Title: Indefinite causal order in quantum mechanics

Date: May 11, 2015 09:15 AM

URL: http://pirsa.org/15050071

Abstract: One of the most deeply rooted concepts in science is causality: the idea that events in the present are caused by events in the past and, in turn, act as causes for what happens in the future. If an event A is a cause of an effect B, then B cannot be a cause of A.
Recently we proposed a framework that assumes that operations in local laboratories are described by quantum mechanics, but where no reference is made to any global causal relations between these operations. The central notion of the formalism is â€œprocessâ€• which is a generalization of the causal notion of â€œquantum stateâ€•. The framework allows for processes in which two operations are neither causally ordered nor in a probabilistic mixture of definite causal orders, i.e. one cannot say that A is before or after B. However a physical interpretation of such correlations was lacking. I will show that the â€œsuperposition of quantum circuitsâ€• â€“ in which the gate ordering is not fixed but controlled by a quantum system â€“ is an example of a process with indefinite causal order. This process provides a reduction in query complexity of certain computational problems and can be realized by placing the laboratories in the gravitational field of a massive object in a spatial superposition.

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Indefinite causal order in quantum mechanics

Caslav Brukner

Mateus Araujo, Cyril Branciard, Fabio Costa, Adrian Feix, Christina Giarmatzi, Ognyan Oreshkov, Magdalena Zych

Conference "Information Theoretic Foundations for Physics",
Perimeter Institute, May 11th 2015

Reconstructions of quantum theory

Ability to account for the origin of the basic principles from which the structure of quantum theory can be derived without invoking mathematical terms such as “rays in Hilbert space”, “self-adjoint operators” etc.

References:

- L. Hardy (2001)
- B. Dakic and C. Brukner (2009)
- L. Masanes and M. Müller, (2011)
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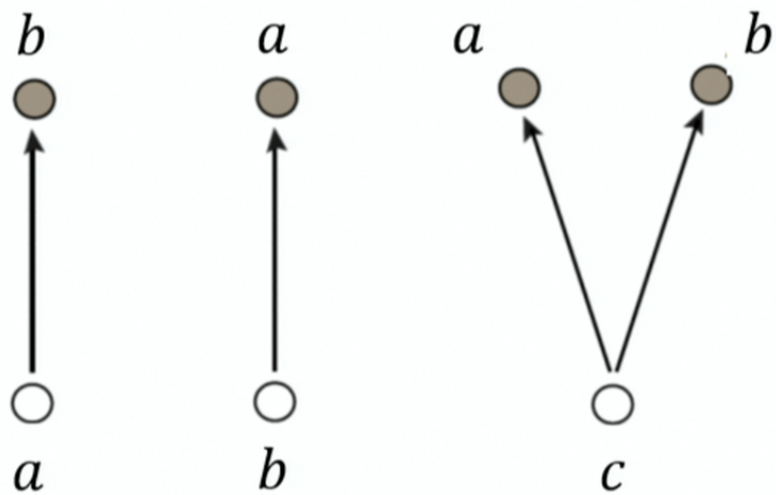
“Correlation does not imply causation”

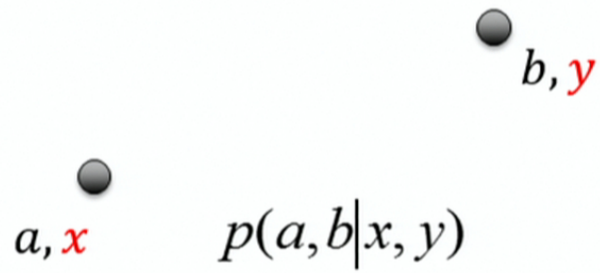


a and b are correlated, but …

The notion of "causation"

Necessity of interventions, or **free variables**, statistically independent of "the rest of the experiment"

The notion of "causation"



$$\sum_a p(a,b|x,y) = p(b|y)$$

$$\sum_b p(a,b|x,y) = p(a|x,y)$$

One-directional signalling

Necessity of interventions, or **free variables**, statistically independent of "the rest of the experiment"

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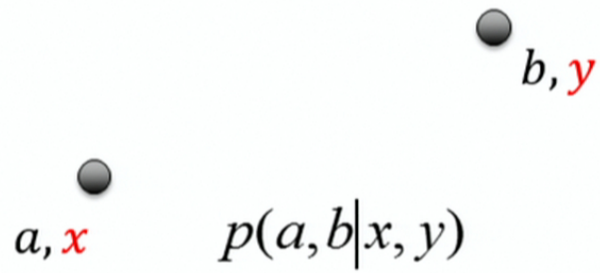

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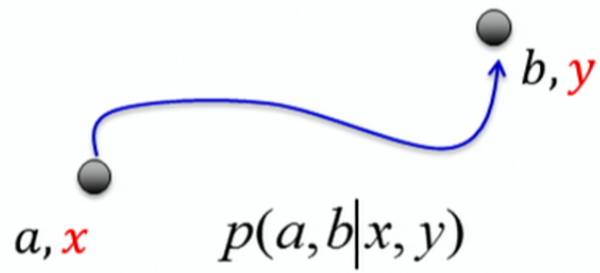

