Title: Quantum theory and spacetime: allies, not enemies

Date: May 12, 2015 09:50 AM

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Abstract: It has become conventional wisdom to say that quantum theory and gravitational physics are conceptually so different, if not incompatible, that it is very hard to unify them. However, in the talk I will argue that the operational view of (quantum) information theory adds a very different twist to this picture: quite on the contrary, quantum theory and space-time are highly fine-tuned to fit to each other.
After a recap of ideas by von Weizsacker, Wootters, and Popescu and Rohrlich, I will show how uncertainty relations, the number of degrees of freedom of the Bloch ball, and the existence of entangled states and possibly the Tsirelson bound can be understood from space-time geometry alone. Conversely, I will show how the 3+1 Lorentz group of space-time can be derived from a purely informational communication scenario of two observers that describe local quantum physics in different Hilbert space bases (joint work with Philipp Hoehn).

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Probabilistic, usually fixed background, …	Deterministic, background-independent, …

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non-renormalizable

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This talk: quantum information adds a different facet to this picture…

This talk

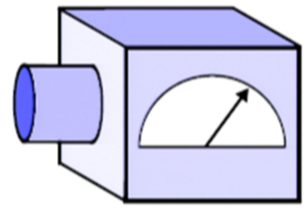
QT **space-time**

They mutually constrain (part of) each other's structure.

They are "strange allies".

Outline

1. A glance beyond quantum theory



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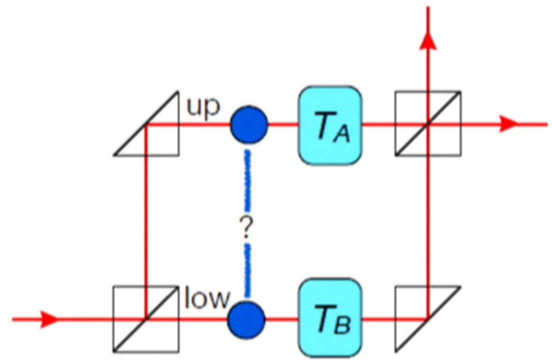

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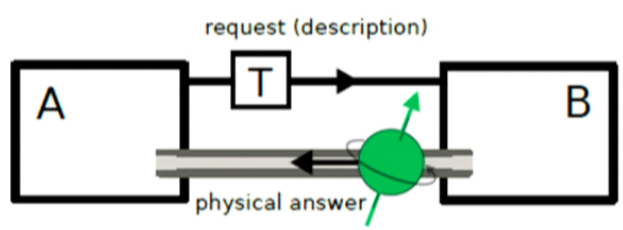

1. A glance beyond QT

Goal: argue counterfactually. "*What if* **XY** *was different?*"

QT **space-time**

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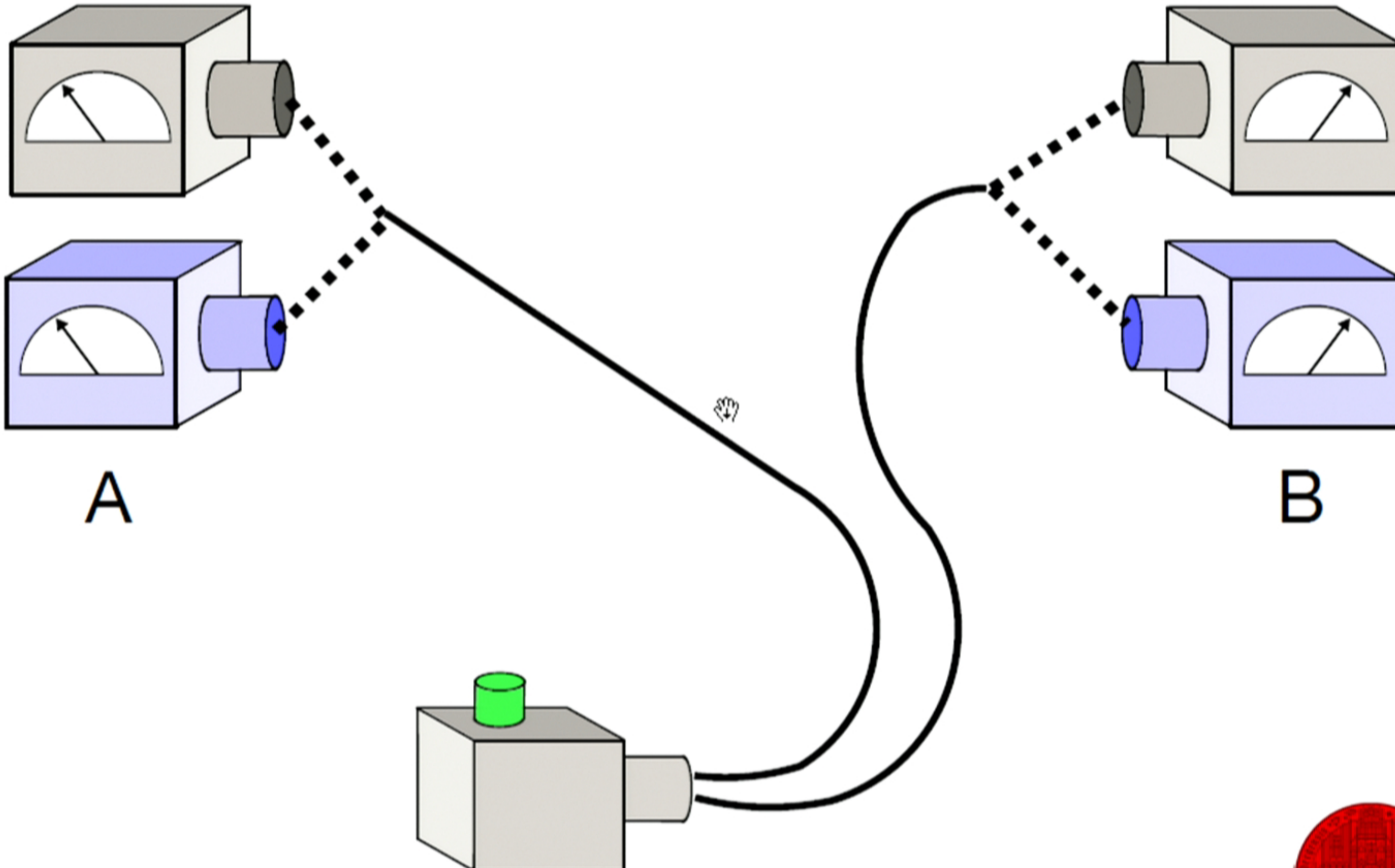
QT ?

space-time Different dimensionality, signature, torsion, discrete graph etc.

Example: Bell-CHSH inequality



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Quantum theory and spacetime: strange allies

Markus P. Müller

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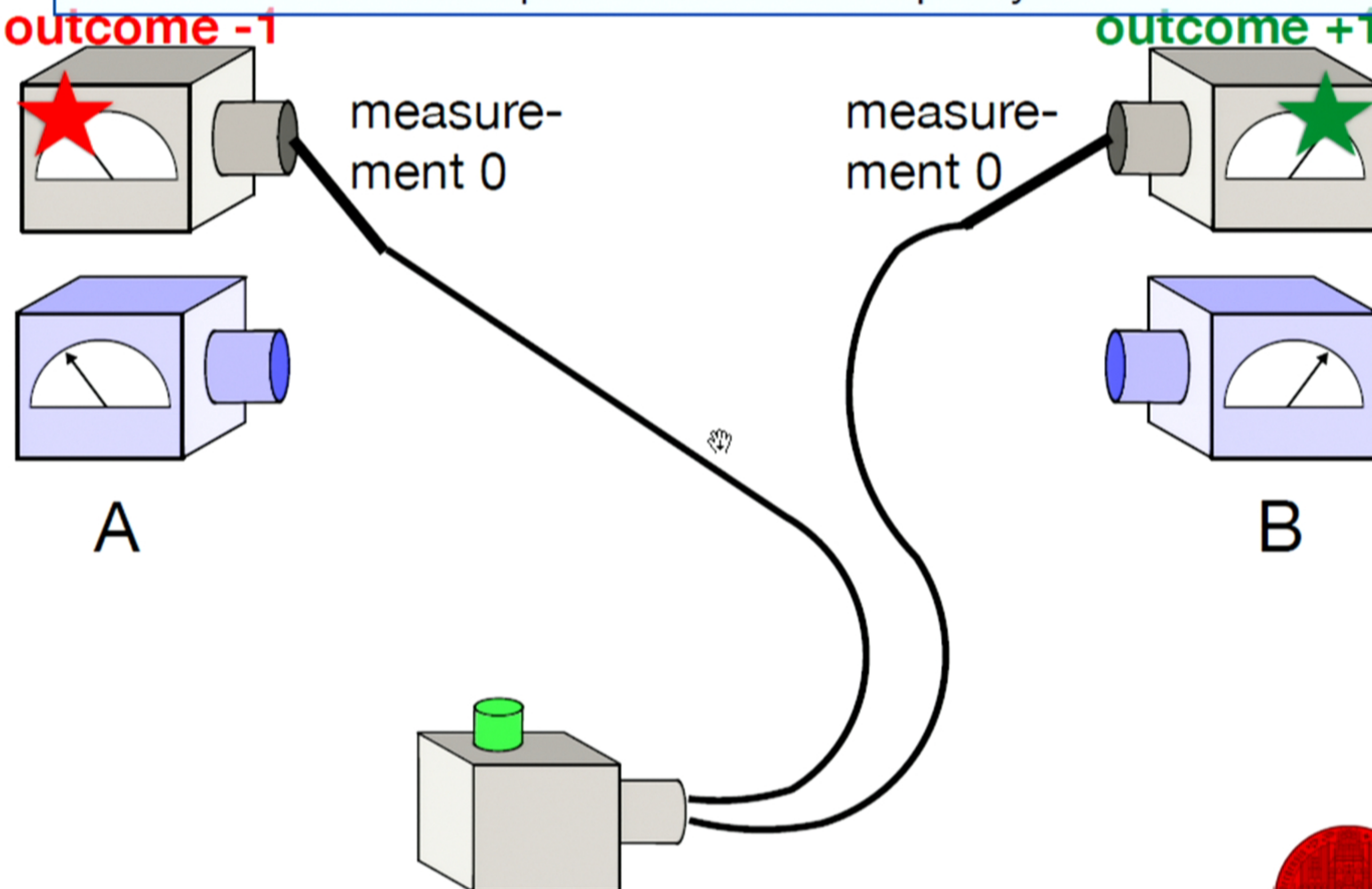

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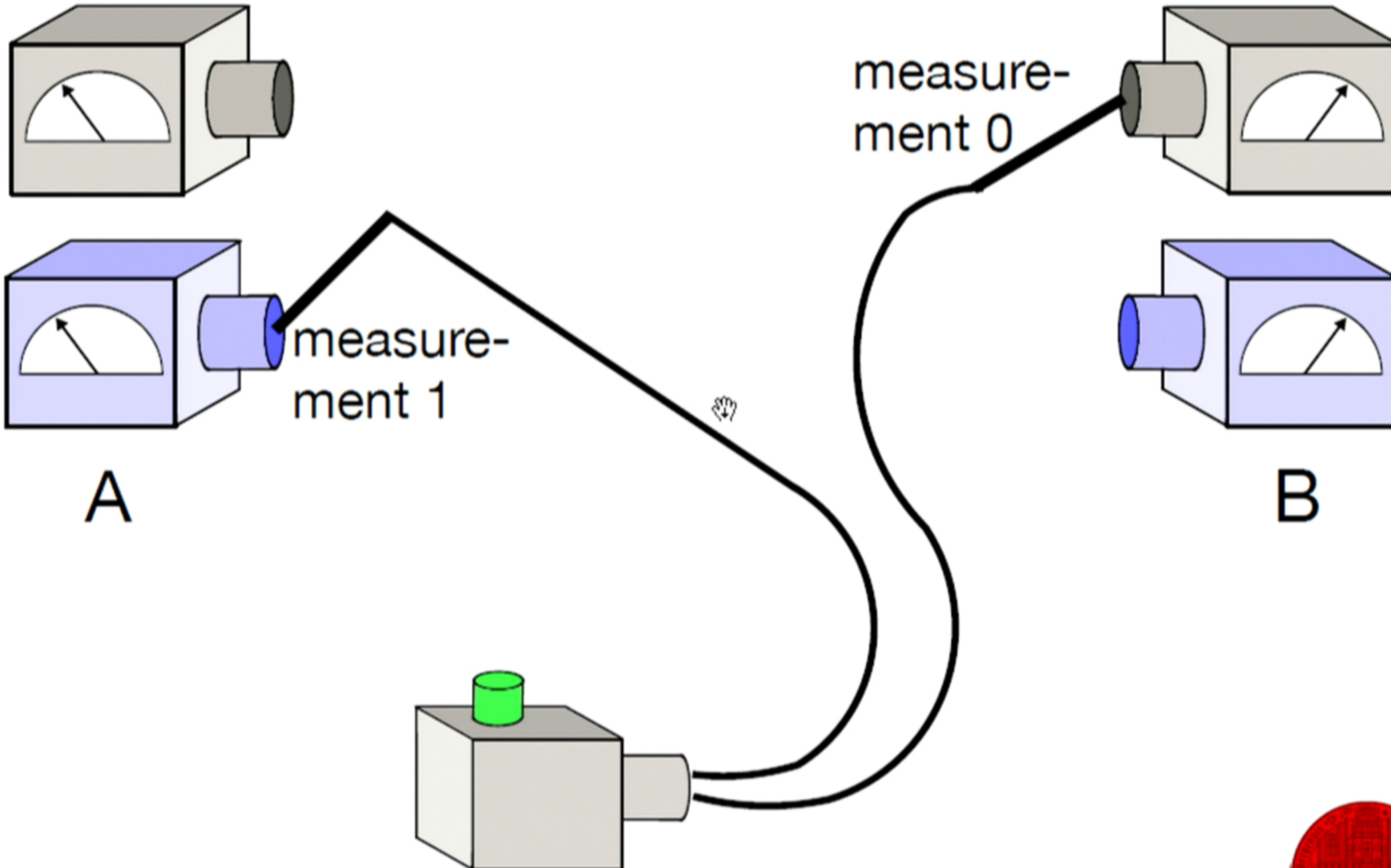

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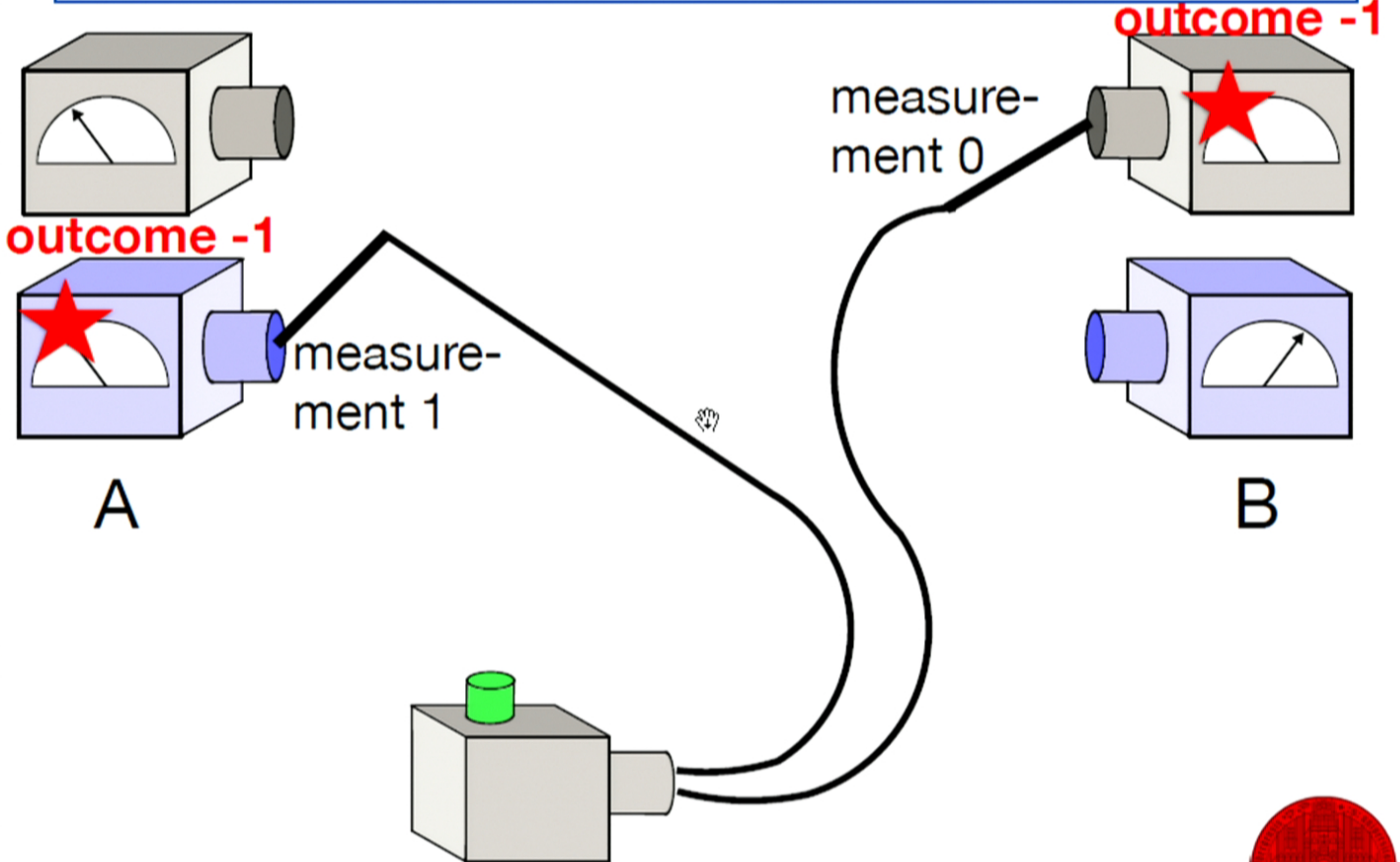

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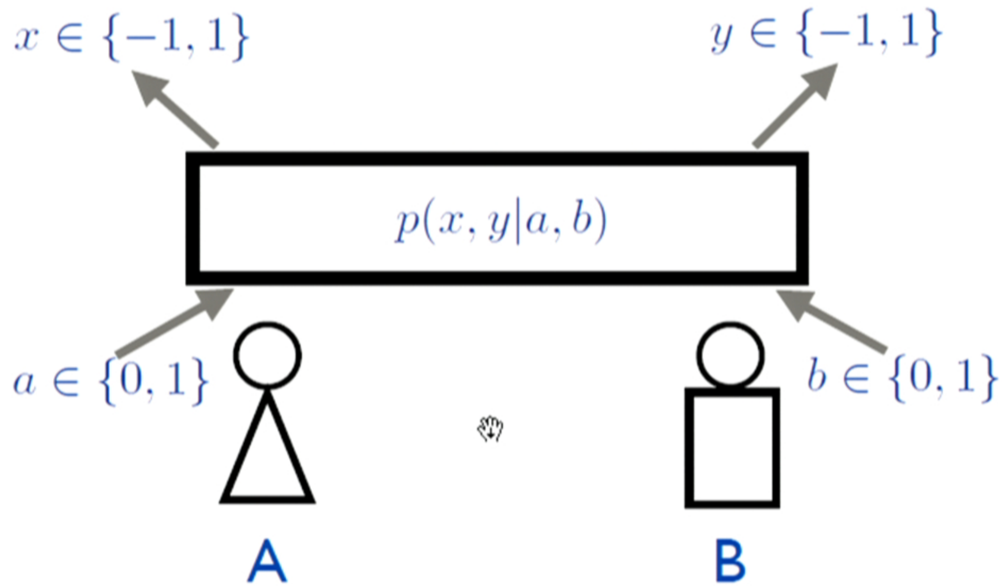

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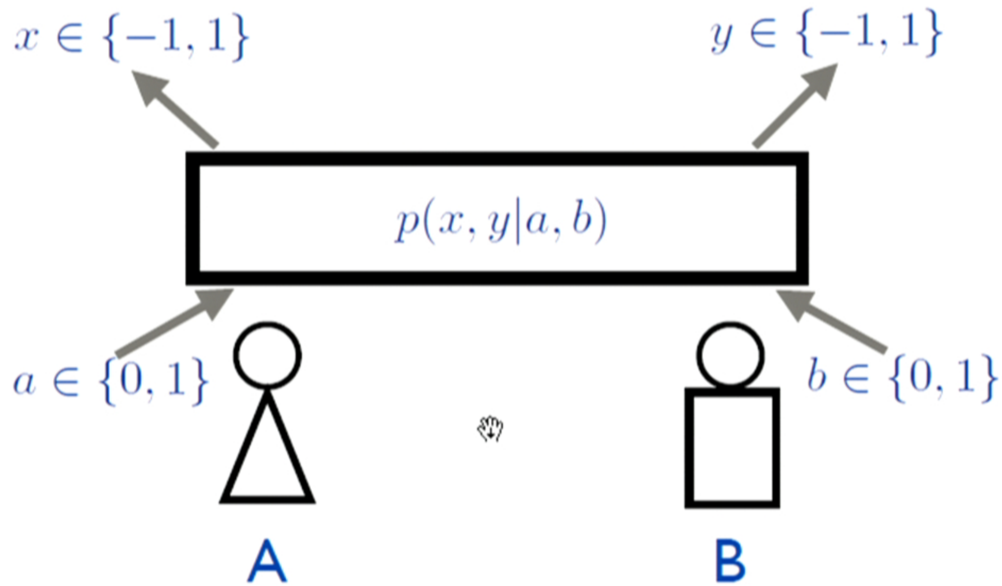

Quantum physics: $p(x, y|a, b) = \langle\psi|\hat{P}_a^x \otimes \hat{P}_b^y|\psi\rangle.$

Classical physics: $p(x, y|a, b)$ conditional prob.

