Title: Towards an information-theoretic foundation of (quantum) thermodynamics

Date: May 11, 2015 11:15 AM

URL: http://pirsa.org/15050072

Abstract: In the classical world of Newton and Laplace, fundamental physics and thermodynamics do not blend well: the former puts forward a picture of nature where states are pure and processes are fundamentally reversible, while the latter deals with scenarios where states are mixed and processes are irreversible. Many attempts have been made at reconciling the two paradigms, but ultimately the source of all troubles remains: if every particle possesses a definite position and a definite velocity, why should experimental data depend on the expectations of agents who have only partial information? Quantum theory offers a way out. Thanks to entanglement, a system and its environment can be jointly in a pure state, while the system alone is in a mixed state. Similarly, the evolution of system and environment can be jointly reversible, but locally irreversible. At the level of axioms, these facts are captured by the Purification Principle [1], from which the whole of quantum theory was derived in Ref. [2].

In this talk I will report on a new research programme, aimed at establishing the Purification Principle as the conceptual foundation for thermodynamics. I will start from the idea that thermodynamical transformations can be mapped into transformations that degrade entanglement. This will lead to a duality between entanglement and thermodynamics, which will be used to define measures of entanglement/entropy and to explore operational tasks like information erasure. At this level, thermodynamical requirements lead directly to requirements on the particular type of Purifications allowed by the theory: for example, forbidding the existence of Entropy Sinks (systems that can absorb entropy from the environment without increasing their internal entropy) is equivalent to imposing the existence of Symmetric Purifications (purifications in which the environment is a mirror image of the system). I will conclude by giving a glimpse on quantitative measures of entanglement/entropy. By adding a requirement about the existence of sharp measurements, I will show that Purification leads directly the an operational version of the spectral theorem, allowing one to define a zoo of well-behaved entropies.

[1] G. Chiribella, G. M. Dâ€™Ariano, and P. Perinotti, Probabilistic Theories with Purification, Phys. Rev. A 81, 062348 (2010)
[2] G. Chiribella, G. M. Dâ€™Ariano, and P. Perinotti, Informational Derivation of Quantum Theory, Phys. Rev. A 84, 012311 (2011)

TOWARDS AN INFORMATION-THEORETIC RECONSTRUCTION OF (QUANTUM) THERMODYNAMICS

Giulio Chiribella and Carlo Maria Scandolo
Institute for Interdisciplinary Information Sciences
Tsinghua University, Beijing

Information-theoretic foundations of physics, PI, May 11-15 2015

National Natural Science Foundation of China

NSFC

FQXi FOUNDATIONAL QUESTIONS INSTITUTE

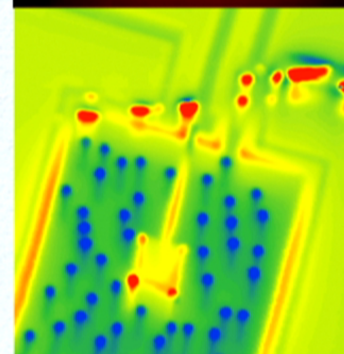
RECRUITMENT PROGRAM OF GLOBAL EXPERTS

THERMODYNAMICS

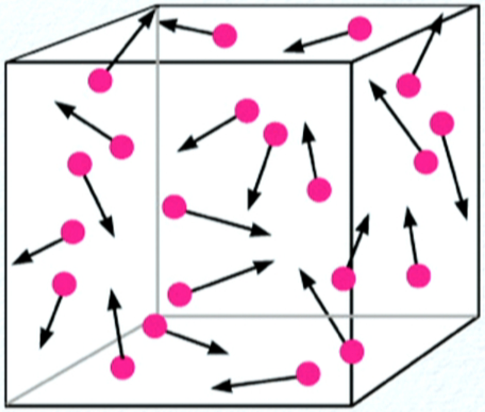
Thermodynamics provides a
very successful paradigm,
with applications in engineering, physics,
biology.

It brought in a number of new notions:
heat, entropy, free energy,
chemical potential.

- are they fundamental or derived?
- how do they fit in the big picture?