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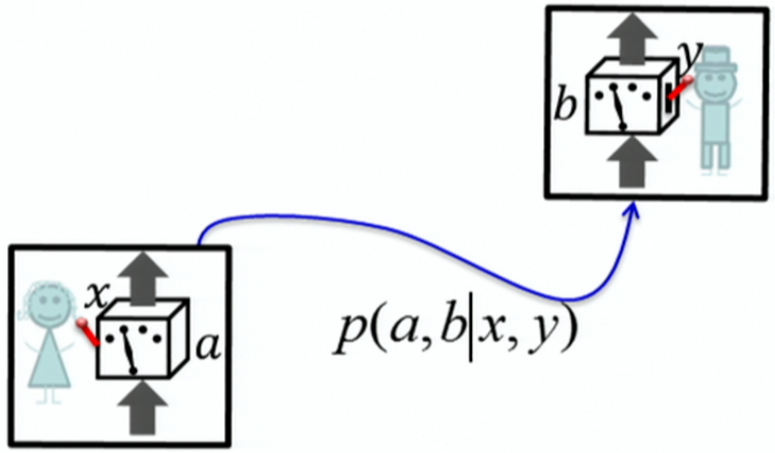

$$p(a,b|x,y)$$

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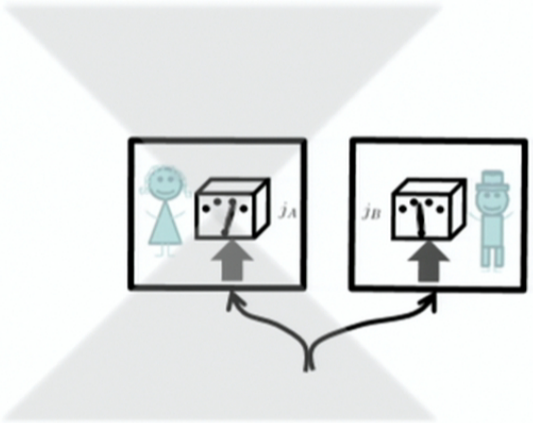
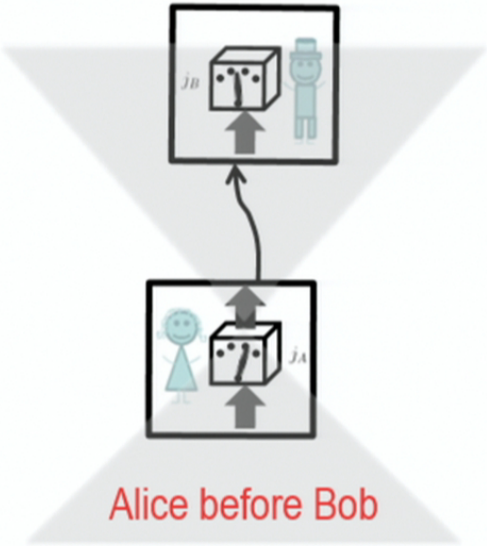
One-directional signalling

Necessity of interventions, or **free variables**, statistically independent of "the rest of the experiment"

Causal order from space-time



Space-like separated



Time-like separated

GR + linearity of QM

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Motivation

- Can one formulate probabilistic theories without the assumption of background space-time or causal structure?

L. Hardy, arXiv:gr-qc/0509120
arXiv:gr-qc/0608043
arXiv:quant-ph/0701019
…

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- Can one formulate probabilistic theories without the assumption of background space-time or causal structure?

- Can a "quantum gravity computer" have greater computational power than (causal) quantum computers?

L. Hardy, arXiv:gr-qc/0509120
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Outline

- "Causal inequalities"
  Device-independent approach to causality

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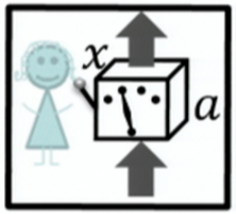
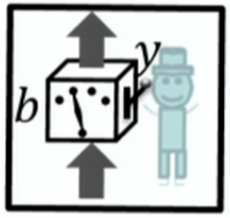
  Device-independent approach to causality

- Framework for quantum mechanics with no assumed global causal structure:
  Device-dependent approach to causality
  Causally non-separable processes

Causal inequalities

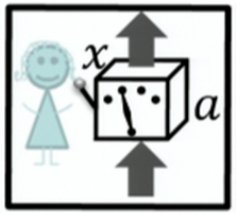
(Device-independent approach to causality)

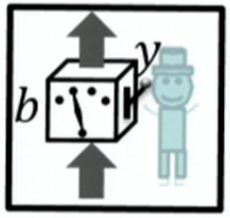




Causal inequalities

(Device-independent approach to causality)





Assumptions:

- System enters each laboratory **only once**.
- The **laboratories are shielded** and interact with "the outside world" only through the system entering and exiting it.

Causal inequalities

(Device-independent approach to causality)

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Causal correlations: either A signals to B or B signals to A, or no-signalling or a convex combination of these situations.

