

Title: Resurgent analysis and its applications to the Witten Laplacian

Date: May 29, 2015 11:00 AM

URL: <http://pirsa.org/15050065>

Abstract: The first lecture will be devoted to the review of the classical theory of the Witten Laplacian, the second -- to the concepts of resurgent analysis. The third -- to applications of the resurgent analysis to the Witten Laplacian. Time permitting, we will touch upon some foundational questions of resurgent analysis.

Witten Laplacian on the circle.

$$(-\hbar^2 \partial_x^2 + (f')^2 - \hbar f'') \psi(x, \hbar) = \hbar E_r \psi(x, \hbar)$$

$x \in \mathbb{R}/\mathbb{Z}$ , periodic b.c. on  $f$

$f$  generic enough poly (Morse+...)

$$E_r \sim e^{-\text{const}/\hbar}$$

$\hbar$  formal parameter


$$E = \hbar E_r$$

$$E_r$$
  
$$\psi$$

$$E_r \sim \sum_j e^{-c_j/h} (a_0 + a_1 h + \dots)$$

$$\psi(x, h)$$

Resurgent fctn  $\psi(h)$

- sectorial germ 
- of exponential growth as  $h \rightarrow 0$
- 

$\psi(x, h)$   
 $e^{-\text{const}/h}$   
 rmed param  
 $= hE_r \quad E_r = O(h)$

$$E_r \sim \sum_j e^{-s_j/h} (a_0 + a_1 h + \dots)$$

$$\psi(x, h)$$

Resurgent fctn  $\varphi(h)$

• sectorial germ

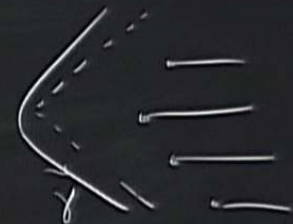


• of exponential growth as  $h \rightarrow 0$

• modulo  $e^{-\infty}$

• s.t.  $\varphi(h) = \int_{\gamma} e^{-s/h} \psi(s) ds$

$\psi$  endlessly an. continuable



CAUTION

CAUTION


$\psi(x, h)$   
 $e^{-\text{const}/h}$   
 small param  
 $= hE_r \quad E_r = O(h)$

