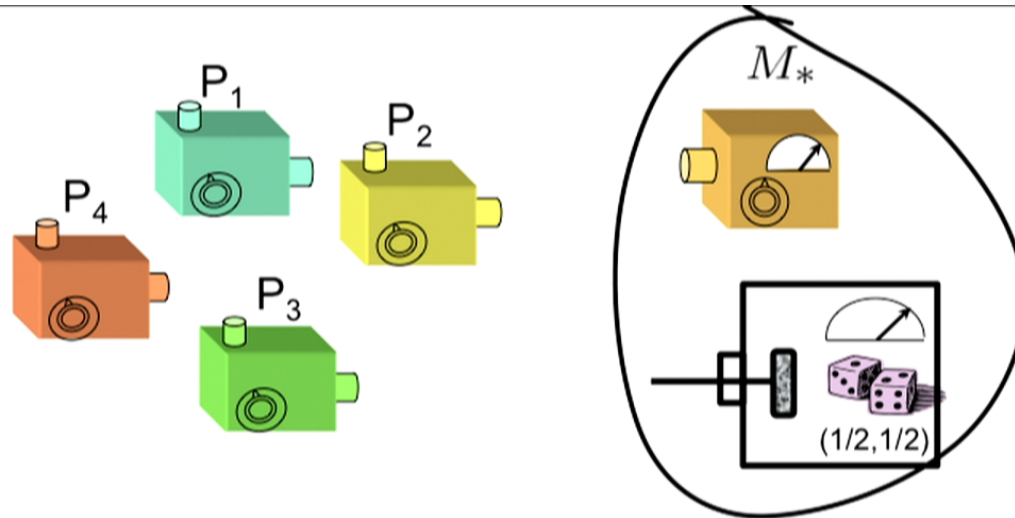


$$e(M_*) = e(\text{coin flip})$$

$$p(X = 0, 1 | M_*, P) = \frac{1}{2}, \forall P \in \mathcal{P}.$$

↓
 Measurement
 noncontextuality

$$\xi(X = 0, 1 | M_*, \lambda) = \frac{1}{2}, \forall \lambda \in \Lambda$$

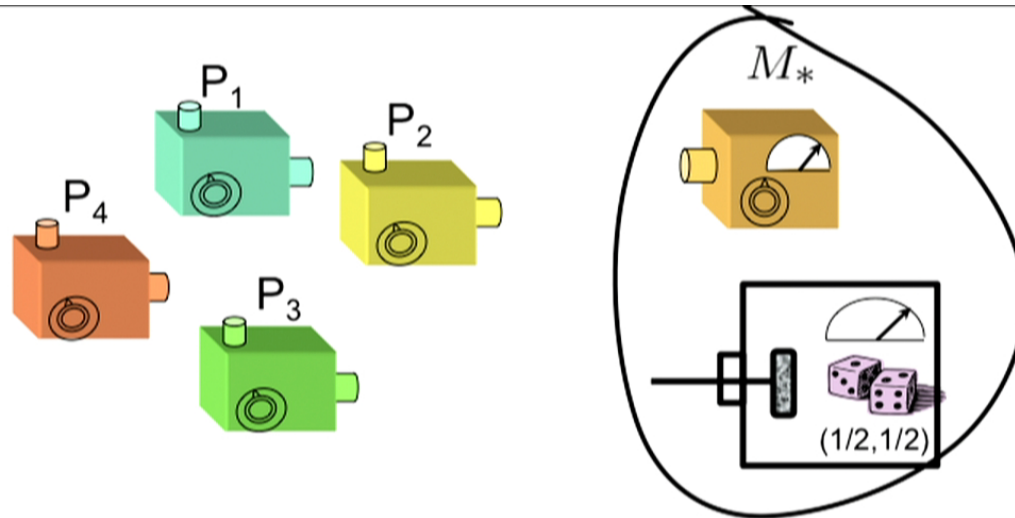


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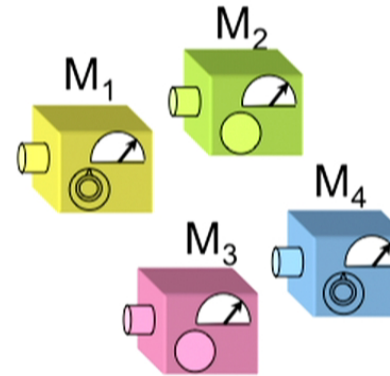
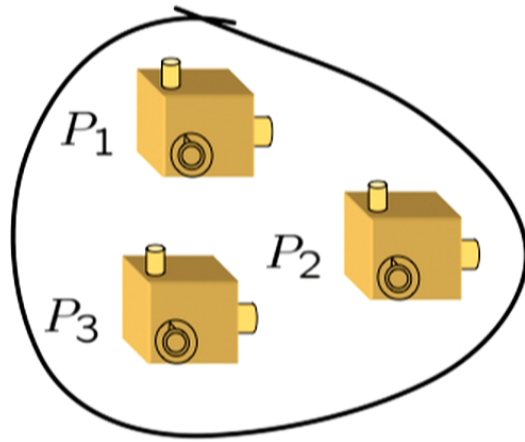


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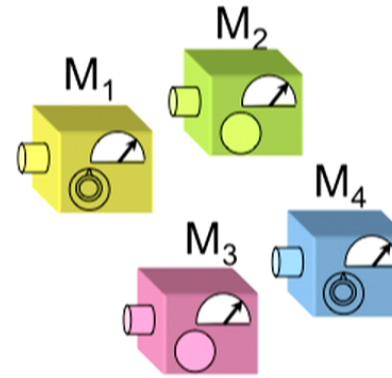
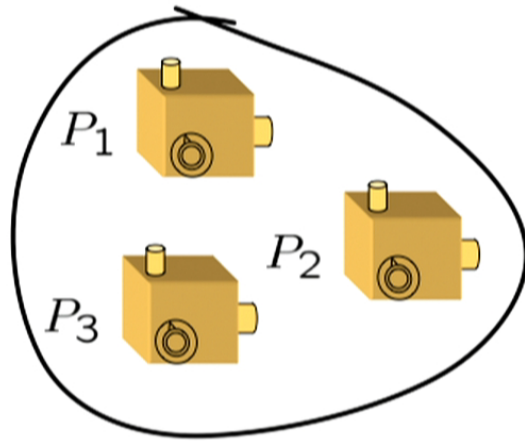


$$e(P_1) = e(P_2) = e(P_3)$$

$$p(X|M, P_1) = p(X|M, P_2) = p(X|M, P_3) \quad \forall M \in \mathcal{M}$$

\downarrow
 Preparation
 noncontextuality

$$\mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3) \quad \forall \lambda \in \Lambda$$



