Title: Bringing General Relativity into the Operational Probabilistic Framework

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Abstract: I will discuss my work (in progress) to formulate General Relativity as an operational theory which includes probabilities and also agency (knob settings). The first step is to find a way to discuss operational elements of GR. For this I adapt an approach due to Westman and Sonego. I assert that all directly observable quantities correspond to coincidences in the values of scalar fields. Next we need to include agency. Usually GR is regarded as a theory in which a solution is simply stated for all space and time (the Block Universe view). Here, instead, we find a way to treat agents as making choices. Finally, we need to incorporate probabilities. For this purpose we take a compositional point of view. We associate a generalized state with regions of the space consisting of coincidences in the values of scalars. We then show how to combine these generalized states to make probabilistic predictions using the duotensor machinery developed previously.

Objective: To develop an operational formulation of General Relativity that accommodates ignorance probabilities and agency: PAGeR.
Homage to the Blackberry pager

Motivation: As a step in developing a theory of Quantum Gravity

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Prelude: composition in physics

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Typically in physics a state pertains to a given time and is used to make predictions for later times. The generalized state is a more general notion than this since we may be making predictions of a more general type.
A key question is how do we calculate the generalized state for a composite object? We propose the following principle.

> **THE COMPOSITION PRINCIPLE:** *The generalized state for a composite object can be calculated from the generalized states for the components by means of a calculation having the same structure as the description of the composition of that object.*

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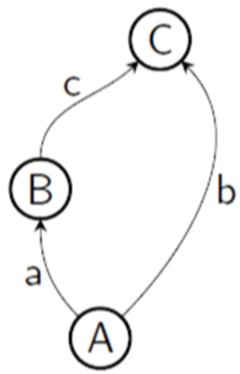
Prelude: example



$$\Leftrightarrow \quad \mathsf{A}^{\mathsf{a}_1\mathsf{b}_2}\mathsf{B}_{\mathsf{a}_1}^{\mathsf{c}_3}\mathsf{C}_{\mathsf{b}_2\mathsf{c}_3}$$

We can associate generalized states

$$\mathsf{A}^{\mathsf{a}_1\mathsf{b}_2} \longrightarrow A^{a_1 b_2}$$
$$\mathsf{B}_{\mathsf{a}_1}^{\mathsf{c}_3} \longrightarrow B_{a_1}^{c_3}$$
$$\mathsf{C}_{\mathsf{b}_2\mathsf{c}_3} \longrightarrow C_{b_2 c_3}$$

with

$$\mathsf{A}^{\mathsf{a}_1\mathsf{b}_2}\mathsf{B}_{\mathsf{a}_1}^{\mathsf{c}_3} \longrightarrow A^{a_1 b_2} B_{a_1}^{c_3}$$

this is in accord with the composition principle.

Earlier work

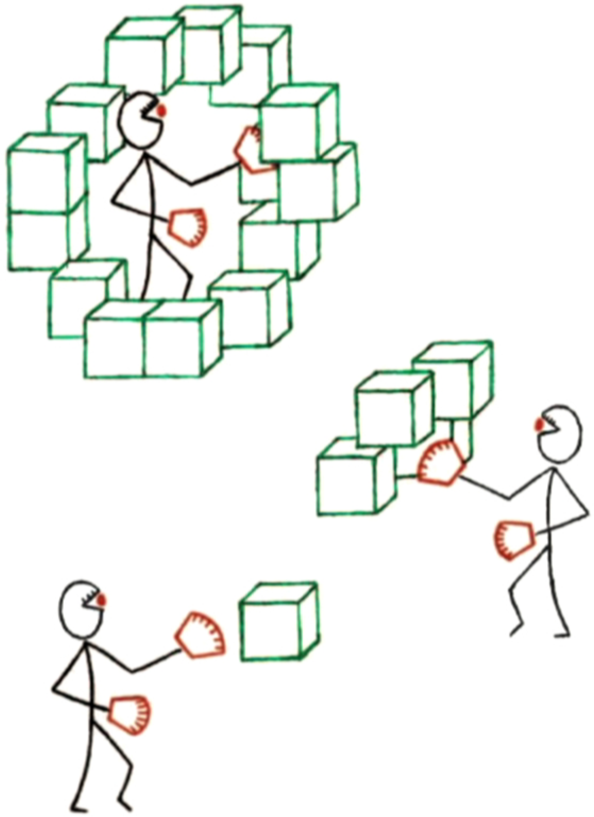
This is part of an ongoing project (papers on the arXiv).