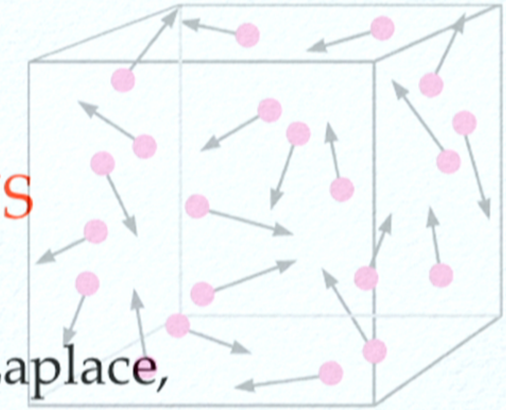
STATISTICAL MECHANICS



Statistical mechanics reduces
thermodynamical to
dynamical + probabilistic notions:

- the initial state of the system is chosen at random according to a probability distribution (typically, invariant under the dynamics)

- thermodynamical quantities are expectation values



IN THE CLASSICAL WORLD:
TENSION AT THE FOUNDATIONS

In the classical world of Newton and Laplace, there is no place for probability at the fundamental level.

- Why should measured quantities depend on the expectations of an agent?

-ergodic theory, symmetries

-max ent principle

-...

THE QUANTUM CASE

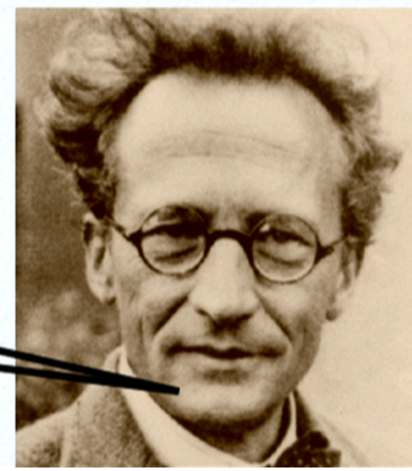
In quantum theory, probability has a different status: mixed states can be seen as marginals of pure states

$$|\Psi\rangle_{AB}$$

ρ_A ρ_B

$$\rho_A = \mathrm{Tr}_B\left[\,|\Psi\rangle\langle\Psi|\,\right]$$

The best possible knowledge of a whole does not necessarily imply the best possible knowledge of its parts.



PURIFICATION AS A FOUNDATION FOR QUANTUM STATISTICAL MECHANICS

Statistical ensembles from
pure states of the system + its environment

Popescu-Short-Winter 2006,
Gemmer-Michel-Mahler 2006,
...

Under various assumptions on the pure state of system + environment, microcanonical and canonical ensembles can be derived as the typical marginal states of the system.

FULL QUANTUM THEORY FROM PURIFICATION

- Causality
- Fine-Grained Composition
- Perfect Distinguishability
- Ideal Compression
- Local tomography

+

PURIFICATION

=

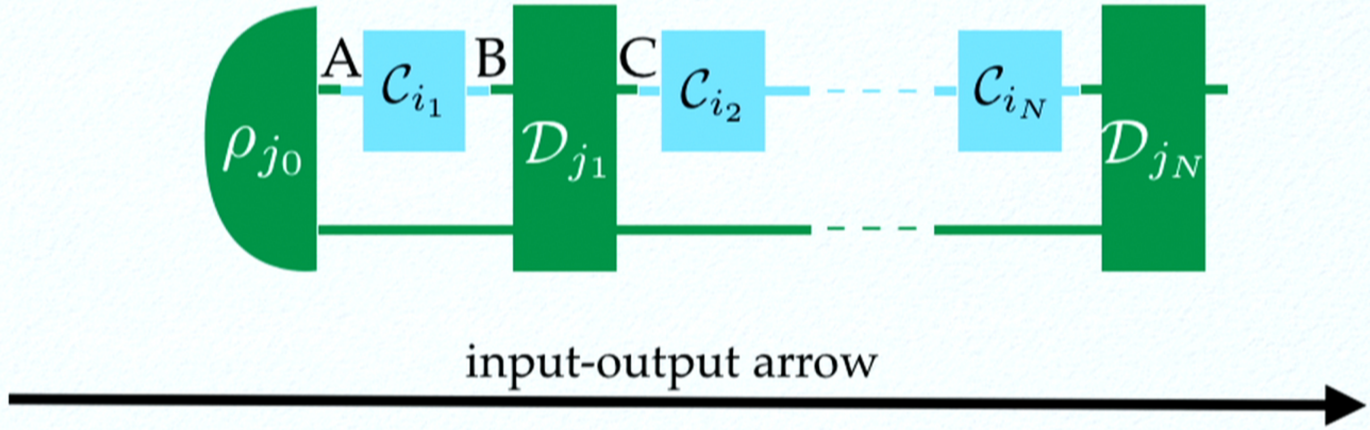
Quantum Theory
(in finite dimensions)

Chiribella, D'Ariano, Perinotti (2010).

