Title: Information and the architecture of quantum theory

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Abstract: I will argue that, apart from their ever growing number of applications to physics, information theoretic concepts also offer a novel perspective on the physical content and architecture of quantum theory and spacetime. As a concrete example, I will discuss how one can derive and understand the formalism of qubit quantum theory by focusing only on what an observer can say about a system and imposing a few simple rules on the observerâ€™s acquisition of information.

Physics and information theory

Information theory in physics: many applications/tools

- horizon entropies, [see Myers and Verlinde talks]
- thermodynamics, [see Chiribella, Oppenheim, Barnum, Jennings, Masanes talks]
- quantum information,...[see ... talks]

But: "Information Theoretic Foundations for Physics?"
⇒ can concepts from information theory also

(a) tell us something about physical content of theories?
(b) be used to build theories?

idea:

operational approach: consider systems, observers and their relations

(im-)possibility of information theoretic activites ⇔ structure of theory

Info foundations for QT or spacetime? ⇒ which activities and relations?

1. QT?
2. Spacetime structure? encoded in relations among observers

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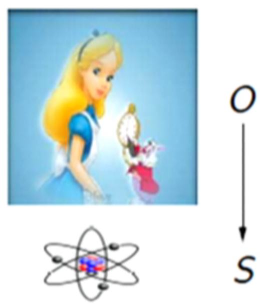
1. QT?
   activity: information acquisition
   relation: observer ⟶ system

2. Spacetime structure? encoded in relations among observers

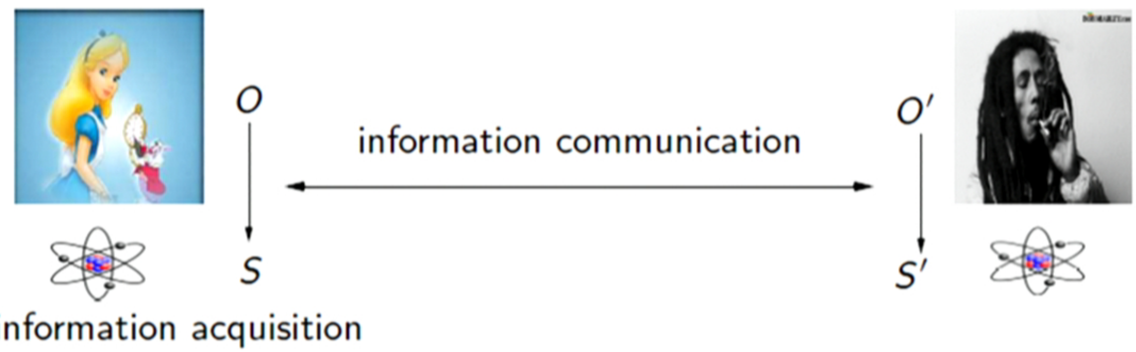


information acquisition

Info foundations for QT or spacetime? $\Rightarrow$ which activities and relations?

1. QT?
   activity: information acquisition
   relation: observer $\longrightarrow$ system

2. Spacetime structure? encoded in relations among observers
   activity: communication
   relation: observer 1 $\longleftrightarrow$ observer 2 $\cdots$



