Title: Ambitwistors-strings and amplitudes

Date: May 30, 2015 09:30 AM

URL: http://pirsa.org/15050068

Abstract: These lectures will focus on the geometry of ambitwistor string theories. These are infinite tension analogues of conventional strings and provide the theory that leads to the remarkable formulae for tree amplitudes that have been developed by Cachazo, He and Yuan based on the scattering equations. Although the bosonic ambitwistor string action is expressed in space-time, it will be seen that its target is classically `ambitwistor space', the space of complexified null geodesics in the complexification of a space-time. The lectures will review Ambitwistor constructions from the 70's and 80's that extend the Penrose-Ward twistor constructions for self-dual Yang-Mills and gravitational fields in four dimensions to arbiitrary fields in general dimension. LeBrun showed that the conformal geometry of a space-time is encoded into the complex structure of ambitwistor space. The linearized version encodes linear fields on space-time into sheaf cohomology classes on ambitwistor space. In the case of momentum eigenstates, these give the `scattering equations' that underly the CHY formulae and the ambitwistor string can be used to compute amplitudes via these formulae. If there is time, the lectures will discuss how different matter theories can be obtained, different geometric realizations of ambitwistor space lead to different formulae, the relationship between the asymptotic symmetries of space-time and Weinberg's soft theorems concerning the behaviour of amplitudes when momenta become small, and/or extensions of the ideas to loop amplitudes.

Lecture 3: A menagerie of ambitwistor strings

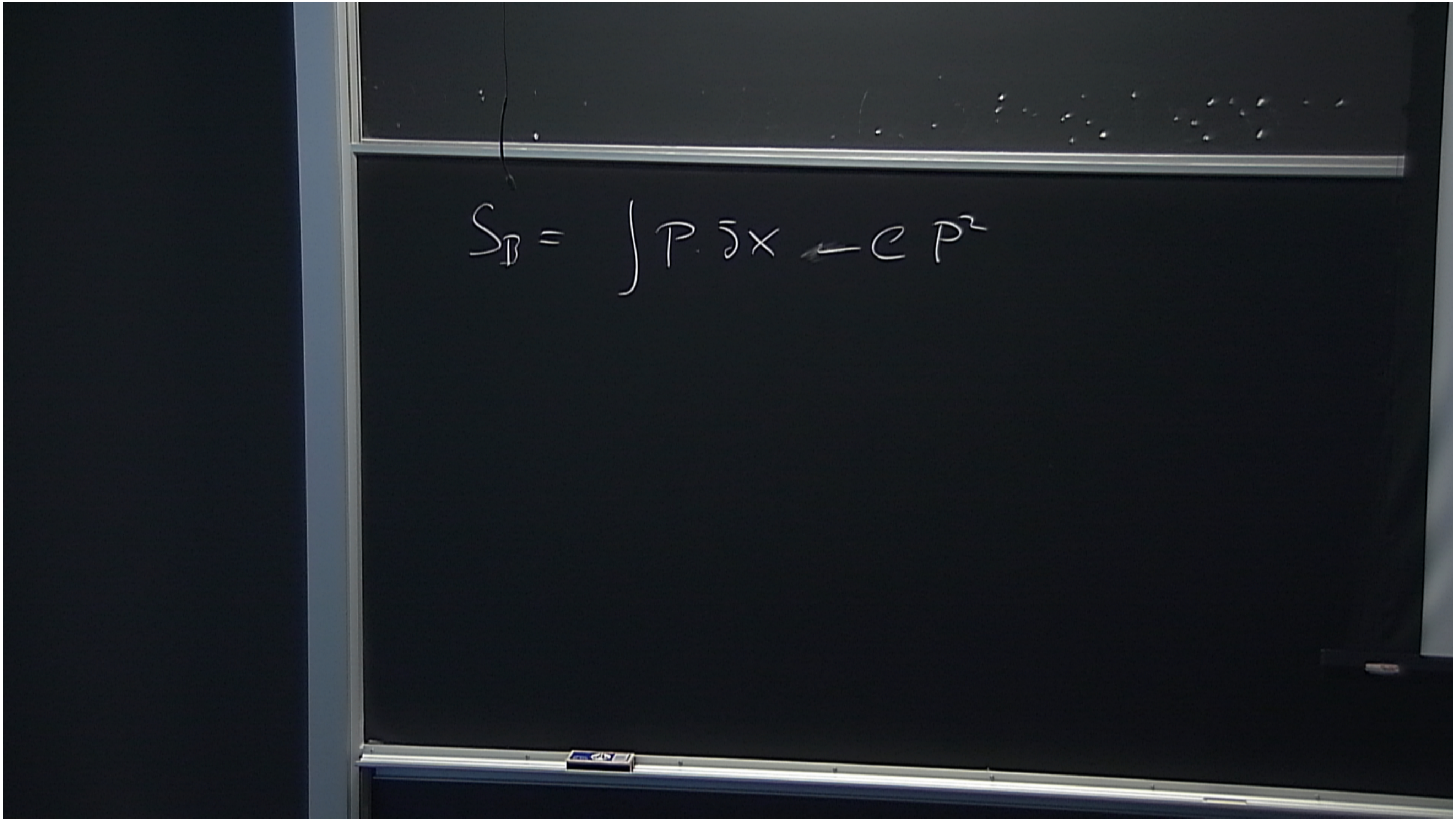
Lionel Mason

The Mathematical Institute, Oxford
lmason@maths.ox.ac.uk

GAP, Perimeter, 29/5/2015

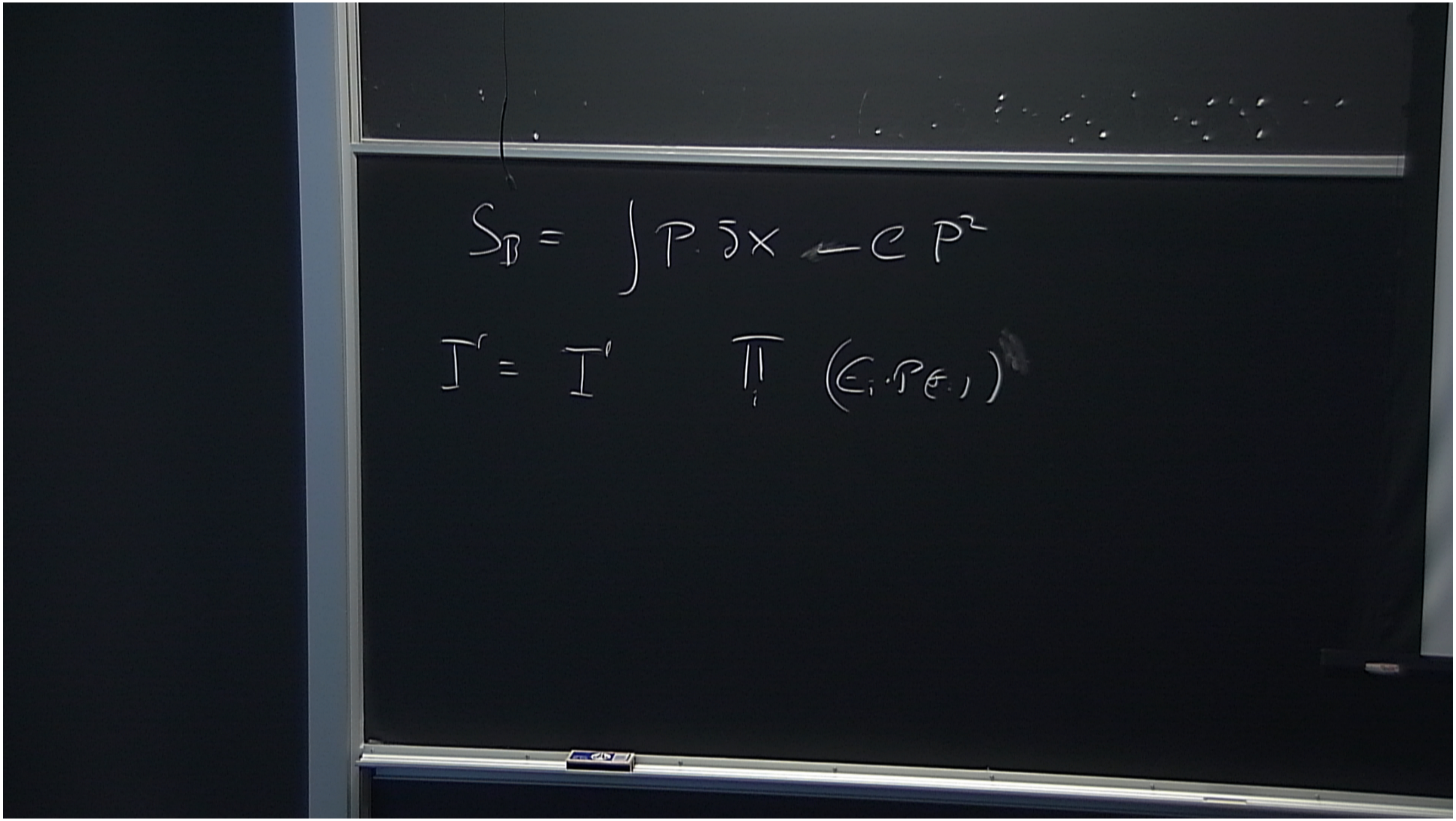
With David Skinner. arxiv:1311.2564, and various collaborations with Tim Adamo, Eduardo Casali, Yvonne Geyer, Arthur Lipstein, Ricardo Monteiro & Kai Roehrig, 1312.3828, 1404.6219, 1405.5122, 1406.1462, CGMMRS 150?.????.