Causal inequalities

(Device-independent approach to causality)

$$p(a,b|x,y) = \lambda p^{A\leq B}(a,b\,|\,x,y) + (1-\lambda)\,p^{B\leq A}(a,b\,|\,x,y)$$

Convex mixture of …

No-signalling

One-directional signalling: Alice to Bob

$$\sum_a p^{A=B}(a,b|x,y) = p^{A=B}(b|y)$$

$$\sum_b p^{A=B}(a,b|x,y) = p^{A=B}(a|x)$$

$$\sum_a p^{A\prec B}(a,b|x,y) = p^{A\prec B}(b|y)$$

$$\sum_b p^{A\prec B}(a,b|x,y) = p^{A\prec B}(a|x,y)$$

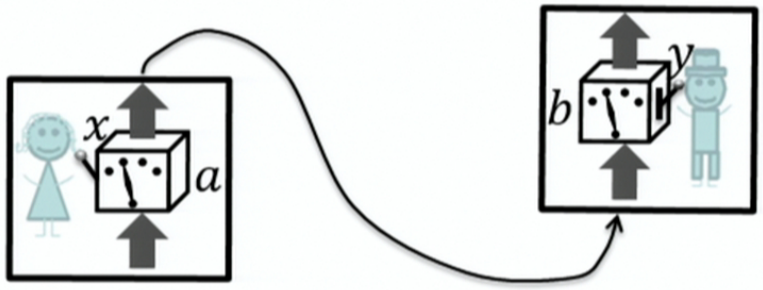
Causal correlations: either A signals to B or B signals to A, or no-signalling or a convex combination of these situations.

The simplest causal polytope

Causal correlations satisfy **causal inequalities**, which are facets of the **causal polytope**

M. Araujo, C. Branciard, A. Feix, F. Costa, C.B. in preparation

The simplest causal polytope



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Causally indefinite correlations

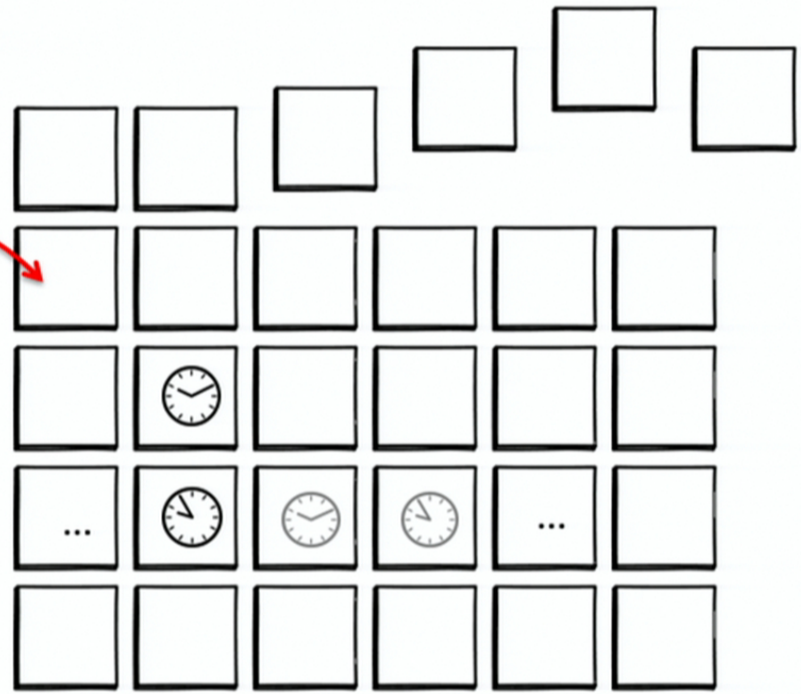
(violating causal inequalities)

Both A signals to B and B signals to A, although the system enters only once the laboratory and the laboratories are shielded.

Opening the box: Quantum laboratories

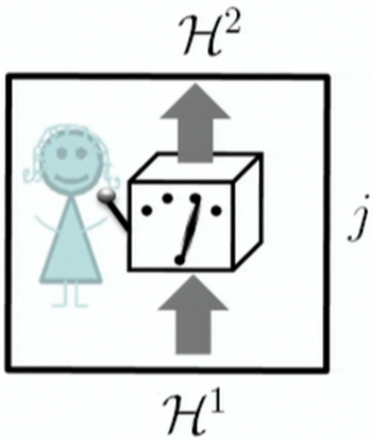
(paching the causal regions)

Locally one has a „causal quantum mechanics“



Main premise:

Local descriptions agree with quantum mechanics



Transformations = completely positive (CP) trace-nonincreasing maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$$

$$\mathcal{M}_{CPTP} = \sum_j \mathcal{M}_j$$

Completely positive trace preserving (CPTP) map

Choi-Jamiołkowski isomorphism

CP maps		Positive matrices
$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