IN SEARCH FOR INFORMATION-THEORETIC AXIOMS FOR (QUANTUM) THERMODYNAMICS


THE FRAMEWORK
Chiribella, D'Ariano, Perinotti 2009

CIRCUITS



A
B
C
$\mathcal{C}_{i_1}$
$\mathcal{C}_{i_2}$
$\mathcal{C}_{i_N}$
ρ_{j_0}
$\mathcal{D}_{j_1}$
$\mathcal{D}_{j_N}$
input-output arrow

PROBABILITIES

- Preparation + measurement = probability distribution

$$= p(a_j, \rho_i)$$

PROBABILITIES

- Preparation + measurement = probability distribution

$$\rho_i \;\mathrm{A}\; a_j \;=\; p(a_j, \rho_i)$$

$$\begin{cases} p(a_j, \rho_i) \geq 0 \\ \displaystyle\sum_{i \in X} \sum_{j \in Y} p(a_j, \rho_i) = 1 \end{cases}$$

OPERATIONAL-PROBABILISTIC THEORIES (OPTS)

$$\boxed{\begin{array}{c}\text{Operational-probabilistic theory}\\ =\\ \text{operational structure}\\ +\\ \text{probabilistic structure}\end{array}}$$

Fact: states span (possibly infinite dimensional) vector spaces, transformations act linearly.

Examples:
-classical theory
-quantum theory
-quantum theory on real Hilbert spaces
-...

DEFINITIONS: PURE AND REVERSIBLE

$\mathrm{A}\, \boxed{\mathcal{T}}\, \mathrm{B}$ is pure iff $\quad \mathrm{A}\, \boxed{\mathcal{T}}\, \mathrm{B} = \sum_x \mathrm{A}\, \boxed{\mathcal{T}_x}\, \mathrm{B}$

implies $\quad \mathrm{A}\, \boxed{\mathcal{T}_x}\, \mathrm{B} \propto \mathrm{A}\, \boxed{\mathcal{T}}\, \mathrm{B} \qquad \forall x$

DEFINITIONS: PURE AND REVERSIBLE

$\xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B}$ is pure iff $\xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B} = \sum_x \xrightarrow{A}\boxed{\mathcal{T}_x}\xrightarrow{B}$

implies $\xrightarrow{A}\boxed{\mathcal{T}_x}\xrightarrow{B} \propto \xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B} \quad \forall x$

$\xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B}$ is reversible iff exists $\xrightarrow{B}\boxed{\mathcal{S}}\xrightarrow{A}$

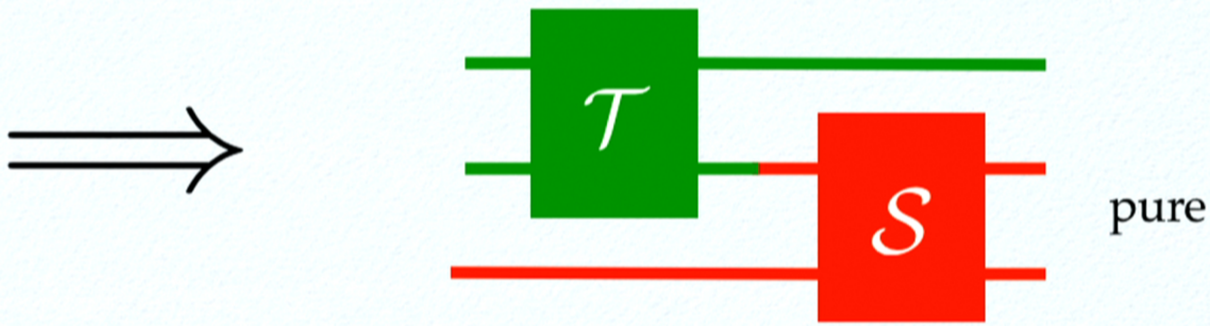
such that $\xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B}\boxed{\mathcal{S}}\xrightarrow{A} = \xrightarrow{A}$

$\xrightarrow{B}\boxed{\mathcal{S}}\xrightarrow{A}\boxed{\mathcal{T}}\xrightarrow{B} = \xrightarrow{B}$

AXIOMS

AXIOM: PURITY PRESERVATION

Purity Preservation: the composition of two transformations in pure



Special case: the product of two pure states is a pure state.