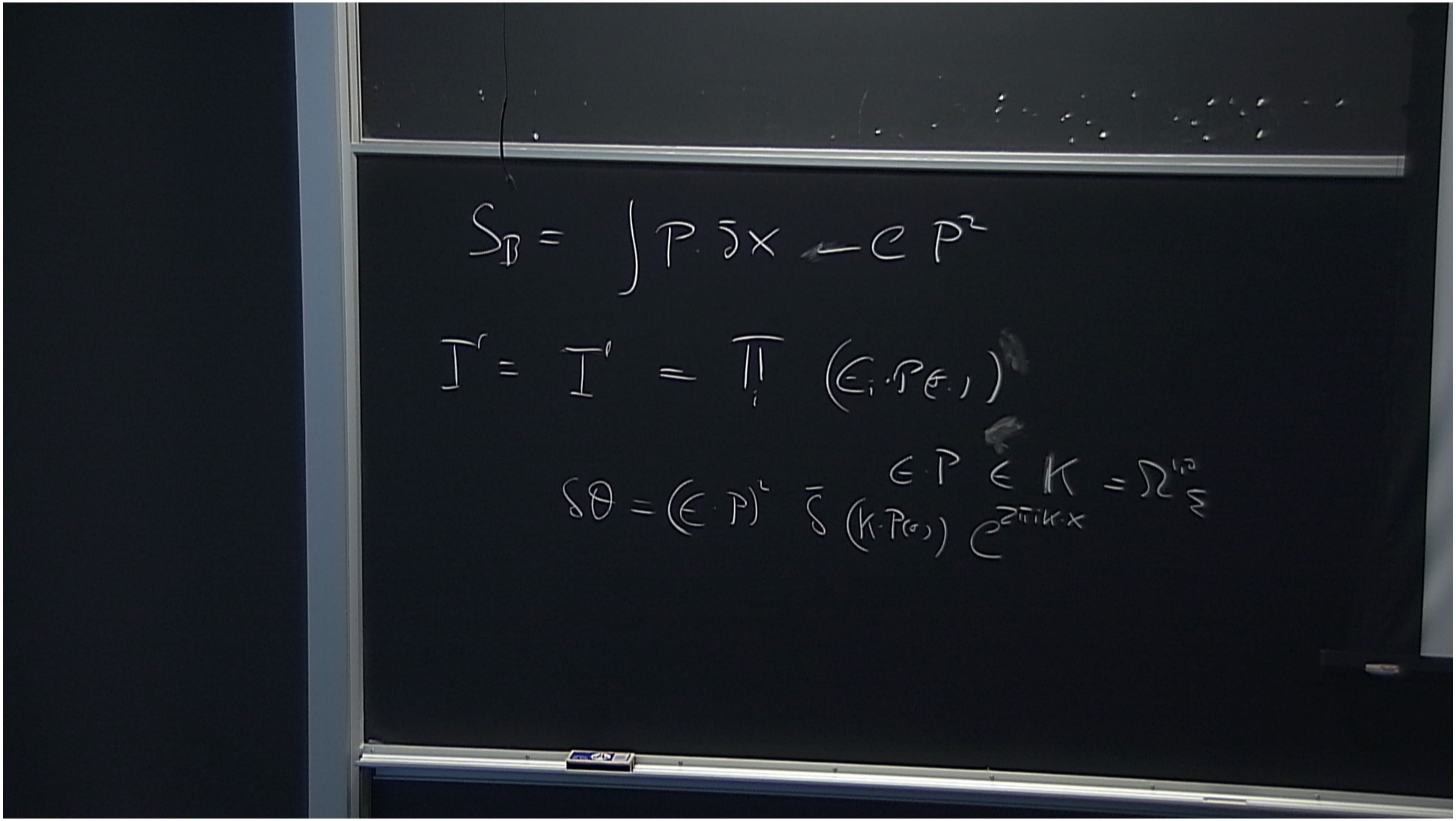
[Cf. also Cachazo, He, Yuan arxiv:1306.2962, 1306.6575, 1307.2199, 1309.0885, 1412.3479]


$S_B = \int P \cdot \partial x - c P^2$

Worldsheet matter

- Decorate null geodesics with spin vectors, vectors for internal degrees of freedom & other holmorphic CFTs.
- Take
$$S = S_B + S^l + S^r$$
where S^l, S^r are some worldsheet matter CFTs.
- Total vertex operators given by
$$v^l v^r \bar{\delta}(k \cdot P)\, \mathrm{e}^{ik \cdot X}$$
with v^l, v^r worldsheet currents from S^l, S^r resp..
- Amplitudes become
$$\mathcal{M}(1, \ldots, n) = \delta^d(\sum_i k_i) \int_{(\mathbb{CP}^1)^n} \frac{I^l I^r \prod_i{}' \bar{\delta}(k_i \cdot P)}{\text{Vol Gauge}}$$
where I^l, I^r are worldsheet correlators of v^ls, v^rs resp..
- In good situations, Q-invariance and discrete symmetries (GSO) rule out unwanted vertex operators.


$S_B = \int P \cdot \partial x - e P^2$
$I' = I' \prod_i (\epsilon_i \cdot P \epsilon_i)$


S_B = ∫ P·∂X − e P^2
Γ' = Γ' = ∏_i (ε_i·P(σ_i))
ε·P ε K = Ω^(2)_Σ
δΘ = (ε·P)^2 δ̄(k·P(σ)) e^(2πik·x)

Amplitude formulae for massless theories.

Theorem (Cachazo, He, Yuan 2013,2014)

Tree-level massless amplitudes in d-dims are integrals/sums

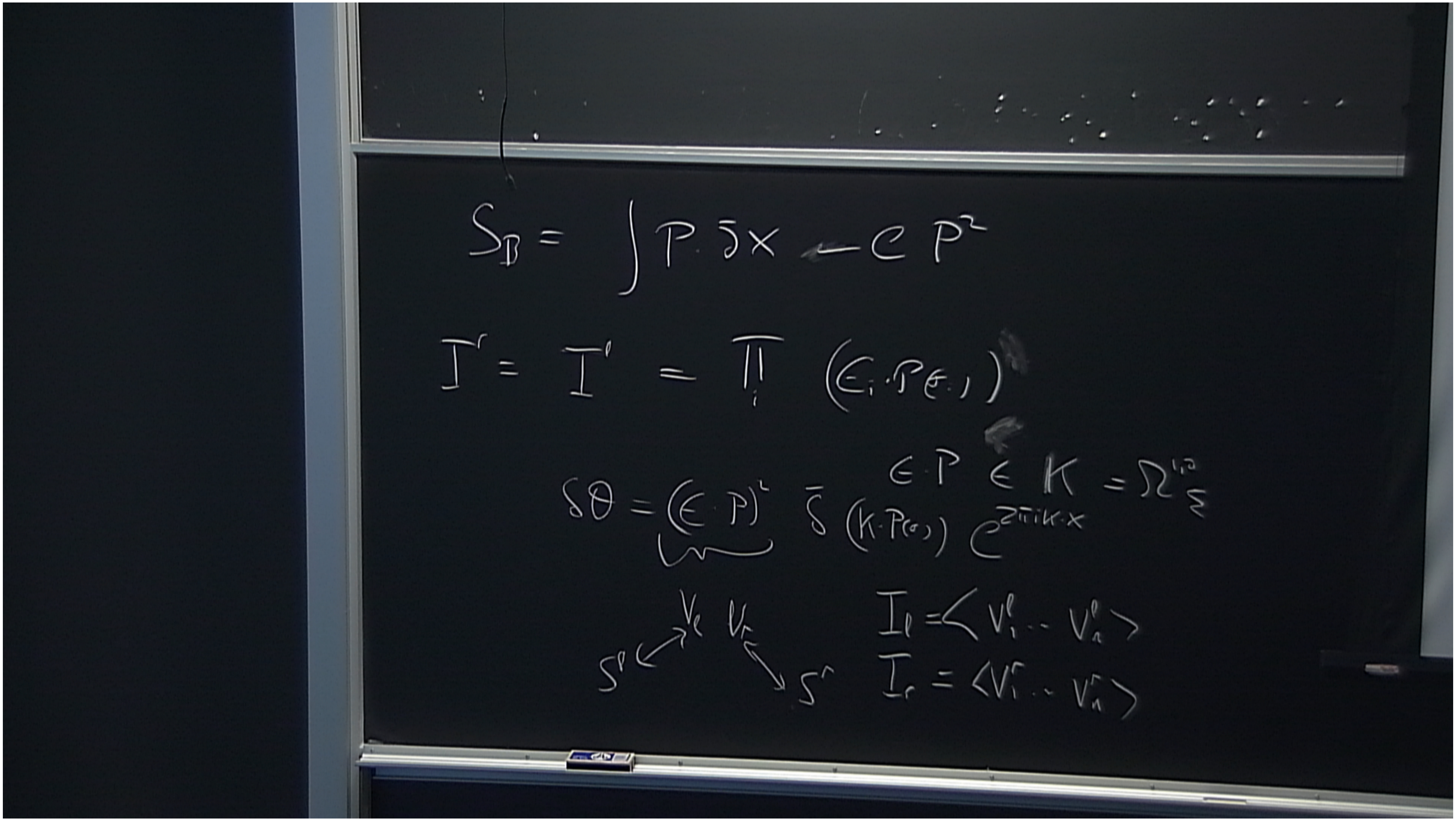
$$\mathcal{M}(1,\dots,n) = \delta^d\left(\sum_i k_i\right)\int_{(\mathbb{CP}^1)^n} \frac{I^l I^r \prod_i \bar\delta(k_i\cdot P(\sigma_i))}{\mathrm{Vol}\ \mathrm{SL}(2,\mathbb{C})\times\mathbb{C}^3}$$

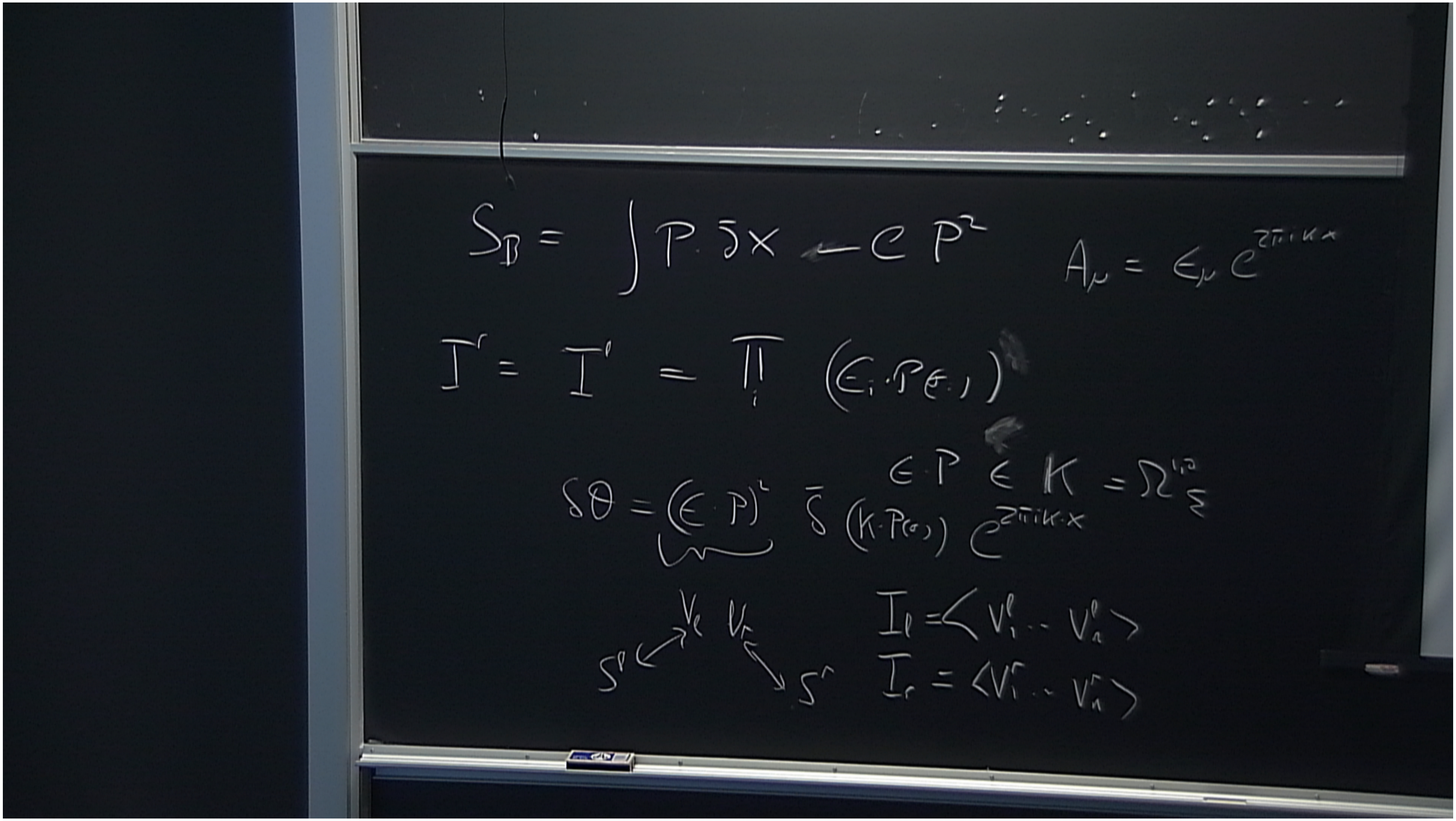
where $I^{l/r} = I^{l/r}(\epsilon_i^{l/r}, k_i, \sigma_i)$ *depend on the theory.*