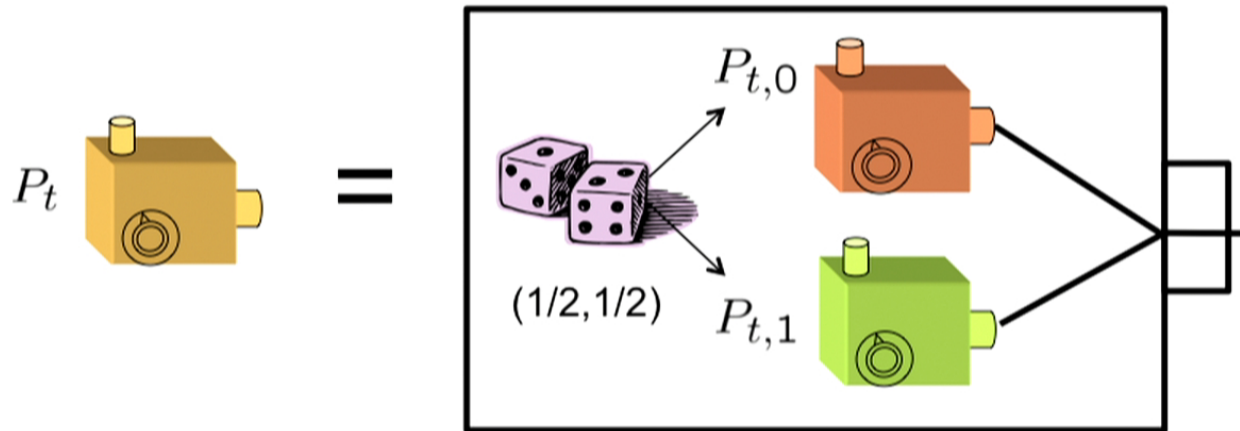
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 Preparation
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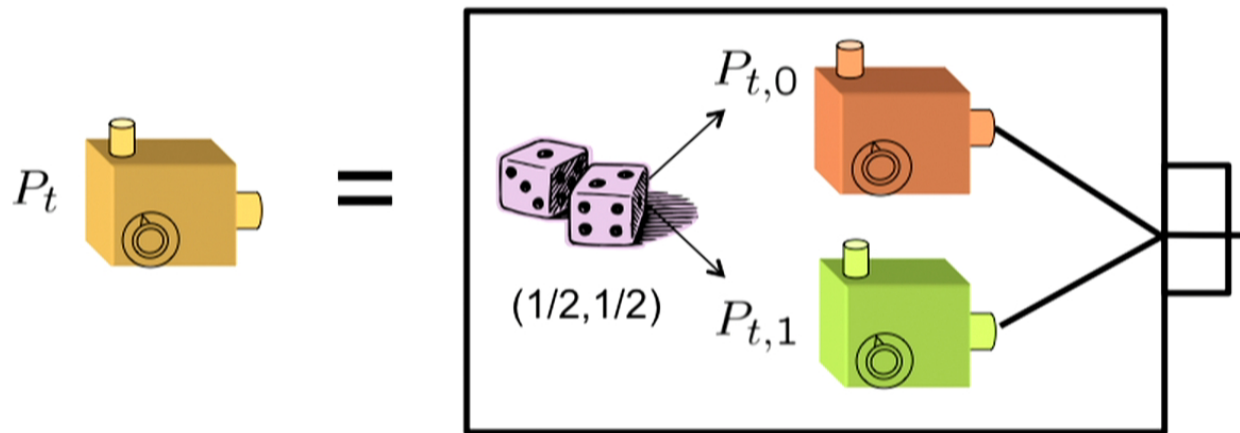
$$\mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3) \quad \forall \lambda \in \Lambda$$

$t \in \{1, 2, 3\}$



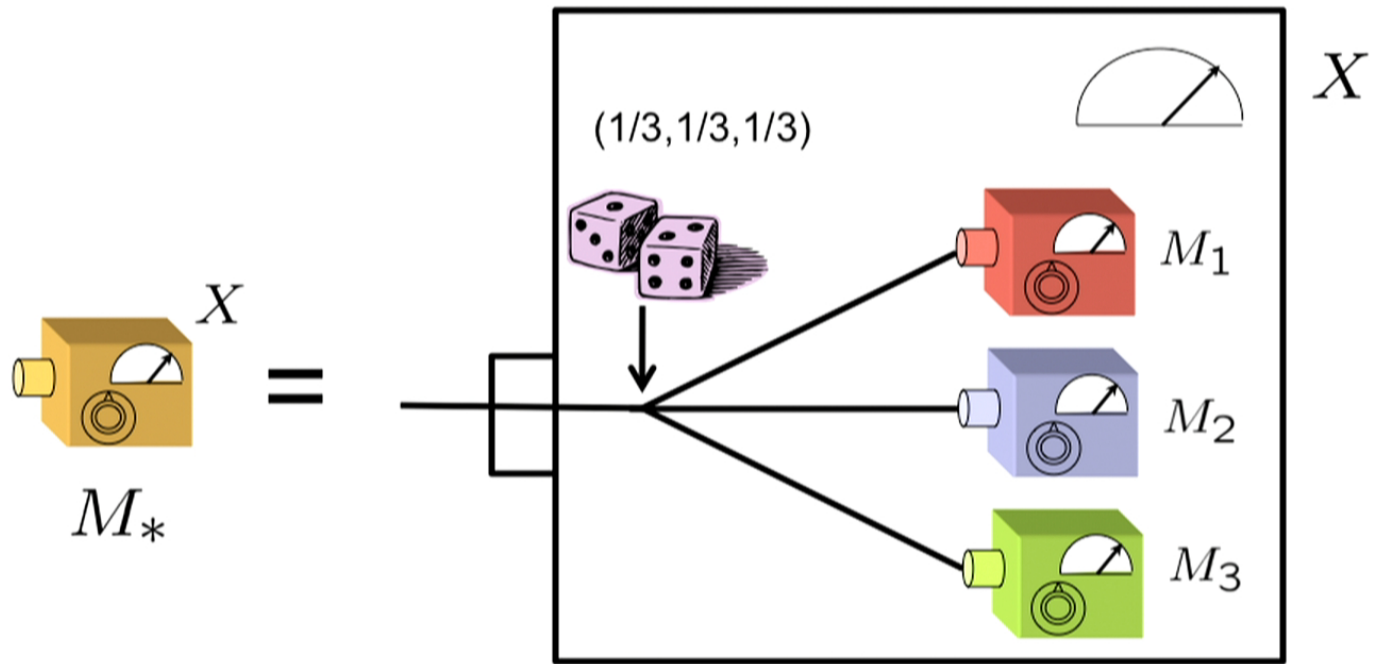
$$\mu(\lambda|P_t) = \frac{1}{2}\mu(\lambda|P_{t,0}) + \frac{1}{2}\mu(\lambda|P_{t,1})$$