- 2005 The causaloid framework: A framework for probabilistic theories with indefinite causal structure
- 2010 The duotensor framework: A way to do probability theory in a manifestly covariant manner (for circuits but applicable to space-time).
- 2011 The Operator Tensor formulation of Quantum Theory: A manifestly covariant way to formulate QT (for circuits but applicable to space-time).
- 2013 Theory of composition in physics: A general framework for thinking about composition in physics.

Related work

- Chris Fuchs, Rüdiger Schack, - Emphasized agent centric approach to Quantum Foundations (QBism)
- Samson Abramsky and Bob Coecke's categorical (pictorial) approach to quantum theory. This emphasizes compositionality.
- Generalized probability theories (going back to Mackey) - much recent work.
- Some space-time approaches to QT:
  - Quantum causal histories - Markopoulou; Dual point of view - Blute, Ivanov and Panangaden;
  - Aharonov and co-workers - multitime states.
  - General Boundary formulation - Oeckl;
  - Quantum combs - Chiribella, D'Ariano, and Perinotti; Oeckl's positive formulation.
  - Various axiomatic approaches to QT (LH, Dakic and Brukner, Masanes Müeller, CDP, ...) (in particular the tomographic locality axiom).
  - Leifer and Spekkens Quantum Bayesian Inference, more recent work by Henson, Lal, and Pusey.
  - Indefinite causal structure: Brukner, Oreshkov, Costa, Cerf

Fuchs Picture for Quantum Cosmology



General Relativity: fields

In GR have a set of fields

$$\mathbf{\Phi} = (g_{\mu\nu}, \text{matter fields})$$

where the matter fields can be things like

- $J^\mu[a]$ - the current of fluid of type a,
- $F^{\mu\nu}$ - the electromagnetic field
- etc.

General Relativity: solutions

We have a set of coupled partial differential equations (including Einstein's field equations). Solve and find a solution

$$\Psi = \left\{ (p, \mathbf{\Phi}) : \forall p \in \mathscr{M} \right\}$$

Note that, if Ψ is a solution, then so is

$$\varphi^* \Psi = \left\{ (p, \varphi^* \mathbf{\Phi}) : \forall p \in \mathscr{M} \right\}$$

(and it has the same physical content). Here φ is a diffeomorphism and it "moves" the fields on $\mathscr{M}$.

Introducing agency

E.g. two fluids *ship* and *wind*. Let

$$G^{\mu}[\text{ship}] = \nabla_{\nu} T^{\mu\nu}[\text{ship}] = \chi^{\mu\alpha} U^{\beta}[\text{ship}] T_{\mu\nu}[\text{wind}]$$

Can think of $\chi^{\mu\nu}$ as depending on sail settings.

Need also a time direction field, $\tau^{\mu}(p)$ (points into forward light cone).

Gauge freedom $\boldsymbol{\tau}' = \sigma\boldsymbol{\tau}$ where σ is a time orientation preserving Lorentz boost from point of view of local inertial frame at p.

Now have

$$\Psi = \left\{(p, \boldsymbol{\Phi}(p), \boldsymbol{\chi}(p), \boldsymbol{\tau}(p)) : \forall p \in \mathscr{M}\right\}$$

Gauge group is now

$$\theta \in G^{+}$$

where $\theta = \sigma\varphi$ (with σ acting only on $\boldsymbol{\tau}$).

Beables in General Relativity

(Usually called observables in GR community)
A beable is given by any function having the property

$$B(\Psi) = B(\theta^* \Psi) \;\; \forall \theta \in G^+$$

Locality is an issue here. No function that depends on fields only in some $\mathscr{A} \subset \mathscr{M}$ can be a beable.

Observables and WS-space

> *Assertion: Observables correspond to scalars having specified values in coincidence.*

Example of scalars.