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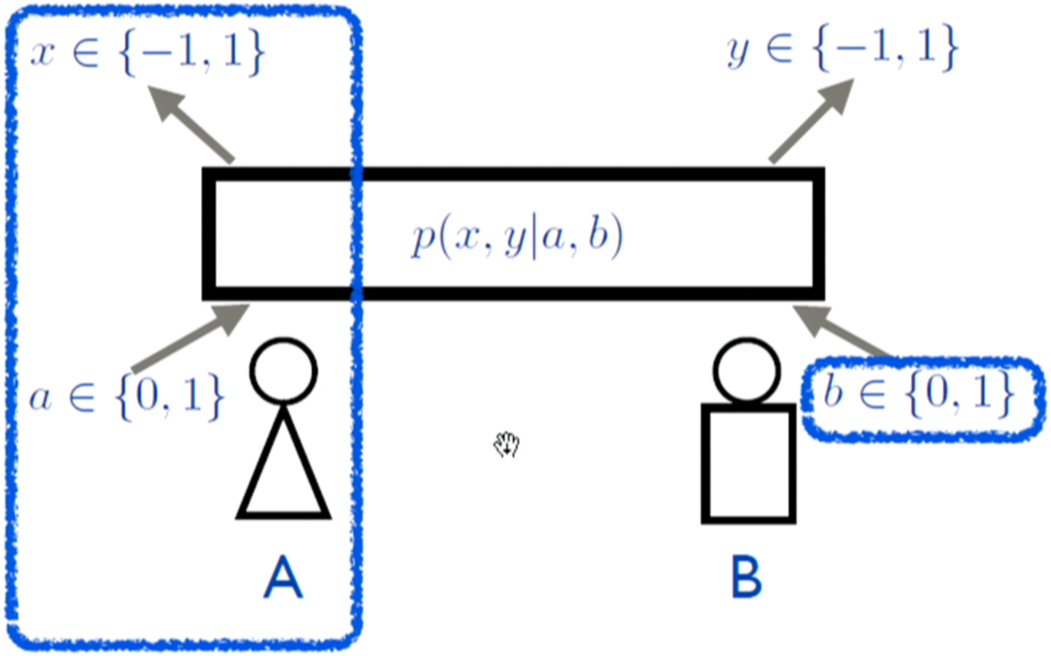

Classical and quantum physics satisfy no-signalling:

$p(x|a)$ does not depend on b (and vice versa).

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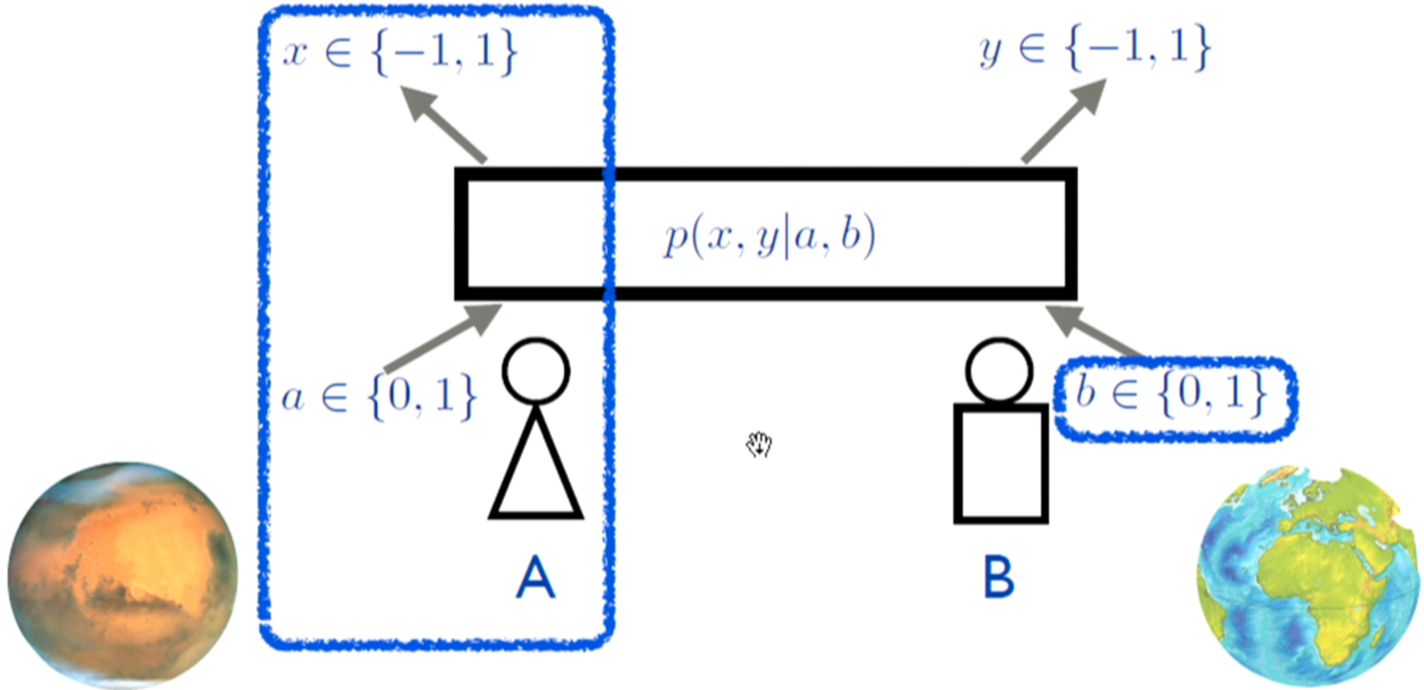

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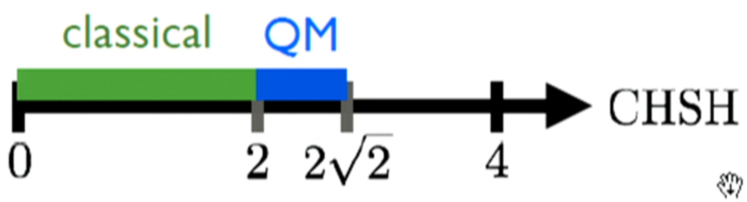
Example: Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\,(x \cdot y | a, b)\,.$$



S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994).

Quantum: Bell inequality violation.

$$\mathrm{CHSH} \leq 2\sqrt{2}.$$

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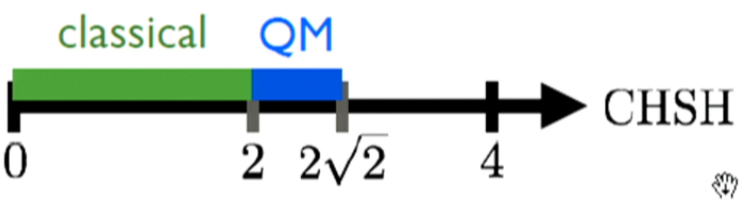

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Answer: **No.** There are ("PR box") correlations that are non-signalling, but not possible in quantum theory.

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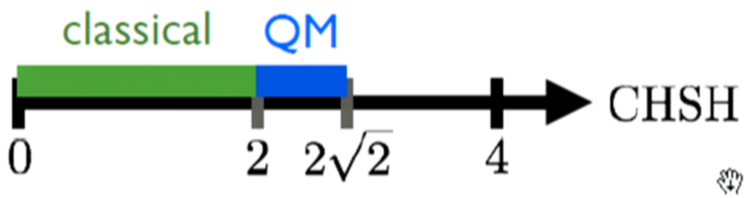

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PR-box correlations: $p(x, y|a, b)$

$$p(+1,+1|a,b) \;=\; p(-1,-1|a,b) = \frac{1}{2}$$

$$\text{if } (a,b) \in \{(0,0),(0,1),(1,0)\}$$

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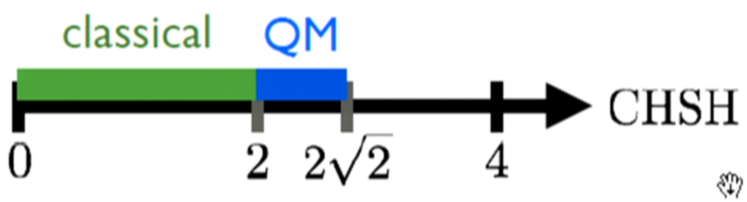

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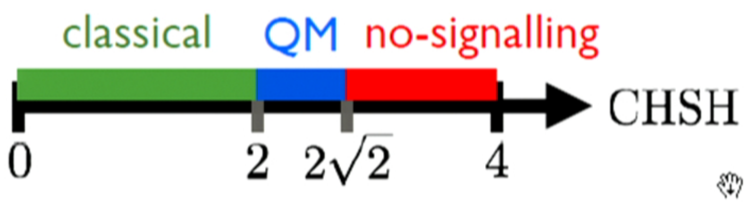

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Conclusion part 1

QT ⟵ **space-time**

Space-time structure enforces no-signalling,
but does not fully determine all of QT

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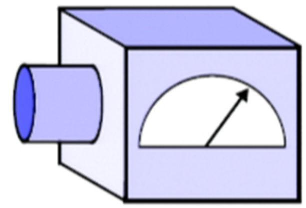

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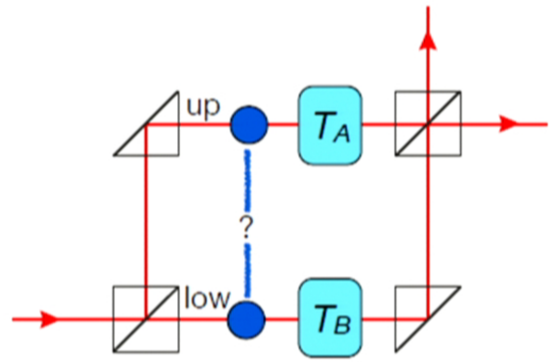

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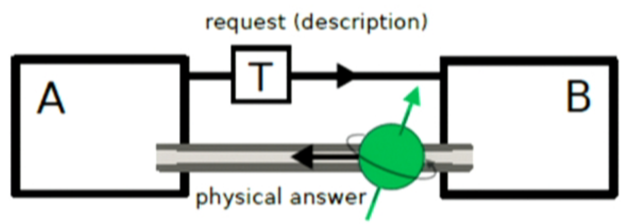

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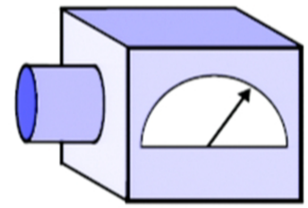

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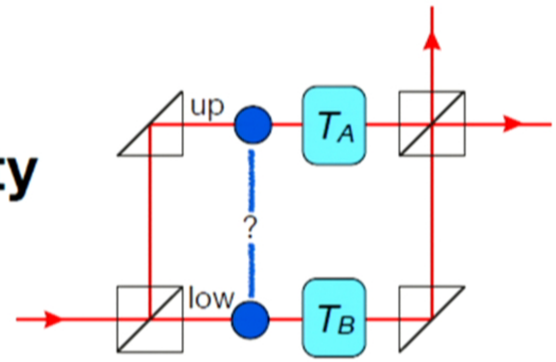

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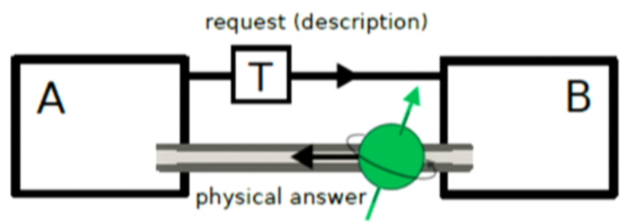
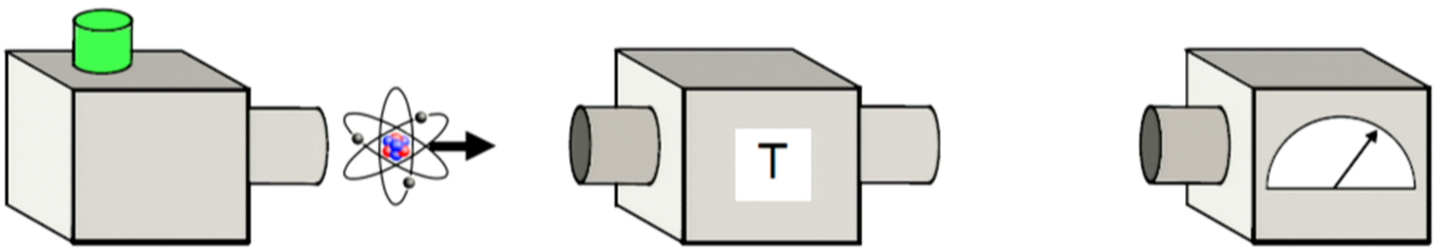


General state spaces

Preparation, transformation, measurement

- On every push of button, the preparation device outputs a physical system in some state.



2. Relativity interferometer

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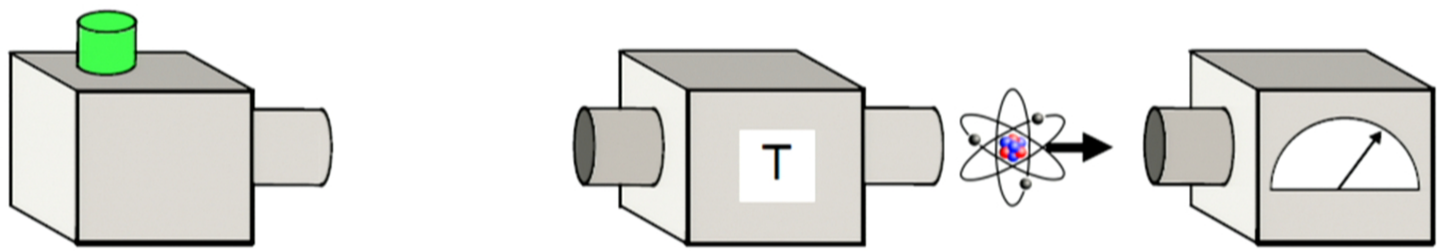

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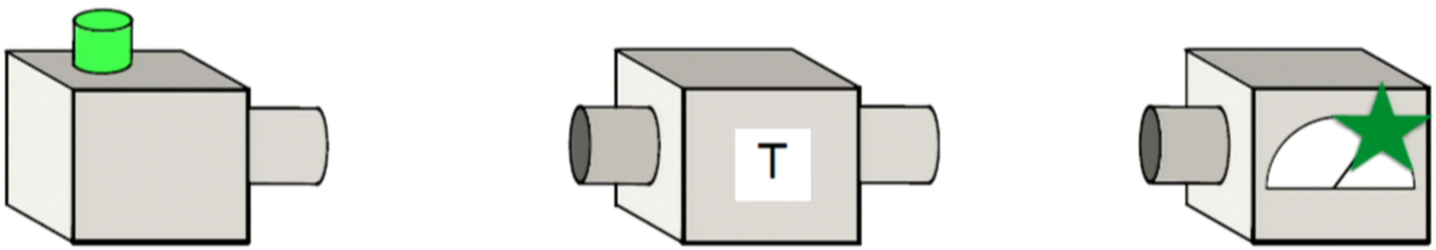
- One of many possible measurements is performed. A definite classical outcome is obtained, with prob. determined by state.

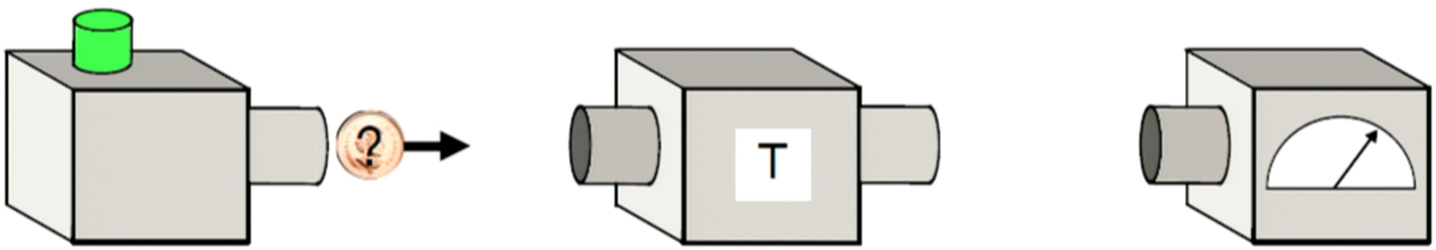


General state spaces

Example: classical coin toss.