$t \in \{1, 2, 3\}$

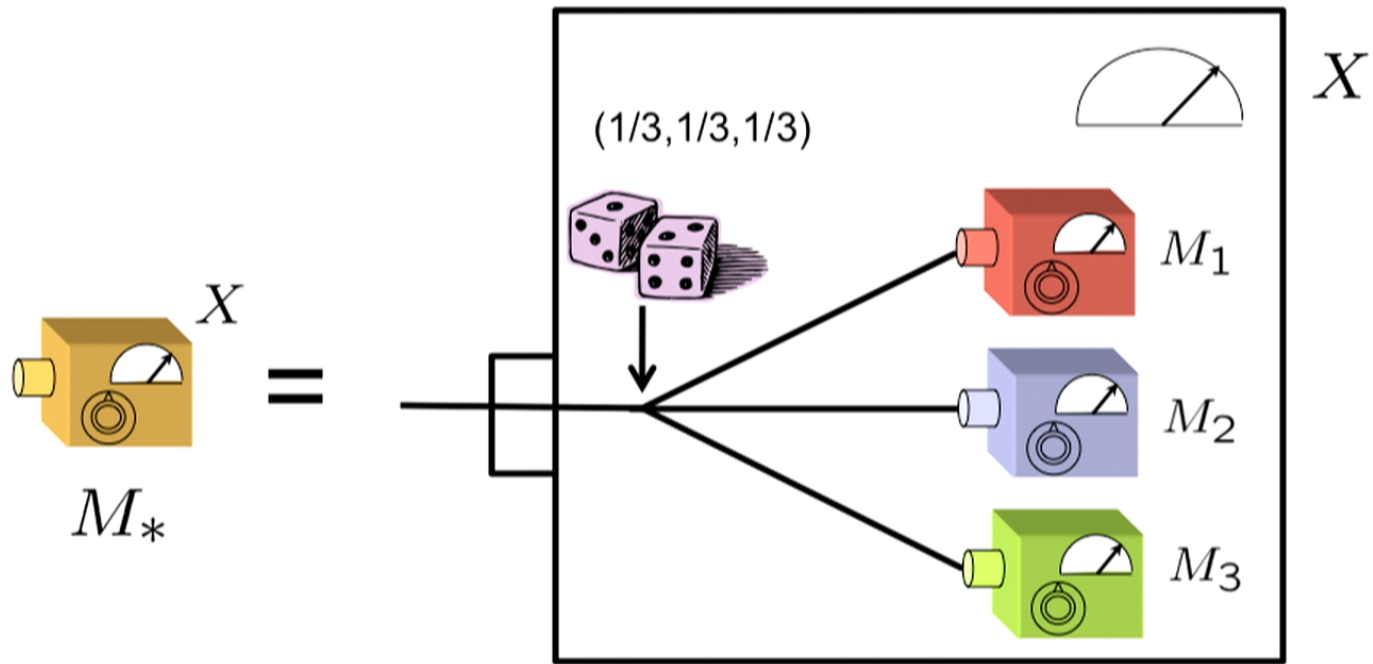


$$\mu(\lambda|P_t) = \frac{1}{2}\mu(\lambda|P_{t,0}) + \frac{1}{2}\mu(\lambda|P_{t,1})$$

$$\frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{1,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{2,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{3,b})$$



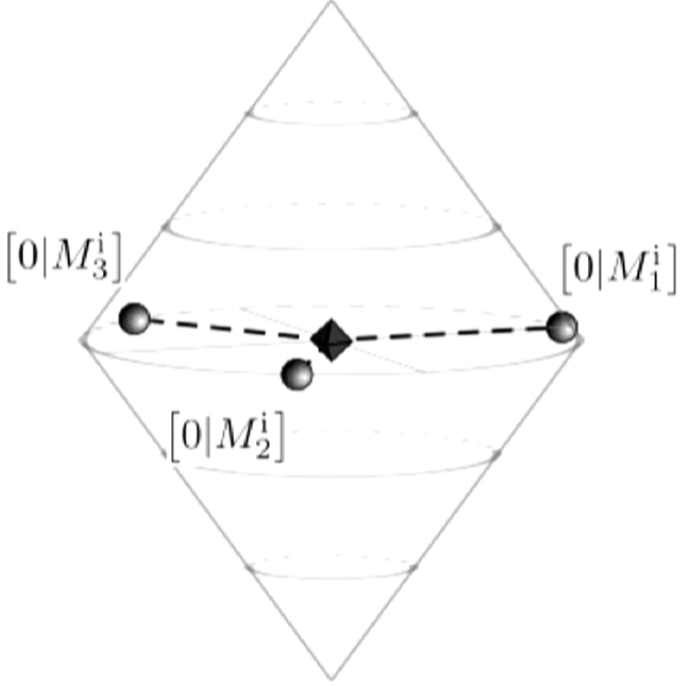
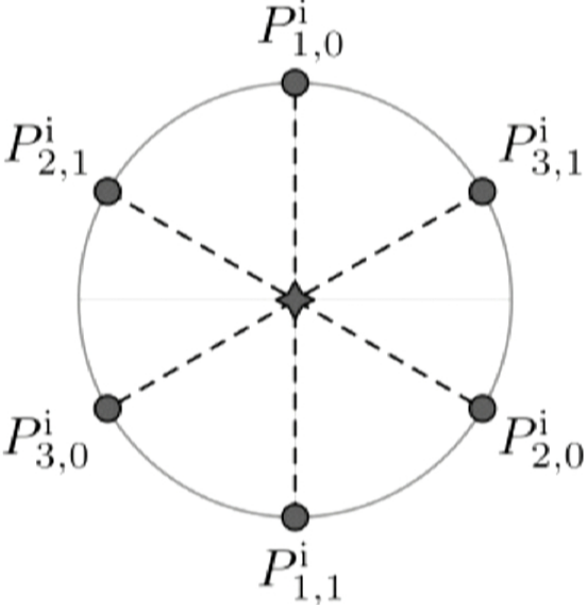
$$\xi(X = b | M_*, \lambda) = \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda)$$



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$$\frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda) = \frac{1}{2}$$

Quantum example



Def'n of average degree of correlation

$$A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b})$$

Theorem: For any $P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1}$
 M_1, M_2, M_3

If $e(P_1) = e(P_2) = e(P_3)$
 $e(M_*) = e(\text{coin flip})$

Then universal noncontextuality implies

$$A \leq \frac{5}{6} \quad \text{A noncontextuality Inequality}$$

Recall $A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b})$

Recall $p(X = b | M_t, P_{t,b}) = \sum_{\lambda} \xi(X = b | M_t, \lambda) \mu(\lambda | P_{t,b})$

Recall $A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b})$

Recall $p(X = b | M_t, P_{t,b}) = \sum_{\lambda} \xi(X = b | M_t, \lambda) \mu(\lambda | P_{t,b})$

$$A = \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} \sum_{\lambda} \xi(X = b | M_t, \lambda) \mu(\lambda | P_{t,b})$$

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$$\xi(X = b | M_t, \lambda) \leq \eta(M_t, \lambda)$$

$$\text{where } \eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda).$$

$$A \leq \frac{1}{3} \sum_{t \in \{1,2,3\}} \sum_{\lambda \in \Lambda} \eta(M_t, \lambda) \left(\frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda | P_{t,b}) \right)$$

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$$A \leq \sum_{\lambda \in \Lambda} \left(\frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right) \nu(\lambda)$$

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$$A \leq \max_{\lambda \in \Lambda} \left(\frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right)$$

where $\eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda)$.

$$A \leq \max_{\lambda \in \Lambda} \left(\frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right)$$