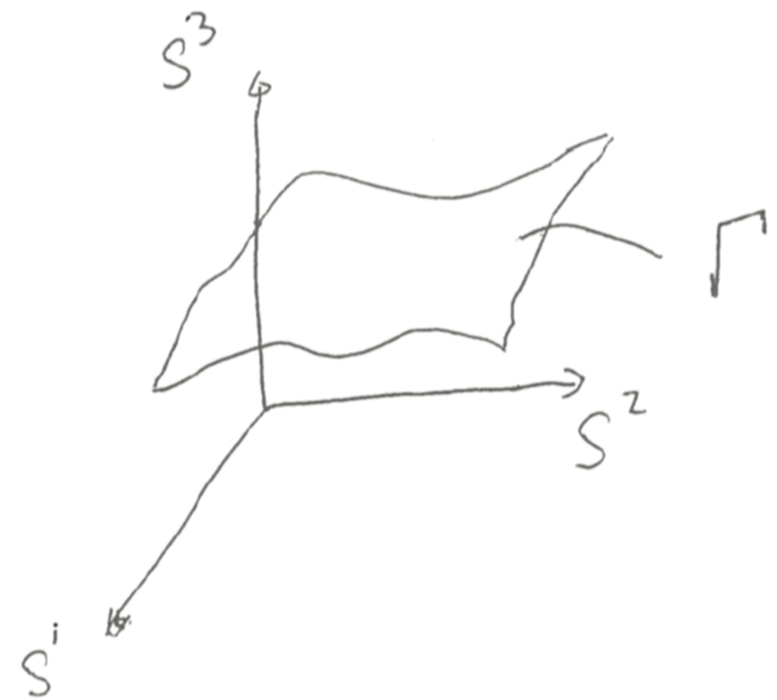
$$S[ab] = g_{\mu\nu} J^{\mu}[a] J^{\nu}[b]$$

We *nominate* a set of scalars generated by $\mathbf{\Phi}$

$$\mathbf{S} = (S^1, S^2, \dots S^K)$$

These should be rich enough to capture our experience.
The Westman-Sonego space (WS-space).
The surface, Γ, of points $\mathbf{S}$ corresponding to a solution Ψ

- Is invariant under $\theta \in G^+$.
- Has intrinsic dimension less than, or equal to $D = \dim(\mathscr{M})$.



S^3
Γ
S^2
S^1

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Agency Again

Can only set what we can observe. Thus, settings should be described by scalars. For example,

$$Q^{kl} = \frac{\partial S^k}{\partial x^\mu}\frac{\partial S^l}{\partial x^\mu}\chi^{\mu\nu}$$

In general,

$$\mathbf{Q} = (Q^1, Q^2, \dots Q^L)$$

This is the setting.
An *Agency strategy* is a choice

$$\mathbf{Q}(\mathbf{S}) \quad \forall S \in \text{WS-space}$$

Specify what would do at every point, $\mathbf{S}$.

Turning a solution inside out

Put

$$(\mathbf{\Phi}(p), \boldsymbol{\chi}(p)) \leftrightarrow (\mathbf{S}(p), \mathbf{Q}(p), \boldsymbol{\omega}(p))$$

Now put

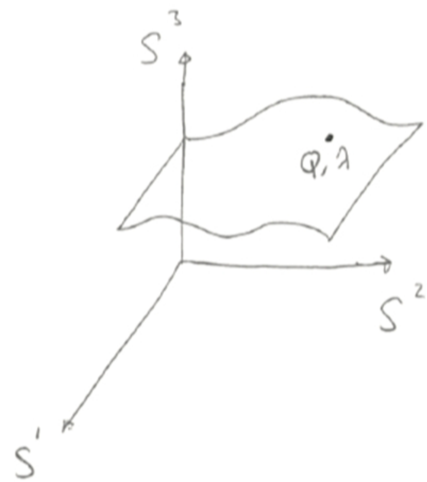
$$\boldsymbol{\lambda}(\mathbf{S}) = \{(p, \boldsymbol{\omega}(p), \boldsymbol{\tau}(p)) : \forall p \in \mathscr{M}_{\mathbf{S}}\}$$

where $\mathscr{M}_{\mathbf{S}}$ is all the points in $\mathscr{M}$ having given value of $\mathbf{S}$.
Write

$$\Psi = \{(\mathbf{S}, \mathbf{Q}, \boldsymbol{\lambda}) : \forall S \in \Gamma\}$$