- The preparation device prepares a physical system in a state ω. Here

$$\omega = \begin{pmatrix} \mathrm{Prob(heads)} \\ \mathrm{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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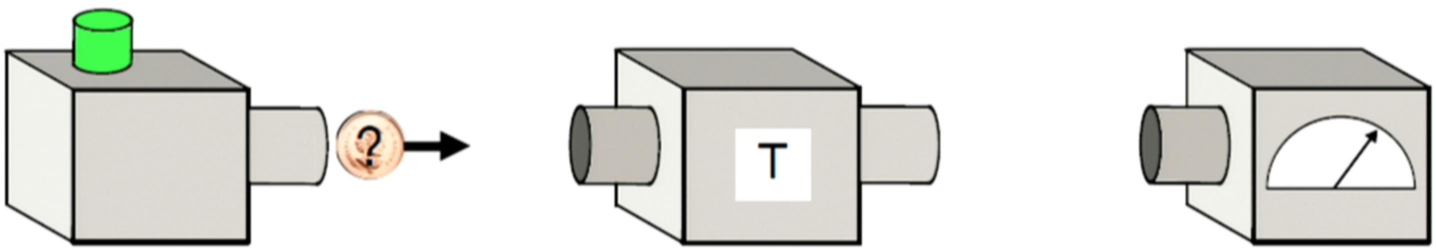

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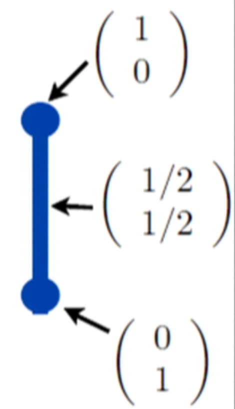
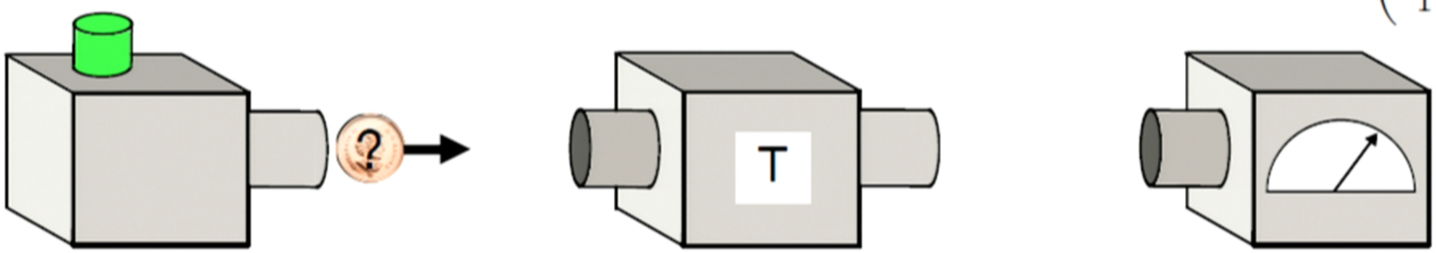
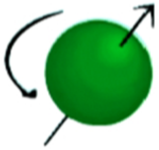


State space Ω: the set of all possible states



General state spaces

Example: quantum spin-1/2 particle.



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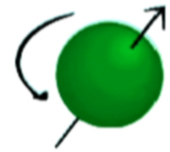
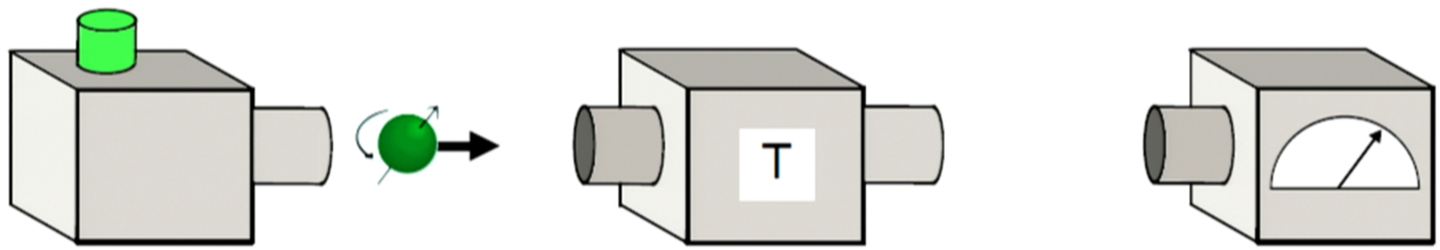


- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: ω is 2x2 density matrix.



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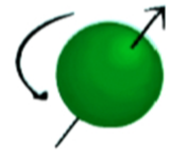

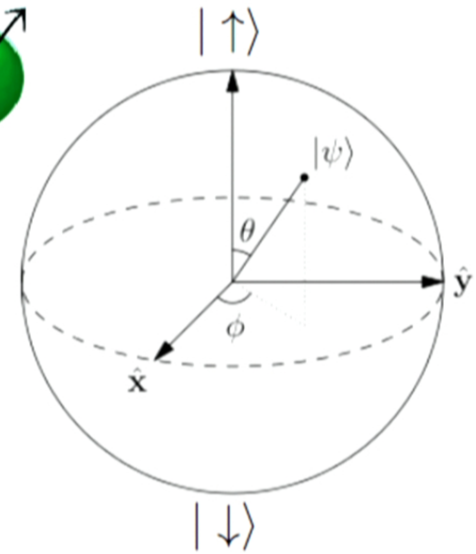

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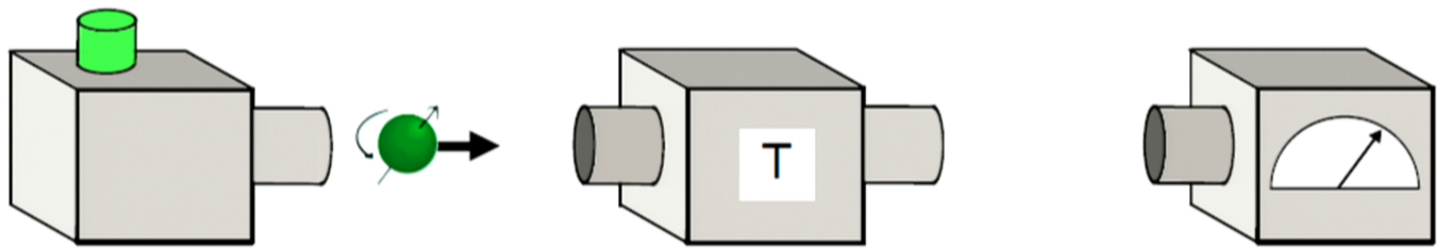

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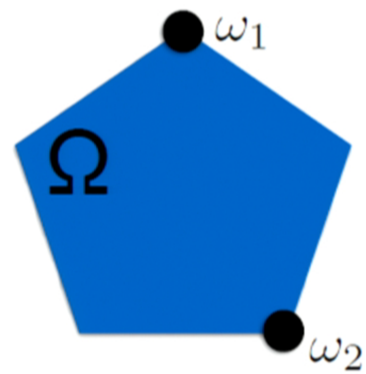
The set of all possible states of a given physical system is called the state space Ω.



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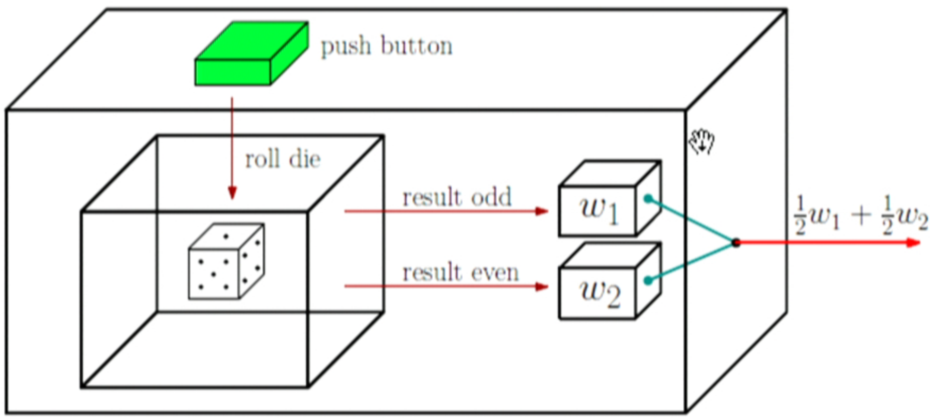

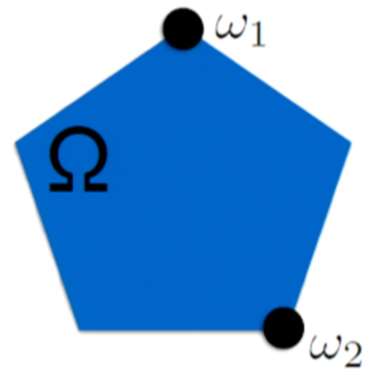

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QT: Ω_N = set of N×N density matrices

CPT: Ω_N = set of prob. distributions $(p_1, \ldots, p_N)$.

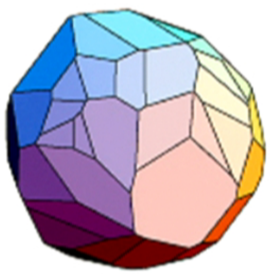
Thus Ω is a convex set.

General state spaces

classical
bit

quantum
bit

"gbit"



Arbitrary convex
state space

Classical "trit"
(3-level-system)

Quantum "trit".
Complicated, 8D!

Relativistic constraints on the state space

Two-level state spaces (“bits“) are naturally ball state spaces:

$d = 1$ classical bit

$d = 2$

$d = 3$ quantum bit

...

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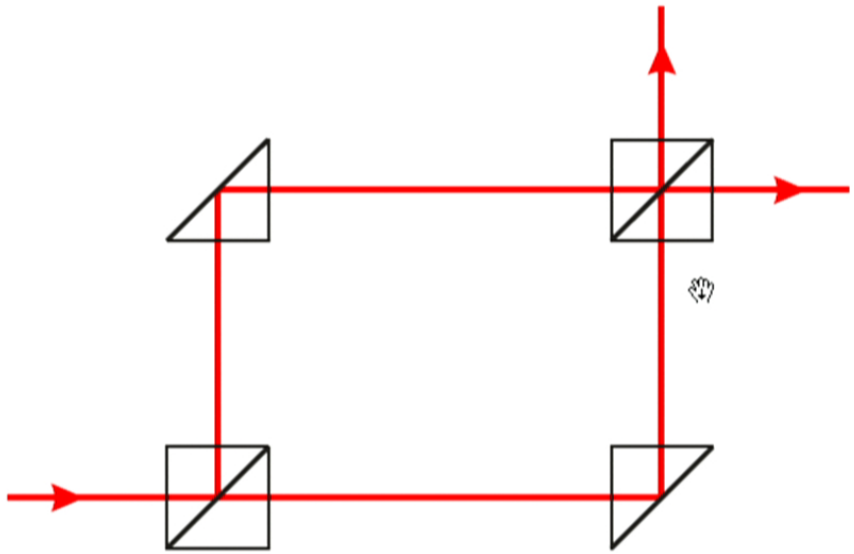
$d = 5$: qubit in quaternionic QT; $d = 9$: octonionic

Relativistic constraints on the state space

A. Garner, MM, O. Dahlsten, arXiv:1412.7112



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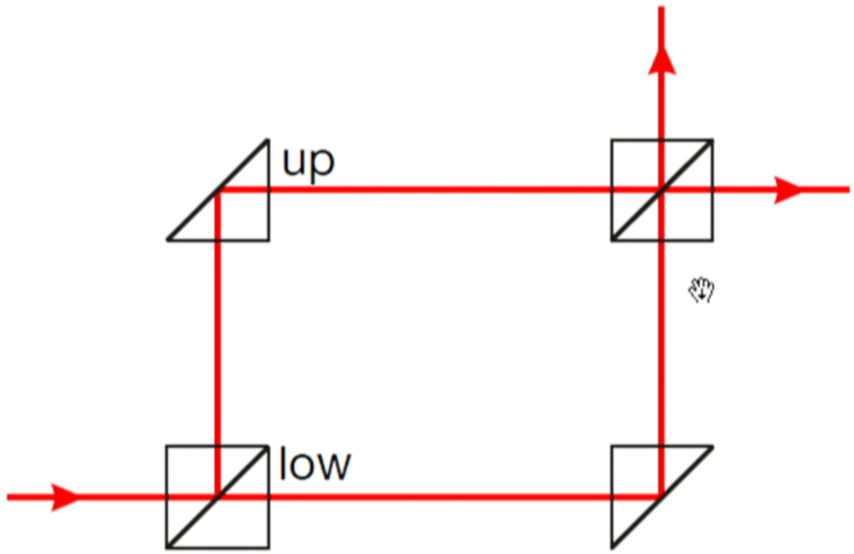

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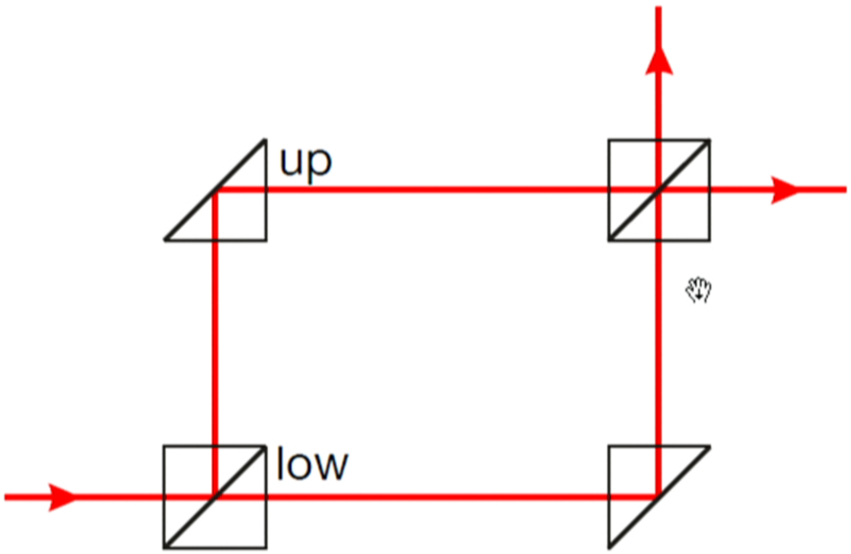

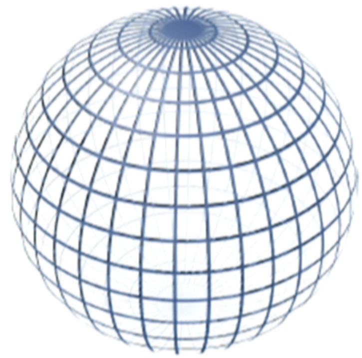

d-dim. "Bloch sphere"

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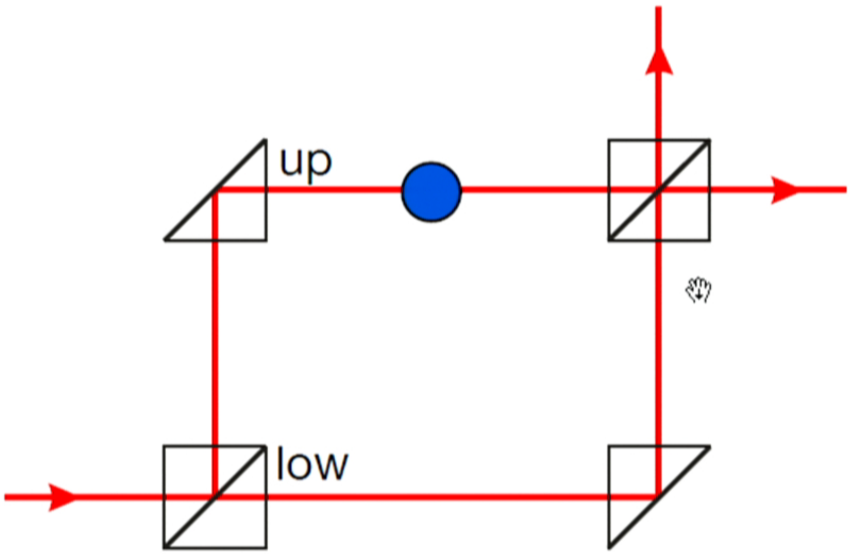

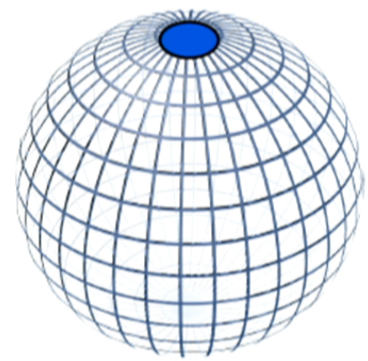

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North-pole state: particle definitely in upper branch.

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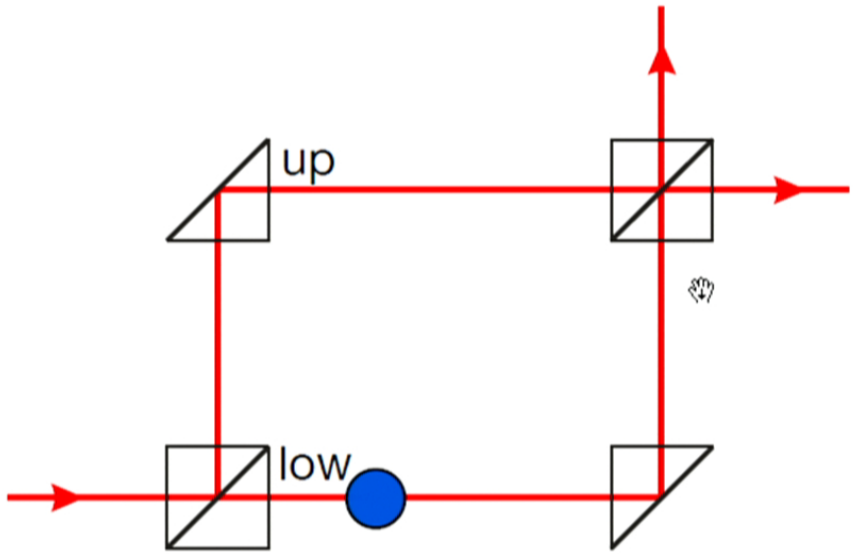

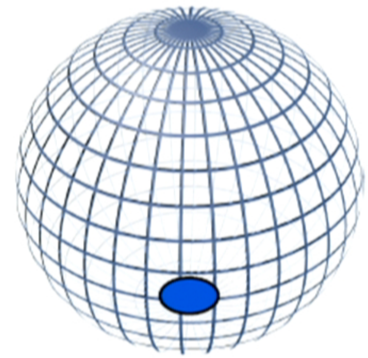

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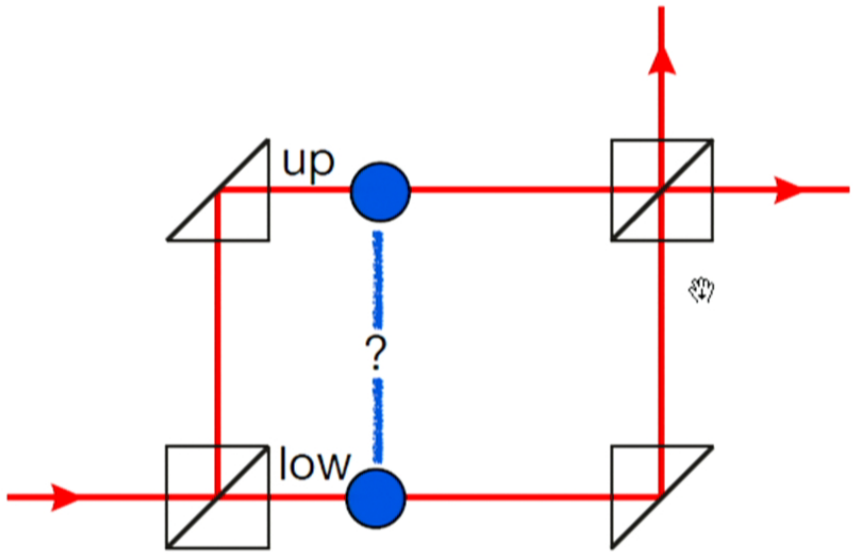

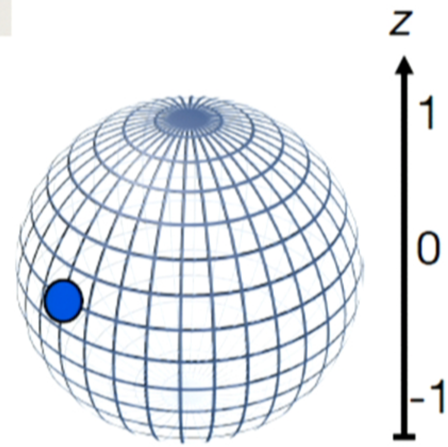

d-dim. "Bloch sphere"

State on equator *z=0*: probability 1/2 for each.