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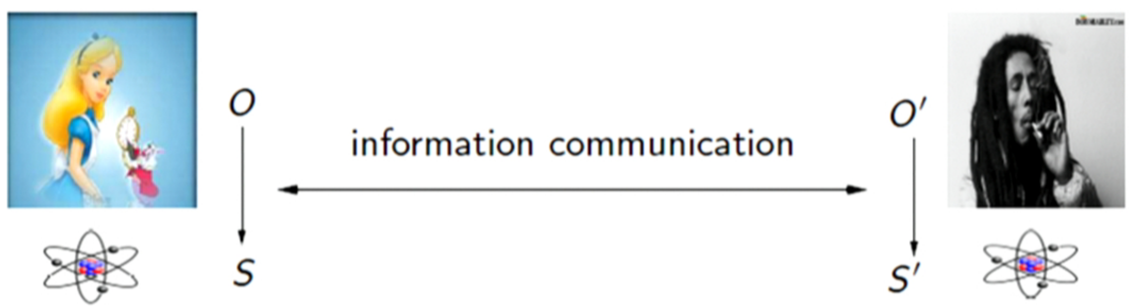
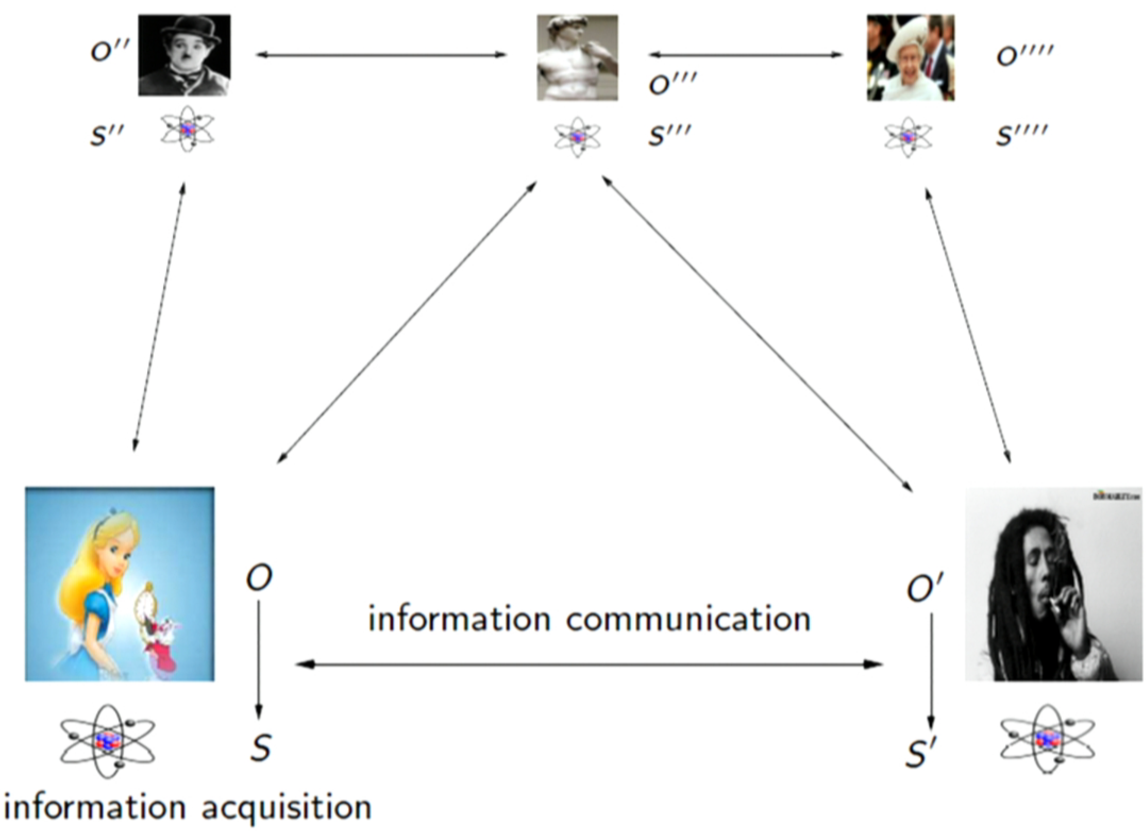


GR has informational essence (e.g., horizons)

- standard: spacetime ⇒ causal structure ⇒ information flow
  causal structure ⇒ spacetime (up to scale) [Hawking, Geroch,...]

Info foundations for QT or spacetime? $\Rightarrow$ which activities and relations?