This is the *inside out form*. Can convert back to

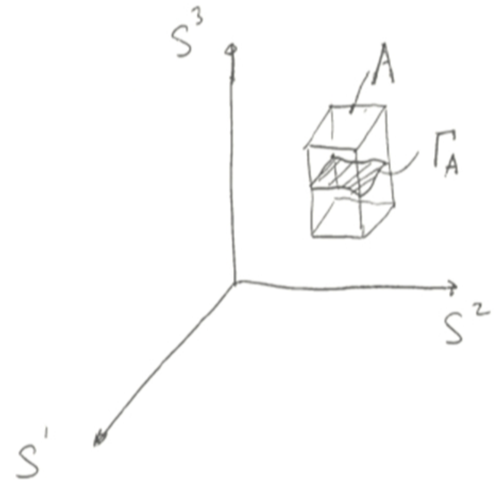
$$\Psi = \left\{(p, \mathbf{\Phi}(p), \boldsymbol{\chi}(p), \boldsymbol{\tau}(p)) : \forall p \in \mathscr{M}\right\}$$



Parts of a solution in WS-space

Consider region, $\mathtt{A}$, of WS-space.
Have

$$\Gamma_{\mathtt{A}} = \Gamma \cap \mathtt{A}$$



In this region, the solution is

$$\Psi_{\mathtt{A}} = \{(\mathbf{S}, \mathbf{Q}, \boldsymbol{\lambda}) : \forall \mathbf{S} \in \Gamma_{\mathtt{A}}\}$$

In region, $\mathtt{A}$, the agent strategy can be written

$$\mathbf{Q}_{\mathtt{A}} = \{(\mathbf{S}, \mathbf{Q}(\mathbf{S})) : \forall \mathbf{S} \in \mathtt{A}\}$$

Propositions for $\mathtt{A}$

Operational Propositions These are the propositions we can directly verify.

- *Basic:* $\text{Prop}_{\mathtt{A}}[\Gamma_{\mathtt{A}}]$
- *Course-grained:* $\text{Prop}[\{\Gamma_{\mathtt{A}}^{\alpha} : \alpha \in O_{\mathtt{A}}\}]$

Ontic Propositions
Define

$$\tilde{\Psi}_{\mathtt{A}} = \{\theta^* \Psi_{\mathtt{A}} : \forall \theta \in G^+\}$$

(Heavy handed way to give gauge invariant presentation of solution.)

- *Basic:* $\text{Prop}[\tilde{\Psi}_{\mathtt{A}}]$
- *Course-grained:* $\text{Prop}[\{\tilde{\Psi}_{\mathtt{A}}^{r} : r \in R_{\mathtt{A}}\}]$

Operational propositions can be written as course-grained ontic propositions.

Composition

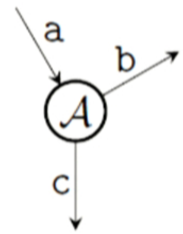
Principle of general compositionality: the laws of physics should be written in such a way that they apply to any compositional description of any object and in terms of a calculation having the same compositional structure as this description.

Compare with *Principle of general covariance* - the laws of physics should be written in such a way that they take the same form in any coordinate system.

Encapsulated propositions, $\mathcal{A}$, $\mathcal{B}$, ...

An encapsulated proposition

$$\mathcal{A}_{\mathrm{a}}^{\mathrm{bc}} = (\mathrm{prop}(\mathcal{A}), \mathbf{Q}_{\mathcal{A}}, (\mathrm{bc}, \mathrm{a})) =$$



Here a, b, ... are directed bounding surfaces.
Can simplify an encapsulated proposition using physics. Let

$$\Omega(\mathcal{A}, \mathbf{Q}_{\mathcal{A}})$$

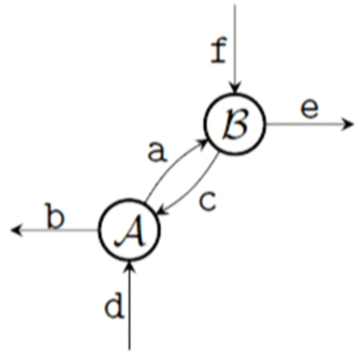
be set of allowed $\tilde{\Psi}_{\mathcal{A}}$. Then can replace

$$\mathrm{prop}(\mathcal{A}) \rightarrow \mathrm{prop}_{\Omega}(\mathcal{A}) = \mathrm{Prop}[\{\tilde{\Psi}^{r}_{\mathcal{A}} : \forall r\} \cap \Omega(\mathcal{A}, \mathbf{Q}_{\mathcal{A}})]$$