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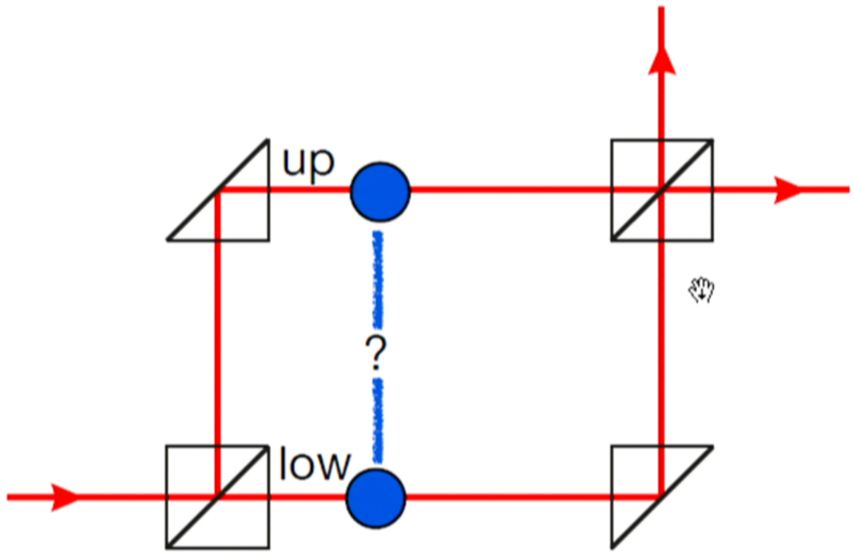

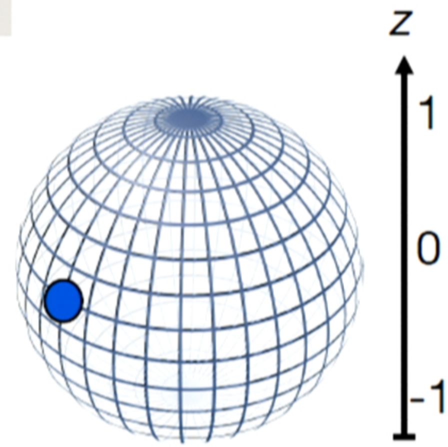

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State on equator *z=0*: probability 1/2 for each.

$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

Relativistic constraints on the state space

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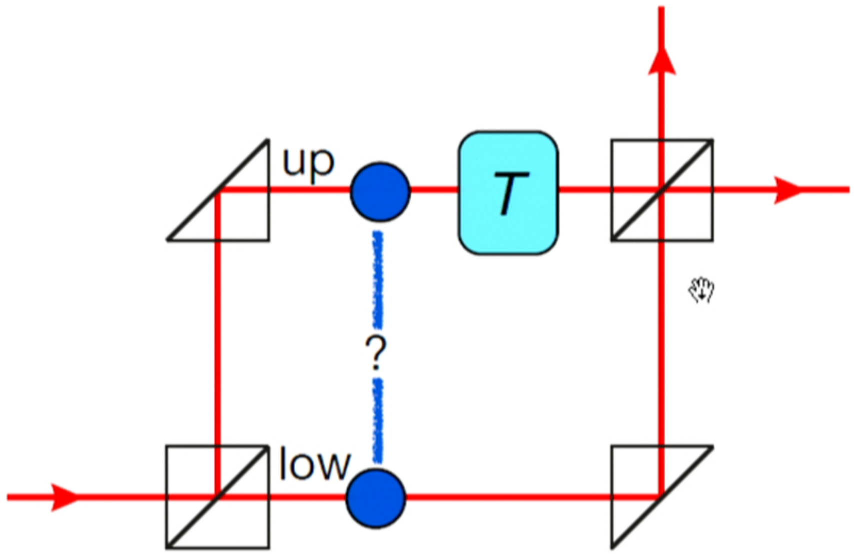

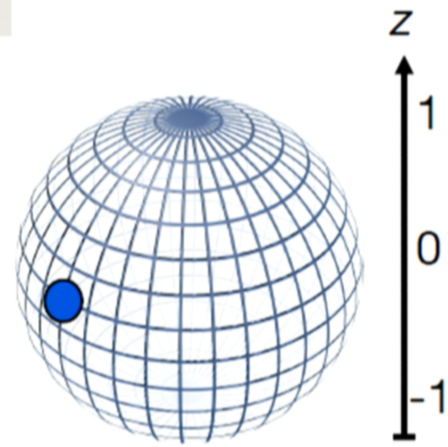

d-dim. "Bloch sphere"

What transformations *T* can we perform locally in one arm...
... without any information loss?

Relativistic constraints on the state space

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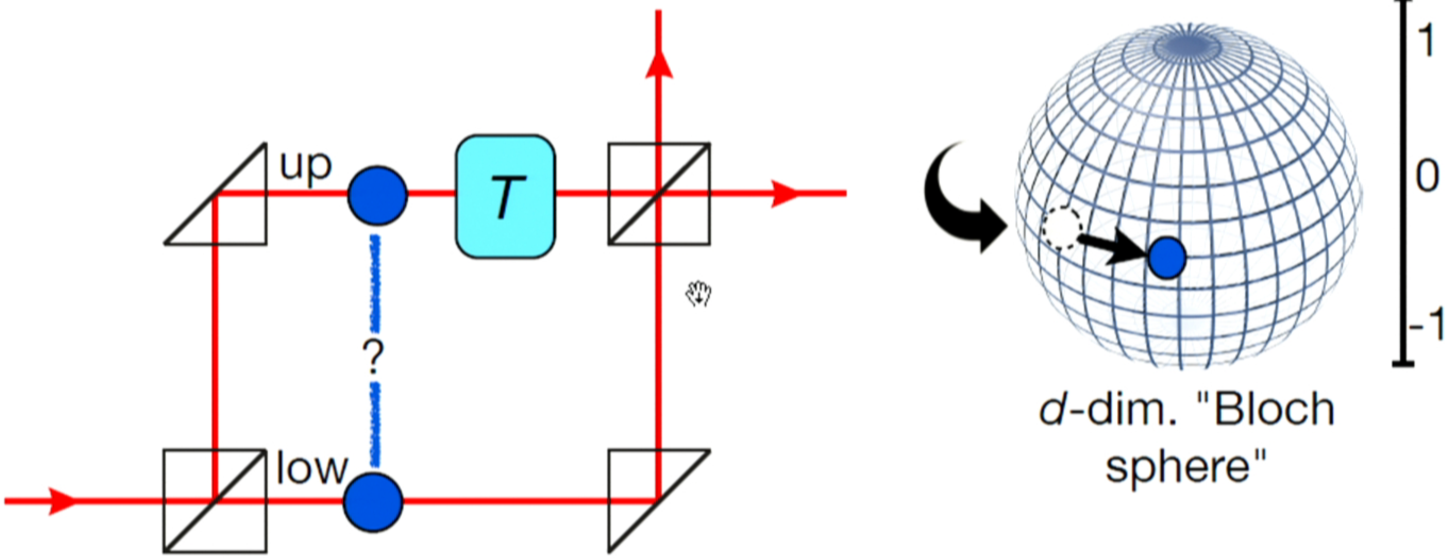
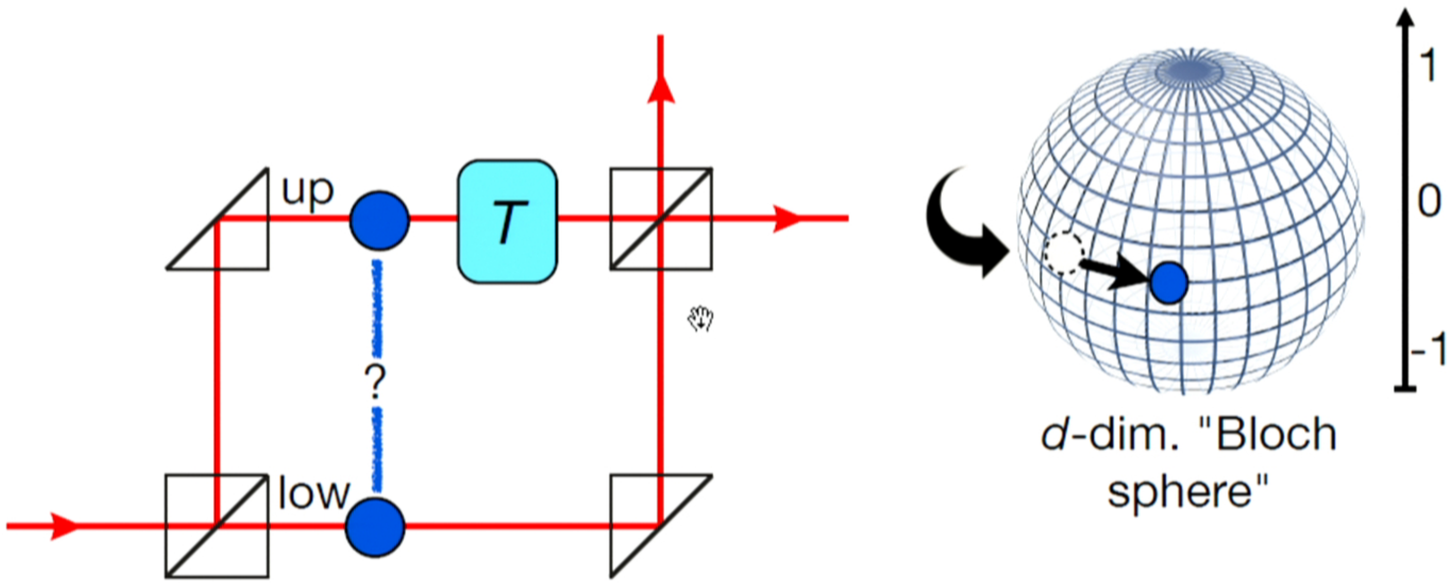


T must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve p(up), i.e. preserve the z-axis.

Relativistic constraints on the state space

Assumption: $\mathcal{G}_{\rm up} = \mathcal{G}_{\rm low} \simeq \mathrm{SO}(d-1)$.



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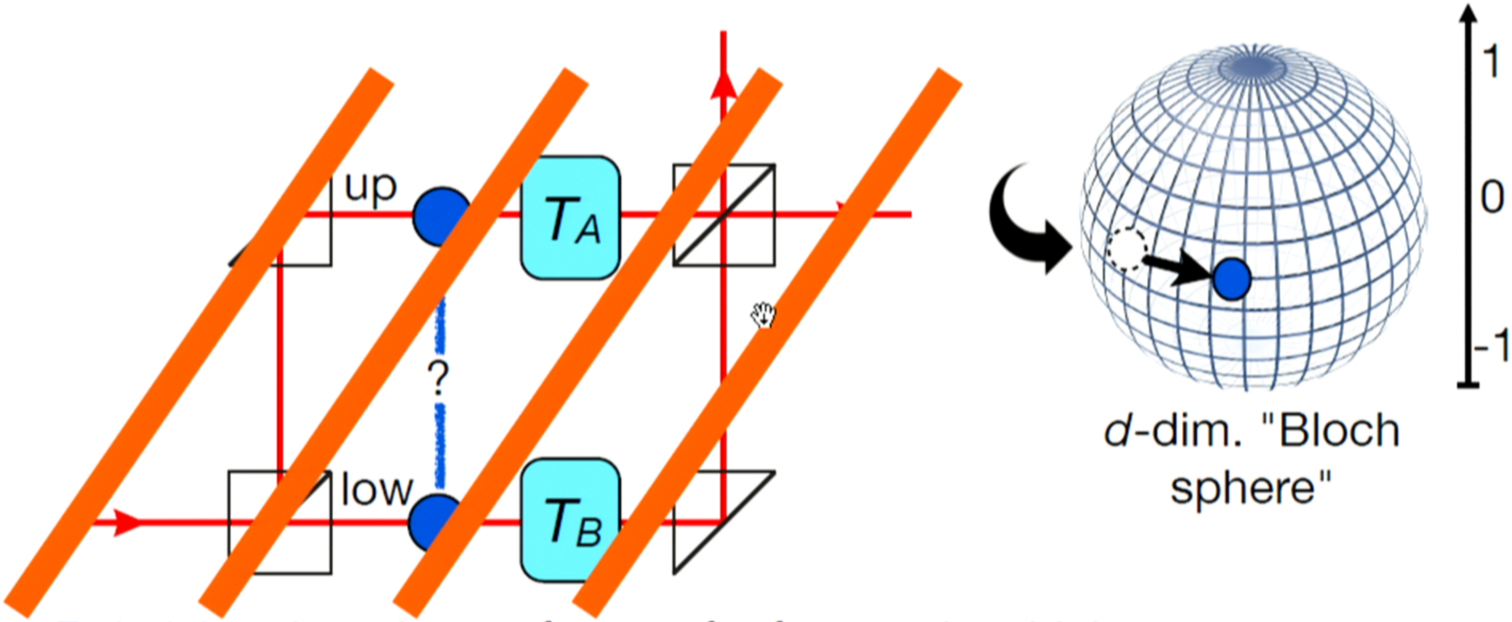
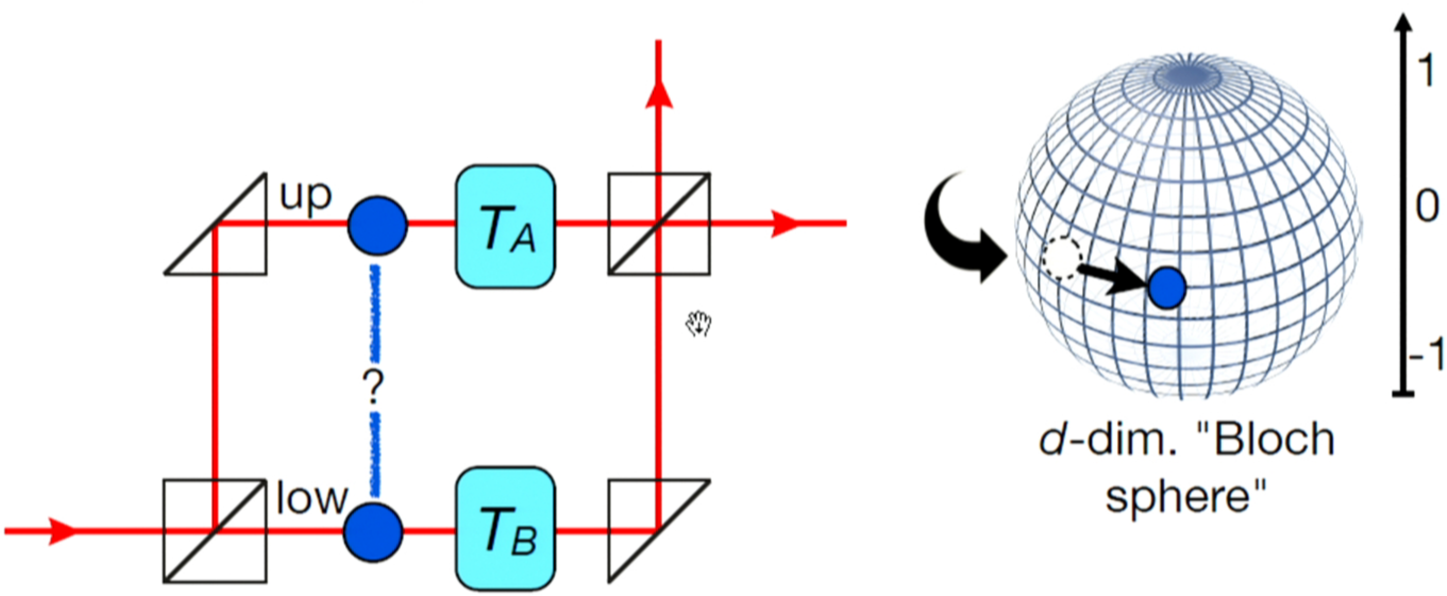


d-dim. "Bloch sphere"

Relativity: there is one frame of reference in which T_A happens first, and then T_B...

Relativistic constraints on the state space

Assumption: $\mathcal{G}_{\rm up} = \mathcal{G}_{\rm low} \simeq \mathrm{SO}(d-1)$.