Choi-Jamiołkowski isomorphism

CP maps

$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$

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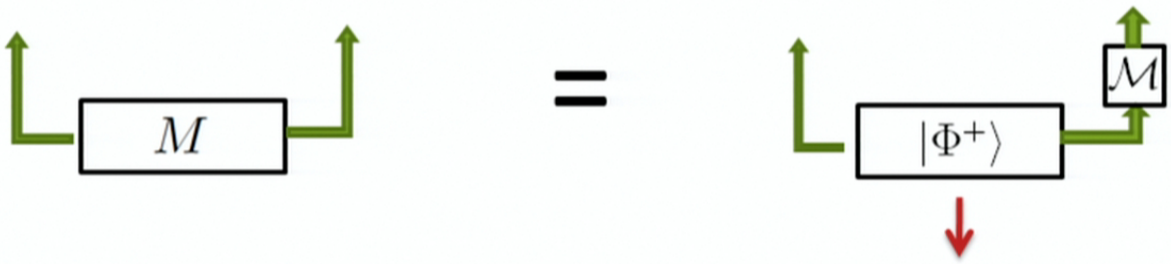

Maximally entangled state

$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$

$|i\rangle \in \mathcal{H}^1$

Choi-Jamiołkowski isomorphism

CP maps

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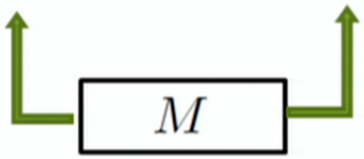

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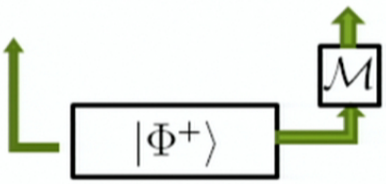

$$M^{12} := \left[\mathcal{I} \otimes \mathcal{M}\left(|\Phi^+\rangle\langle\Phi^+|\right)\right]^{\mathrm{T}}$$

Maximally entangled state

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$$

$$|i\rangle \in \mathcal{H}^1$$

Two (or more) parties

Probabilities are bilinear functions of the CP maps

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2} \left(M^{A_1 A_2} \otimes M^{B_1 B_2} \right) \right]$$

Bipartite probabilities

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1. Nonnegative probabilities:

(ancillary entangled states do not fix causal order)

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$$W^{A_1 A_2 B_1 B_2} \geq 0$$

2. Probability is 1 for all CPTP maps.

Bipartite probabilities

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(ancillary entangled states do not fix causal order)

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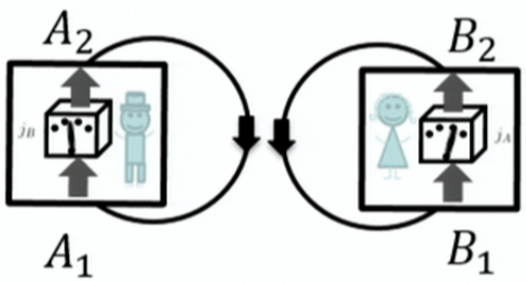
$$\mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(M_{\mathrm{CPTP}}^{A_1A_2} \otimes M_{\mathrm{CPTP}}^{B_1B_2}\right)\right] = 1,$$

Forbidden processes

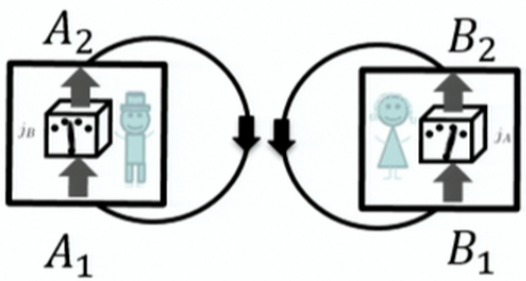
$$W^{A_1A_2} \otimes W^{B_1B_2}$$



Single Loops

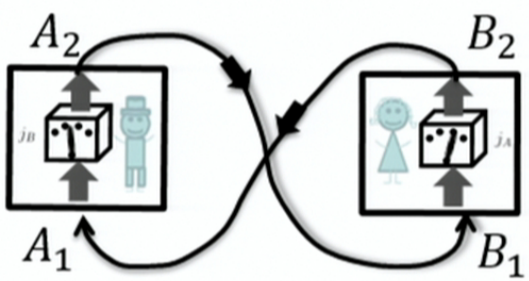
Forbidden processes

$W^{A_1A_2} \otimes W^{B_1B_2}$



Single Loops

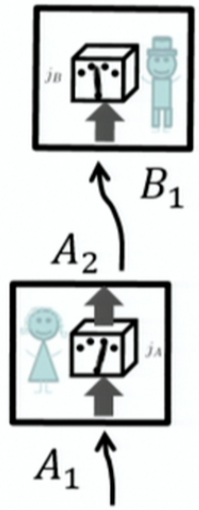
$W^{A_1B_2} \otimes W^{A_2B_1}$



Double Loops

Admissible processes

$$W^{A_1 A_2 B_1} \otimes \mathrm{I}^{B_2}$$



Channels from Alice to Bob

Time-like separated

Causally separable processes

Ordered processes:

$W^{A \preccurlyeq B}$ A signals to B, or no signaling

$W^{B \preccurlyeq A}$ B signals to A, or no signaling

Causally separable processes are those which can be written as **convex mixtures of ordered processes**:

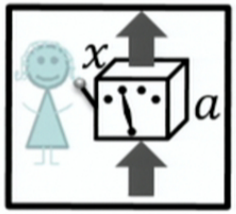
$$W^{A_1 A_2 B_1 B_2} = \lambda W^{A \preccurlyeq B} + (1-\lambda) W^{B \preccurlyeq A}$$

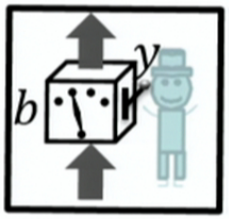
Not all processes are causally separable
"Superpositions of causal orders"?