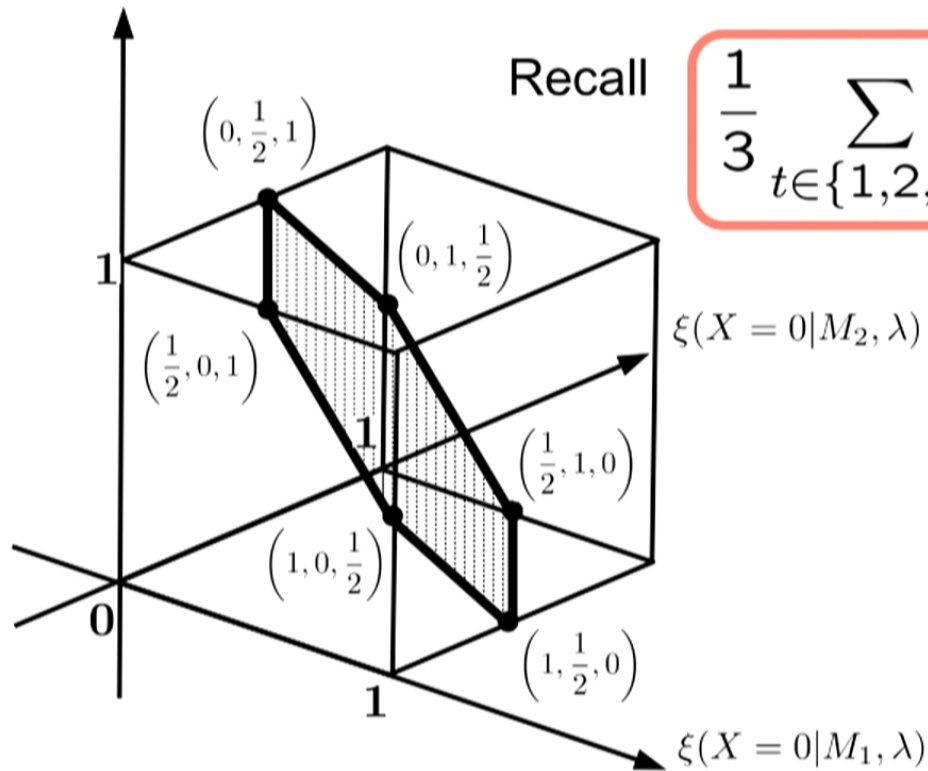
where $\eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda)$.

Recall $\frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda) = \frac{1}{2}$

$$A \leq \max_{\lambda \in \Lambda} \left(\frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right)$$

where $\eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda)$.

$\xi(X = 0 | M_3, \lambda)$

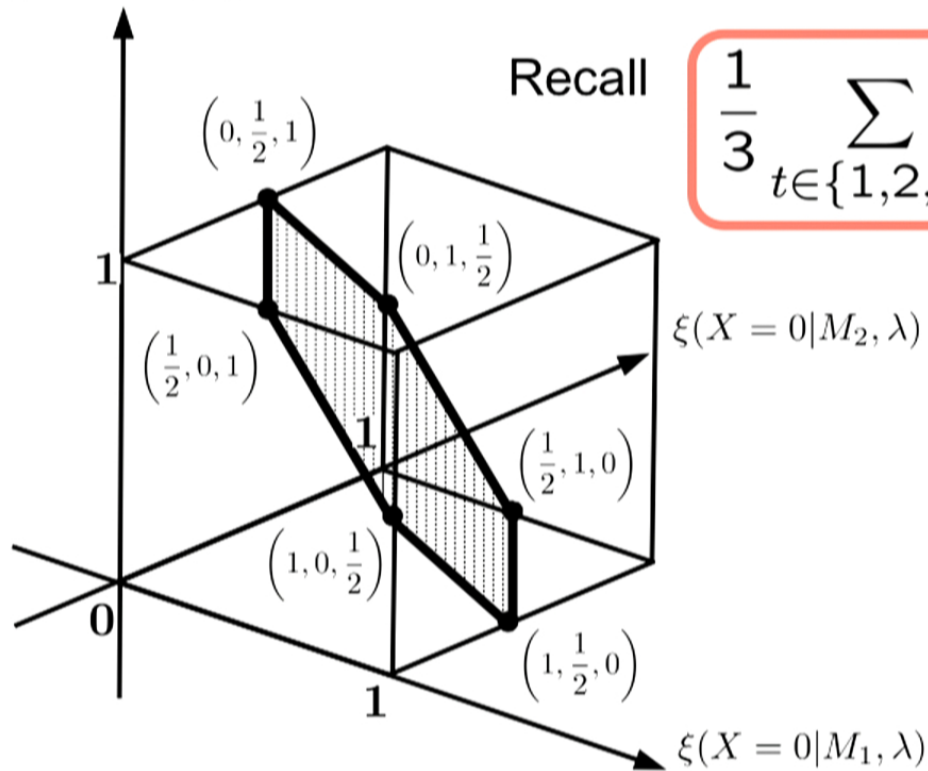


$$\frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda) = \frac{1}{2}$$

$$A \leq \max_{\lambda \in \Lambda} \left(\frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right)$$

where $\eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda)$.

$\xi(X = 0 | M_3, \lambda)$

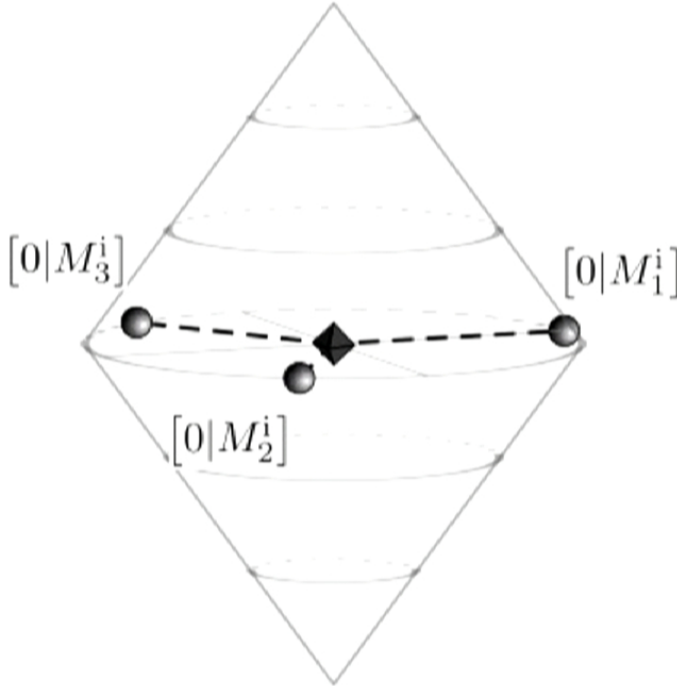
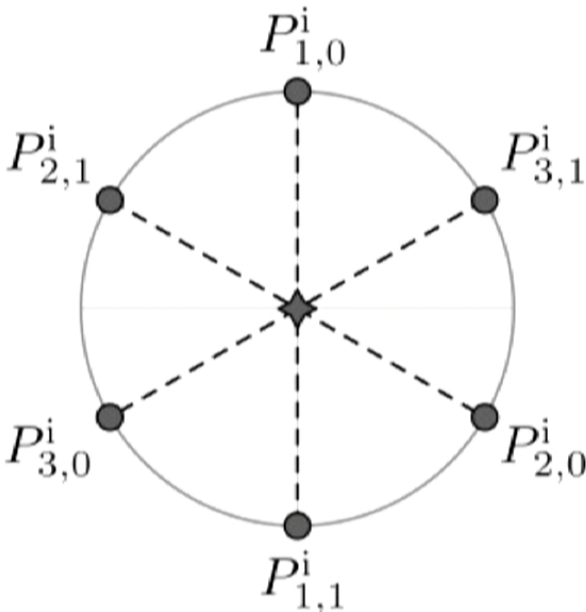