AXIOM: CAUSALITY

Operations performed in the future cannot affect the probability of outcomes of experiments done in the present

Equivalently: there is only one way to discard a system

AXIOM: CAUSALITY

Operations performed in the future cannot affect the probability of outcomes of experiments done in the present

Equivalently: there is only one way to discard a system

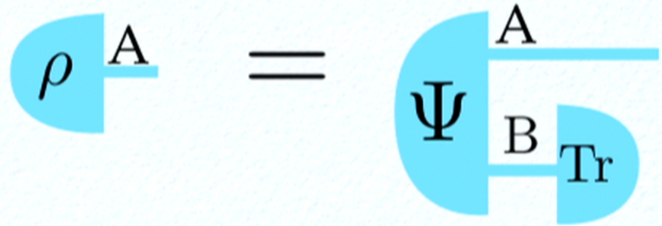
e.g. in QT:

$= \mathrm{Tr}[\rho]$

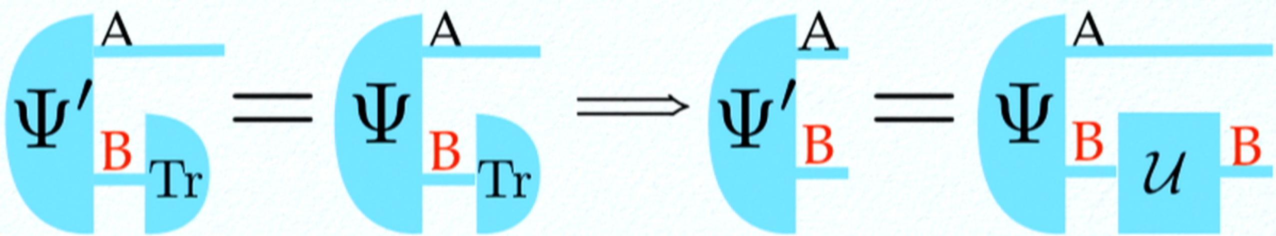
$$\rho_A := \mathrm{Tr}_A[\rho_{AB}]$$

AXIOM: PURIFICATION

- **Existence:** For every state ρ of A
  there is a system B and a pure state Ψ of $A \otimes B$
  such that

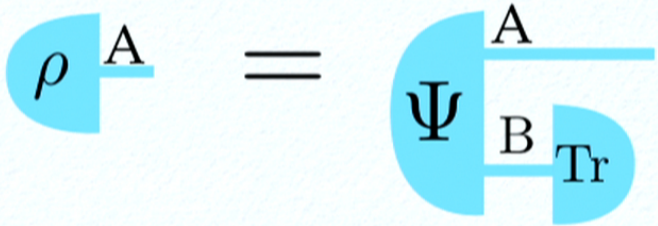


- **Uniqueness:** two purifications of the same state are equivalent
  up to a reversible transformation

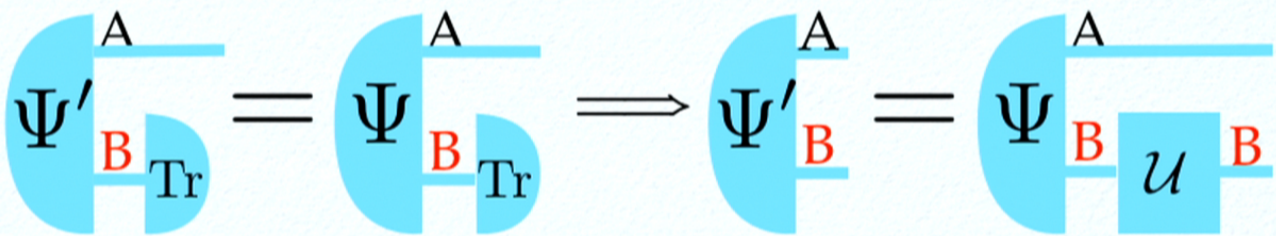


AXIOM: PURIFICATION

- Existence: For every state ρ of A
  there is a system B and a pure state Ψ of $A \otimes B$
  such that