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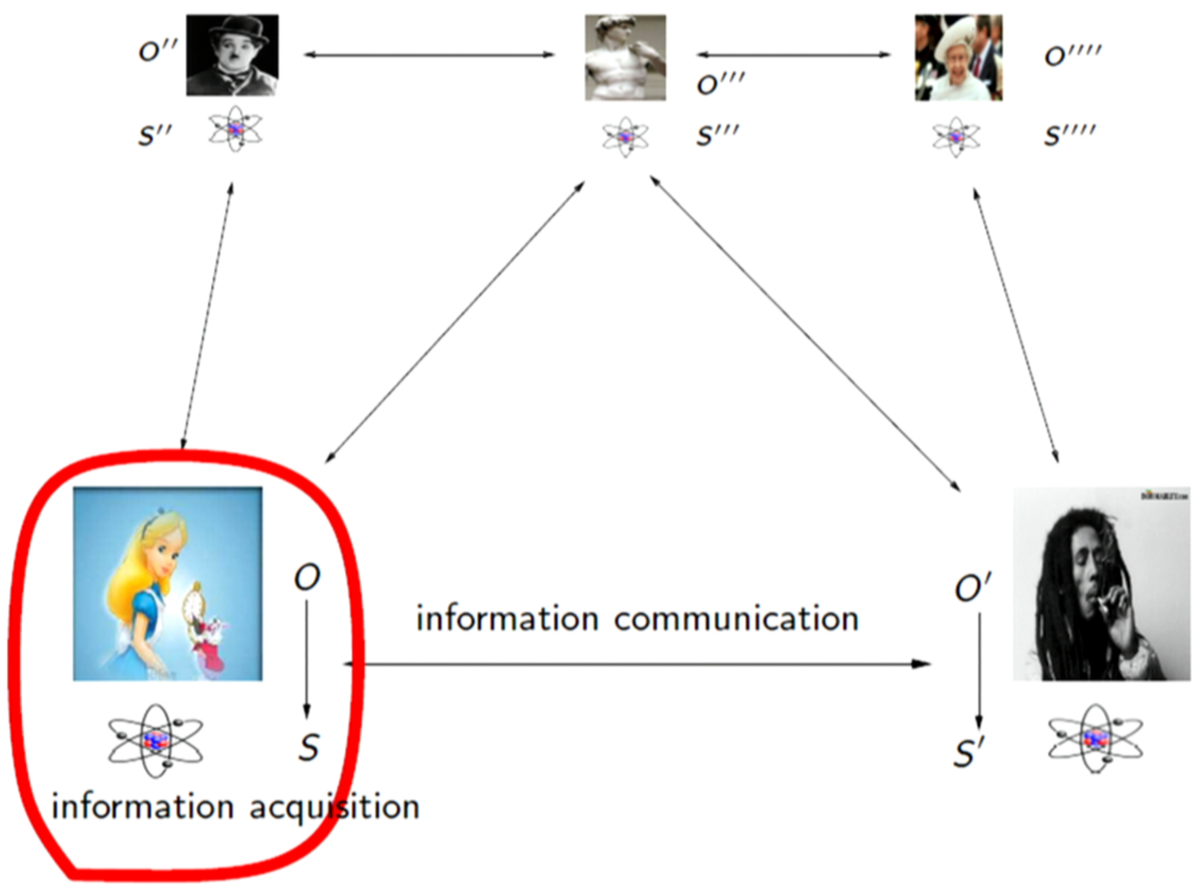
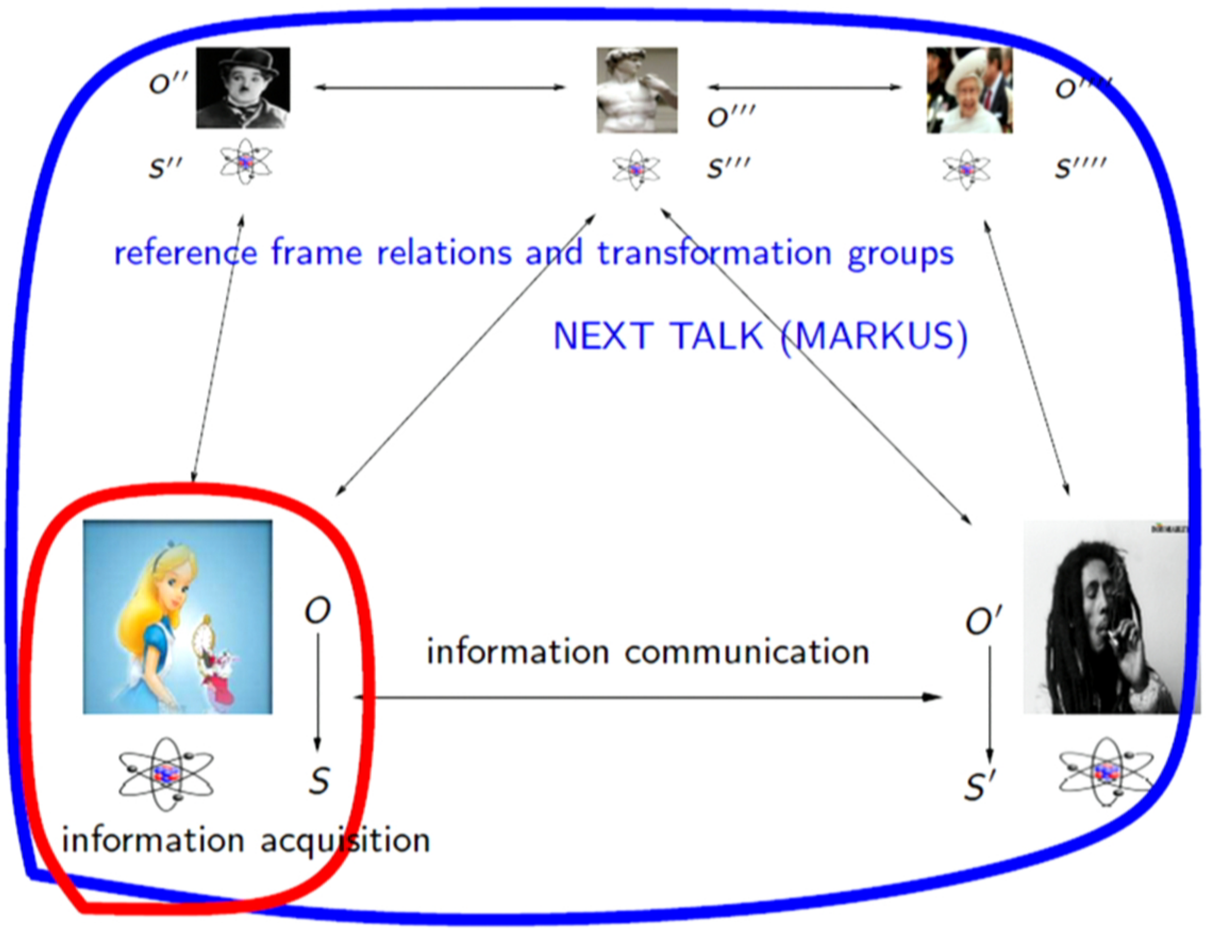


from information acquisition to quantum theory $\Rightarrow$ THIS TALK

Info foundations for QT or spacetime? ⇒ which activities and relations?



from information acquisition to quantum theory ⇒ THIS TALK

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What makes QT special?