$$\frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda) = \frac{1}{2}$$

$$A \leq \frac{1}{3} \left(1 + 1 + \frac{1}{2} \right) = \frac{5}{6}$$

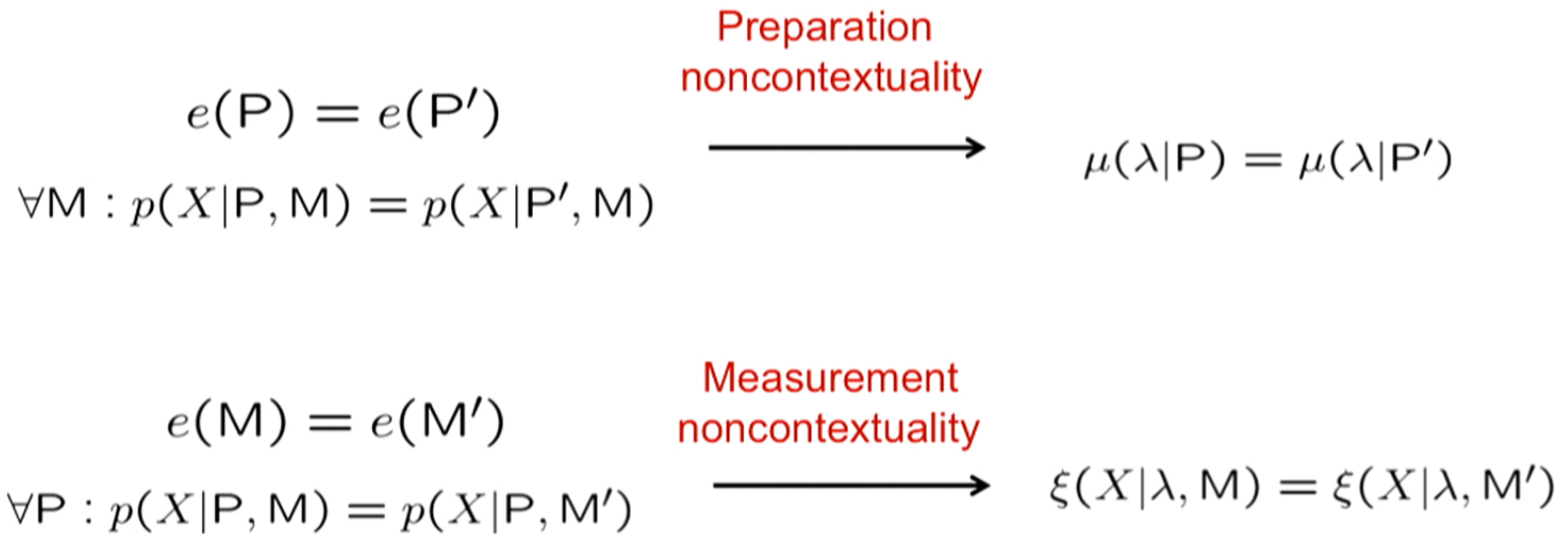
Quantum violation

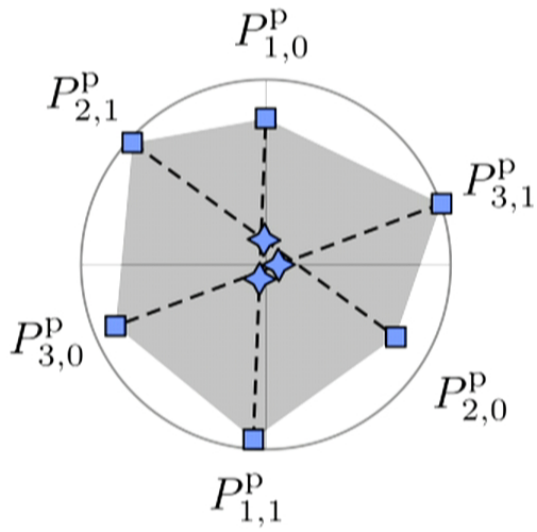
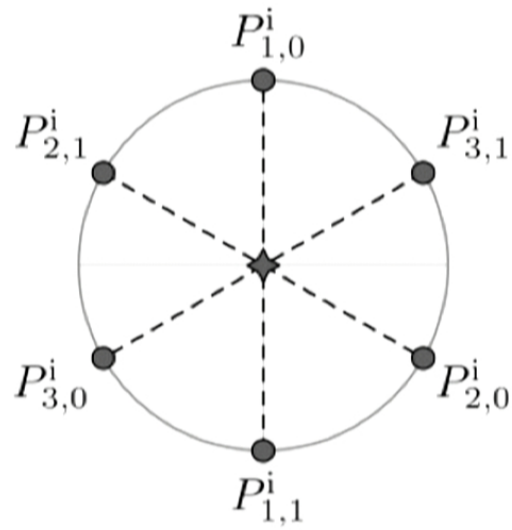
$$A = 1$$

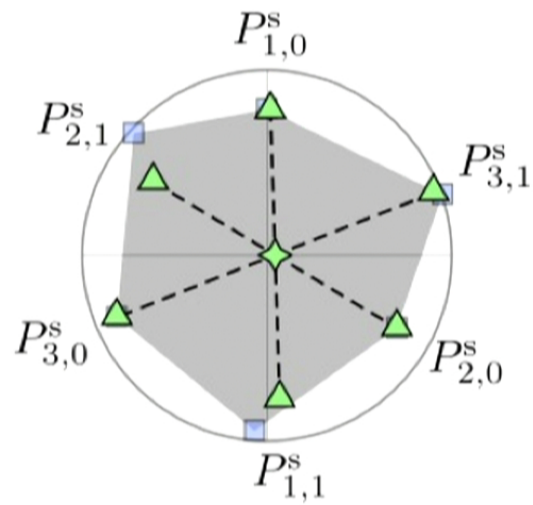
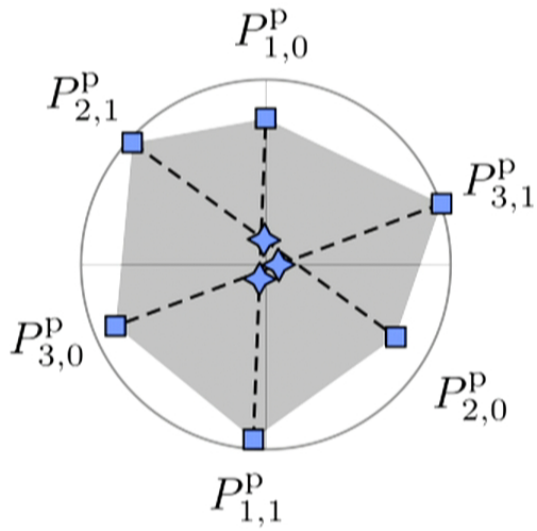
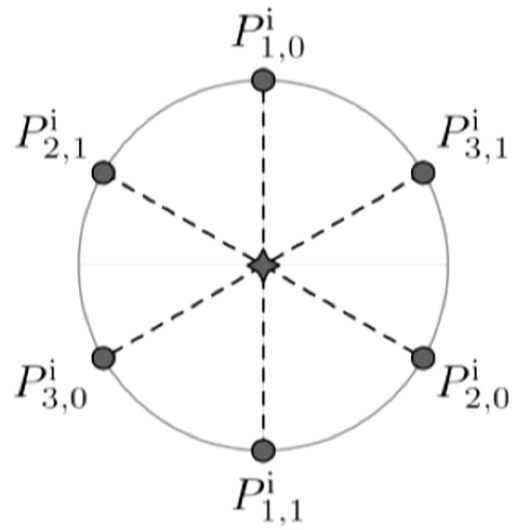