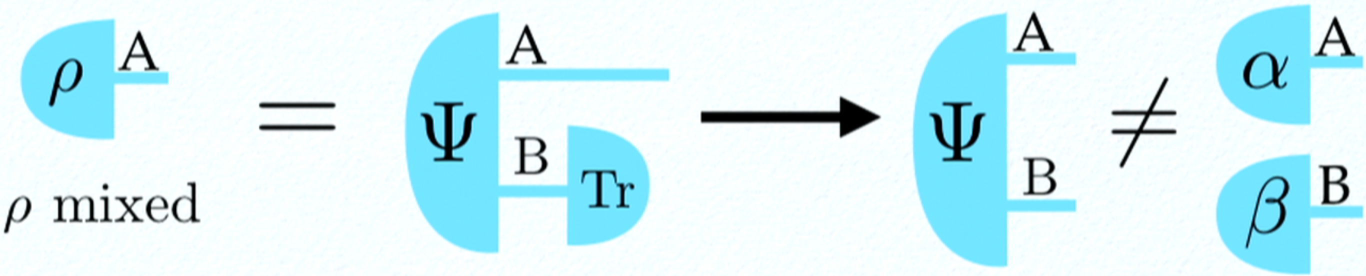


- Uniqueness: two purifications of the same state are equivalent
  up to a reversible transformation



CONSEQUENCES OF PURIFICATION

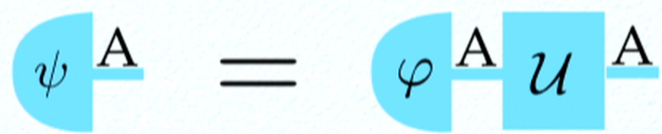
- The purification of a mixed state is entangled



ρ mixed

- Every two pure states of a given system are connected by a reversible transformation

$\forall \varphi, \psi \; \exists \mathcal{U}$:

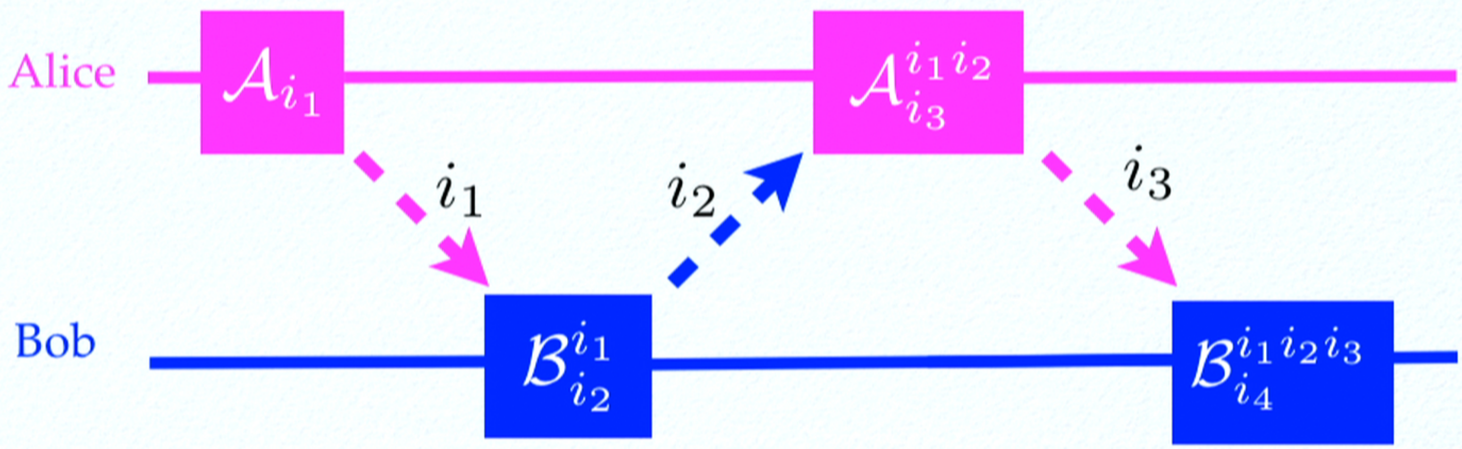


$\mathcal{U}$ reversible

Transitivity: cf. Hardy, Dakic-Brukner, Masanes-Müller, ...

THE LOCC PARADIGM

Two (or more) parties perform
local operations in their laboratories
and communicate classical data to each other.