- polarizations ϵ^l_μ for spin 1, $\epsilon^l_\mu\otimes\epsilon^r_\nu$ for spin-2, $(k\cdot\epsilon = 0\dots)$.
- Introduce skew $2n\times 2n$ matrices $M = \begin{pmatrix} A & C\\ -C^t & B\end{pmatrix}$,

  $$A_{ij} = \frac{k_i\cdot k_j}{\sigma_i-\sigma_j},,\quad B_{ij} = \frac{\epsilon_i\cdot\epsilon_j}{\sigma_i-\sigma_j},\quad C_{ij} = \frac{k_i\cdot\epsilon_j}{\sigma_i-\sigma_j},\quad \text{for } i\neq j$$

  and $A_{ii} = B_{ii} = 0$, $C_{ii} = \epsilon_i\cdot P(\sigma_i)$.
- For YM, $I^l = Pf'(M)$, $I^r = \prod_i \frac{1}{\sigma_i-\sigma_{i-1}}$.
- For GR $I^l = Pf'(M^l)$, $I^r = Pf'(M^r)$


S_B = ∫ P·∂X − e P²
I^r = I^l = ∏_i (ε_i·P(σ_i))
ε·P ε K = Ω^{1,2}_Σ
δθ = (ε·P)² δ̄ (K·P(σ)) e^{2πiK·X}
V_l V_r
S^l ⟷ S^r
I_l = ⟨V^l_1 ... V^l_n⟩
I_r = ⟨V^r_1 ... V^r_n⟩


$S_B = \int P \cdot \partial X - e P^2$
$A_\mu = \epsilon_\mu e^{2\pi i k \cdot x}$
$I^r = I^l = \prod_i (\epsilon_i \cdot P(\sigma_i))$
$\epsilon \cdot P \quad \epsilon \quad K = \Omega^{1,2}_{\Sigma}$
$\delta\theta = (\epsilon \cdot P)^2 \bar{\delta}(k \cdot P(\sigma)) e^{2\pi i k \cdot x}$
$V_l \quad V_r$
$S^l \leftrightarrow V_l \quad V_r \leftrightarrow S^r$
$I_l = \langle V_1^l \dots V_n^l \rangle$
$I_r = \langle V_1^r \dots V_n^r \rangle$

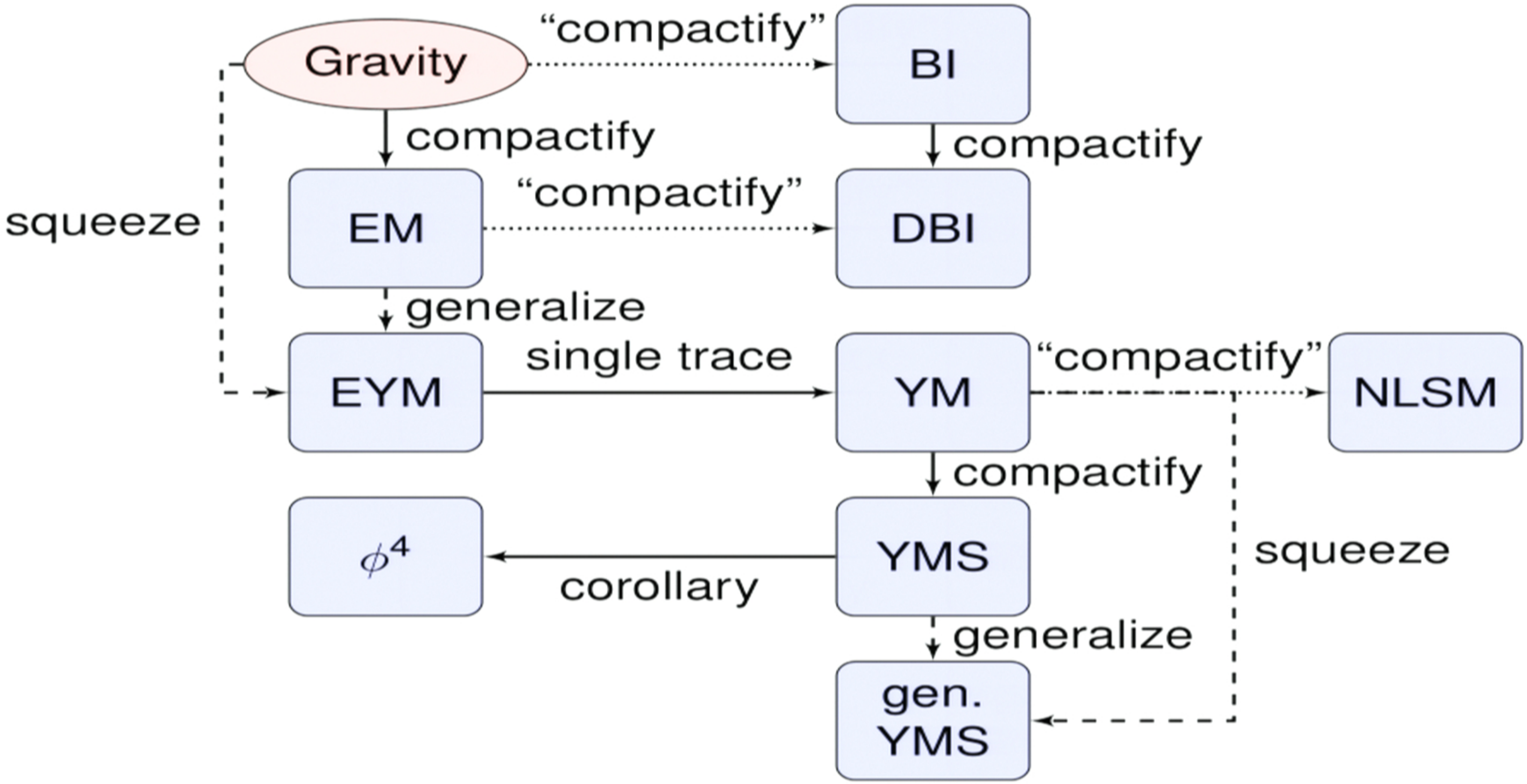


Figure: Theories studied by CHY and operations relating them.

Spinning light rays and worldsheet SUSY, S_Ψ

Let $\Psi^\mu \in K^{1/2}$, spin 1/2 fermions on Σ,

$$S_\Psi = \int g_{\mu\nu}\Psi^\mu \bar\partial \Psi^\nu - \chi P_\mu \Psi^\mu$$

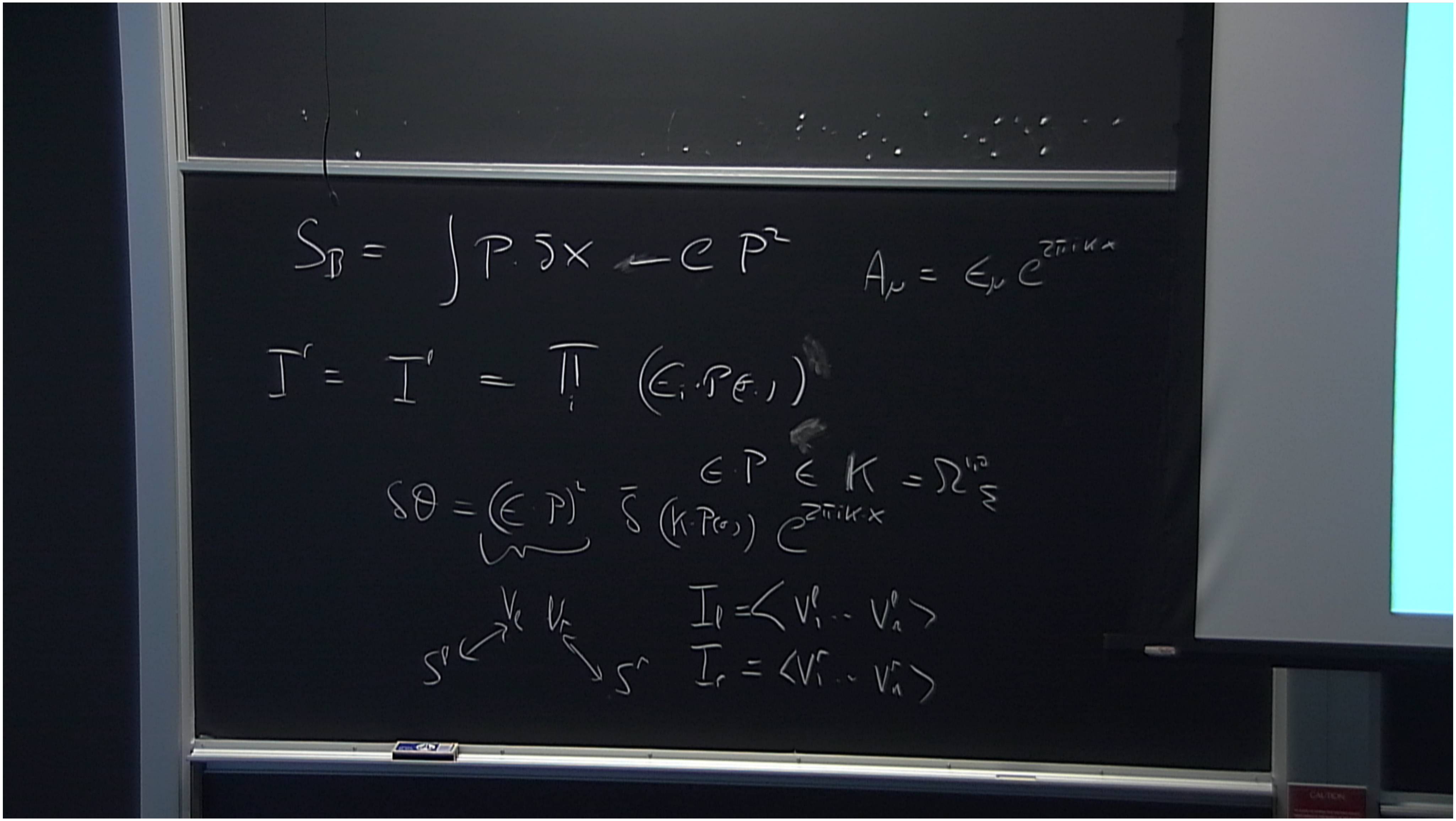
$\chi \rightsquigarrow$ constraints $P \cdot \Psi = 0$ and gauge field for worldline susy