O. Oreshkov, F. Costa, Č.B., Nature Communication **3**: 1092 (2012)

Violation of causal inequalities

1 bit input,
1 bit output





1 bit input,
1 bit output

Using a see-saw algorithm: both causal inequalities are violated by causally non-separable processes.

Guess Your Neighbour's Input (GYNI) game:

$$p(x=a, y=b) \approx 0.5694 > \frac{1}{2}$$

Lazy version of the GYNI game:

$$p(a(x \oplus b)=0, b(y \oplus a)=0) \approx 0.8194 > \frac{3}{4}$$

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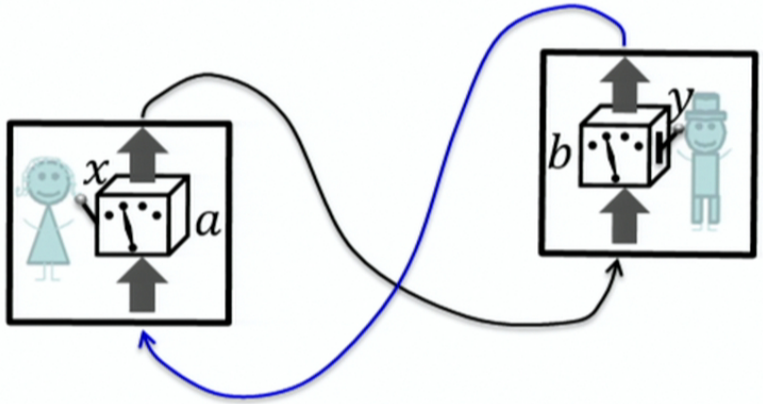
Process violating causal inequalities

$$W^{A_1A_2B_1B_2} = \frac{\mathbb{I}^{A_1A_2B_1B_2}}{4} + \frac{1}{4\sqrt{2}}\left(Z^{A_1}\mathbb{I}^{A_2}X^{B_1}X^{B_2} + Z^{A_1}Z^{A_2}Z^{B_1}\mathbb{I}^{B_2}\right)$$

Process violating causal inequalities



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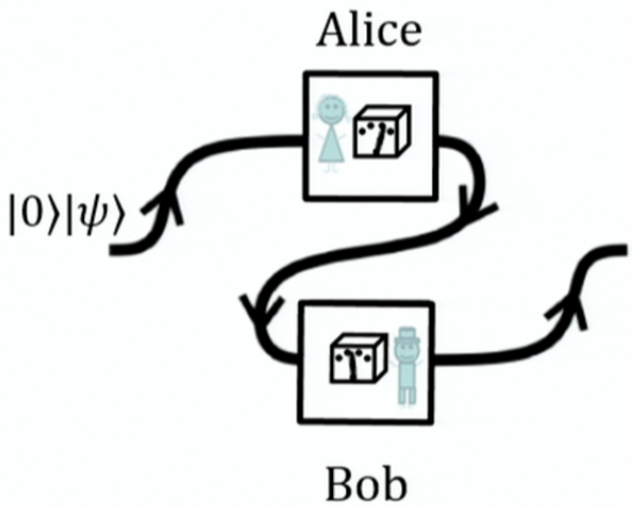
? Physical realization of processes violating the causal inequalities

Quantum switch –
quantum control of causal order



Channel from Alice to Bob

Quantum switch –
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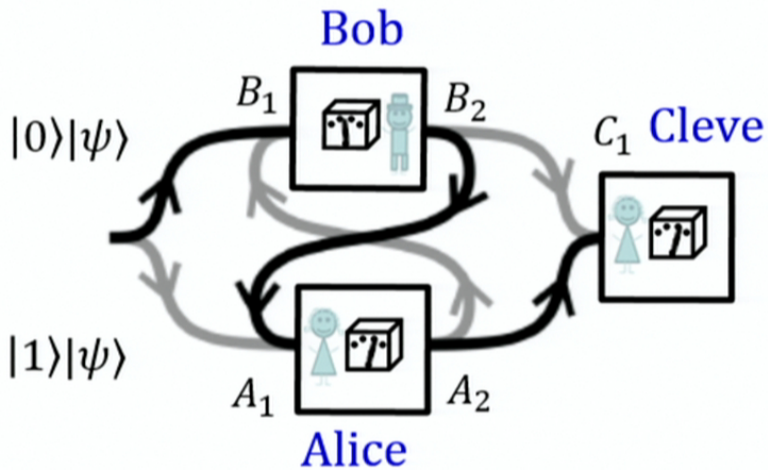