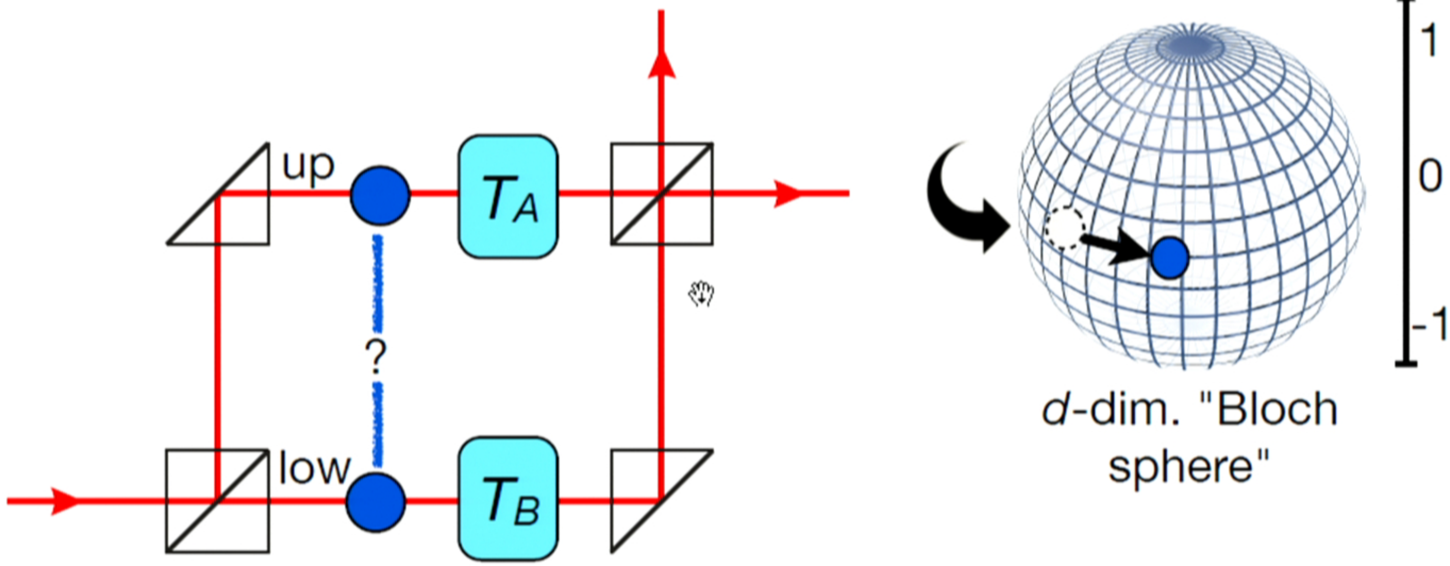


Detector click statistics is Lorentz-invariant

$\Rightarrow T_A T_B = T_B T_A$ for all $T_A, T_B \in \mathrm{SO}(d-1)$.

Relativistic constraints on the state space

$\Rightarrow \quad d \leq 3$ (In fact, *d*=3, otherwise these transformations are all trivial.)



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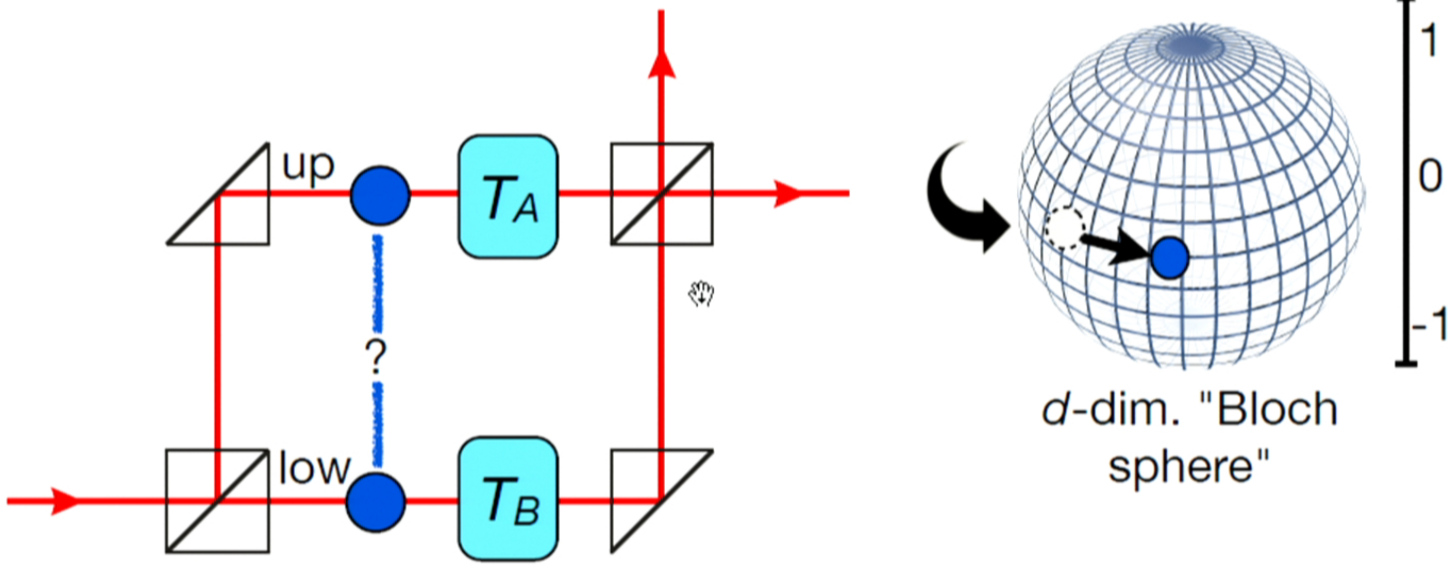

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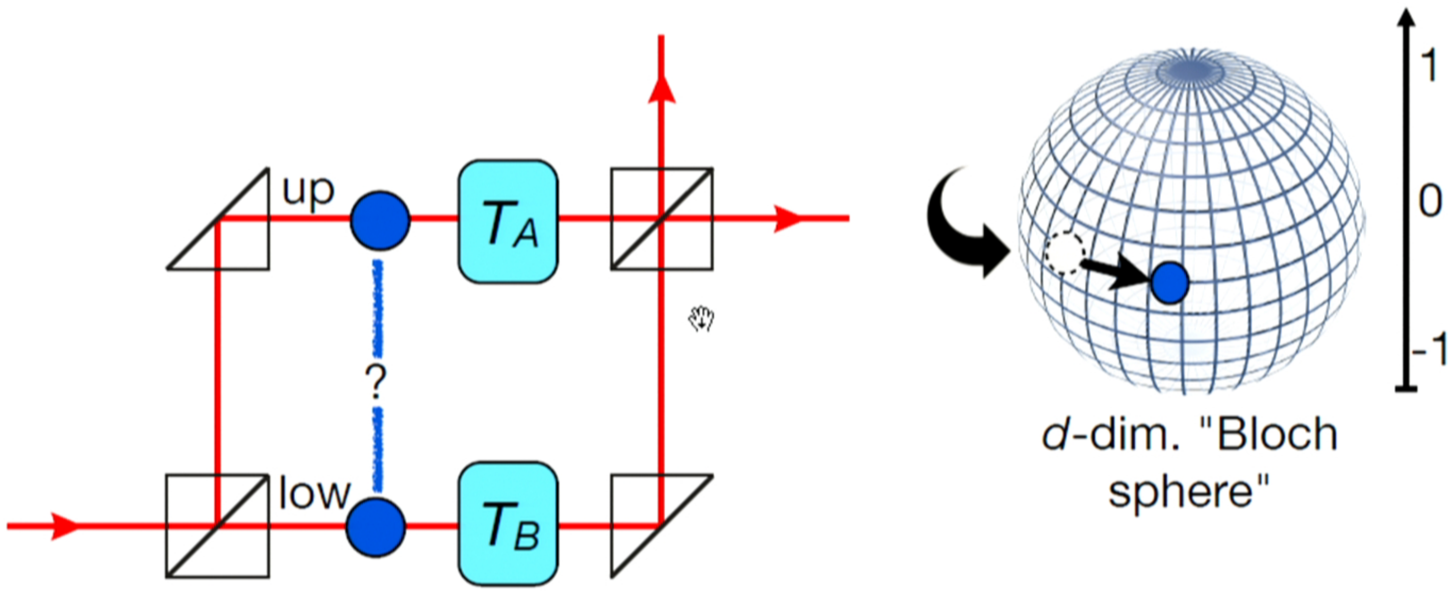
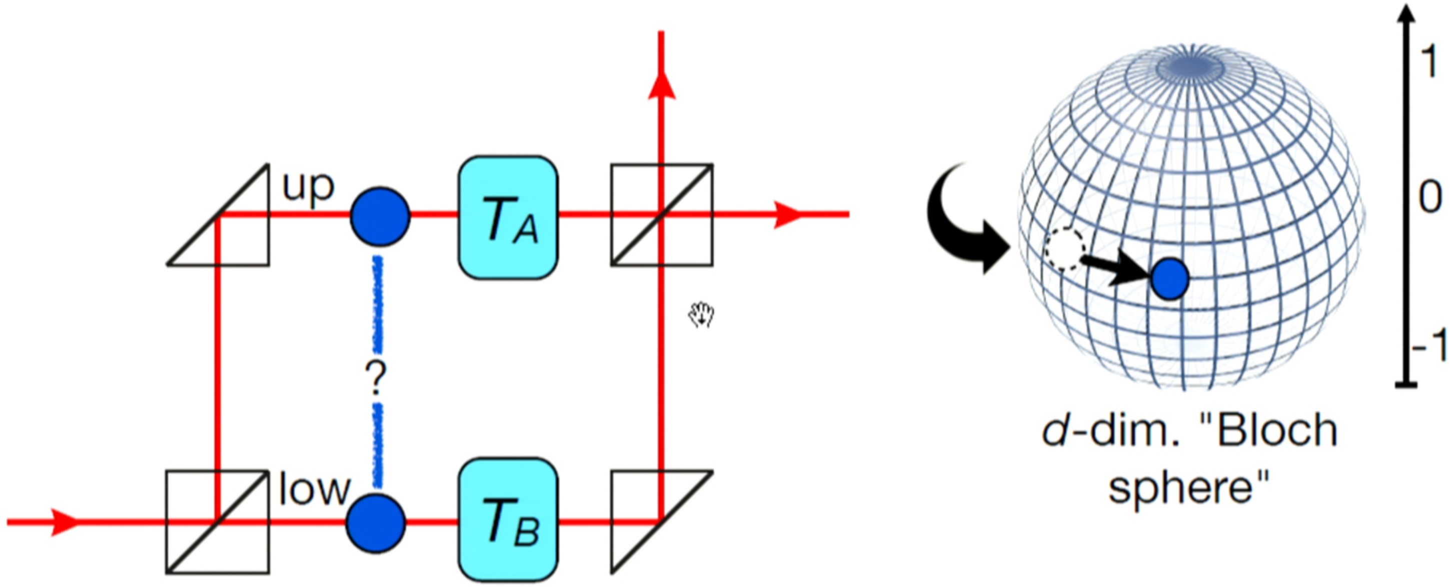
Relativistic constraints on the state space

Weaker assumption: $\mathcal{G}_{\rm up}$ and $\mathcal{G}_{\rm low}$ isomorphic



Relativistic constraints on the state space

Weaker assumption: $\mathcal{G}_{\mathrm{up}}$ and $\mathcal{G}_{\mathrm{low}}$ isomorphic



$\Rightarrow \quad d \leq 5.$ Quaternionic QM survives.

Conclusion part 2

QT ⟵ **space-time**

Space-time structure constrains QT's state space:
upper bound on # of incompatible qubit observables.

2. Relativity interferometer

Conclusion part 2

Consequences for actual interference experiments:

PHYSICAL REVIEW LETTERS

VOLUME 42 12 MARCH 1979 NUMBER 11

Proposed Test for Complex versus Quaternion Quantum Theory

Asher Peres

Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel

(Received 7 December 1978)

If scattering amplitudes are ordinary complex numbers (not quaternions) then there is a universal algebraic relationship between the six coherent cross sections of any three scatterers (taken singly and pairwise). A violation of this relationship would indicate either that scattering amplitudes are quaternions, or that the superposition principle fails. Some experimental tests are proposed, involving neutron diffraction by crystals made of three different isotopes, neutron interferometry, and K_S-meson regeneration.

Conclusion part 2

Consequences for actual interference experiments:

- Generalized Peres Test
  - Quaternion quantum mechanics?
  - Octonion quantum mechanics?

PHYSICAL REVIEW LETTERS

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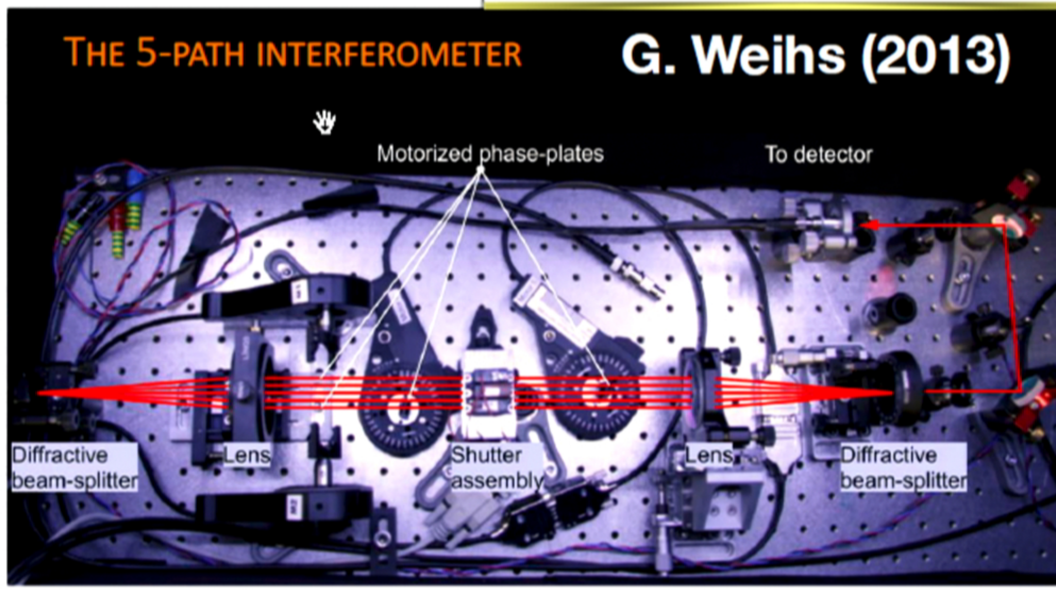

Could it also go the other way?

QT → **space-time**

		3. SO(3,1)	

Quantum theory and spacetime: strange allies

Markus P. Müller

Hints for QT → space-time

Spin: representation of rotation group SO(d)

reversible transformation G_R of state space

$\Updownarrow$ 1:1

spatial rotation R

Hints for QT → space-time

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Impossible for most state spaces!

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$\updownarrow$ 1:1

linear structure of tangent space

Hints for QT → space-time

Spin: representation of rotation group SO(d)

linear structure of statistical mixtures

$\updownarrow$ 1:1

linear structure of tangent space

→ only possible for standard qubit and $d=3$.

But the 3D-Bloch ball can be derived from information-theoretic postulates!

(Cf. talk by Philipp Hoehn; Lucien Hardy 2001; Dakic, Brukner 2009; Masanes, Müller 2011; Chribella, D'Ariano, Perinotto 2011, uvm.)

Hints for QT → space-time

Recall Philipp's talk:

GR: has informational essence (e.g., horizons)

- standard: spacetime $\Rightarrow$ causal structure $\Rightarrow$ information flow
- alternative: information flow $\Rightarrow$ causal structure $\Rightarrow$ spacetime (up to scale)

  [Hawking, Geroch,...]

Hints for QT → space-time

Recall Philipp's talk:

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- alternative: information flow $\Rightarrow$ causal structure $\Rightarrow$ spacetime (up to scale)

[Hawking, Geroch,...]

Can we infer (local) space-time symmetry from informational relations between observers?

The general setup

Philipp Hoehn and MM, arXiv:1412.8462

Related (but different): B. Dakic and C. Brukner, arXiv:1307.3984 (!!)

MM and Ll. Masanes, New J. Phys. **15**, 053040 (2013)

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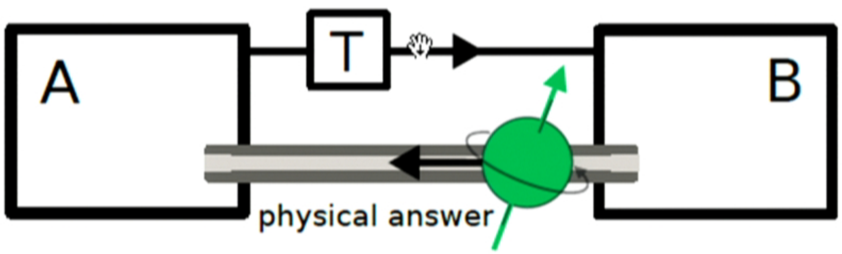

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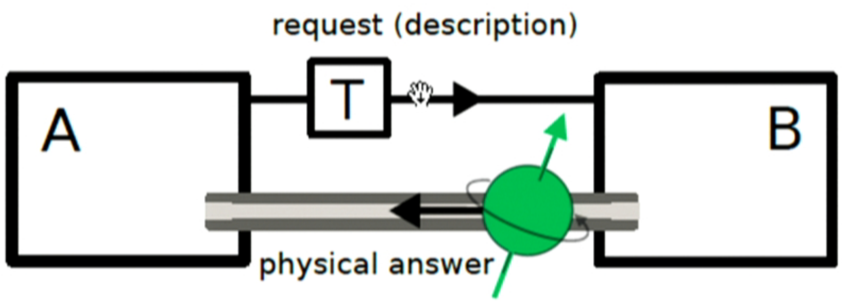
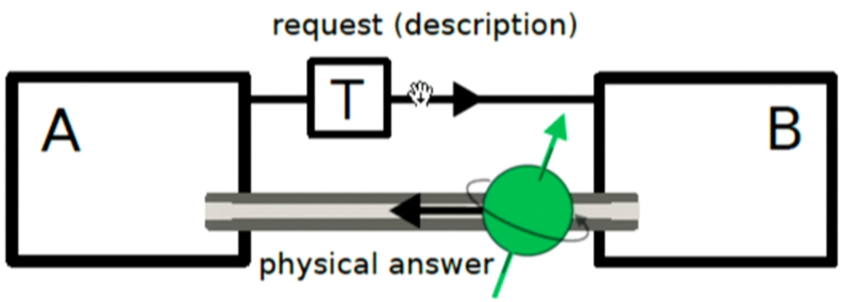


Two distant observers (Alice, Bob) who have never met.

Alice (on phone): "Please send the following object: *[description]*"
Bob: "Ok!", and sends some physical object back.

The general setup

Problem: A+B have never met
→ lack of common reference frame



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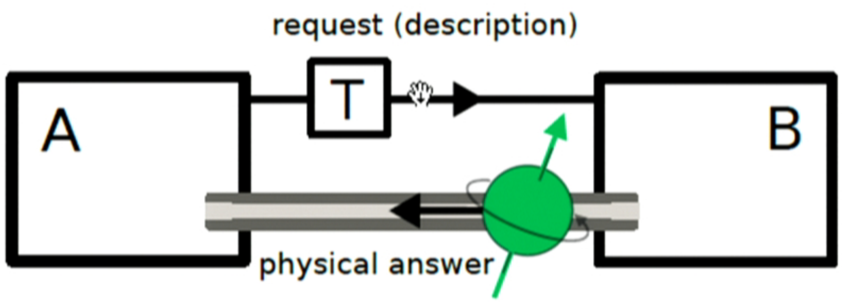
The general setup

Bob's description of physical objects

Problem: A+B have never m...
→ lack of common reference frame
→ need correcting transformation $T = \varphi_B \circ \varphi_A^{-1}$.