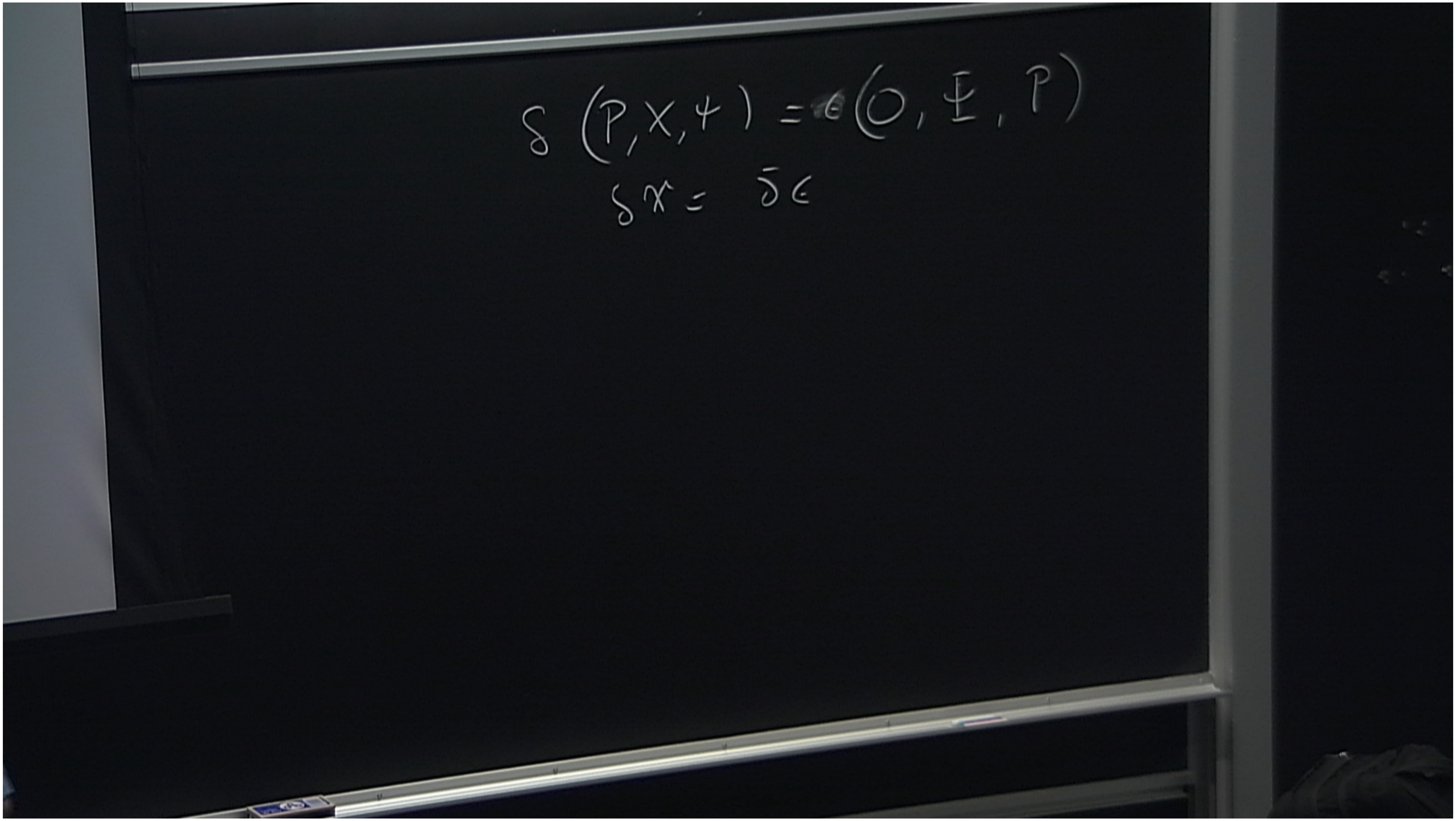
$$D = \Psi \cdot \frac{\partial}{\partial X} + P \cdot \frac{\partial}{\partial \Psi}\,, \qquad \{D, D\} = P \cdot \frac{\partial}{\partial X}\,.$$

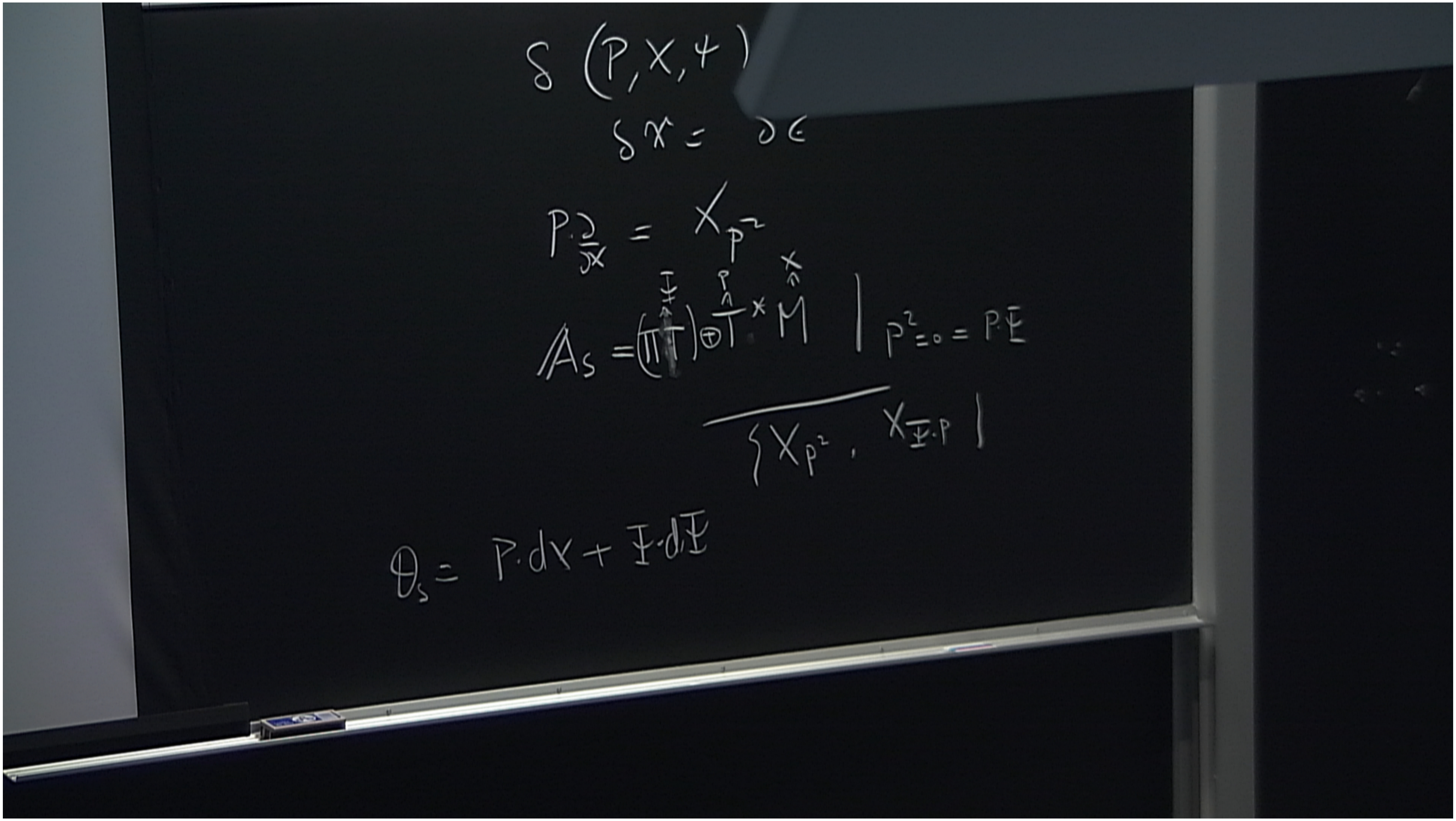
- Gauge fix $\chi = 0 \rightsquigarrow$ bosonic ghosts $(\beta, \gamma) \in (K^{3/2}, T^{1/2})$.
- Extend BRST operator with $Q_\Psi = \int \gamma P \cdot \Psi$.
- For v^l or v^r, must replace $\epsilon \cdot P$ by

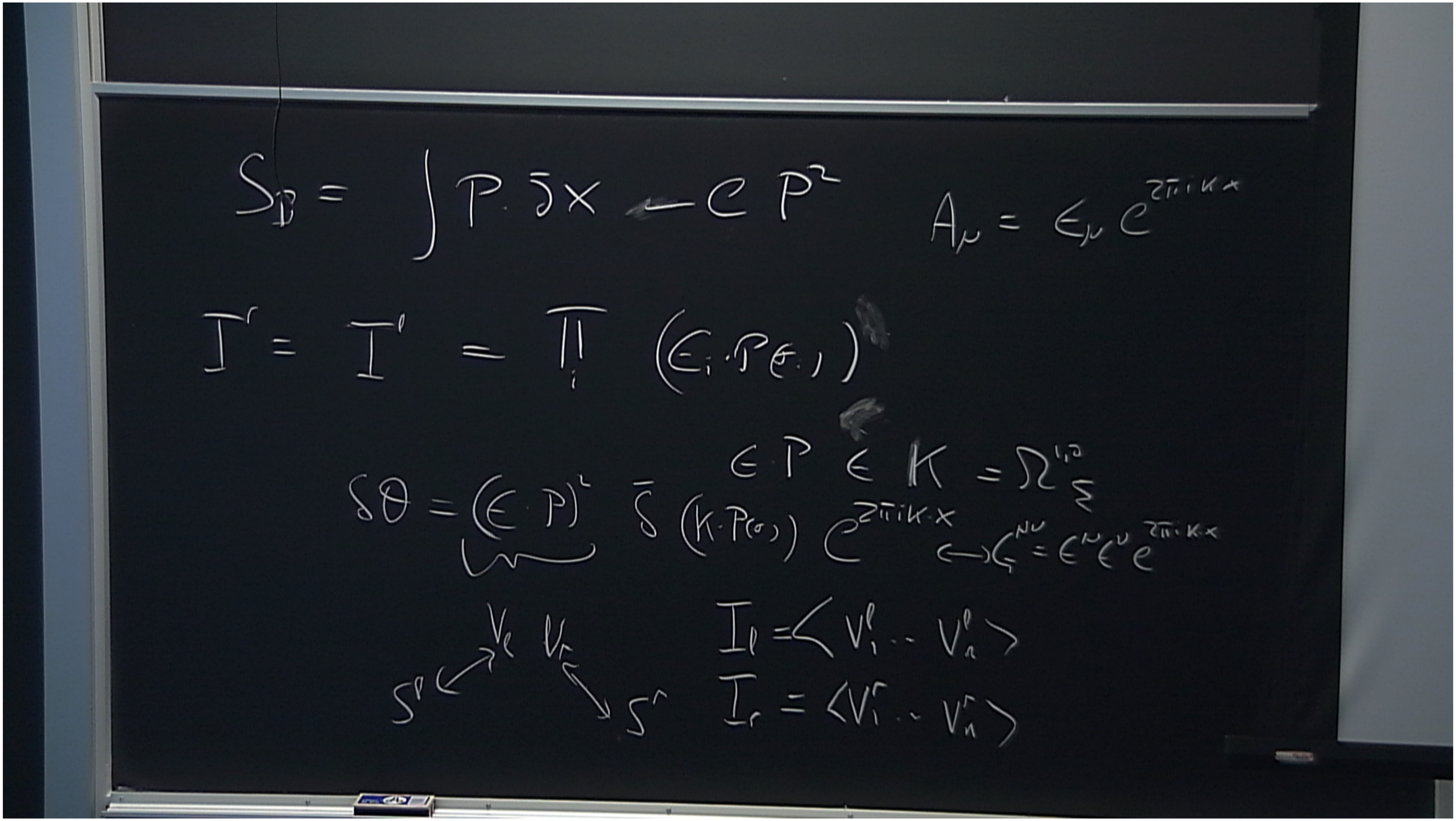
$$u = \delta(\gamma)\epsilon \cdot \Psi \quad \text{(fixed)}, \qquad v = \epsilon \cdot P + \epsilon \cdot \Psi k \cdot \Psi\,.$$