QT hugely successful, but

(A) defined by operationally obscure axioms rather than physical statements

(B) evaded commonly accepted apprehension of its physical content

$\Rightarrow$ in contrast to relativity

advent of fundamental theories (convincing conceptual scheme for QG?):

- what are characterizing phys. features of QT? (how could the world be different if dropped?....)
- what does QT tell us about Nature or about what we can say about Nature?

$\Rightarrow$ improve understanding with operational axiomatization

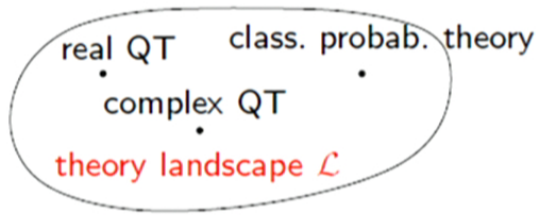
Why (re-)constructing QT?

(i) give operational sense to usual textbook axioms (why $\mathcal{H}$, $\otimes$, $\mathbb{C}$, U...?)
(ii) better understand QT within larger context of alternatives
(iii) better understand physical content of QT
(iv) 'intuitive' explanations for typical quantum phenomena
(v) gain new structural insights?

$\exists$ successful reconstructions in *generalized or operational probabilistic theories*
['01-'14 Hardy, Dakic, Brukner, Masanes, Müller, D'Ariano, Chiribella, Perinotti, Barnum, Ududec......]

$\Rightarrow$ here: different reconstruction with focus on information acquisition
[motivation from: Rovelli, Brukner, Zeilinger, Spekkens, Fuchs,......]

Outline for the remainder



Steps

1. define landscape $\mathcal{L}$ of theories
2. postulates for qubit QT within $\mathcal{L}$
3. summary of derivation of quantum formalism

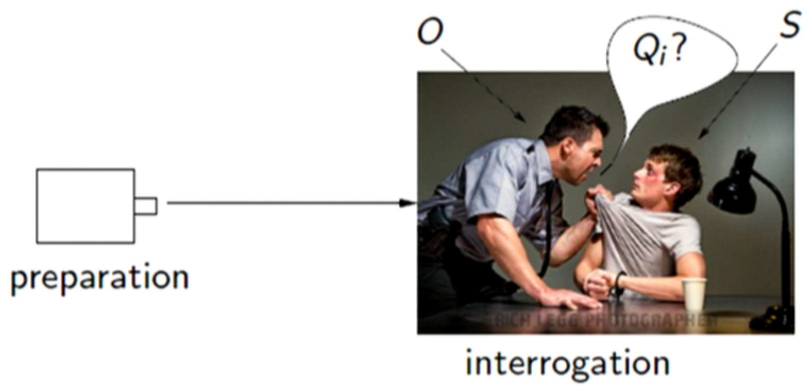
   (architecture of theory: state spaces, time evol. group, measurements...)

Specifying the landscape of inference theories [PH '14]

focus: information acquisition of observer O about system S

premise: speak only about info that O has access to (purely operational)

Setup: O interrogates S with *binary*, repeatable questions Q_i, $i = 1, \dots$



Basic ingredients:

$\mathcal{Q}$: set of binary Qs that O may ask S

Σ: set of all possible answer statistics (every prep. to produce specific answer statistic)

- assume: O has theoretical model for $\mathcal{Q}, \Sigma$

$\Rightarrow$ what is this model?

Specifying the landscape of inference theories II [PH '14]

- Bayesian viewpoint: for specific S, O assigns probabilities p_i to Q_i accord. to his info about
  1. Σ
  2. particular S

- p_i encode all O can say about $S \Rightarrow$ state of S: collection of p_i

  $\Rightarrow$ state space: Σ (to be convex)

  $\Rightarrow$ state is O's 'catalogue of knowledge' about S [see also Rovelli, QBism,...]

- assume: $\exists$ state of no information $p_i = \frac{1}{2}\ \forall\, i$ (as prior)

- single vs. multiple shot interrogation:

single: determine single system state $\Rightarrow$ 'collapse' if info gain vs. prior

multiple: estimate ensemble state (Bayesian updating)

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Specifying the landscape of inference theories III [PH '14]

Q_1, Q_2 are maximally

independent if, rel. to state of no info, answer to only Q_1 gives no info about Q_2: $p(Q_1, Q_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ factorizes

compatible if simultaneously knowable: $\exists$ state s.t. p_1, p_2 simult. $0, 1$

complementary if $p_1 = 0, 1$, then $p_2 = 1/2$ $\forall$ states (and vice versa)