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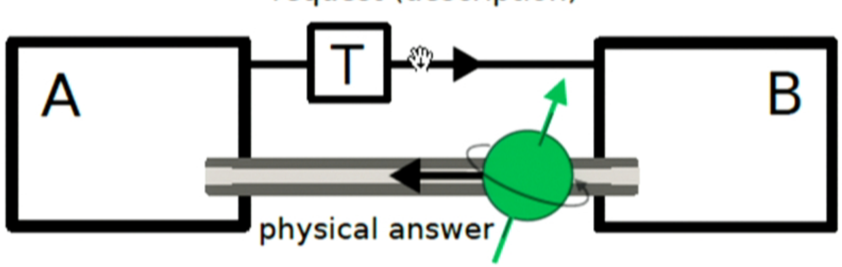

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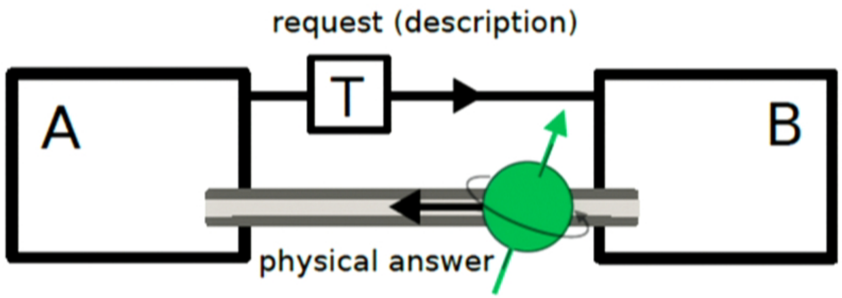
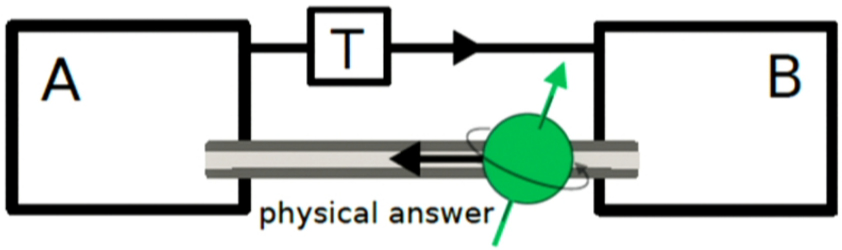


Example: assume classical mechanics;
physical objects= billard balls with specified v.

The general setup



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A and B could choose arbitrary continuous encodings φ_A, φ_B.

$$\varphi : \text{physical velocities} \to \mathbb{R}^3.$$

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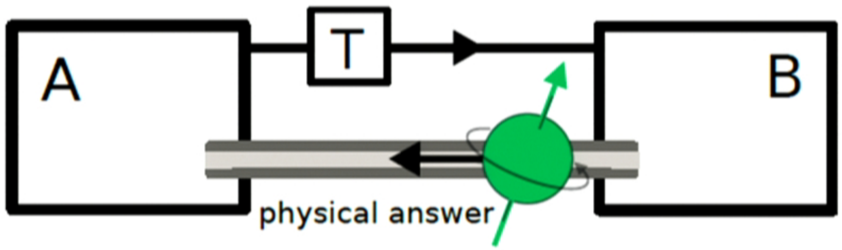

Example: assume classical mechanics;
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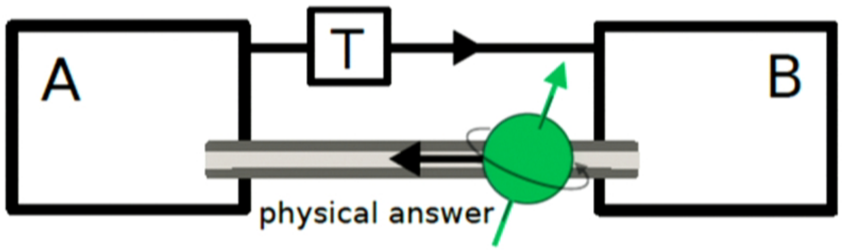
A and B could choose arbitrary continuous encodings φ_A, φ_B.

$$\varphi : \text{physical velocities} \to \mathbb{R}^3.$$

Then there would always be an element $T \in \mathcal{G}$ that does the job,

$$\mathcal{G} = \{\text{homeomorphisms of } \mathbb{R}^3\}.$$

The general setup



Example: assume classical mechanics;
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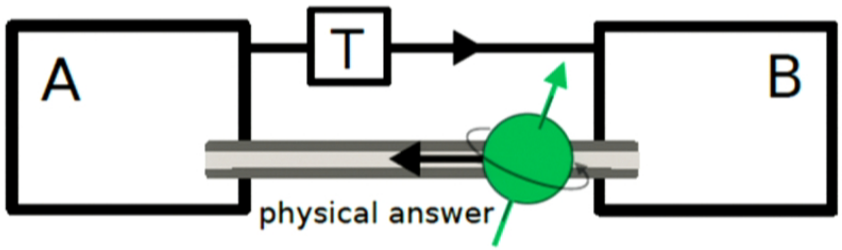
A and B could choose inertial frame encodings φ_A, φ_B.

Then $T \in \mathcal{G}'$ is sufficient, where
$\mathcal{G}' = \mathrm{Aff}(3, \mathbb{R})$, the affine group.

The general setup



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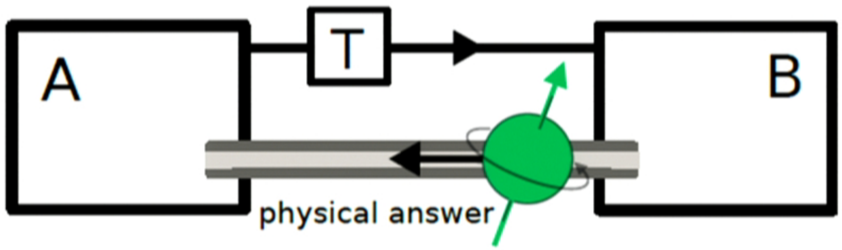
Then $\mathcal{G}' \subsetneq \mathcal{G}$, so this is "better" than before.

They cannot do better: $\mathcal{G}_{\min} = \mathrm{Aff}(3, \mathbb{R})$.

The general setup



Choice of objects to be sent

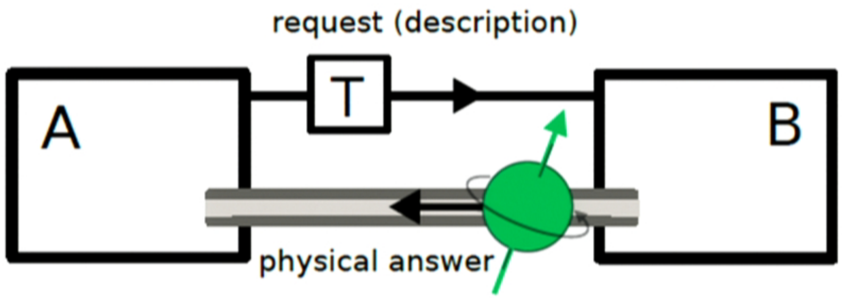
+

Physical / operational background assumptions

↓

Minimal group $\mathcal{G}_{\min}$.

The general setup



Choice of objects to be sent

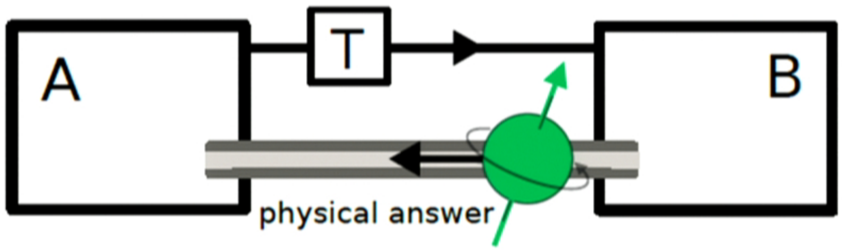
+

Physical / operational background assumptions

Minimal group $\mathcal{G}_{\min}$.

Theorem: If $\mathcal{G}_{\min}$ exists then it is unique (as an abstract group).

Sending finite-dimensional quantum states



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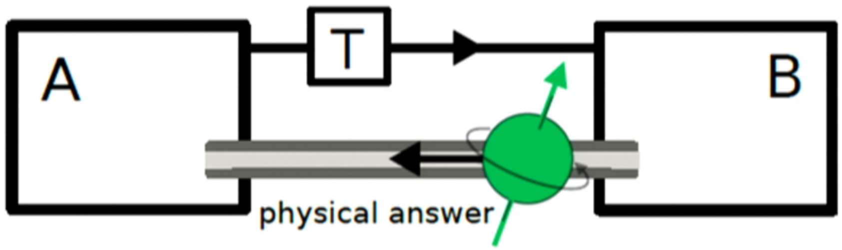


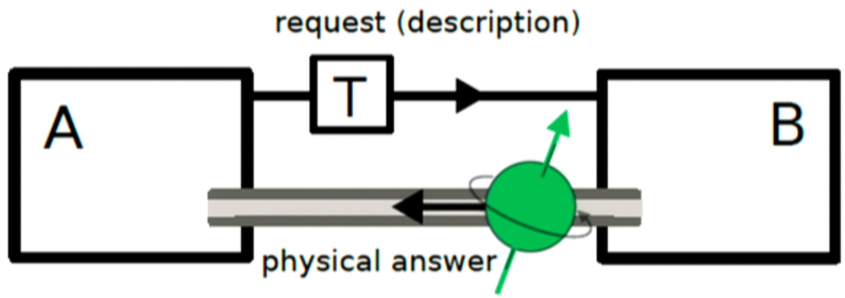
Quantum states $\rho_{n\times n}$ for some $n \in N$

+

No (helpful) additional assumptions at all

Minimal group $\mathcal{G}_{\min}$.

Sending finite-dimensional quantum states



Quantum states $\rho_{n\times n}$ for some $n \in N$

+

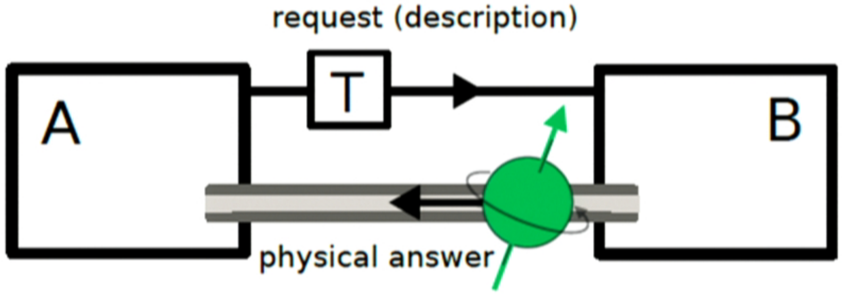
No (helpful) additional assumptions at all

Huge minimal group $\mathcal{G}_{\min} = \left\{ \bigotimes_{n\in\mathbb{N}} \bigotimes_{\mathcal{H}_n} (U_{\mathcal{H}_n} \bullet U_{\mathcal{H}_n}^{-1}) \right\}$,
one unitary / antiunitary U for each Hilbert space.

3. SO(3,1)

Quantum theory and spacetime: strange allies — Markus P. Müller

Sending finite-dimensional quantum states



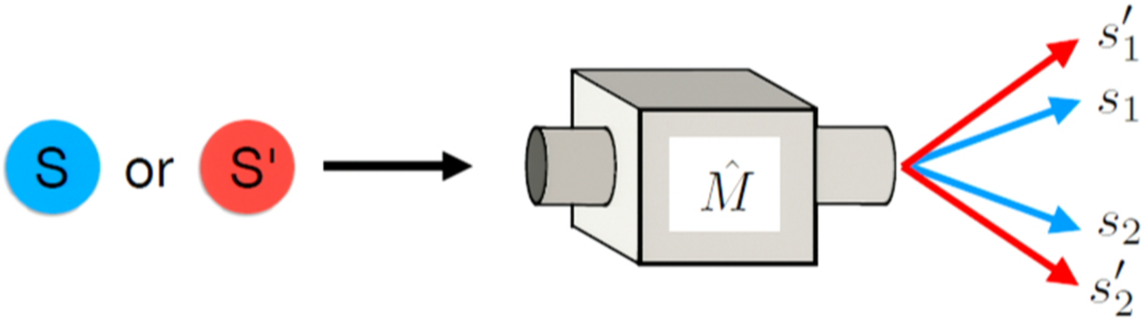
In our universe, we have additional structure:

Can measure "the same" observable on different Hilbert spaces.

Does this help to get a smaller $\mathcal{G}_{\min}$?

Yes!

Same observable on different quantum systems S



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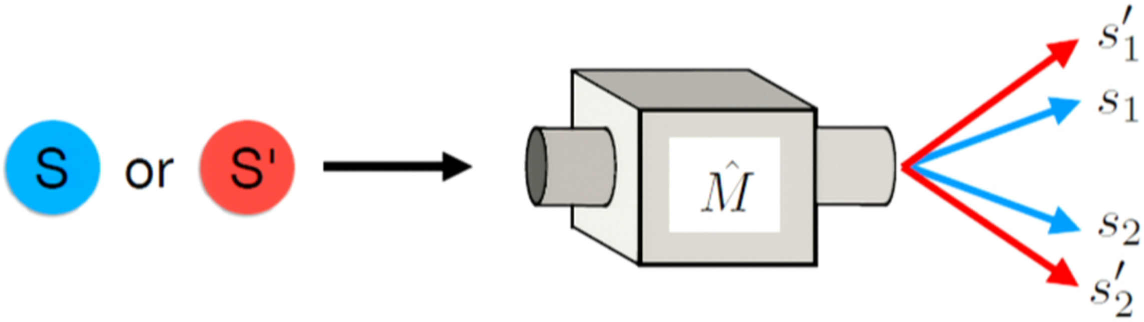

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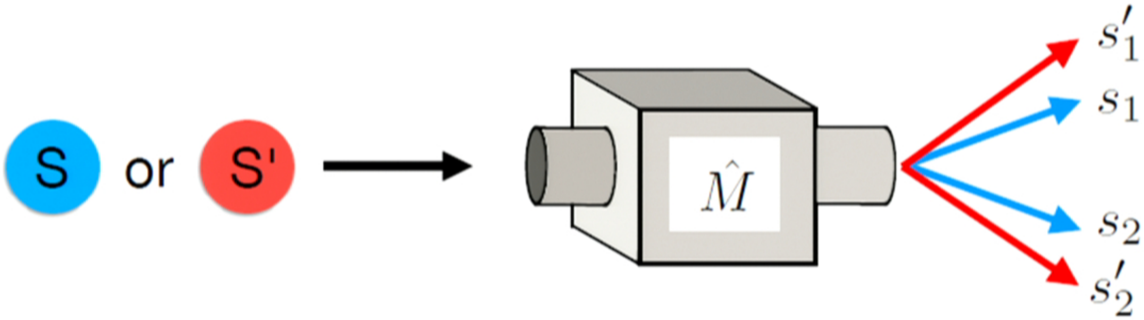
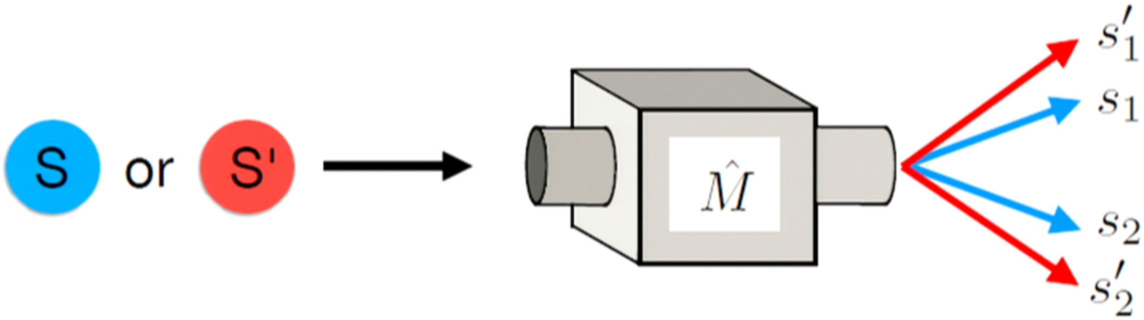


Physical quantity $\hat{M}$ is called (S,S')-co-measurable.

Same observable on different quantum systems S



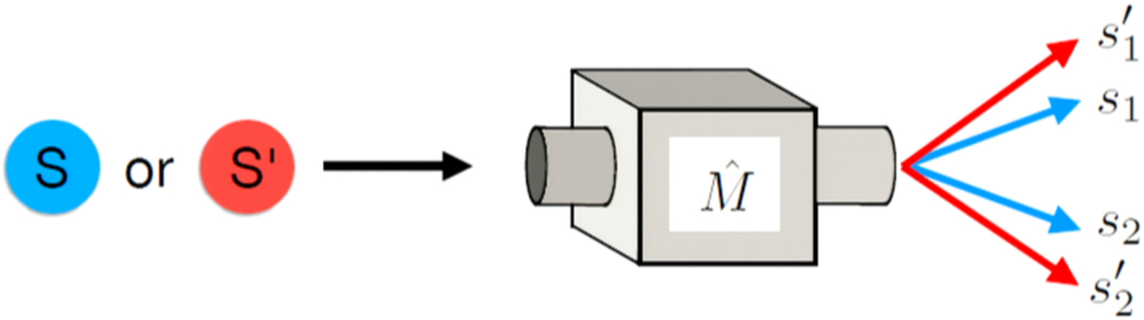
Physical quantity $\hat{M}$ is called (S,S')-co-measurable.

Example: S=spin-1/2 qubit, S'=spin-1 qutrit

$\hat{M} = S_z$, spin in z-direction

$$\hat{M}(S) = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}, \qquad \hat{M}(S') = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Same observable on different quantum systems S



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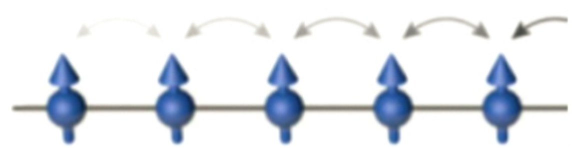
Example: S=spin-1/2 qubit, S'=spin-1 qutrit

$$\hat{M} = S_z,$$ spin in z-direction

Possible origin: "natural interaction" between S and S'?

$\hat{M}$ conserved quantity?

Build measurement device from many S-systems?