LOCC
transformation:

$$\mathcal{L} = \sum_{i_1, i_2, i_3, i_4} \left(\mathcal{A}^{i_1 i_2}_{i_3} \mathcal{A}_{i_1} \otimes \mathcal{B}^{i_1 i_2 i_3}_{i_4} \mathcal{B}^{i_1}_{i_2} \right)$$

ENTANGLEMENT ORDERING

Definition: ρ_{AB} more entangled than $\sigma_{A'B'}$

iff $\rho_{AB} \xrightarrow{\text{LOCC}} \sigma_{A'B'}$

ENTANGLEMENT ORDERING

Definition: 

more entangled than 

iff 

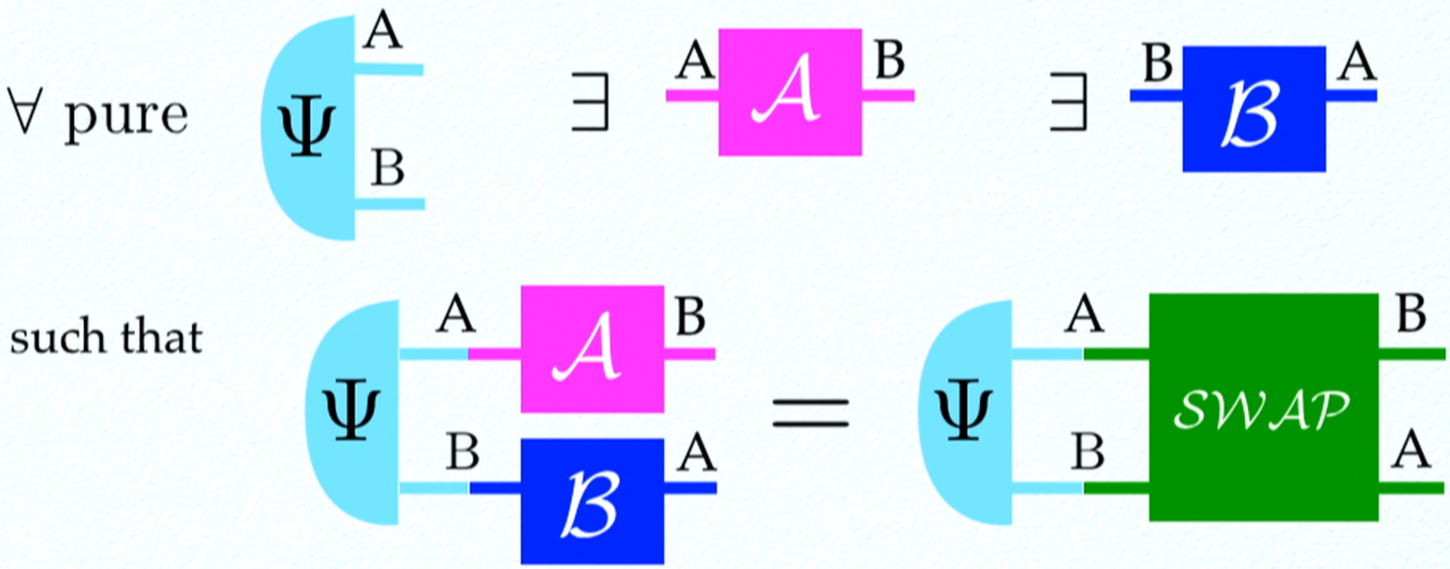
$\xrightarrow{\text{LOCC}}$ 

Least entangled states:
those that can be generated from nothing by LOCC

$$\rho = \sum_x p_x \, \rho_{Ax} \otimes \rho_{B_x}$$

AXIOM: LOCAL EXCHANGEABILITY

Local Exchangeability: every pure state of two systems can be swapped by local operations



Satisfied by quantum and classical theories
and by (all known states in) the theory of no-signalling boxes.

THE RARE PARADIGM

An agent has partial control on the parameters of a reversible dynamics:

$$p_x$$

As a result, she implements a Random Reversible (RaRe) transformation

$$\mathcal{R} = \sum_x p_x \mathcal{U}_x$$

THE RARE PARADIGM

~~Prepare, Discard~~

An agent has partial control on the parameters of a reversible dynamics:

p_x

As a result, she implements a
Random Reversible (RaRe) transformation

$$\mathcal{R} = \sum_x p_x \mathcal{U}_x$$

THE CONTROLLABILITY ORDERING

Definition: ρ^{A} more controllable than σ^{A}

iff $\rho^{A} \xrightarrow{\text{RaRe}} \sigma^{A}$

cf. group majorization by Masanes & Müller

Pure states are maximally controllable:

no mixed state can be more controllable than a pure state.