- assumption: state parametrized by max. set of pairwise indep. Q_i

$$\vec{P} = \begin{pmatrix} p_1 \\ \vdots \\ p_D \end{pmatrix}, \quad p_i \text{ prob. that } Q_i = \text{'yes'}$$

- ansatz: O's info about Q_i: $0 \leq \alpha(p_i) \leq 1$ bit $\Rightarrow$ total info:

$$I(\vec{P}) = \sum_{i=1}^{D} \alpha(p_i)$$

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Rules for information acquisition for N qubits [PH '14]

(first two motivated from Rovelli, Zeilinger, Brukner)

P1: (limited information) "O can acquire maximally $N \in \mathbb{N}$ independent bits of information about S at the same time."
$\exists\, Q_i,\ i = 1, \ldots, N$ (mutually) independent compatible

P2: (complementarity) "O can always get up to N new (independent) bits of information about S. Whenever O asks a new question he experiences no net loss of information."

P3: (completeness) "O's info about S can be distributed over all Q's in any way consistent with P1 and P2."

P4: (preservation) "O's total amount of information about S preserved between interrogations".

P5: (time evolution) Evolution of $\vec{P}$ continuous, can be non-trivial

P6: (locality) "O can determine state of composite system by only interrogating its constituents."

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$\exists\, Q_i'$, $i = 1, \ldots, N$ (mutually) independent compatible but $Q_i, Q'_{j=i}$ complementary

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P4: (preservation) "O's total amount of information about S preserved between interrogations".
$I = \sum_{i=1}^{D_N} \alpha_i(p_i)$ constant in time between interrogations

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has 2 solutions: qubits & rebits (i.e., complex and real QT)

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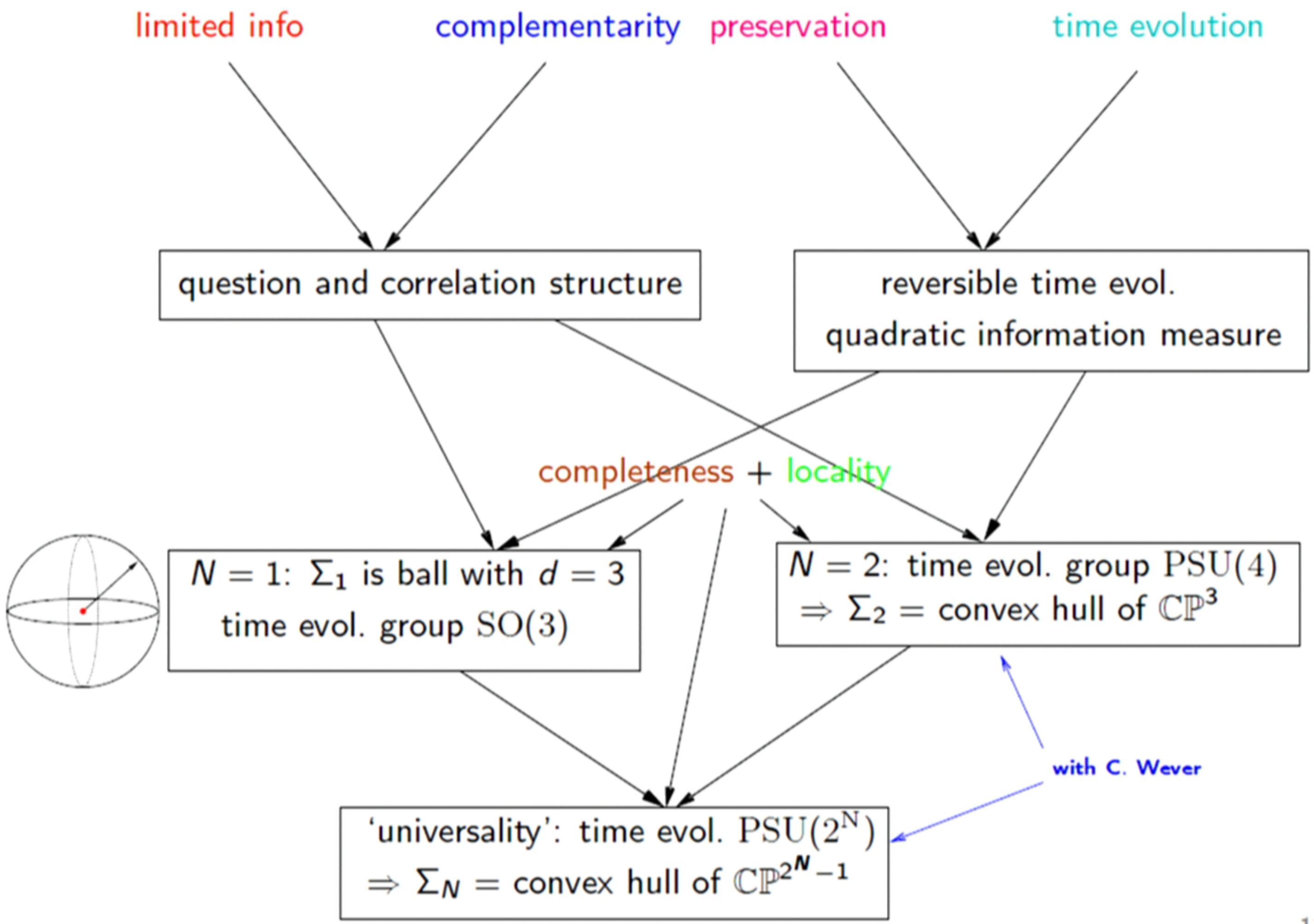
Strategy