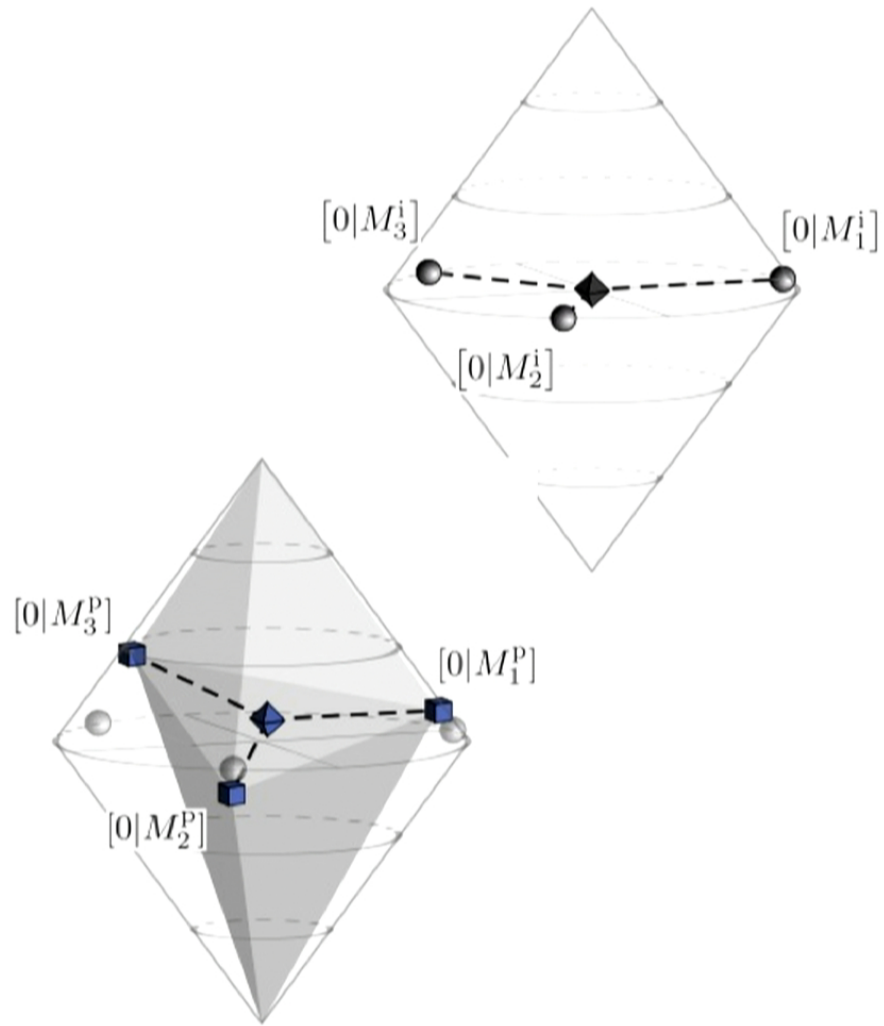
Robust to noise

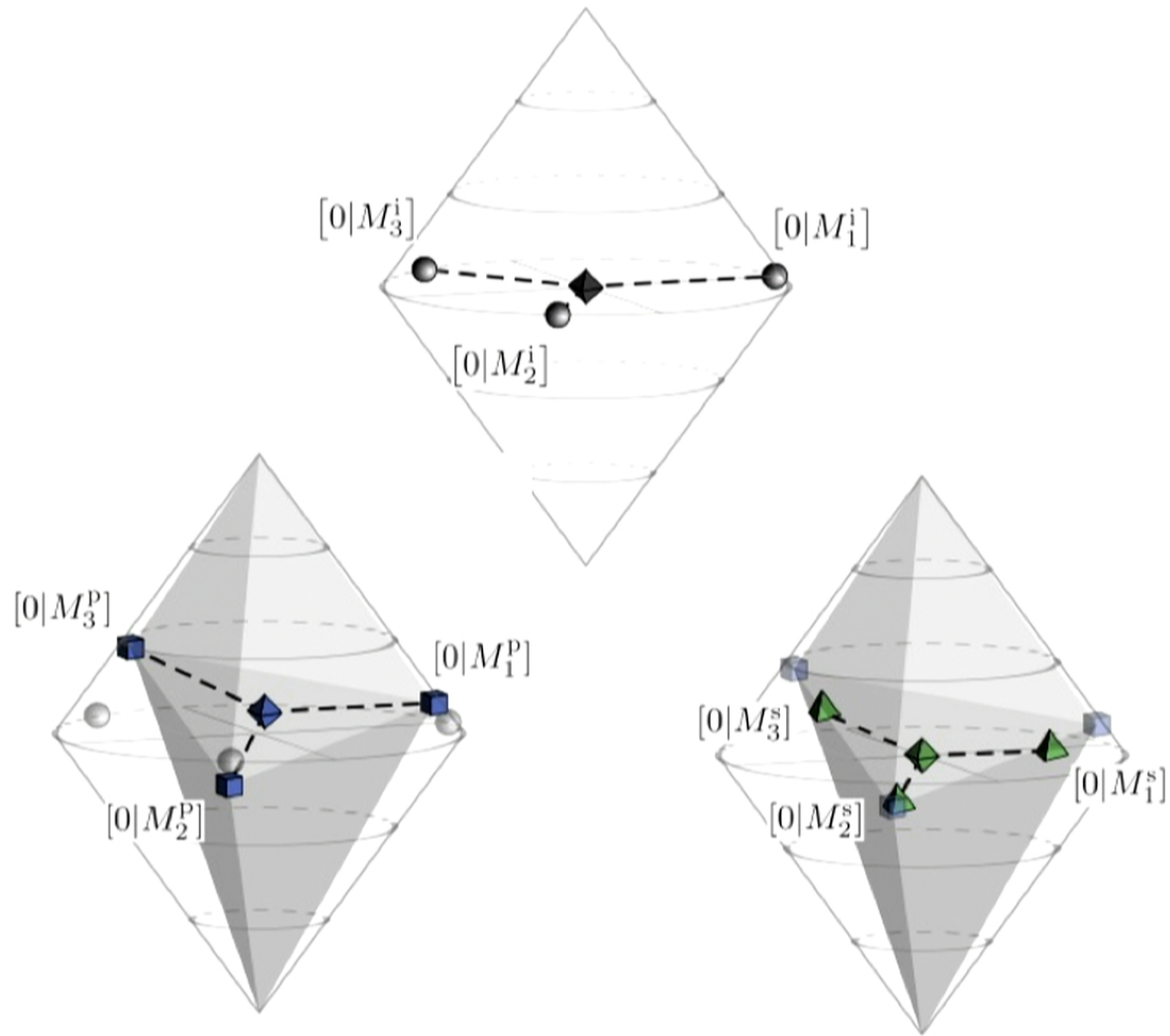
Obstacle #2: Lack of exact operational equivalence