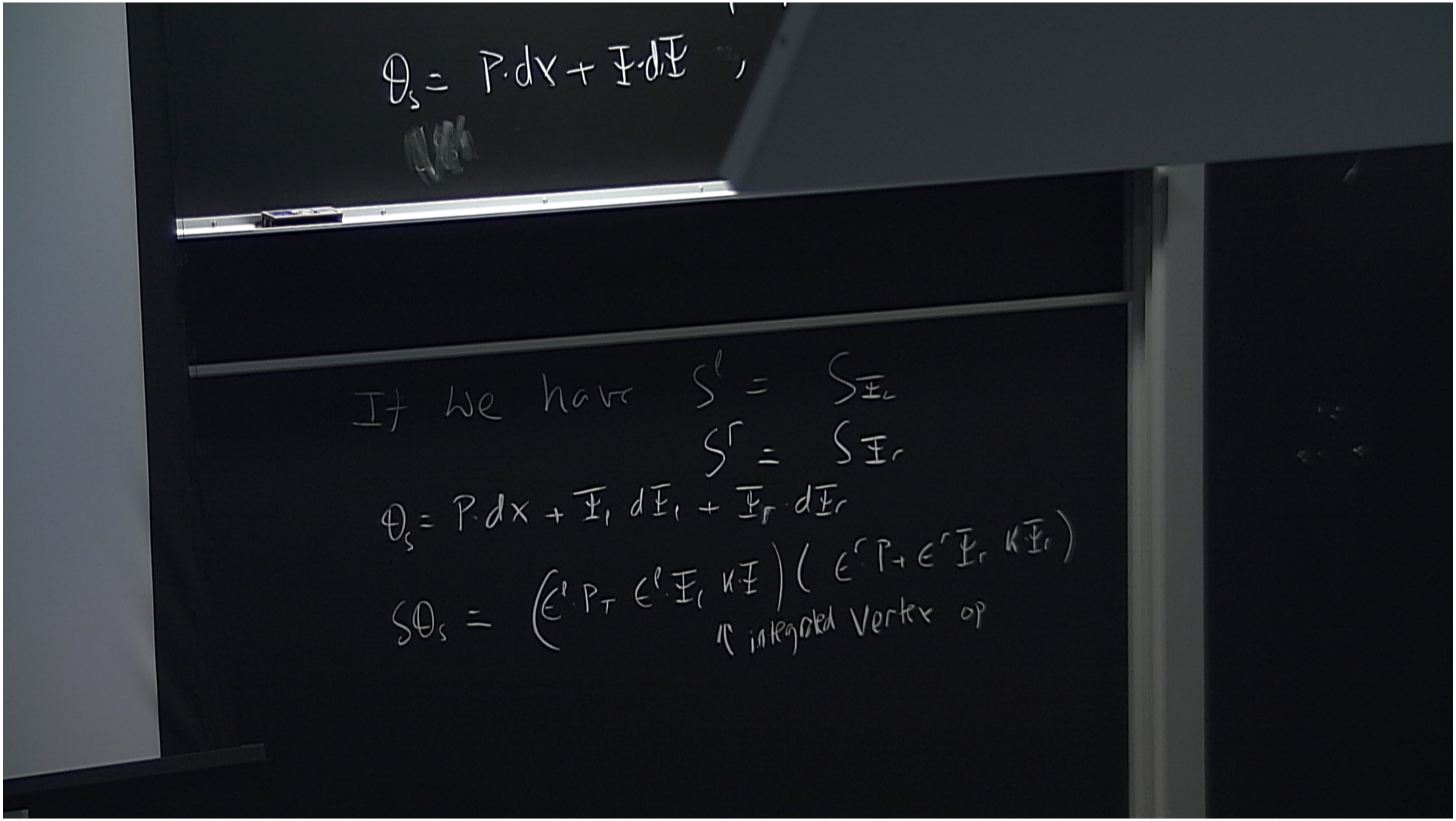
- Worldsheet correlator $\langle u_1 u_2 v_3 \ldots v_n \rangle = Pf'(M)$ for I^l or I^r.
- GSO symmetry $(\Psi, \gamma, \beta) \to (-\Psi, -\gamma, -\beta) \Rightarrow$ need u or v.


S_B = ∫ P · ∂̄x − e P²
A_μ = ε_μ e^{2πik·x}
I' = I' = ∏_i (ε_i · P(σ_i))
ε · P ε K = Ω
δθ = (ε · P)² ∂̄ (K · P(σ)) e^{2πik·x}
V_l V_r
S^l S^r
I_l = ⟨V_1^l .. V_n^l⟩
I_r = ⟨V_1^r .. V_n^r⟩


δ (P, X, ψ) = ε (0, Σ, P)
δ x = ∂̄ ε


S (P, X, Ψ)
δx = ∂ε
P·∂/∂x = X_{p²}
A_s =
p² = 0 = P·Ψ
{X_{p²}, X_{Ψ·P}}
θ_s = P·dx + Ψ·dΨ


S_B = ∫ P · ∂X − e P²
A_μ = ε_μ e^{2πi k·x}
I^r = I^l = ∏_i (ε_i · P(σ_i))
ε · P ε K = Ω^{1,0}_Σ
δθ = (ε · P)² δ̄ (K · P(σ)) e^{2πi k·x} ⟷ ζ^{μν} = ε^μ ε^ν e^{2πi k·x}
V_l V_r
S^l S^r
I_l = ⟨V_1^l … V_n^l⟩
I_r = ⟨V_1^r … V_n^r⟩


Θs = P·dx + Ψ·dΨ
If we have S^l = S_Ψl
S^r = S_Ψr
Θs = P·dx + Ψl dΨl + Ψr·dΨr
δΘs = (ε^l·P + ε^l·Ψl κΨ)(ε^r P + ε^r Ψr κΨr)
integrated Vertex op

Spinning light rays and worldsheet SUSY, S_Ψ

Let $\Psi^\mu \in K^{1/2}$, spin 1/2 fermions on Σ,

$$S_\Psi = \int g_{\mu\nu}\Psi^\mu \bar\partial \Psi^\nu - \chi P_\mu \Psi^\mu$$

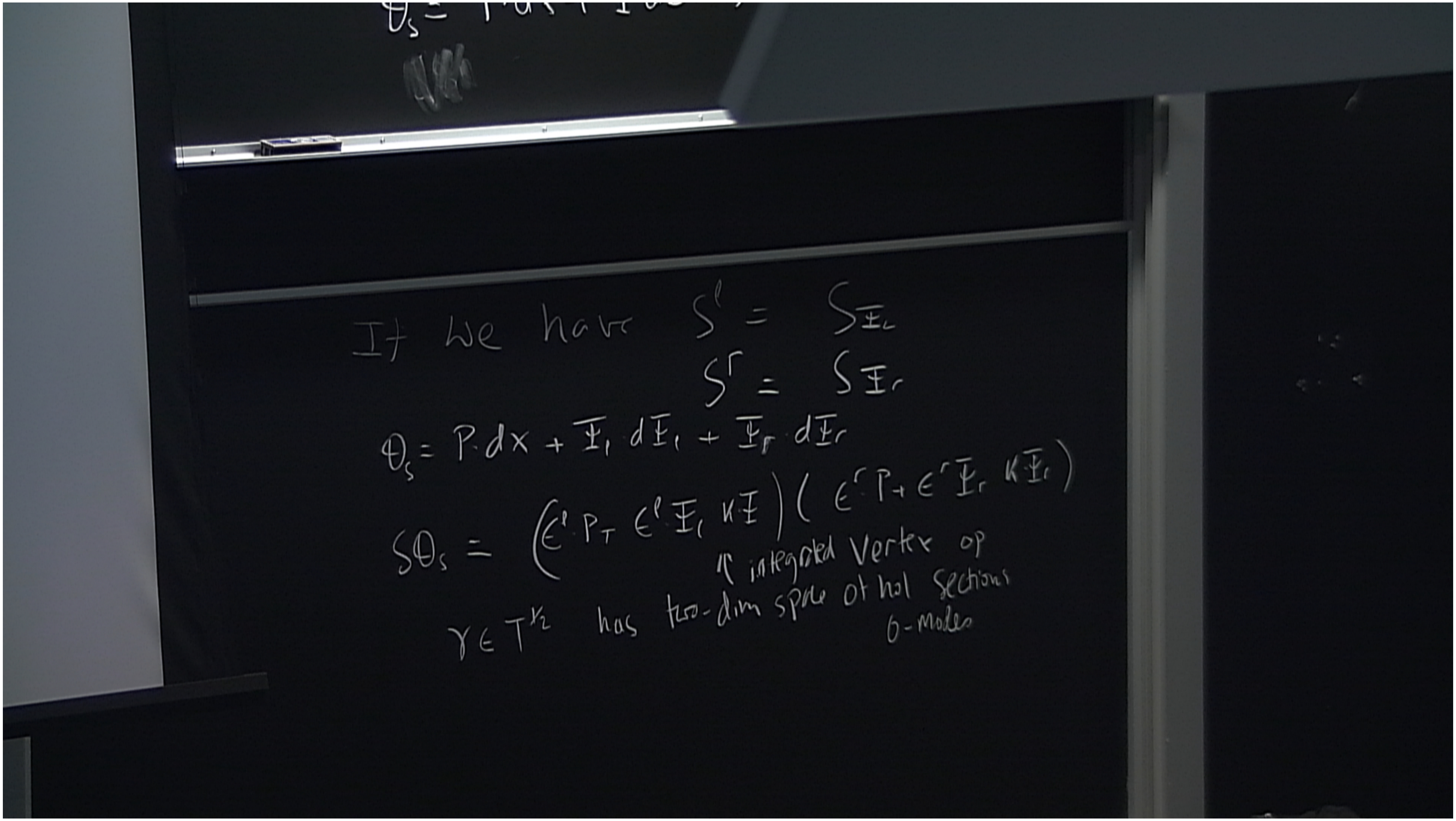
$\chi \rightsquigarrow$ constraints $P \cdot \Psi = 0$ and gauge field for worldline susy