FROM CONTROLLABILITY TO PURITY

Thm: The following are equivalent

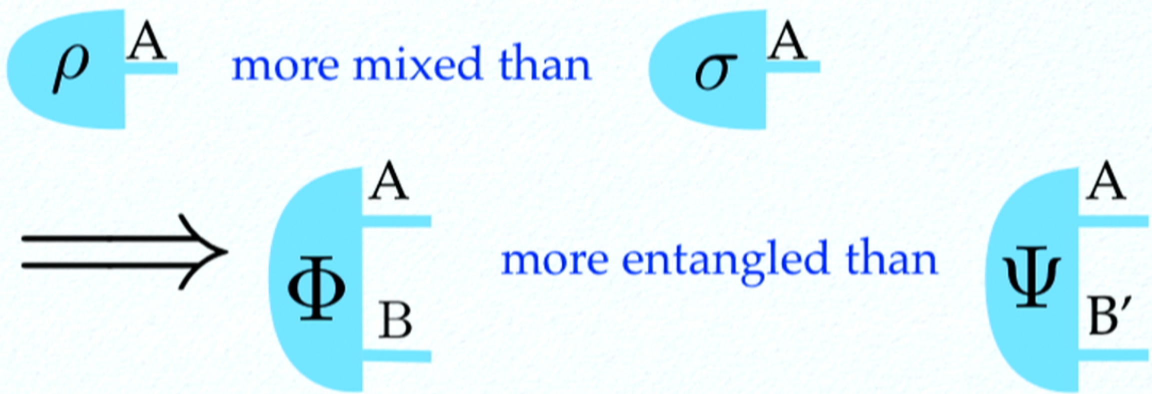
- existence of a maximum: for every system, there exists a state more controllable than every other state

- transitivity: for every system, every two pure states are connected by a reversible transformation

Transitivity of reversible transformations =
necessary condition for a sensible "theory of purity"

MORE MIXED IMPLIES MORE ENTANGLED

Thm 1: Assuming Purification,
one has the following implication:



$\Phi\,,\Psi =$ purifications of $\rho\,,\sigma$

AXIOM: NO ENTROPY SINKS

For every reasonable notion of entropy
an entropy sink should have "infinite entropy"

Let us demand that such states do not exist:

No Entropy Sinks: no physical state can act as an entropy sink

No Entanglement Sources: no physical state can be used as a catalyst to generate infinite copies of an entangled state