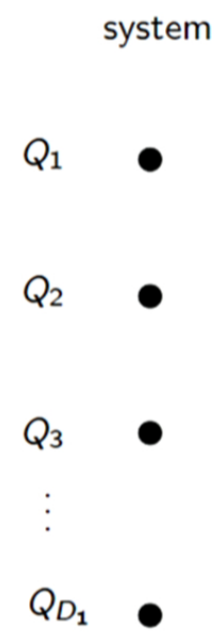

limited info
complementarity
preservation
time evolution
question and correlation structure
reversible time evol.
quadratic information measure
completeness + locality
$N = 1$: Σ_1 is ball with $d = 3$
time evol. group $\mathrm{SO}(3)$
$N = 2$: time evol. group $\mathrm{PSU}(4)$
$\Rightarrow \Sigma_2 =$ convex hull of $\mathbb{CP}^3$
with C. Wever
'universality': time evol. $\mathrm{PSU}(2^{\mathrm{N}})$
$\Rightarrow \Sigma_N =$ convex hull of $\mathbb{CP}^{2^N-1}$

Compatibility and independence structure of questions [PH '14]

$N = 1$: only individual Q_i, $i = 1, \ldots, D_1 \Rightarrow D_1 =?$ (know $D_1 \geq 2$)

$N = 2$: $2D_1$ individual Q_i

vertex: individual question Q_i



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$N = 2$: $2D_1$ individual Q_i + D_1^2 composite questions:

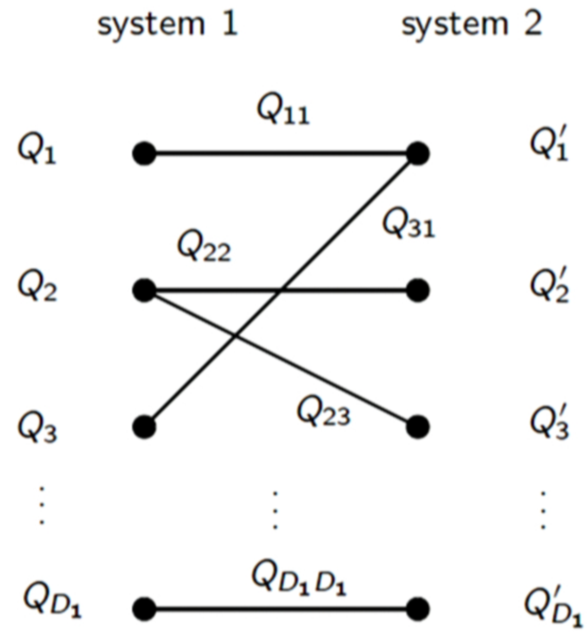
$Q_{ij} := Q_i \leftrightarrow Q'_j$ "Are answers to Q_i and Q'_j the same?"

vertex: individual question Q_i, Q'_j

edge: composite question Q_{ij}

can show: Q_{ij}

1. pairwise indep.
2. complementary if corresp. edges intersect (e.g., Q_{11}, Q_{31})
3. compatible if corresp. edges non-intersecting (e.g., Q_{11}, Q_{22})



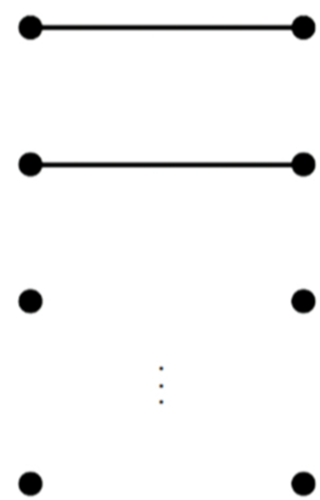
Entanglement and monogamy from complementarity [PH '14]

- O can spend max. amount of 2 indep. `bits` (c.f. limited info) over composite Qs

  ⇒ O has no individual info

  ⇒ entanglement (due to complementarity)

  > *"...the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts..." (Schrödinger, 1935)*



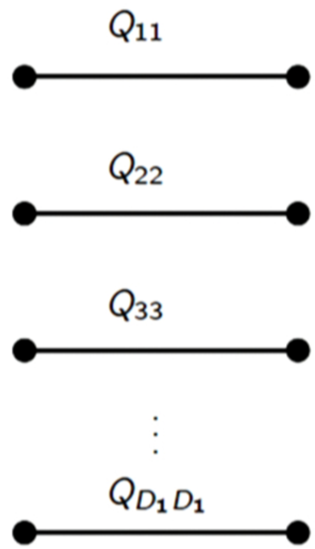
- define entanglement: > 1 bit in Q_{ij}
  [see Brukner, Zeilinger]