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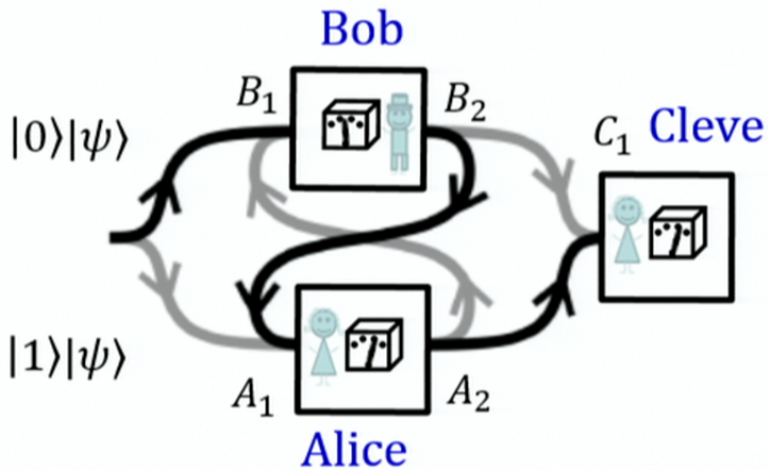
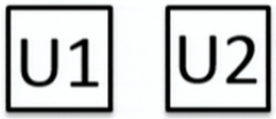


$$|w\rangle = 1/\sqrt{2}(|0\rangle|\psi\rangle^{B_1}|\phi^+\rangle^{B_2A_1}|\phi^+\rangle^{A_2C_1} + |1\rangle|\psi\rangle^{A_1}|\phi^+\rangle^{A_2B_1}|\phi^+\rangle^{B_2C_1})$$

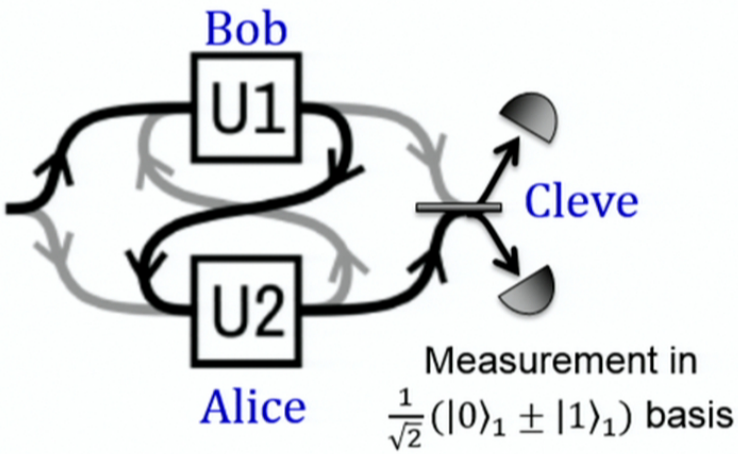
- Process is pure
- Process allows two-way signalling
- C is the last

M. Araujo, C. Branciard, A. Feix, F. Costa, C, Giarmatzi, C.B. in preparation

Computational advantage

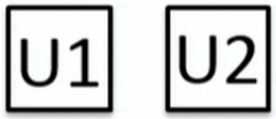


Promise:
Either $[U_1, U_2] = 0$ or $\{U_1, U_2\} = 0$



G. Chiribella, Phys. Rev. A **86,** 040301(R) (2012)
G. Chiribella, G.M. D'Ariano, P. Perinotti, & B. Valiron, Phys. Rev. A **88**, 022318 (2013)

Computational advantage



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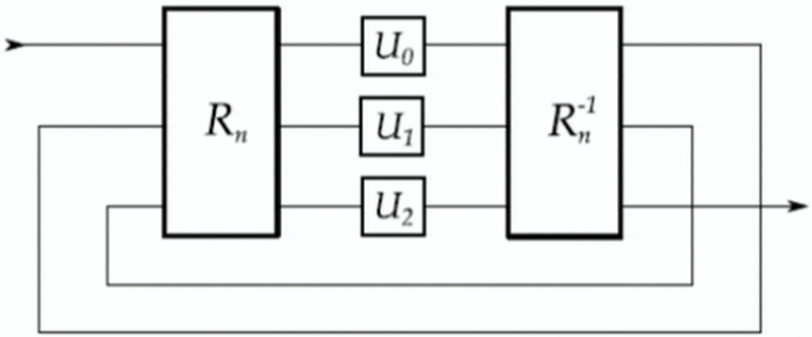
Causal quantum algorithms require at least **two** queries of one of the gates

G. Chiribella, Phys. Rev. A **86,** 040301(R) (2012)
G. Chiribella, G.M. D'Ariano, P. Perinotti, & B. Valiron, Phys. Rev. A 88, 022318 (2013)