$$E_r \sim \sum_0^\infty e^{-cs/h} (a_0 + a_1 h + \dots)$$

$$\psi(x, h) \sim \begin{cases} \log h \\ e^{-1/\sqrt{h}} \end{cases}$$

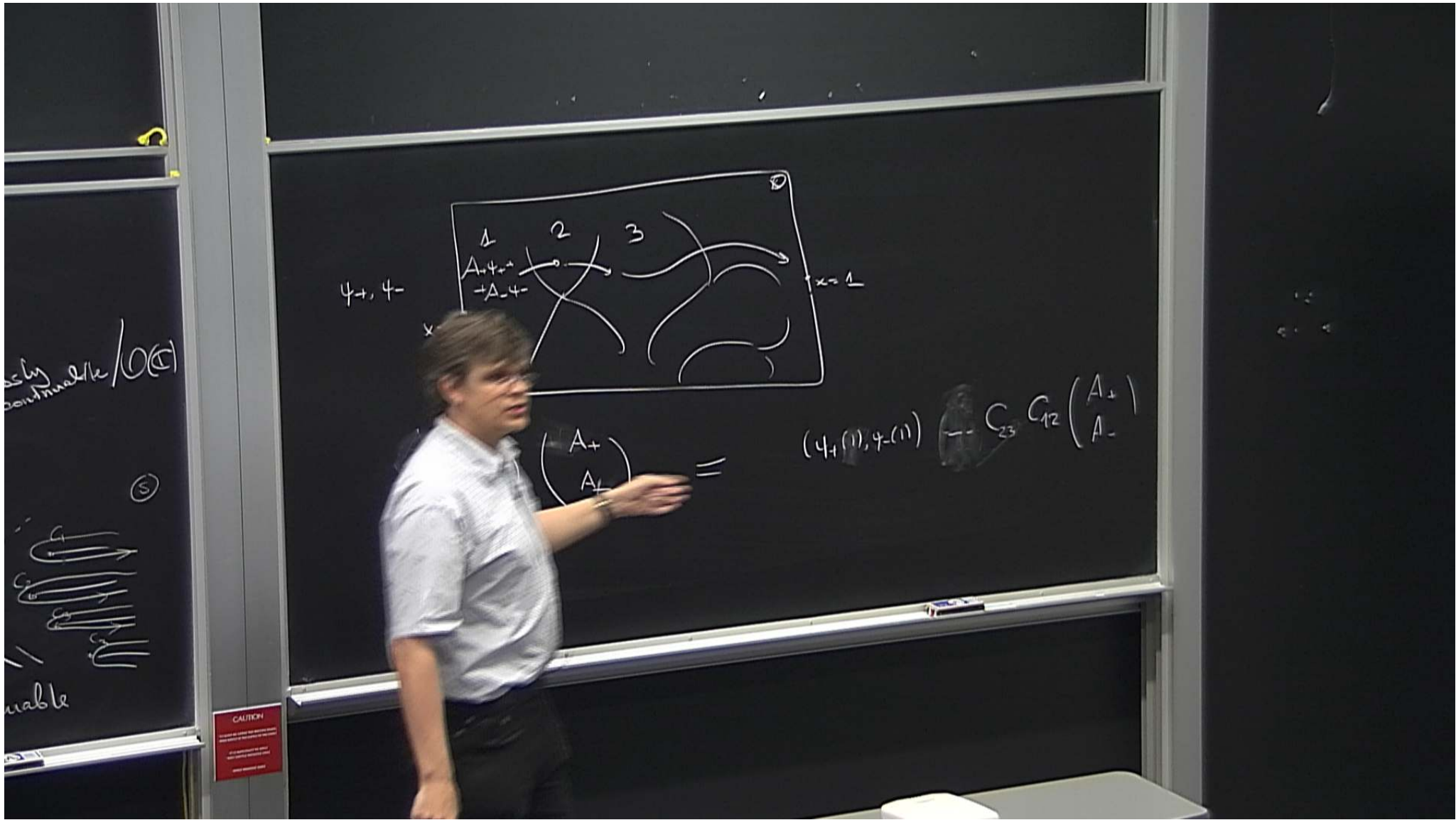
$\psi$  endlessly an. continuable /  $O(h)$