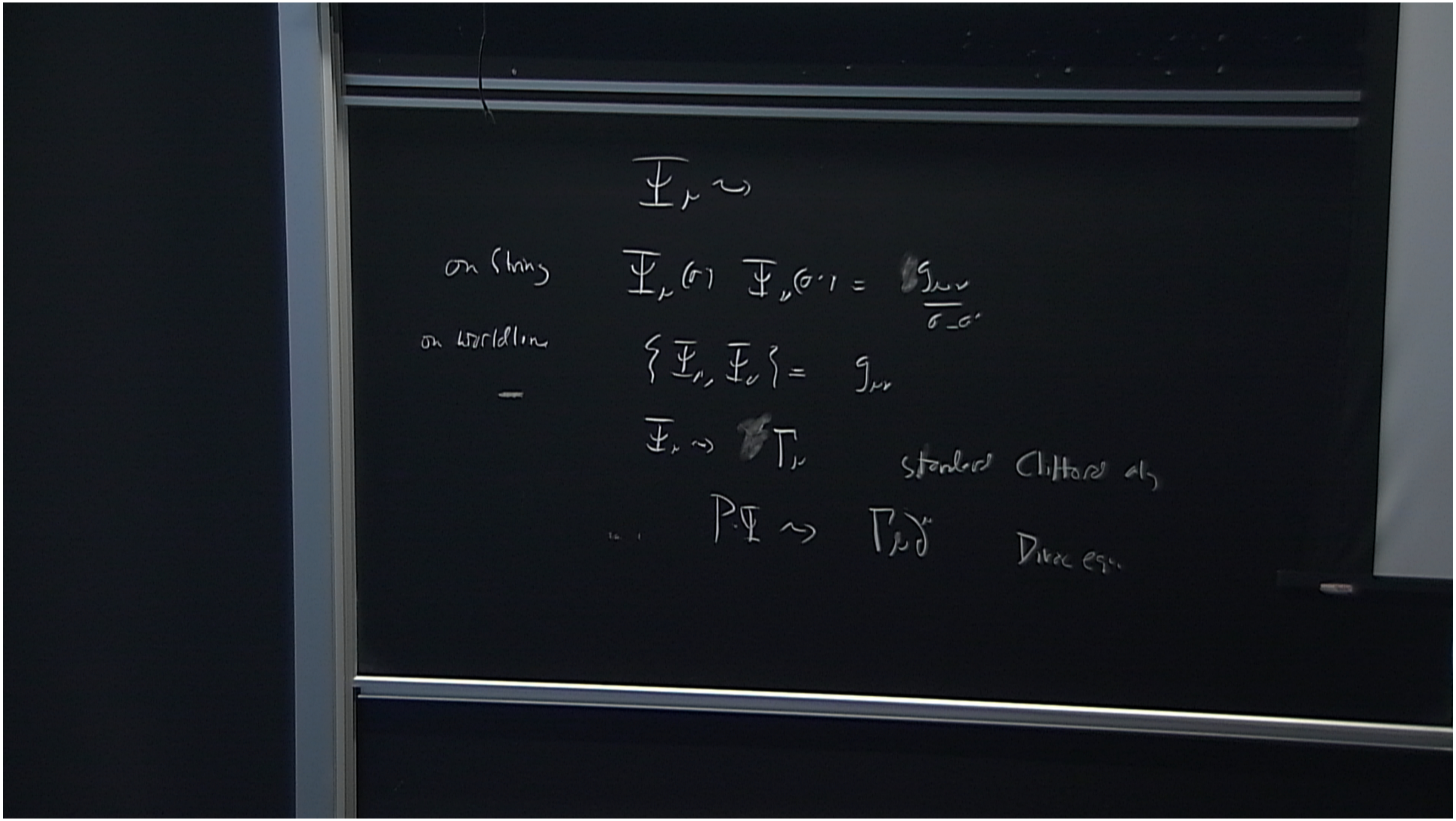
$$D = \Psi \cdot \frac{\partial}{\partial X} + P \cdot \frac{\partial}{\partial \Psi}, \qquad \{D, D\} = P \cdot \frac{\partial}{\partial X}.$$

- Gauge fix $\chi = 0 \rightsquigarrow$ bosonic ghosts $(\beta, \gamma) \in (K^{3/2}, T^{1/2})$.
- Extend BRST operator with $Q_\Psi = \int \gamma P \cdot \Psi$.
- For v^l or v^r, must replace $\epsilon \cdot P$ by

$$u = \delta(\gamma)\epsilon \cdot \Psi \quad \text{(fixed)}, \qquad v = \epsilon \cdot P + \epsilon \cdot \Psi k \cdot \Psi.$$

- Worldsheet correlator $\langle u_1 u_2 v_3 \ldots v_n \rangle = Pf'(M)$ for I^l or I^r.
- GSO symmetry $(\Psi, \gamma, \beta) \to (-\Psi, -\gamma, -\beta) \Rightarrow$ need u or v.


If we have $S^l = S_{\Psi_l}$
$S^r = S_{\Psi_r}$
$\Theta_s = P \cdot dx + \Psi_l \, d\Psi_l + \Psi_r \cdot d\Psi_r$
$S\Theta_s = \left(\epsilon^l \cdot P + \epsilon^l \Psi_l \, K\Psi\right)\left(\epsilon^r P + \epsilon^r \Psi_r \, K\Psi_r\right)$
integrated Vertex op
$\gamma \in T^{1/2}$ has two-dim space of hol sections
0-modes


on string
on worldline
standard Clifford alg
Dirac eq.

Amplitude formulae for massless theories.

Theorem (Cachazo, He, Yuan 2013,2014)

Tree-level massless amplitudes in d-dims are integrals/sums

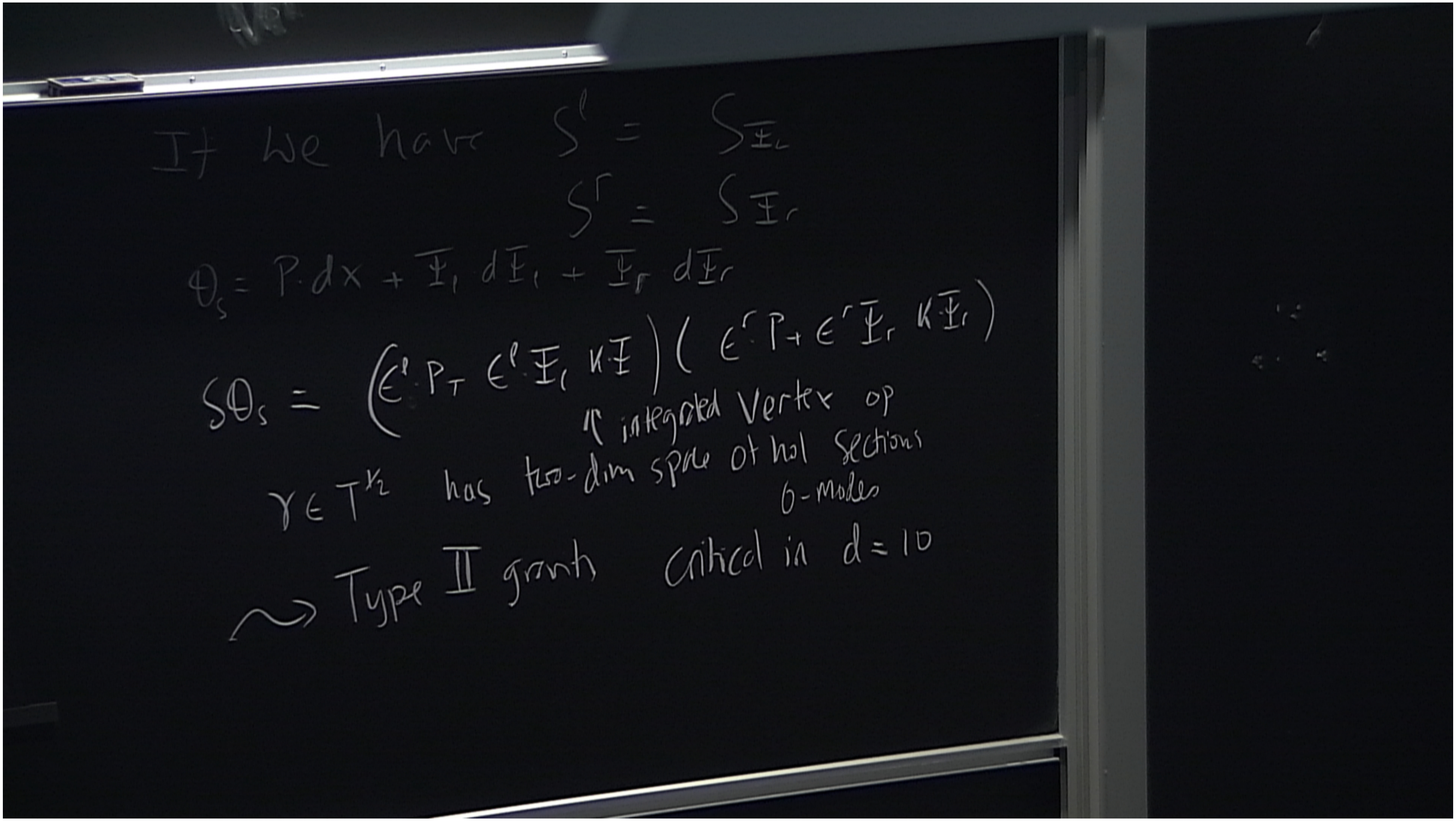
$$\mathcal{M}(1,\ldots,n) = \delta^d\left(\sum_i k_i\right)\int_{(\mathbb{CP}^1)^n} \frac{I^l I^r \prod_i \bar{\delta}(k_i\cdot P(\sigma_i))}{\text{Vol SL}(2,\mathbb{C})\times\mathbb{C}^3}$$

where $I^{l/r} = I^{l/r}(\epsilon_i^{l/r}, k_i, \sigma_i)$ *depend on the theory.*

- polarizations ϵ^l_μ for spin 1, $\epsilon^l_\mu\otimes\epsilon^r_\nu$ for spin-2, $(k\cdot\epsilon = 0\ldots)$.
- Introduce skew $2n\times 2n$ matrices $M = \begin{pmatrix} A & C \\ -C^t & B\end{pmatrix}$,

$$A_{ij} = \frac{k_i\cdot k_j}{\sigma_i-\sigma_j},, \quad B_{ij} = \frac{\epsilon_i\cdot\epsilon_j}{\sigma_i-\sigma_j}, \quad C_{ij} = \frac{k_i\cdot\epsilon_j}{\sigma_i-\sigma_j}, \quad \text{for } i\neq j$$

  and $A_{ii} = B_{ii} = 0$, $C_{ii} = \epsilon_i\cdot P(\sigma_i)$.
- For YM, $I^l = Pf'(M)$, $I^r = \prod_i \frac{1}{\sigma_i - \sigma_{i-1}}$.
- For GR $I^l = Pf'(M^l)$, $I^r = Pf'(M^r)$


If we have $S^l = S_{\Psi_l}$
$S^r = S_{\Psi_r}$
$\Theta_s = P\cdot dx + \Psi_l\, d\Psi_l + \Psi_r\, d\Psi_r$
$\delta\Theta_s = (\epsilon^l\cdot P + \epsilon^l\cdot\Psi_l\, K\cdot\Psi)(\epsilon^r\cdot P + \epsilon^r\cdot\Psi_r\, K\cdot\Psi_r)$
↑ integrated vertex op
$\gamma \in T^{1/2}$ has two-dim space of hol sections
0-modes
⇝ Type II gravity critical in $d = 10$

Free Fermions and current algebras

- Free 'real' Fermions $\rho^a \in \mathbb{C}^m \otimes K^{1/2}$

$$S_\rho = \int_\Sigma \delta_{ab}\rho^a\bar\partial\rho^b\,, \qquad a = 1, \ldots m,$$

- These generate current algebra e.g., if $\mathfrak{g}$ is Lie algebra with structure constants f^{abc}, $m = \dim\mathfrak{g}$ then

$$j^a = f^{abc}\rho^b\rho^c\,, \qquad j^a(\sigma)j^b(0) = \frac{k\delta^{ab}}{\sigma^2} + \frac{f^{abc}j^c}{\sigma} + \ldots .$$

Here level $k = C$.