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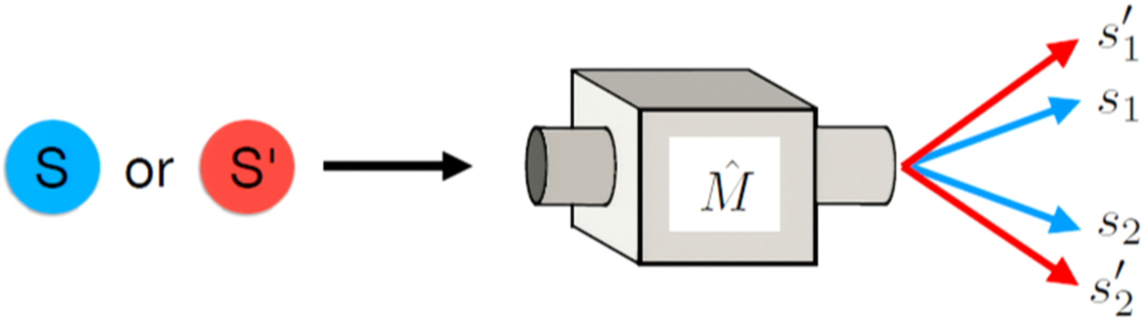

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$$\hat{M} = S_z, \text{ spin in } z\text{-direction}$$

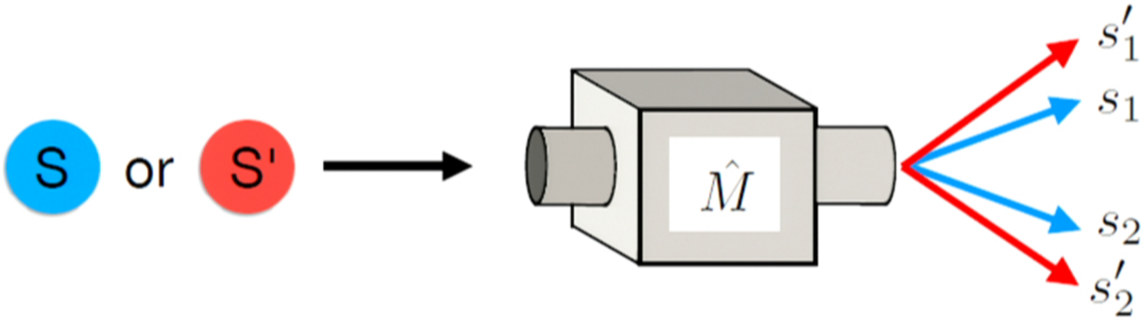
Possible origin: "natural inter... tween S and S'?

$\hat{M}$ conserved qu...

Build ... from ... S-systems?

Doesn't matter for us.

Same observable on different quantum systems S



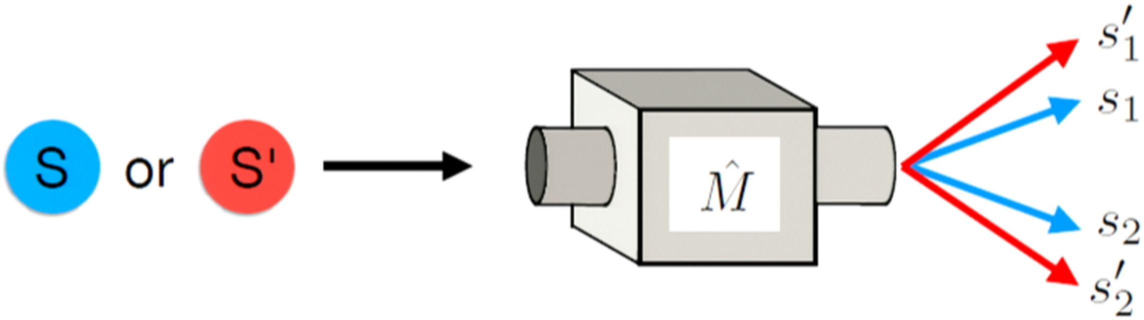
Physical quantity $\hat{M}$ is called (S,S')-co-measurable.

Set of co-measurable observables: consistency conditions.

$$\hat{M}_1 = \hat{S}_z; \quad \hat{M}_2 = \begin{cases} \hat{S}_z & \text{if spin-}1/2\text{ sent in} \\ \hat{S}_x & \text{otherwise} \end{cases}$$

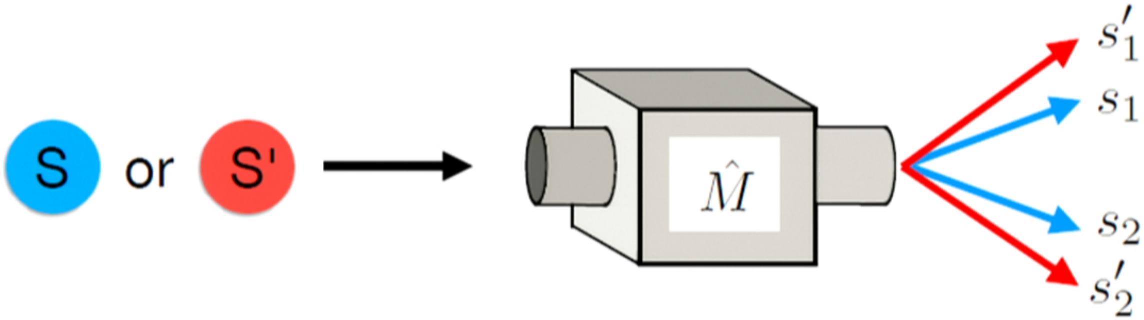
cannot both be co-measurable.

Same observable on different quantum systems S



Consistency conditions for (S,S')-co-measurable obs.:

Same observable on different quantum systems S

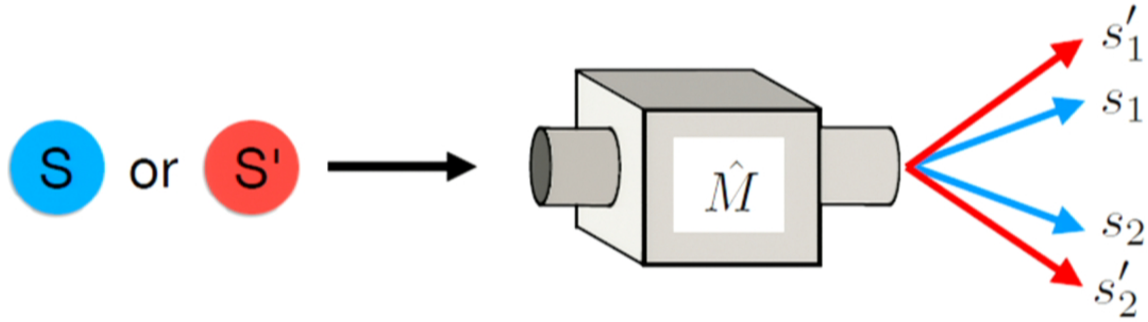


Consistency conditions for (S,S')-co-measurable obs.:

- $\hat{M}_n(S) \stackrel{n\to\infty}{\longrightarrow} \hat{M}(S) \quad \Rightarrow \quad \hat{M}_n(S') \stackrel{n\to\infty}{\longrightarrow} \hat{M}(S')$.
- $\hat{M}_1(S) \leq \hat{M}_2(S) \quad \Rightarrow \quad \hat{M}_1(S') \leq \hat{M}_2(S')$.
- $\hat{M}(S) = 0 \quad \Rightarrow \quad \hat{M}(S') = 0$.

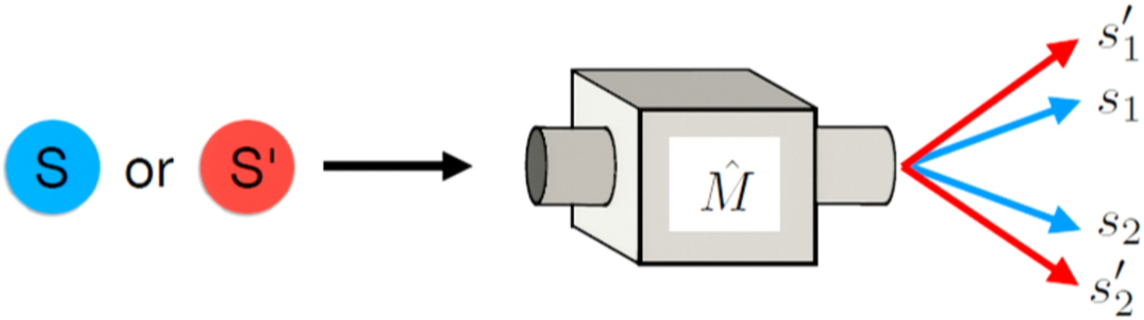
3. SO(3,1)

Same observable on different quantum systems S



Call S and S' *equivalent* if all their observables are (S,S')-co-measurable. Equivalence classes: **S**.

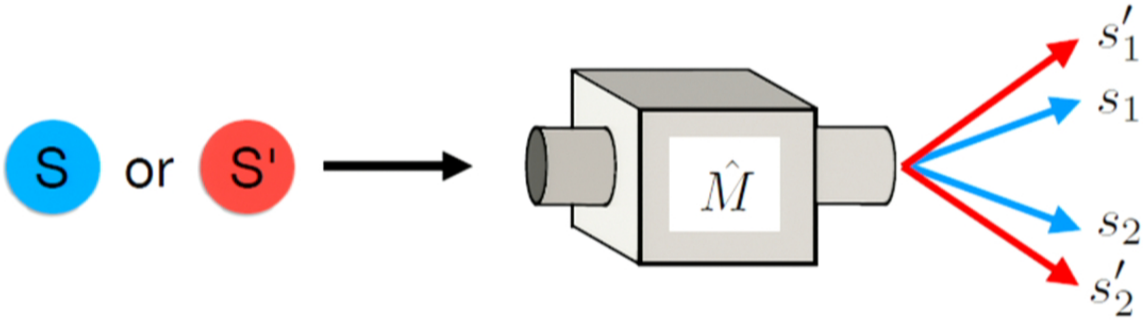
Same observable on different quantum systems S



Consistency conditions for (S,S')-co-measurable obs.:

- $\hat{M}_n(S) \stackrel{n\to\infty}{\longrightarrow} \hat{M}(S) \quad \Rightarrow \quad \hat{M}_n(S') \stackrel{n\to\infty}{\longrightarrow} \hat{M}(S')$.
- $\hat{M}_1(S) \leq \hat{M}_2(S) \quad \Rightarrow \quad \hat{M}_1(S') \leq \hat{M}_2(S')$.
- $\hat{M}(S) = 0 \quad \Rightarrow \quad \hat{M}(S') = 0$.

Same observable on different quantum systems S



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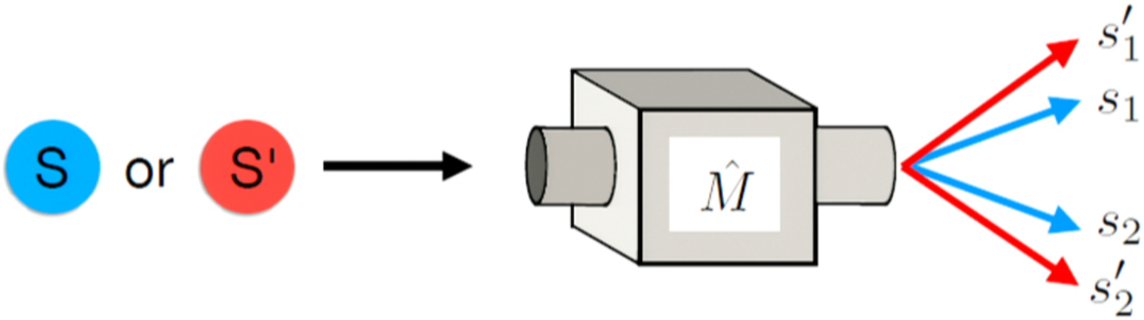
3. SO(3,1)

Same observable on different quantum systems S



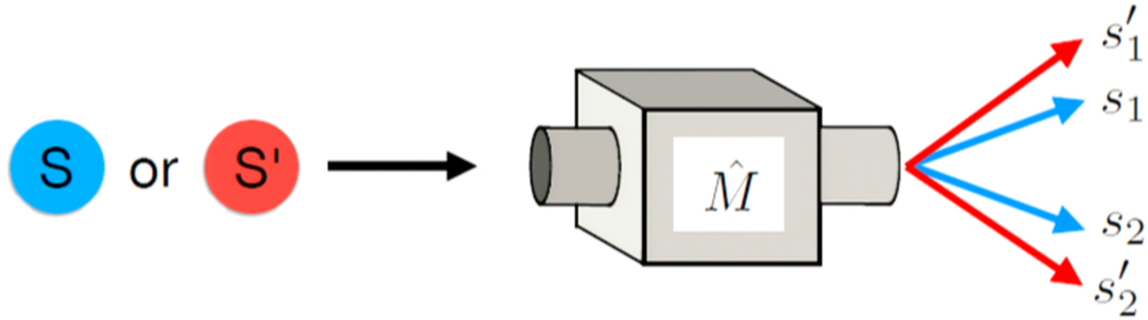
Call S and S' *equivalent* if all their observables are (S,S')-co-measurable. Equivalence classes: **S**.

invertible

Theorem: Then *dim* S=*dim* S', and $\hat{M}(S) = X\hat{M}(S')^T X^\dagger$.

possibly transpose

Same observable on different quantum systems *S*



Call *S* and *S'* *equivalent* if all their observables are (*S*,*S'*)-co-measurable. Equivalence classes: **S.**

invertible

Theorem: Then *dim S*=*dim S'*, and $\hat{M}(S) = X\hat{M}(S')X^\dagger$.

Ex.:
- *S*= *S'* up to choice of basis (⇒ *X* unitary),
- *S* and *S'* spins of electrons with different momenta.

Interaction graph

Draw an arrow from **S** to **S'** if the set of (**S**,**S'**)-co-measurable observables is tomographically complete on **S'**.

Interaction graph

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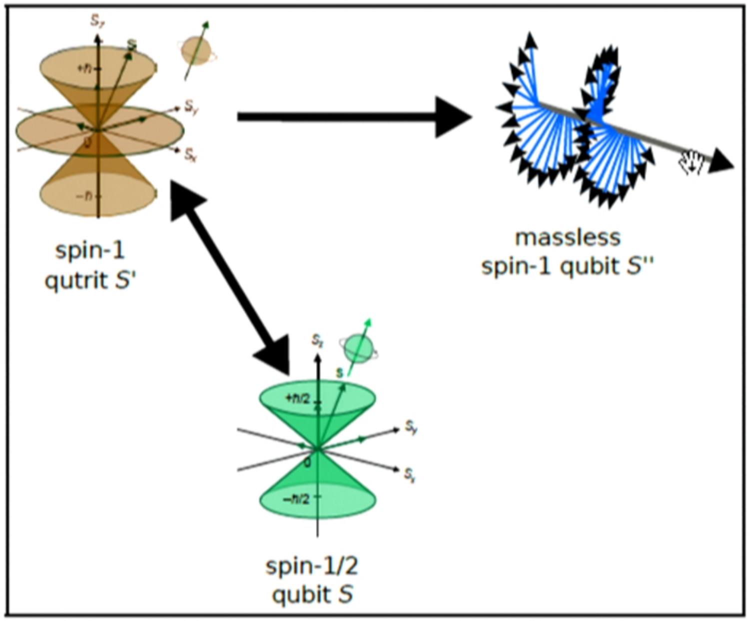
In our specific physical world:



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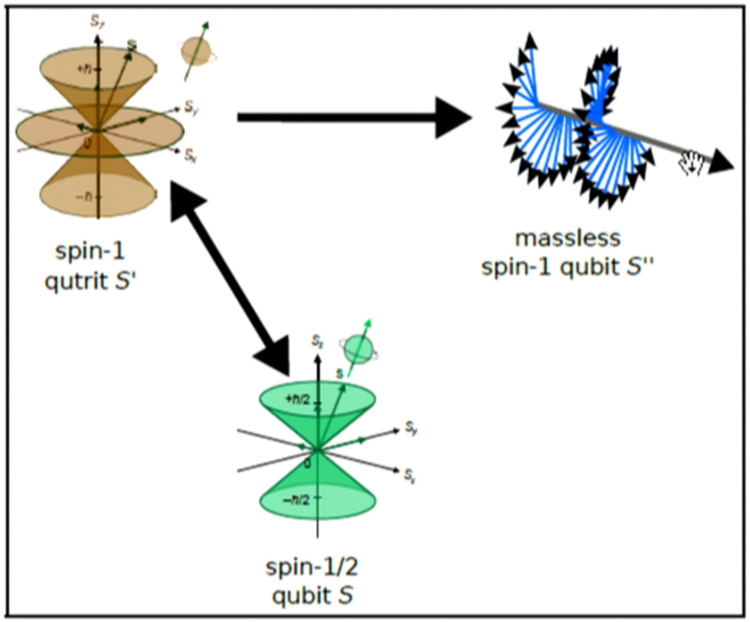
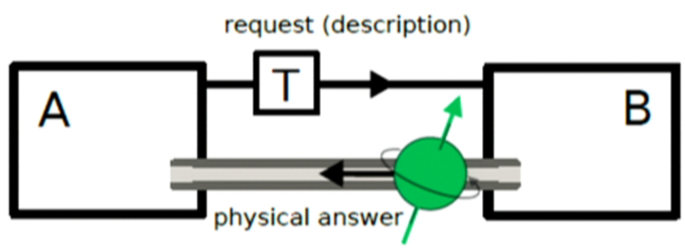
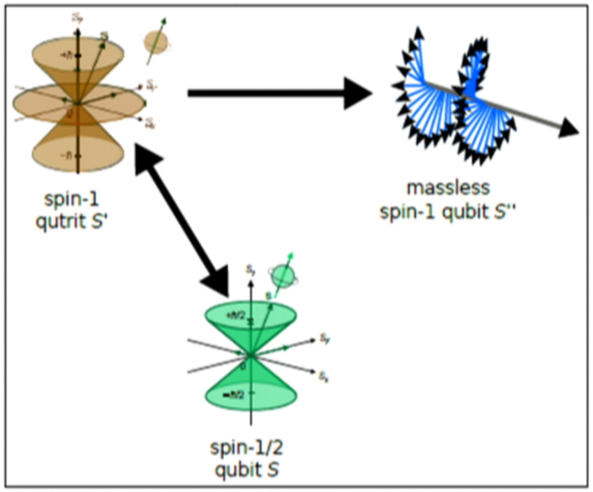


Interpretation:
Alice and Bob can lift their agreement on description of **S** to description of **S'**.



Interaction graph

A: Dear Bob, please send me the qutrit state $|\psi\rangle = (1, 0, -1)/\sqrt{2}$!