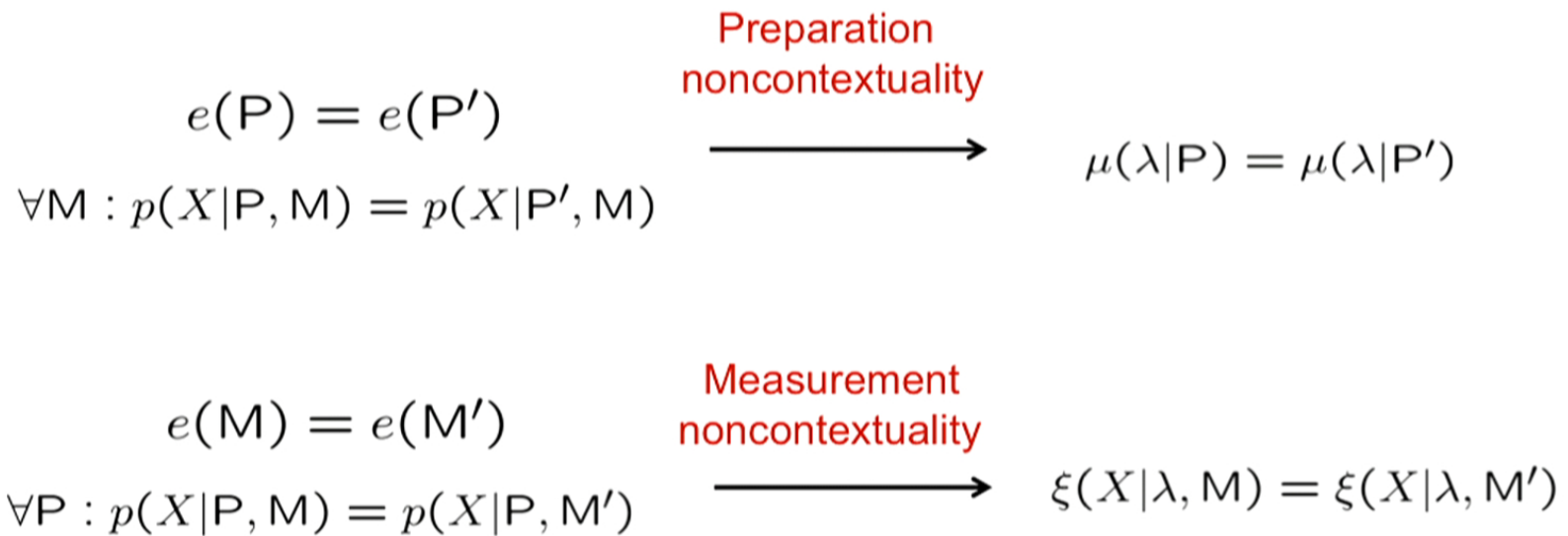




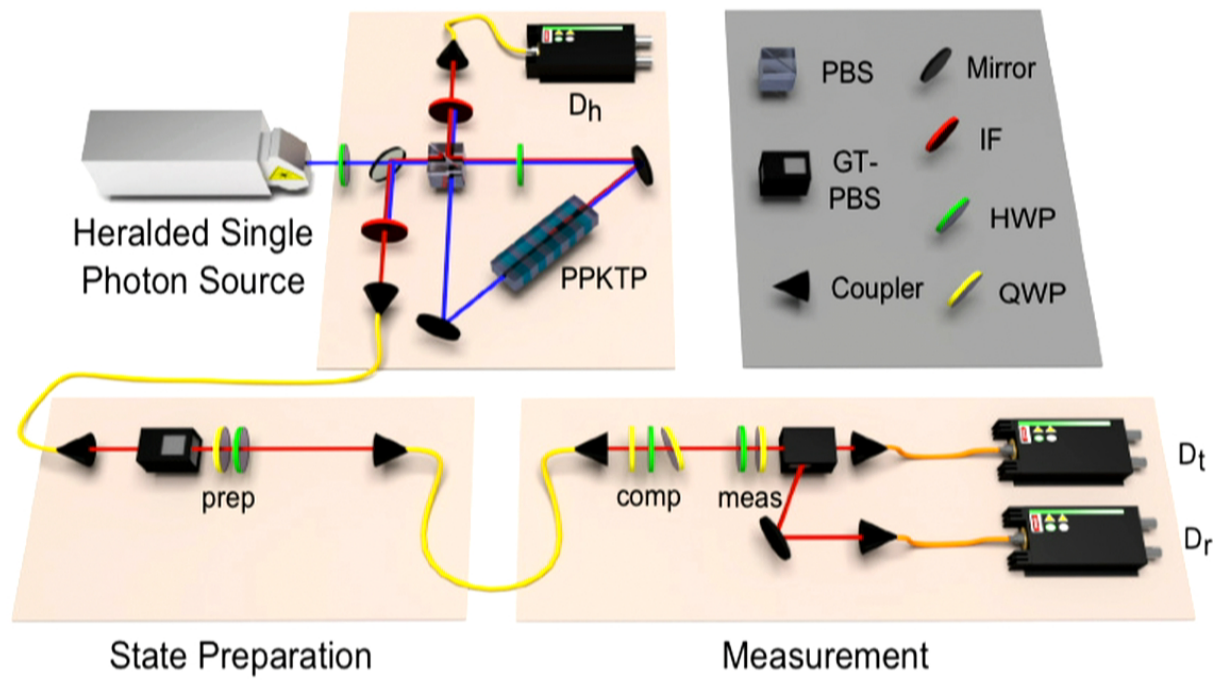


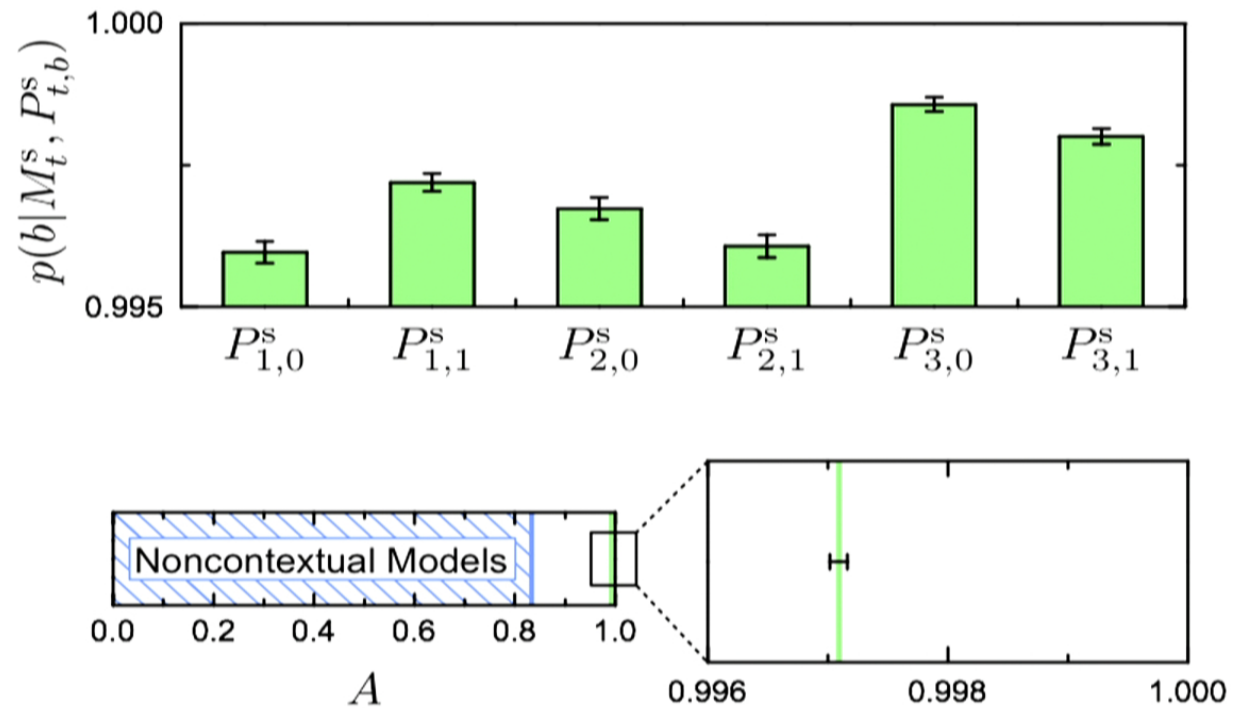


A remaining issue:
How to verify that a given set of operations is
tomographically complete?



This is the new frontier for experimental tests of noncontextuality

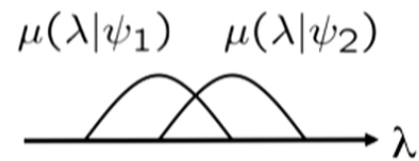
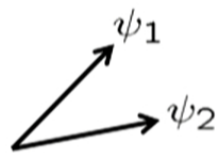




$A = 0.99709 \pm 0.00007$
 violating the noncontextual bound by 2300σ

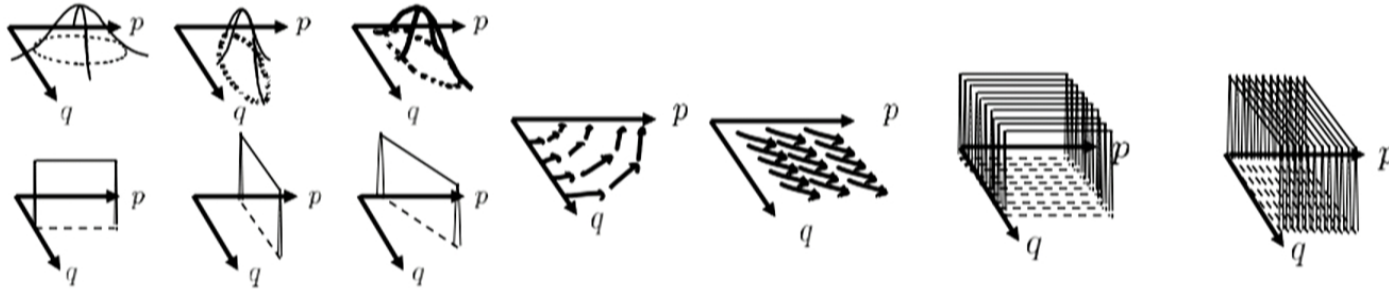
Significance for characterizing nonclassicality

Epistemically restricted classical theories



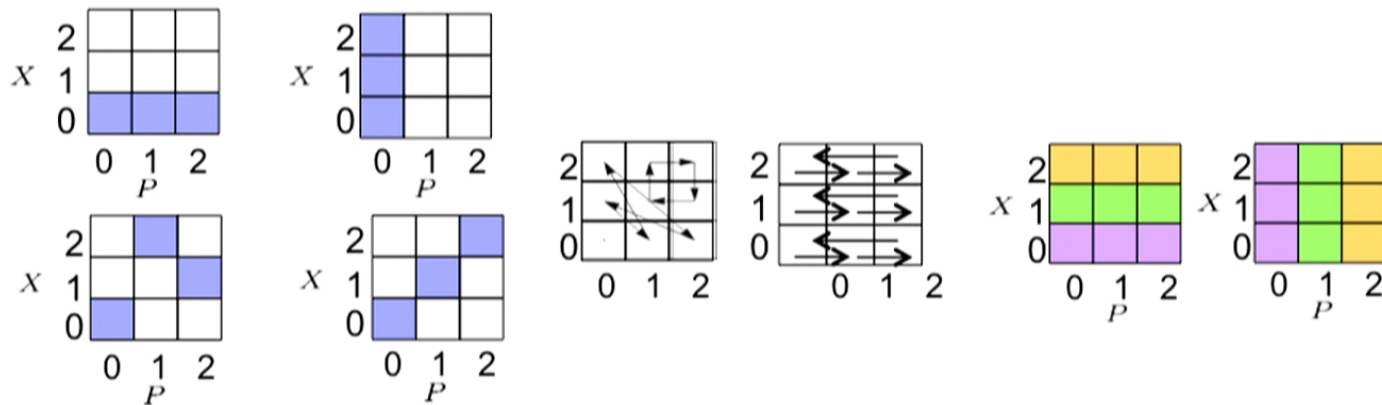
Can recover Gaussian quantum mechanics

Bartlett, Rudolph, RWS, 2011



Can recover the stabilizer theory of qutrits

RWS, arXiv:1409.5041



Categorizing nonclassical phenomena

Those arising in epistemically
restricted classical theories

Interference
Noncommutativity
Entanglement
Collapse
No perfect state discrimination
No cloning
Steering
Teleportation
Tunneling
Improvements in metrology
Pre and post-selection effects
Key distribution
Others...

Weakly nonclassical

Those not arising in epistemically
restricted classical theories

Noncontextuality inequality violations
Bell inequality violations
Computational speed-up
Certain aspects of items on the left