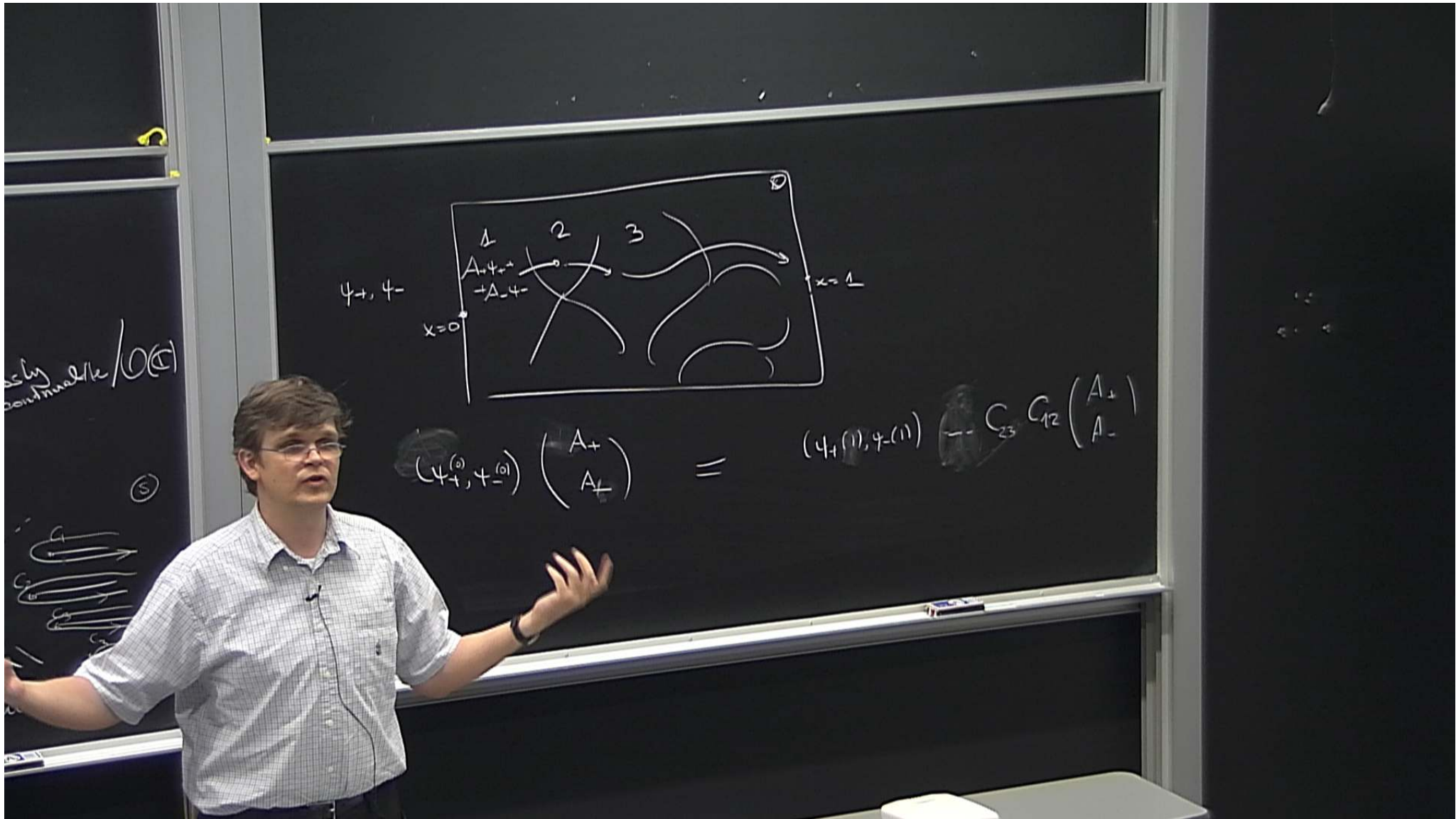
Resurgent fctn  $\varphi(h)$

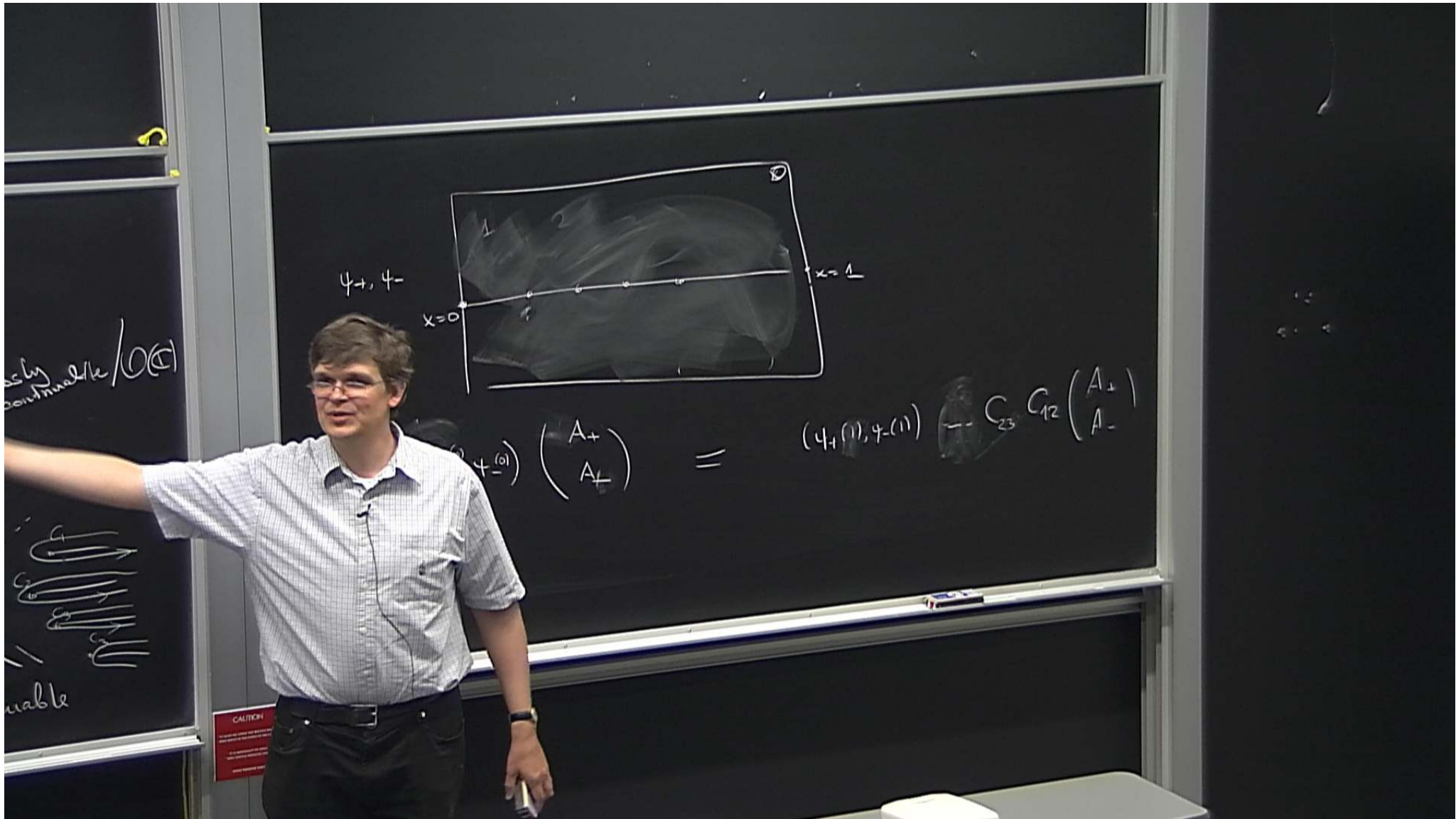
- sectorial germ 
- of exponential growth as  $h \rightarrow 0$
- moduls  $e^{-\infty}$   $e^{-\text{const}/h}$