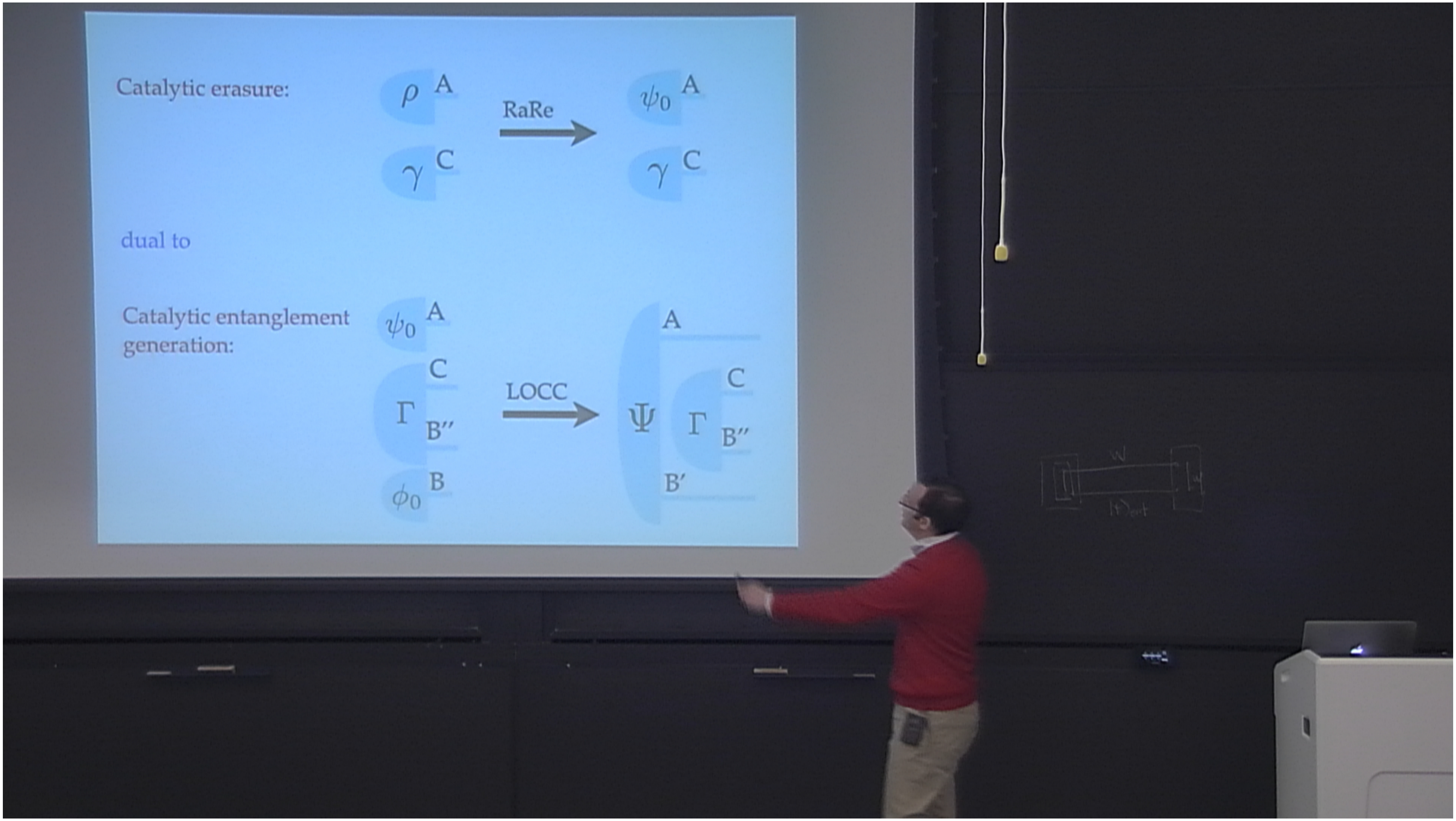
DIAGONALIZATION
AND
SPECTRAL ENTROPIES

REFERENCES FOR THIS TALK

- Chiribella, D'Ariano, Perinotti
  Quantum Theory from First Principles,
  CUP, forthcoming.
- Probabilistic Theories with Purification
  Phys. Rev. A 81, 062348 (2010)
- Informational Derivation of Quantum Theory
  Phys. Rev. A 84, 012311 (2011)

- Chiribella and Scandolo
  Entanglement and thermodynamics in general probabilistic theories
  arXiv 1504.07045
- Towards an information-theoretic foundation of (quantum) thermodynamics,
  in preparation.

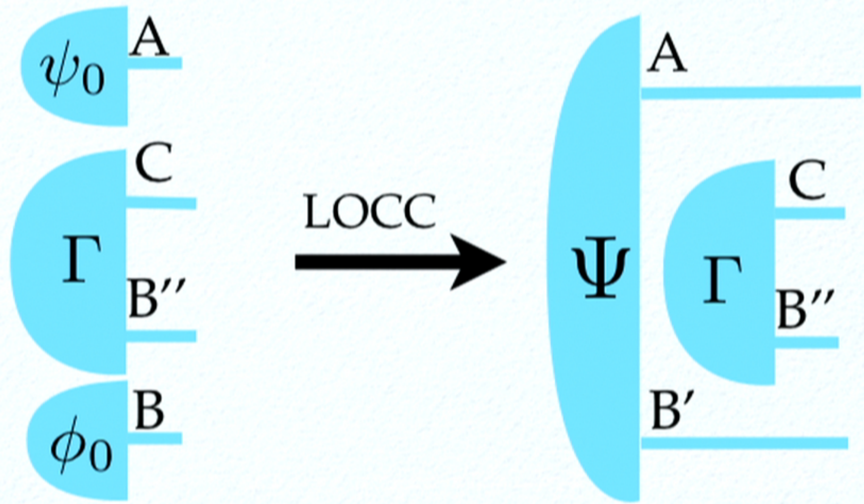
- Chiribella and Spekkens (eds), Quantum Theory: information-theoretic foundations and foils, Springer Verlag, in press


Catalytic erasure:
ρ A
γ C
RaRe
ψ0 A
γ C
dual to
Catalytic entanglement generation:
ψ0 A
C
Γ B″
ϕ0 B
LOCC
Ψ A
Γ C
B″
B′

Catalytic erasure:

dual to

Catalytic entanglement generation:



5 axioms for quantum thermodynamics

- Causality
- Purity Preservation
- Purification
- Pure Sharpness
- Reversible Permutability

Do they imply quantum theory as well?

5 axioms for quantum thermodynamics

- Causality
- Purity Preservation
- Purification
- Pure Sharpness
- Reversible Permutability

Do they imply quantum theory as well?