- Current algebra gives Vertex ops

$$v = t\cdot j\,.$$

- Correlators give 'Parke-Taylor' factors + unwanted multi-trace terms

$$\langle v_1 \ldots v_n\rangle = \frac{\mathrm{tr}(t_1 \ldots t_n)}{\sigma_{12}\sigma_{23}\cdots\sigma_{n1}} + \ldots$$

where $\sigma_{ij} = \sigma_i - \sigma_j$.

Comb system [Casali-Skinner]

- To avoid multitrace terms, take current algebra level $k = 0$.
- But, then there is no trace $\langle v_1 \ldots v_n \rangle = 0$.
- Gauge fermionic spin 3/2 current to give two fixed vertex operators to end chain of structure contants 'comb'.
- Use fermions $\tilde{\rho}^a, \rho^a \in \mathfrak{g} \otimes K^{1/2}$, bosons $q^a, y^a \in \mathfrak{g} \otimes K^{1/2}$

$$S_{CS} = \int_\Sigma \tilde{\rho}_a \bar{\partial} \rho^a + q_a \bar{\partial} y^a + \chi \, \mathrm{tr} \rho \left(\frac{[\tilde{\rho}, \rho]}{2} + [q, y] \right) .$$

- Gauge fix $\chi = 0 \rightsquigarrow$ ghosts (β, γ)
- Gives vertex operators

$$u = \delta(\gamma) t \cdot \rho \quad \tilde{u} = \delta(\gamma) t \cdot \tilde{\rho} , \qquad v = t \cdot [\rho, \rho] , \quad \tilde{v} = [\tilde{\rho}, \rho] + [q, y] .$$

- To be nontrivial, correlator must have just one untilded VO

$$\langle u_1 \tilde{u}_2 \tilde{v}_3 \ldots \tilde{v}_n \rangle = \mathcal{C}(1, \ldots, n) := \frac{\mathrm{tr}(t_1 [t_2, [t_3, \ldots [t_{n-1}, t_n] \ldots])}{\sigma_{12} \cdots \sigma_{n1}} .$$

Comb system [Casali-Skinner]

- To avoid multitrace terms, take current algebra level $k = 0$.
- But, then there is no trace $\langle v_1 \ldots v_n \rangle = 0$.
- Gauge fermionic spin 3/2 current to give two fixed vertex operators to end chain of structure contants 'comb'.
- Use fermions $\tilde{\rho}^a, \rho^a \in \mathfrak{g} \otimes K^{1/2}$, bosons $q^a, y^a \in \mathfrak{g} \otimes K^{1/2}$

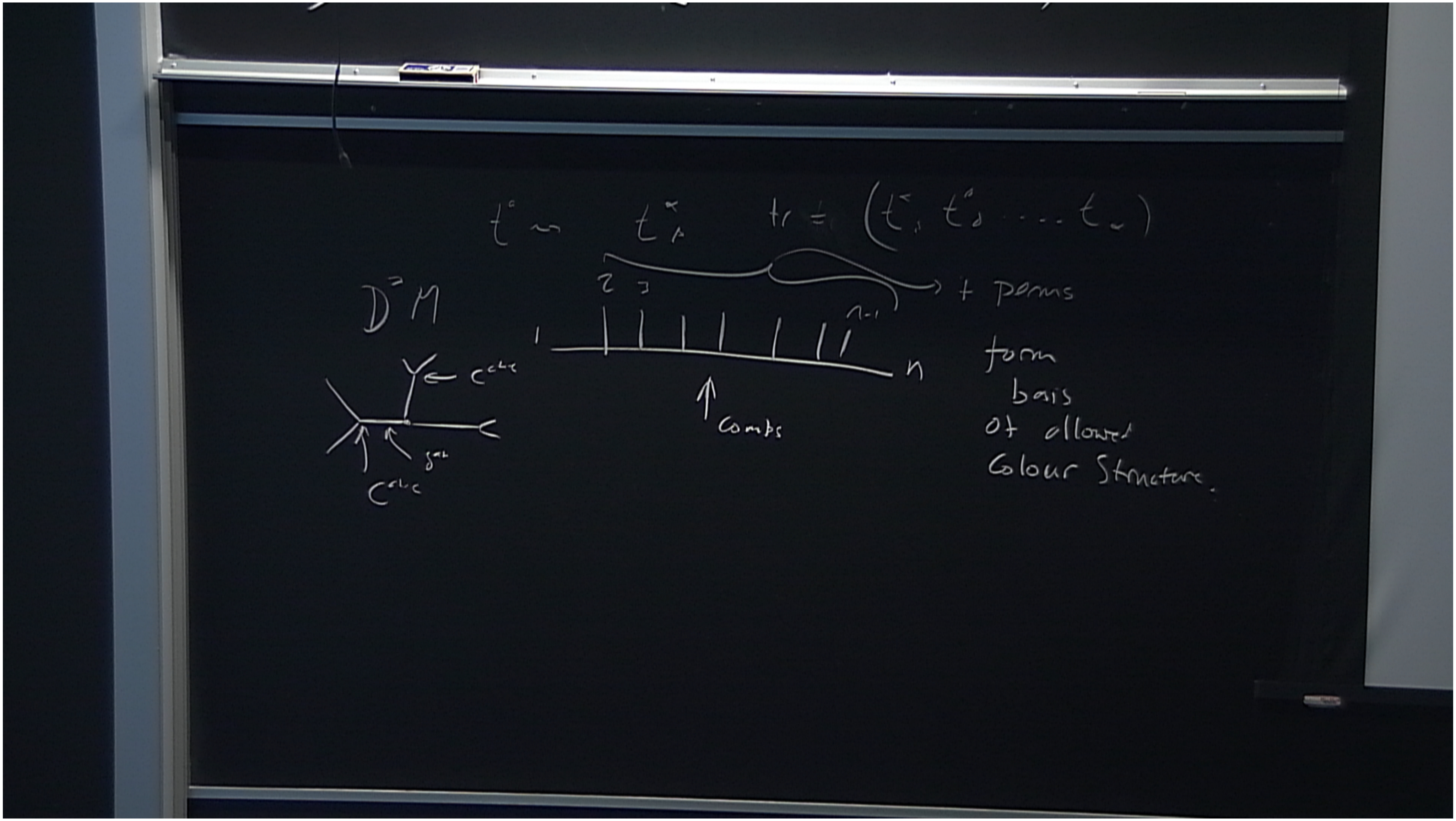
$$S_{CS} = \int_\Sigma \tilde{\rho}_a \bar{\partial} \rho^a + q_a \bar{\partial} y^a + \chi \,\mathrm{tr} \rho \left(\frac{[\tilde{\rho}, \rho]}{2} + [q, y] \right) .$$

- Gauge fix $\chi = 0 \rightsquigarrow$ ghosts (β, γ)
- Gives vertex operators

$$u = \delta(\gamma) t \cdot \rho \qquad \tilde{u} = \delta(\gamma) t \cdot \tilde{\rho} , \qquad v = t \cdot [\rho, \rho] , \qquad \tilde{v} = [\tilde{\rho}, \rho] + [q, y] .$$

- To be nontrivial, correlator must have just one untilded VO

$$\langle u_1 \tilde{u}_2 \tilde{v}_3 \ldots \tilde{v}_n \rangle = \mathcal{C}(1, \ldots, n) := \frac{\mathrm{tr}(t_1 [t_2, [t_3, \ldots [t_{n-1}, t_n] \ldots])}{\sigma_{12} \cdots \sigma_{n1}} .$$


+ perms
form
basis
of allowed
Colour Structure.
combs

The 2013 CHY formulae & ambitwistor models

Above lead essentially to original models & formulae:

- $(S^l, S^r) = (S_{\tilde{\Psi}}, S_{\Psi}) \rightsquigarrow$ type II gravity,
- $(S^l, S^r) = (S_{CS}, S_{\Psi}) \rightsquigarrow$ heterotic with YM,
- $(S^l, S^r) = (S_{CS}, S_{CS}) \rightsquigarrow$ bi-adjoint scalar.

The first two are critical in $d = 10$ (with appropriate gauge group in heterotic case).
The latter two come with unphysical gravity.

S_{CS} improves on current algebras in avoiding multi-trace terms.

Combined matter systems

$S_{\Psi_1,\Psi_2} = S_{\Psi_1} + S_{\Psi_2}$ two worldsheet susy's for S^l or S^r. This is maximum. It gives VO currents

$$u = \delta(\gamma_1) k \cdot \Psi_2 \,, \qquad v = k \cdot \Psi_1 k \cdot \Psi_2 \,.$$

$S_{\Psi,\rho} = S_\Psi + S_\rho$ combines 'real' Fermions with susy, $\rightsquigarrow$ VO currents as usual for S_Ψ and

$$u_t = \delta(\gamma) t \cdot \rho \,, \qquad v_t = k \cdot \Psi t \cdot \rho \,.$$

$$S_{\Psi,CS} = \int_\Sigma \Psi \cdot \bar\partial \Psi + \tilde\rho_a \bar\partial \rho^a + q_a \bar\partial y^a + \chi \left(P \cdot \Psi + \operatorname{tr}\rho \left(\frac{[\tilde\rho,\rho]}{2} + [q,y] \right) \right) .$$

With ghosts etc., the VO currents are those for S_Ψ and

$$\begin{aligned} \tilde u_t &= \delta(\gamma) t \cdot \tilde\rho \,, \qquad u_t = \delta(\gamma) t \cdot \rho \,, \\ \tilde v_t &= k \cdot \Psi t \cdot \tilde\rho + t \cdot ([\tilde\rho, \rho] + [q, y]) \,, \quad v_t = k \cdot \Psi t \cdot \rho + t \cdot [\rho, \rho] \,. \end{aligned}$$

GSO now reverses signs of all fields in matter system.

Einstein-T^*Yang-Mills

With $(S^l, S^r) = (S_{\Psi,CS}, S_{\tilde{\Psi}})$ we obtain Einstein-T^*YM.