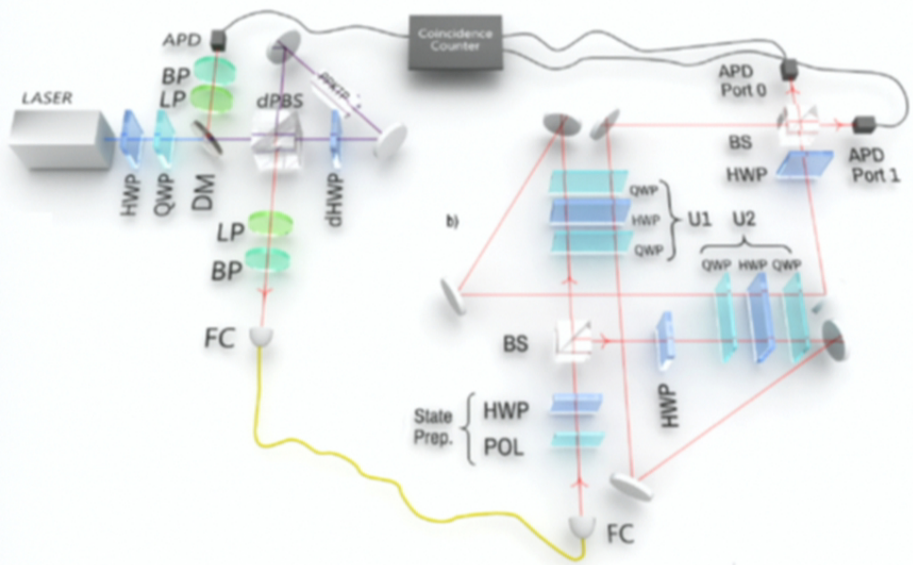
Reduction of query complexity



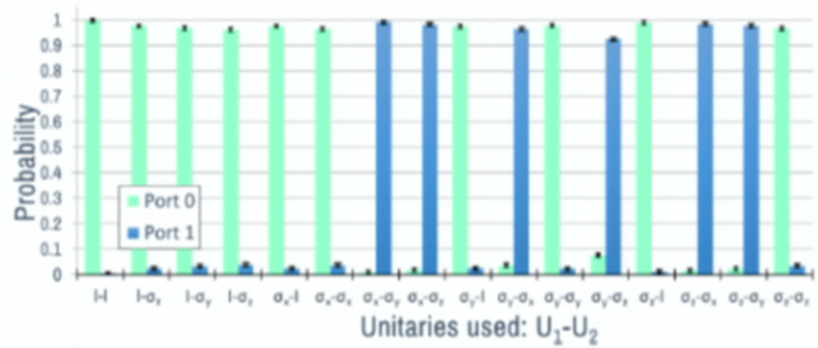
Reduction of the **query complexity** from $O(n^2)$ to $O(n)$ for n gates using quantum controlled ordering of gates.

M. Araujo, F. Costa, C.B., Phys. Rev. Lett. **113**, 250402 (2015) (Editor's Choice)

Experimental Demonstration

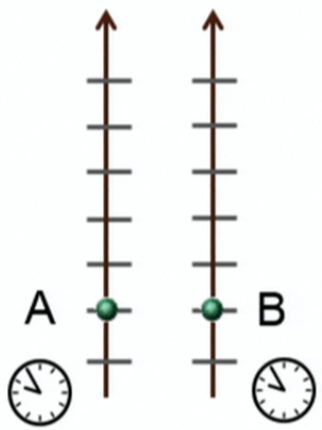


Group of P. Walther,
University of Vienna,
arXiv:1412.4006



Dynamical causal relations in GR

Events are defined by physical processes
e.g. *"the second tick of clock A"*



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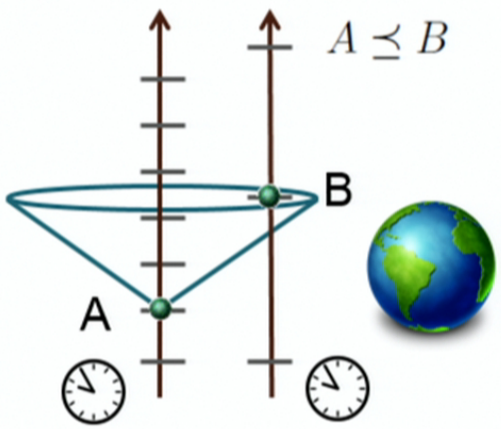