Strongly nonclassical

Operational theories

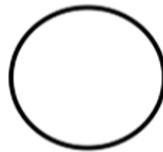


$$p(X|M, P)$$

Classical



Nonclassical



“Nonsimplicial”

Nonsimplicial but noncontextual

Remote steering

Einstein 1935; Caves-Fuchs-Schack 2000; Harrigan-RWS 2010

Three box paradox

Leifer-RWS 2004

Quantum multiplexing

RWS 2004

No error-free discrimination of nonorthogonal states

RWS 2004

Nonzero probability of wavepacket tunneling through a barrier

Bartlett-Rowe 1999

Failure of noncontextuality

Failure of preparation

noncontextuality = BI violation

Bell 1964; Barrett 2006 unpublished; Liang-RWS-Wiseman 2010

Anomalous weak values

Pusey 2015

Probability of success in parity-oblivious multiplexing

RWS-Buzacott-Kheenn-Pryde-Toner 2008

Precise tradeoff of probability of discrimination with nonorthogonality

RWS-Wolfe (work in progress)

Precise dependence of tunneling probability on wavepacket width

RWS (work in progress)

Nonsimplicial but noncontextual

Failure of noncontextuality

Teleportation

???

No cloning

???

Various quantum information
processing protocols

???

Interference phenomena
Leifer-RWS unpublished

???

Quantum vacuum phenomena

???

Existence of path integral
expression for unitary dynamics
Koh-Penney-RWS (unpublished)

???

Thermodynamic phenomena

???