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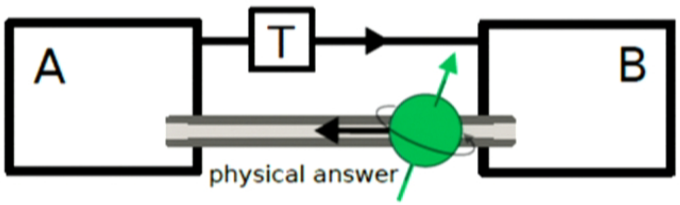

		3. SO(3,1)	

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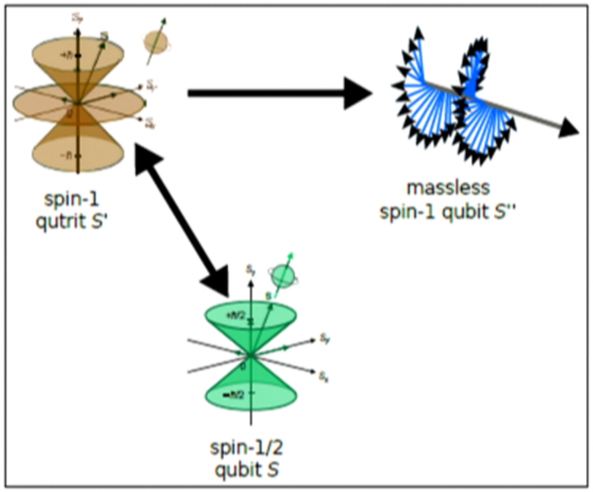

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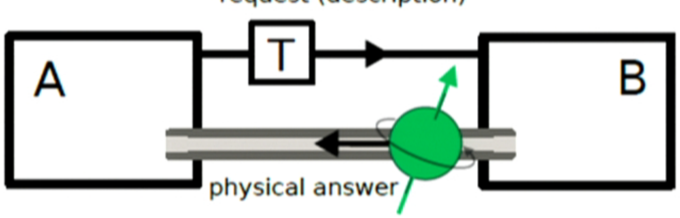
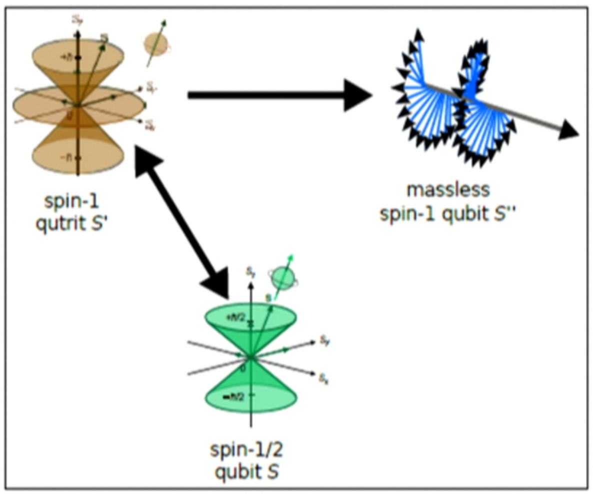


Interaction graph

A: Dear Bob, please send me the qutrit state $|\psi\rangle = (1, 0, -1)/\sqrt{2}$!

B: Don't know how to do this… haven't set an **S'**- reference frame!

A: But we have for **S**! So please build the measurement devices with $\hat{M}_i(\mathbf{S}) =$*[descriptions]*. Then I mean the state $|\psi\rangle$ with the following outcome probabilities in these device: *[numbers].*



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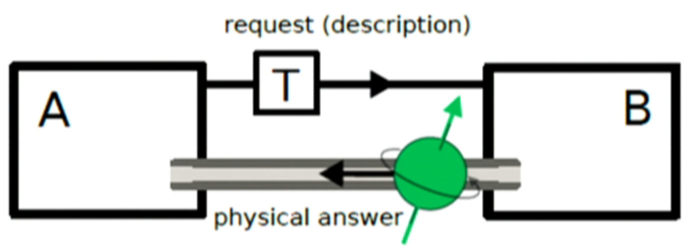
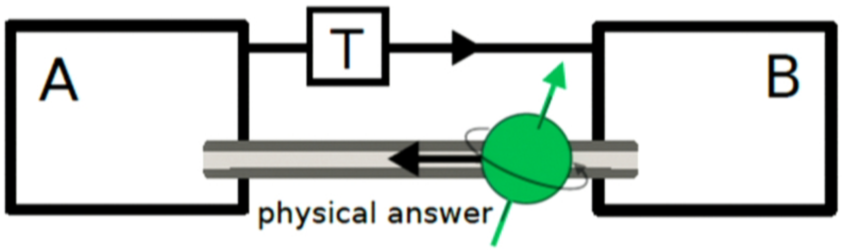


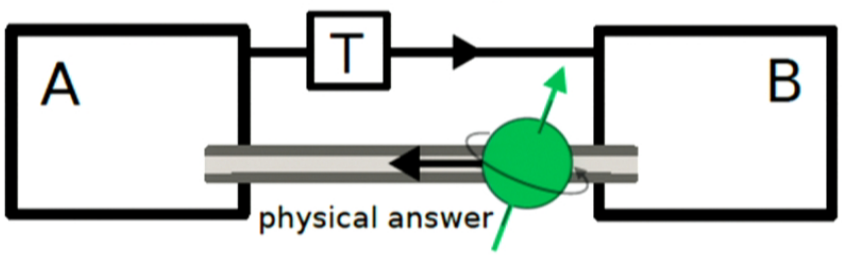
The communication task revisited

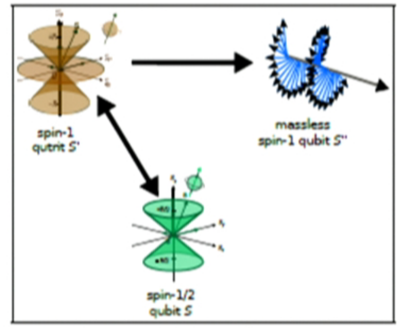


Quantum states ρ on any system S

+

The communication task revisited



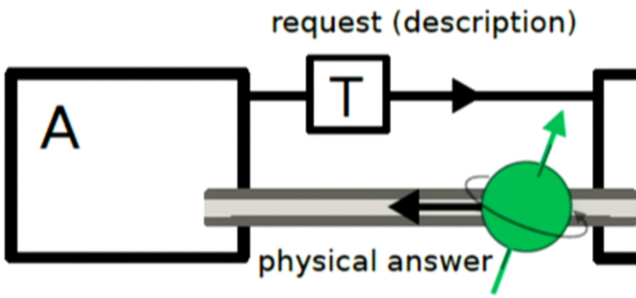


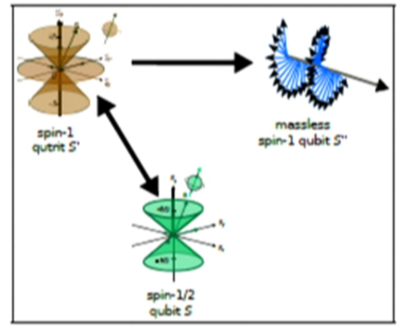
Quantum states ρ on any system S

+

Lots of co-meaurable observables:
interaction graph is *connected*

The communication task revisited





Quantum states ρ on any system S

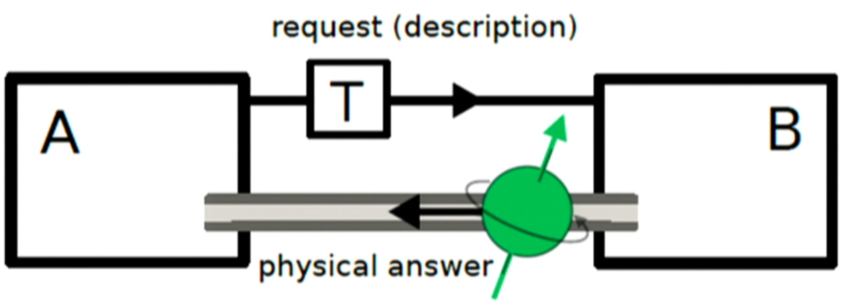
+

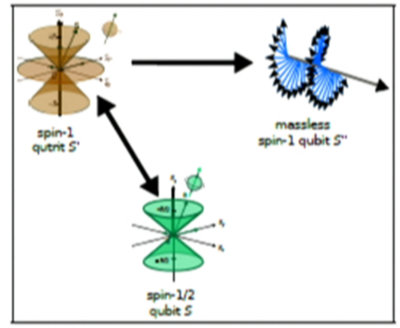
Lots of co-meaurable observables: interaction graph is *connected*

↓

$\mathcal{G}_{\rm min}$ is the symmetry group of the smallest-dimensional root.

The communication task revisited





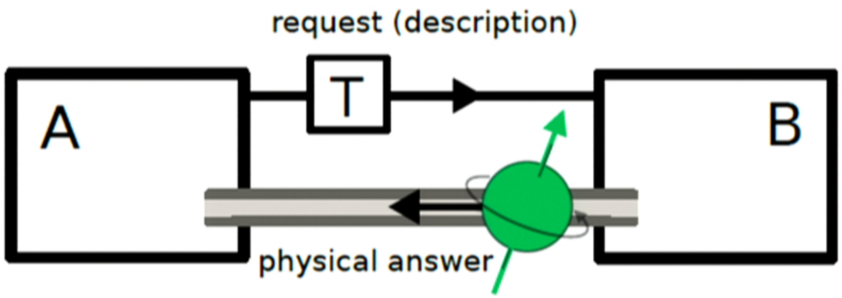
Quantum states ρ on any system S

+

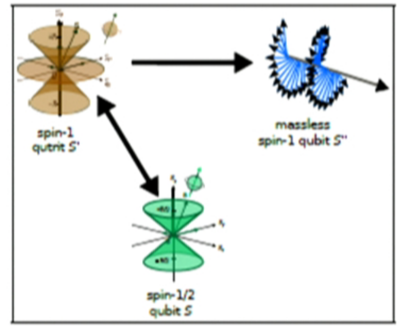
Minimality: interaction graph *connected*, and there is a **qubit root**

$$
\begin{aligned}
\mathcal{G}_{\min} &= \left\{\hat{M} \mapsto X\hat{M}X^\dagger \text{ or } X\hat{M}^T X^\dagger \mid X \in \mathrm{GL}(2,\mathbb{C})\right\} \\
&\simeq \mathbb{R}^+ \times \mathrm{O}^+(3,1).
\end{aligned}
$$

Representations of the Lorentz group



$$\mathcal{G}_{\min} = \mathbb{R}^+ \times \mathrm{O}^+(3,1).$$



Theorem: Let **S** be the "root qubit", and **S'** any system.

- If all **S**-observables are (**S**,**S'**)-co-measurable, then **S'** carries a projective rep. of $\mathrm{SO}^+(3,1)$.

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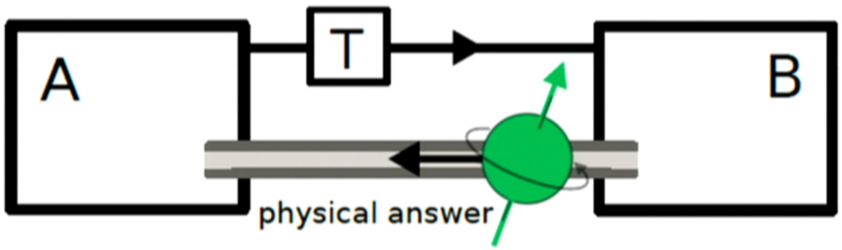

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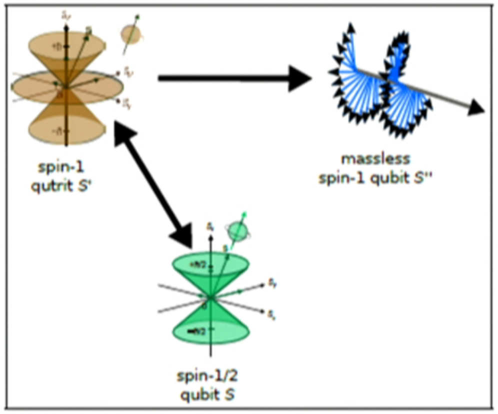
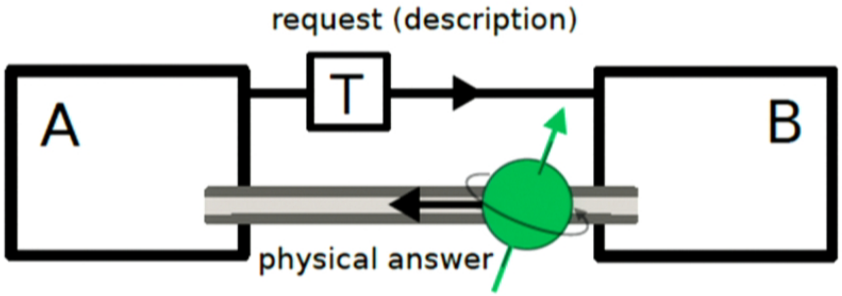
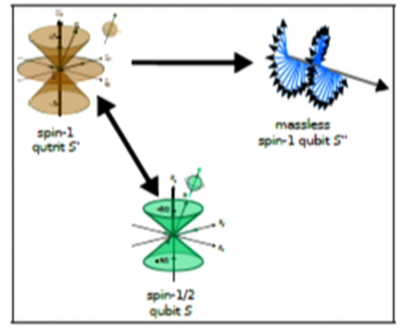


Theorem: Let **S** be the "root qubit", and **S'** any system.

- If all **S**-observables are (**S**,**S'**)-co-measurable, then **S'** carries a projective rep. of $\mathrm{SO}^+(3,1)$.
- All other **S'** carry a proj. rep. of the subgroup that preserves the (**S**,**S'**)-co-measurable observables ("Wigner little groups").

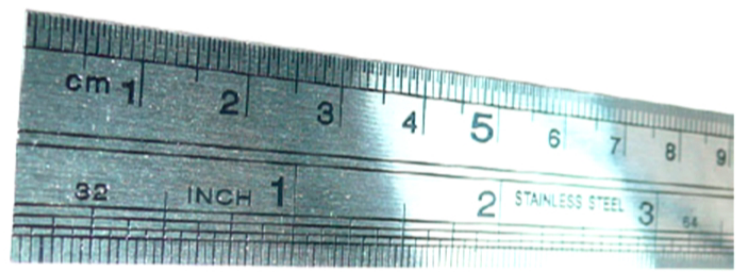
Interpretation of $\mathcal{G}_{\min}$





$$\mathcal{G}_{\min} = \mathbb{R}^+ \times \mathrm{O}^+(3,1).$$

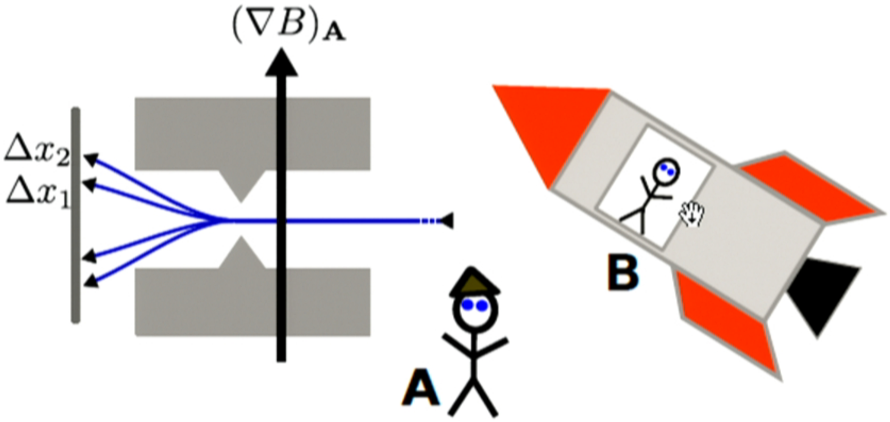
Interpretation of $\mathbb{R}^+$-factor: Alice and Bob choose different units.



Relativistic Stern-Gerlach measurements

M. C. Palmer, M. Takahashi and H. F. Westman, *WKB analysis of relativistic Stern-Gerlach measurements*, Annals of Physics **336**, 505–516 (2013); [arXiv:1208.6434].

P. Hoehn and MM, in preparation



momentum p

$$\mathcal{H}_p \simeq \mathbb{C}^2$$

$$S_\Lambda : \mathcal{H}_p \to \mathcal{H}_{\Lambda p} \text{ isometry}$$

$$\hat{M}_B = S_\Lambda \hat{M}_A S_\Lambda^\dagger$$

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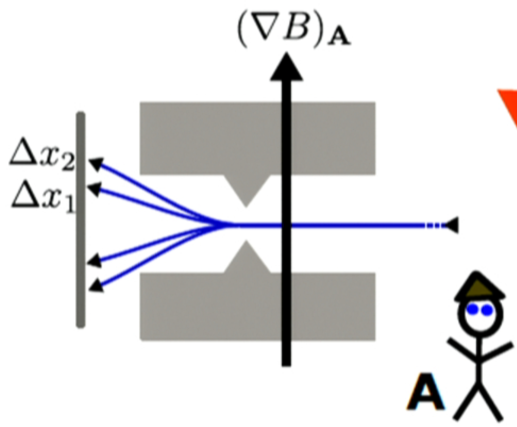
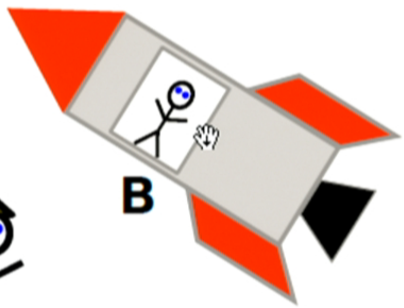




momentum p

$\mathcal{H}_p \simeq \mathbb{C}^2$

$S_\Lambda : \mathcal{H}_p \to \mathcal{H}_{\Lambda p}$ isometry

$\hat{M}_B = S_\Lambda \hat{M}_A S_\Lambda^\dagger$

Observed acceleration of wave packets:

$G^{\mathrm{A,B}} = (G_0^{A,B}, \vec{G}^{A,B})$

Eigenvalues of $\hat{M}$: $G_0^{A,B} \pm |\vec{G}^{A,B}|$

Rest frame of particle $G_0^{A,B} = 0$, but boosted observer sees asymmetric deflections → two different eigenvalues

3. SO(3,1)

Reversal of standard viewpoint

Standard point of view:

space-time symmetries → representations,
operational properties

Here:

Operational properties → representations,
space-time (?) symmetries

Conclusions

QT ⟷ **space-time**

Interesting insights into architecture of physics; quantum gravity?
Common link: information theory / operationalism.

Conclusions

QT ⟷ space-time

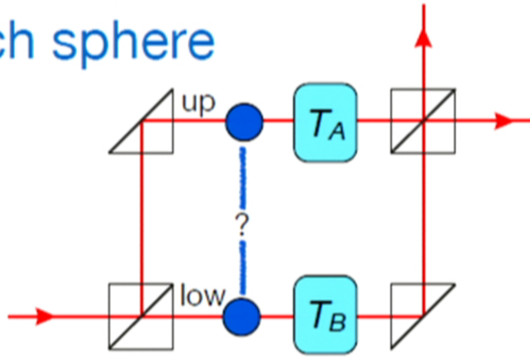
Interesting insights into architecture of physics; quantum gravity?
Common link: information theory / operationalism.

Lots of related work (Q-reference frames, QIT↔QG, …)

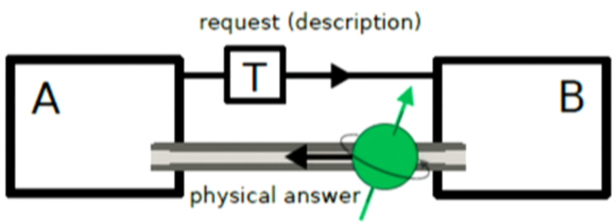
Conclusions

- Relativity of simultaneity on interferometer implies $d \leq 3$ / $d \leq 5$ for the Bloch sphere

Andy Garner, MM, Oscar Dahlsten, arXiv:1412.7112



- Quantum communication task with operational background assumptions (satisfied in our world) yields the Lorentz transformations (and rep's)



Philipp Hoehn, MM, arXiv:1412.8462.