- Graviton vertex operators now come from

$$u^l u^r = \delta(\gamma)\epsilon\cdot\Psi\,\delta(\tilde{\gamma})\tilde{\epsilon}\cdot\tilde{\Psi}\,, \quad v^l v^r = \epsilon\cdot(P+\Psi k\Psi)\,\tilde{\epsilon}\cdot(P+\Psi k\cdot\Psi)\,.$$

  Worldsheet correlators give $Pf'(M)Pf'(\widetilde{M})$.
- Two types of gluon VOs from $u^l_t u^r, v^l_t v^r$ and $\tilde{u}^l_t u^r, \tilde{v}^l_t v^r$.
- With gluons, worldheet correlator gives sum of

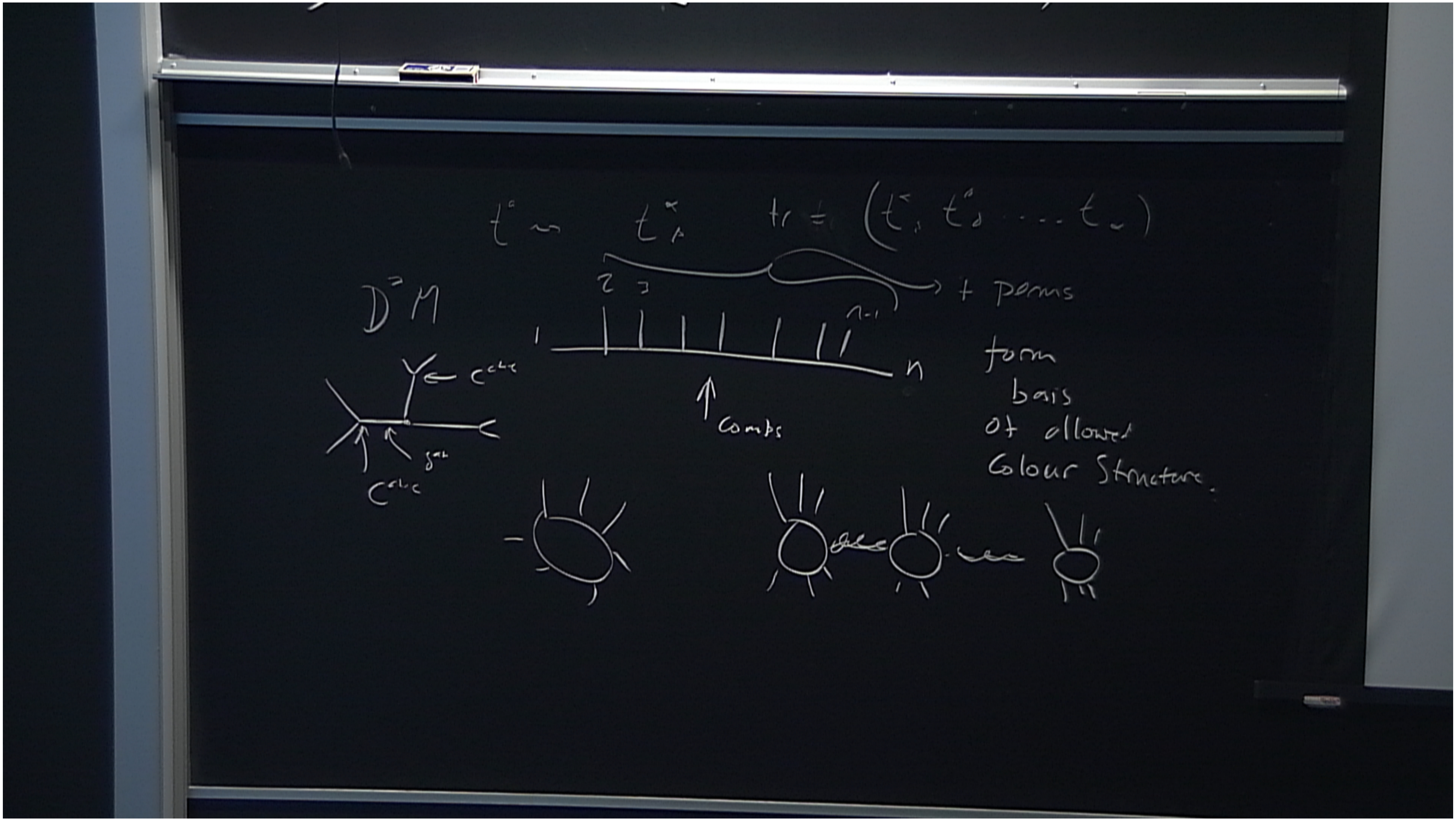
$$\mathcal{C}(T_1)\ldots\mathcal{C}(T_r)Pf'(\Pi)Pf'(\tilde{M})$$

  where $(T_1, \ldots, T_r)$ is a partition of gluons, Π from CHY.

Theory is linear YM on full YM background i.e., T^*YM + gravity

$$\int_M R_{NS}\,\mathrm{dvol} + \mathrm{tr}(A\wedge D_{\tilde{A}}{}^*F_{\tilde{A}})$$

with $A \leftrightarrow u^l_t u^r, v^l_t v^r$, $\tilde{A} \leftrightarrow \tilde{u}^l_t u^r, \tilde{v}^l_t v^r$.
Can replace S_{CS} with anomalous S_{YM} with single gluon type.



+ perms
form basis of allowed Colour Structure.
combs

Ambitwistor strings with combined matter

S^l \ S^r	S_Ψ	S_{Ψ_1,Ψ_2}	$S_{\rho,\Psi}^{(\tilde{m})}$	$S_{CS,\Psi}^{(\tilde{N})}$	$S_{CS}^{(\tilde{N})}$
S_Ψ	E				
S_{Ψ_1,Ψ_2}	BI	Galileon			
$S_{\rho,\Psi}^{(m)}$	*EM* $\mathrm{U}(1)^m$	DBI	*EMS* $\mathrm{U}(1)^m \times \mathrm{U}(1)^{\tilde{m}}$		
$S_{CS,\Psi}^{(N)}$	EYM	ext. DBI	*EYMS* $\mathrm{SU}(N) \times \mathrm{U}(1)^{\tilde{m}}$	*EYMS* $\mathrm{SU}(N) \times \mathrm{SU}(\tilde{N})$	
$S_{CS}^{(N)}$	YM	Nonlinear σ	*EYMS* $\mathrm{SU}(N) \times \mathrm{U}(1)^{\tilde{m}}$	*gen. YMS* $\mathrm{SU}(N) \times \mathrm{SU}(\tilde{N})$	*Biadjoint Scalar* $\mathrm{SU}(N) \times \mathrm{SU}(\tilde{N})$

Table: Theories arising from the different choices of matter models.

EM-EYM-ambi.pdf × ambi-gap.pdf ×

Page: 18 (20 / 30) Find:

Theories	Integrated vertex operators	Central charge c	Lagrangian
E	$V_h = (\epsilon \cdot P + k \cdot \Psi \epsilon \cdot \Psi)\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$	$3(d-10)$	
EM	V_h, V_γ $V_\gamma = (k \cdot \Psi\, t \cdot \rho)\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$	$3(d - 10 + \frac{m}{6})$	
EMS	$V_h, V_\gamma, V_{\tilde{\gamma}}, V_S$ $V_S = (k \cdot \Psi\, t \cdot \rho)\left(k \cdot \tilde{\Psi} \tilde{\rho} \cdot t\right) e^{ik \cdot X}$	$3(d - 10 + \frac{m+\tilde{m}}{6})$	
BI	$V_{BI} = (k \cdot \Psi_1 k \cdot \Psi_2)\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$	$\frac{1}{2}(7d - 38)$	$\mathcal{L} = \ell^{-2}\left(\sqrt{-\det(\eta_{\mu\nu} - \ell^2 F_{\mu\nu})} - 1\right)$
Galileon	$V_G = (k \cdot \Psi_1 k \cdot \Psi_2)\left(k \cdot \tilde{\Psi}_1 k \cdot \tilde{\Psi}_2\right) e^{ik \cdot X}$	$4d - 8$	$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi + \sum_{m=3}^{\infty} g_m \phi \det\{\partial^{\mu_i} \partial_{\nu_j} \phi\}_{i,j=1}^{m-1}$
DBI	$V_{BI}, V_{S_{BI}}$ $V_{S_{BI}} = (k \cdot \Psi_1 k \cdot \Psi_2)\left(k \cdot \tilde{\Psi}\, t \cdot \tilde{\rho}\right) e^{ik \cdot X}$	$\frac{1}{2}(7d + m - 38)$	$\mathcal{L} = \ell^{-2}\left(\sqrt{-\det(\eta_{\mu\nu} - \ell^2 \partial_\mu \phi^a \partial_\nu \phi^a - \ell F_{\mu\nu})} - 1\right)$
"little" YM	$V_g = (\frac{1}{2} t \cdot [\rho, \rho])\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$ $V_{\tilde{g}} = (t \cdot ([\rho, \tilde{\rho}] + [q, y]))\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$	$\frac{5}{2}(d - 12)$	
"little" EYM	$V_h, V_g, V_{\tilde{g}}$ $V_g = (k \cdot \Psi\, t \cdot \rho + \frac{1}{2} t \cdot [\rho, \rho])\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$ $V_{\tilde{g}} = (k \cdot \Psi\, t \cdot \tilde{\rho} + t \cdot ([\rho, \tilde{\rho}] + [q, y]))\left(\tilde{\epsilon} \cdot P + k \cdot \tilde{\Psi} \tilde{\epsilon} \cdot \tilde{\Psi}\right) e^{ik \cdot X}$	$3(d - 10)$	
NLSM	$V = (\frac{1}{2} t \cdot [\rho, \rho])\left(k \cdot \tilde{\Psi}_1 k \cdot \tilde{\Psi}_2\right) e^{ik \cdot X}$ $\tilde{V} = (t \cdot ([\rho, \tilde{\rho}] + [q, y]))\left(k \cdot \tilde{\Psi}_1 k \cdot \tilde{\Psi}_2\right) e^{ik \cdot X}$	$3d - 19$	$\mathcal{L} = -\frac{1}{2} \mathrm{tr}\left((\mathbb{1} - \lambda^2 \Phi)^{-1} \partial_\mu \Phi (\mathbb{1} - \lambda^2 \Phi)^{-1} \partial^\mu \Phi\right)$ $\Phi = \phi^a t^a$

Table 4. Table of the different theories and their integrated vertex operators.

ENG 15:30

Summary & Outlook

Chiral $\alpha' = 0$ ambitwistor strings use LeBrun's correspondence to give theories underlying CHY formulae old & new.

- Incorporates colour/kinematics Yang-Mills/gravity ideas. Any insight into geometry of BCJ?
- Quantization ties scattering of null geodesics into that for gravitational waves.
- Critical models extend to loops .
- Does new representation give new insights into loop integrands?
- Insight into nonperturbative phenomena? Why do we need a Riemann surface to formulate Yang-Mills/Gravity? Curved Backrounds?

The quantum gravity loop integrand

[Adamo, Casali, Skinner 2013, Casali Tourkine 2015 Geyer, M., Monteiro, Tourkine. . .]]

10d type II gravity model is critical so extends to higher genus:

- At genus g, P is a 1-form and acquires dg zero-modes.
- These are the loop momenta for g-loops.
- Standard string technology can be adapted at all g.
- E.g., at 1-loop, $n = 4$, obtain modular invariant sum over spin structures (α, β) for (Ψ_1, Ψ_2) of

$$\mathcal{M}^{(1)}_{\alpha;\beta}(1, \ldots, 4) = \delta^{10}\left(\sum_i k_i\right) \int \mathrm{d}^{10}p \wedge \mathrm{d}\tau \wedge \bar{\delta}\left(P^2(\sigma_1; \tau)\right)$$

$$\prod_{j=2}^{4} d\sigma_j \, \bar{\delta}(k_j \cdot P(\sigma_j)) \, \frac{\vartheta_\alpha(\tau)^4 \vartheta_\beta(\tau)^4}{\eta(\tau)^{24}} \, \mathrm{Pf}(M_\alpha) \, \mathrm{Pf}(\widetilde{M}_\beta)$$

Now checked in many ways and related to standard integrand.

Is power counting better than that from space-time?