The presence of a mass slows down the rate of nearby clocks

Dynamical causal relations in GR

Events are defined by physical processes
e.g. *"the second tick of clock A"*

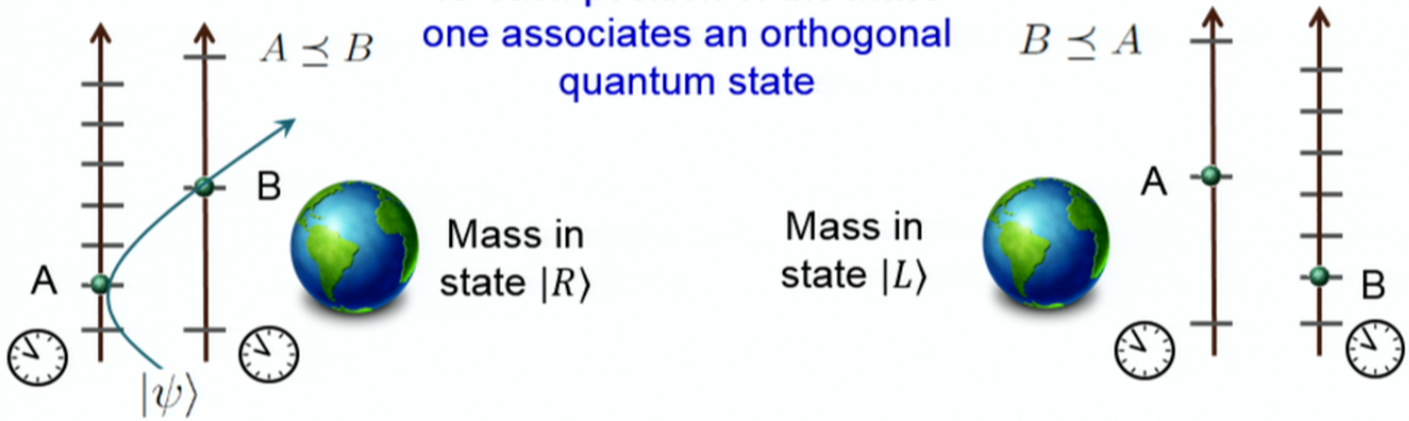
The presence of a mass slows down the rate of nearby clocks

The causal relation between events can depend on the state of a massive system

Quantum switch

via superposition of masses

To each position of the mass one associates an orthogonal quantum state



$$|R\rangle|\psi\rangle \to |R\rangle U_B U_A |\psi\rangle$$

M. Zych, F. Costa, I. Pikovski, C.B., in preparation

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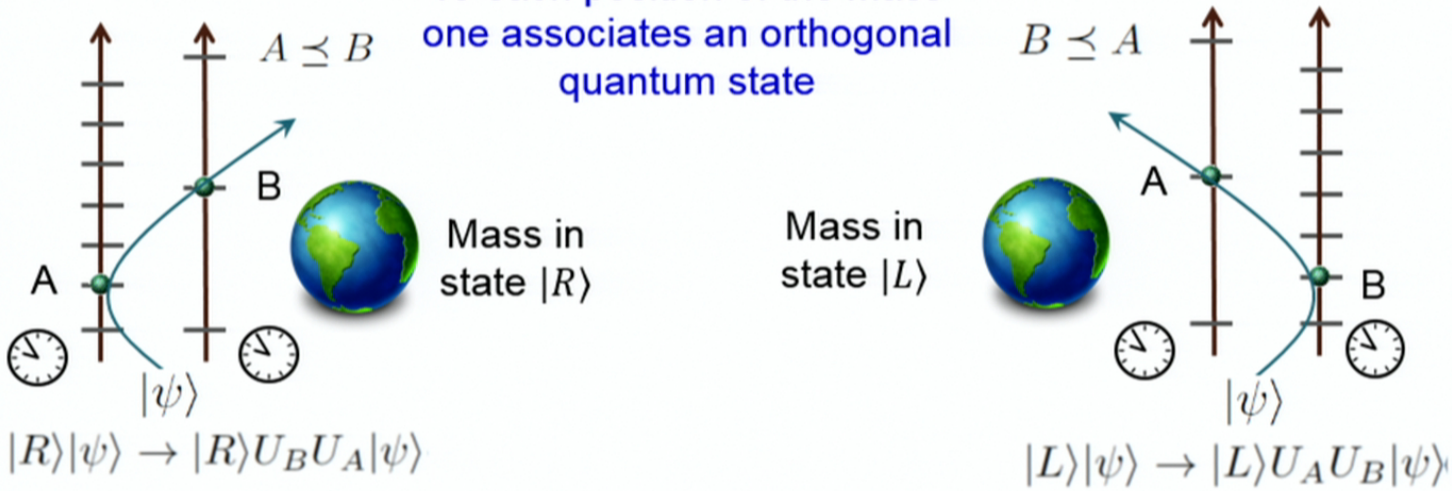

Mass in superposition: $(|L\rangle_M + |R\rangle_M)|S\rangle \to |L\rangle_M \hat{U}_A \hat{U}_B |S\rangle + |R\rangle_M \hat{U}_B \hat{U}_A |S\rangle$

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Summary

- Process formalism: unified framework for signalling (time-like) and no-signalling (space-like) quantum correlations.

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- Process formalism: unified framework for signalling (time-like) and no-signalling (space-like) quantum correlations.
- Situations where the causal order between laboratory operations is not definite → global causal order need not be a necessary element of quantum theory.
- Indefinite causal structures as a new resource for quantum information processing
- Can we realize causal processes violating the causal inequalities in the lab?

FWF Der Wissenschaftsfonds.

CoQuS Complex Quantum Systems

FQXi

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RAQUE Randomness and Quantum Entanglement