Composing encapsulated propositions

Can consider two encapsulated propositions joined as follows

$$\mathcal{C}^{\mathtt{ab}}_{\mathtt{cd}}\mathcal{B}^{\mathtt{ec}}_{\mathtt{af}} \iff$$



Can simplify using physical matching. Prune so as to only keep cases that match. This is subtle because of the G^+ group.
For each element of $\theta_{\mathcal{B}}$ and $\theta_{\mathcal{C}}$ on each side, have a condition

$$\mathsf{cond}[\mathsf{ac}]$$

that must take the same value on each side. For these cases, the joint solution

$$\Psi^q_{\mathcal{A}} \cup \Psi^r_{\mathcal{B}}$$

is a solution for joint region.

Equivalence classes under composition

Let
$$a = \left\{\theta^* \mathsf{cond}[a] : \forall\theta \in G^+\right\}$$
(Heavy handed). If there exists a $\Psi_{\mathcal{A}} \in \tilde{\Psi}_{\mathcal{A}}$ satisfying one of these conditions then there will be one satisfying every one of these conditions. Then
$$\mathcal{A}_{\mathsf{a}}^{\mathsf{bc}} \quad \text{and} \quad \mathcal{A}'^{\mathsf{bc}}_{\mathsf{a}}$$
belong to equivalence class under composition if they have same (bc, a) at boundary.

Probabilities

Assume that (for whatever reason) we have probabilities for different ontic encapsulated propositions associated with

$$\mathcal{A}_{\mathsf{a}}^{\mathsf{bc}}$$

Then we write

$$A_a^{bc} = \text{probability density of matching } (bc, a) \text{ at boundary}$$

Need a measure, da, to talk of probability density.

Can now avail ourselves of the duotensor machinery (with continuous rather than discrete indices).

Principle of general compositionality

$$\mathcal{A}_{\mathsf{a}}^{\mathsf{bc}} \longrightarrow A_a^{bc}$$
$$\mathcal{B}_{\mathsf{be}}^{\mathsf{d}} \longrightarrow B_{be}^{d}$$
$$\mathcal{A}_{\mathsf{a}}^{\mathsf{bc}} \mathcal{B}_{\mathsf{be}}^{\mathsf{d}} \longrightarrow A_a^{bc} B_{be}^{d}$$

Repeated index implies integration (rather than summation) using measure $\mathrm{d}b$.

F-locality

Relative probability

$$\frac{\mathsf{Prob}(\mathcal{A}^{\mathsf{a}}_{\mathsf{bc}})}{\mathsf{Prob}(\mathcal{A}'^{\mathsf{a}}_{\mathsf{bc}})}$$

well conditioned iff generalized states are proportional

$$A^a_{bc} = kA'^a_{bc}$$

Then relative probability is equal to k.
Allows us to do calculations in face of indefinite causal structure.

Causality

Special to General Relativity

$$[(g_{\mu\nu} = \eta_{\mu\nu}] \rightarrow [g_{\mu\nu} \text{ satisfies } G_{\mu\nu} = 8\pi T_{\mu\nu}]$$

Compare with

$$[\text{Pavia causality condition}] \rightarrow [???]$$

Can demand that a deterministic effect is unique employing $\boldsymbol{\tau}$ field. Does this fully characterize generalized states?

Conclusions

- Have indicated route to an operational probabilistic formulation of GR
- Various challenges remain: measure da, causality condition, constraints on generalized states, ...
- Will introduce fiducials and free encapsulated propositions (not tied to a particular region of WS-space).
- Can sketch route to QG.

Plan of attack on Quantum Gravity

$$\begin{tikzcd} \mathsf{OpQT} \arrow[r, "\mathsf{GRize}"] & \mathsf{QuAGeR} \\ \mathsf{CProbT} \arrow[u, "\mathsf{quantize}"] \arrow[r, "\mathsf{GRize}"'] & \mathsf{PAGeR} \arrow[u, "\mathsf{quantize}"'] \end{tikzcd}$$

- Quantization: simplex $\Longrightarrow$ curved convex set from a Hilbert space.
- GRization: fixed causal structure $\Longrightarrow$ fuzzy causal structure.