- intuitive explanation for monogamy:
  - A,B max. entangled
  - ⇒ 2 indep. bits over A,B spent
  - ⇒ O cannot know anything else about A,B (incl. correl. with C)

What is the dimension D_1 of the Bloch sphere? [PH '14]

- Q_{ii}, $i = 1, \ldots, D_1$ pairwise independent, compatible
- O can acquire answers to all D_1 composites Q_{ii} simultaneously (Specker's principle)
- limited info: O cannot know more than $N = 2$ indep. bits about S

$\Rightarrow$ answers to any two Q_{ii} determine answers to all other Q_{jj}



- e.g., truth table for any three Q_{ii} $(a \neq b)$: $\Rightarrow Q_{33} = Q_{11} \leftrightarrow Q_{22}$ or $\neg(Q_{11} \leftrightarrow Q_{22})$

Q_{11}	Q_{22}	Q_{33}
0	1	a
1	0	a
1	1	b
0	0	b

$\Rightarrow$ holds for all compatible sets of Q_{ij}: $2 \leq D_1 \leq 3$

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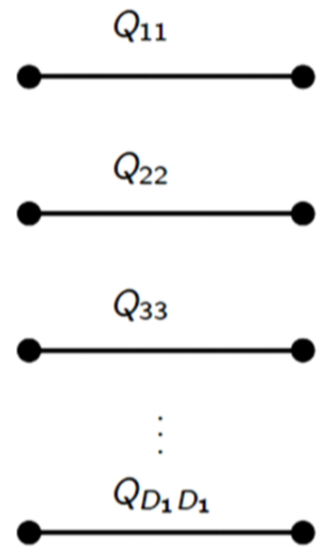

Q_{11}	Q_{22}	Q_{33}
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1	1	b
0	0	b

Information measure [PH '14]

preservation and time evolution imply:

1. reversible time evolution $T \in$ some 1-param. group

$$\vec{r}(t) = T(t) \cdot \vec{r}(0)$$

with 'Bloch' vector $\vec{r} = 2\,\vec{P} - \vec{1}$

2. O's info about Q_i $\alpha_i = (2p_i - 1)^2 \Rightarrow O$'s total info about S:

$$I = ||2\,\vec{P} - \vec{1}||^2 = \sum_{i=1}^{D_N} (2\,p_i - 1)^2 = \sum_{i=1}^{D_N} r_i^2$$

[from different perspective also proposed by Brukner, Zeilinger]

$\Rightarrow$

"(Bloch vector length)^2 = (# of answered Qs)"

3. $\{$all possible time evolutions$\} \subset \mathrm{SO}(\mathrm{D_N})$

$\Rightarrow$ total info I 'conserved charge' of time evol.

$N = 1$ and the Bloch ball [PH '14]

argued before: $D_1 = 3 \Rightarrow$ have: $\vec{P} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$

- **pure states:**

$$I = (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 1\,\texttt{bit}$$

- **mixed states:**

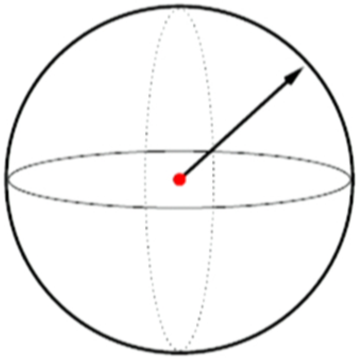
$$0\,\texttt{bit} < (2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 < 1\,\texttt{bit}$$

- **completely mixed state:**

$$(2\,p_1 - 1)^2 + (2p_2 - 1)^2 + (2p_3 - 1)^2 = 0\,\texttt{bit}$$

using completeness axiom:

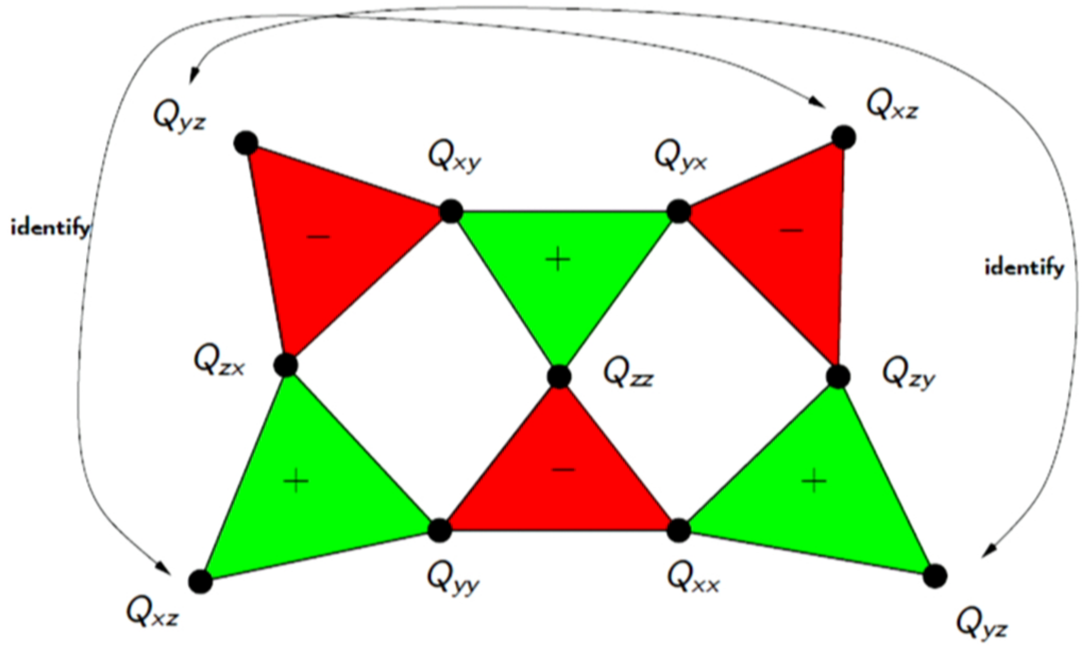
1. Σ_1 = Bloch ball ✓
2. $\{$all time evolutions $T\} = \mathrm{SO}(3)$ ✓

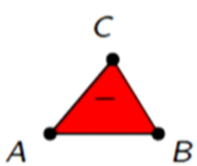


Correlation structure for $N = 2$ qubits [PH '14]

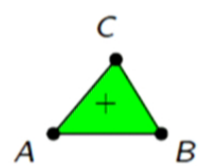
Compatibility structure of Qs $\Rightarrow$ correlation structure for 2 qubits in QT

Q, Q' compatible if connected by edge, otherwise complementary





$\Leftrightarrow$ odd correlation
$A = \neg(B \leftrightarrow C)$,
etc...



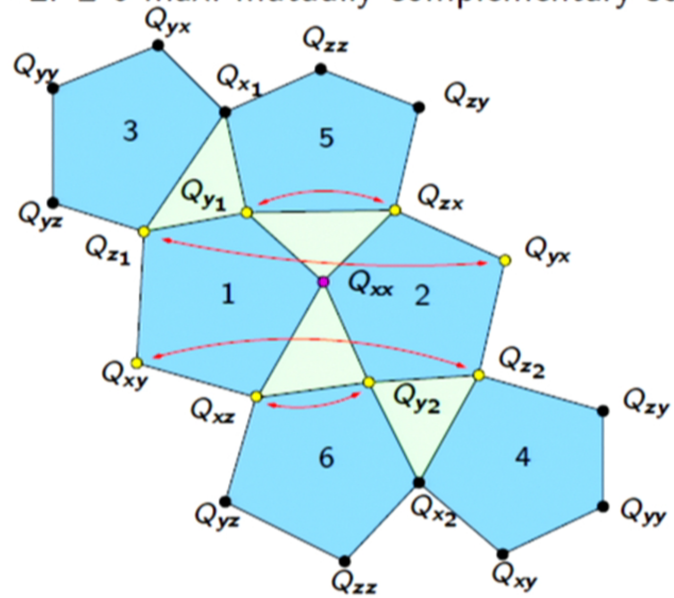
$\Leftrightarrow$ even correlation
$A = B \leftrightarrow C$,
etc...

$N = 2$: complementarity, unitarity and pure states [with C. Wever, to appear]

- pure states for $N = 1$: max. set of mutually compl. Qs carries 1 bit

$$r_x^2 + r_y^2 + r_z^2 = 1\,\mathrm{bit}$$

- generalizes to $N = 2$: $\exists$ 6 max. mutually complementary sets of 5 Qs,



- pure states have "conserved charges": $\mathrm{Info}(\mathrm{Pent}_i) = 1$ bit, $i = 1, \ldots, 6$,

$$e.g., \qquad \mathrm{Info}(\mathrm{Pent}_1) = r_{y_1}^2 + r_{z_1}^2 + r_{xx}^2 + r_{xy}^2 + r_{xz}^2 = 1\,\mathrm{bit}$$

$\Rightarrow$ define unitary group $\mathrm{PSU}(4)$ and pure state space $\mathbb{C}P^3$

Conclusions

Rules on O's acquisition of information about S yield formalism of QT

state as 'catalogue of knowledge'

$\Rightarrow$ sufficient to speak only about O's info!

Bohr:

"It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature..."

novel constructive perspective on:

- dimensionality of state spaces
- entanglement and correlation structure
- monogamy
- quantifying O's info
- origin of unitary group from complementarity and 'conserved info charges'

further reading: PH arXiv:1412.8323 (revision coming!), PH and C. Wever (forthcoming)