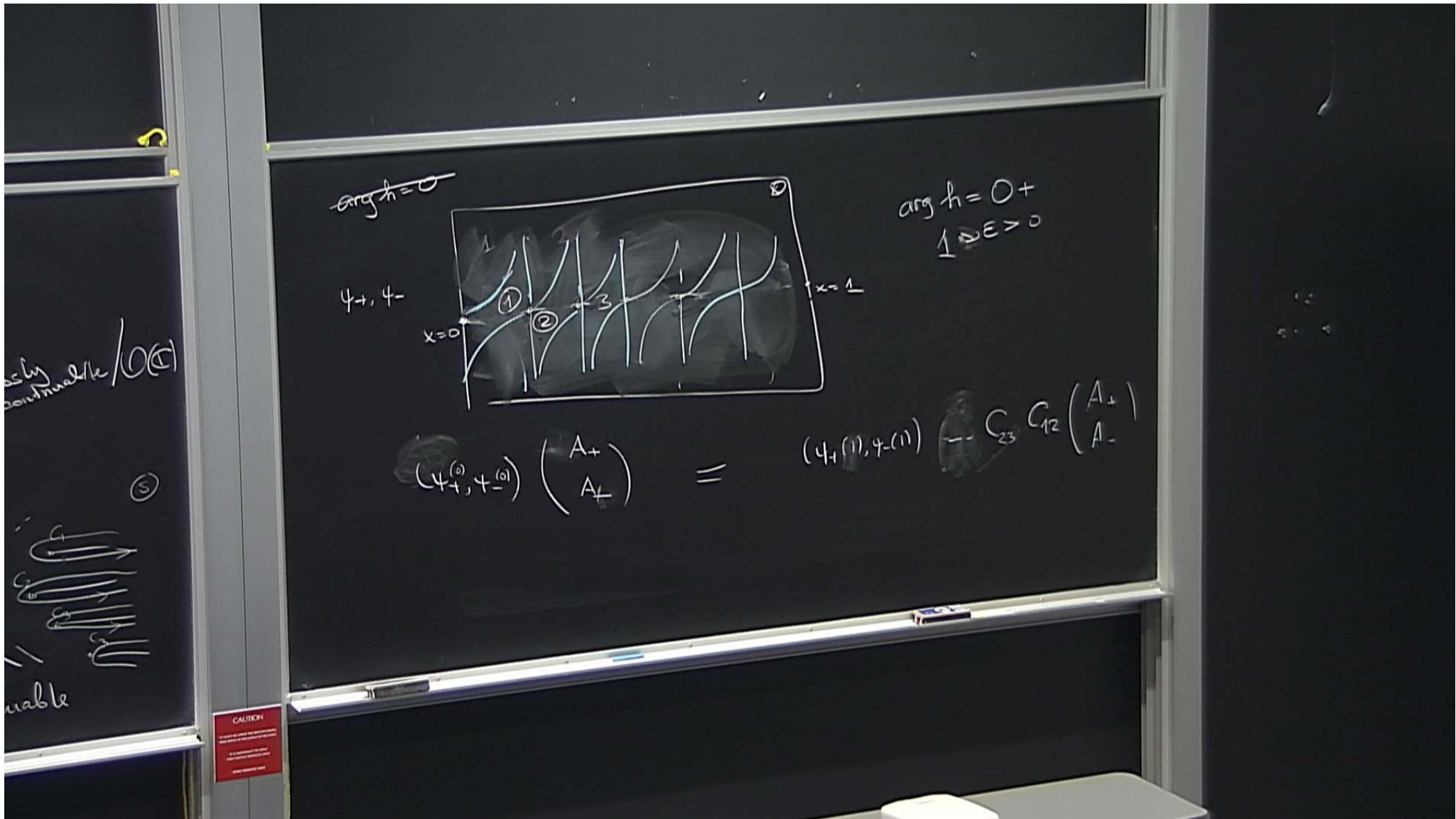
• s.t.h  $\varphi(h) = \int_\gamma e^{-s/h} \psi(s) ds$   $\psi$  endlessly an. continuable





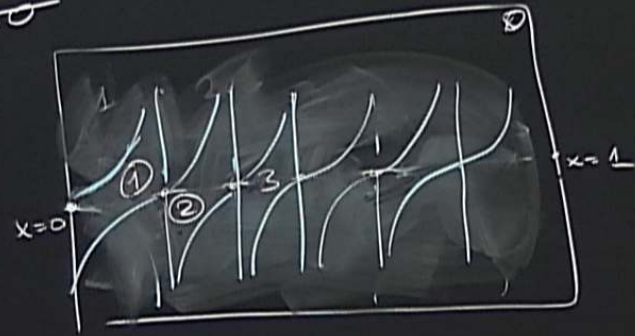






$\arg h = 0$

$\psi_+, \psi_-$



$\arg h = 0+$   
 $\downarrow \Rightarrow E > 0$

$$(\psi_+^{(0)}, \psi_-^{(0)}) \begin{pmatrix} A_+ \\ A_- \end{pmatrix} =$$

$$(\psi_+(1), \psi_-(1)) \dots C_{23} C_{12} \begin{pmatrix} A_+ \\ A_- \end{pmatrix}$$

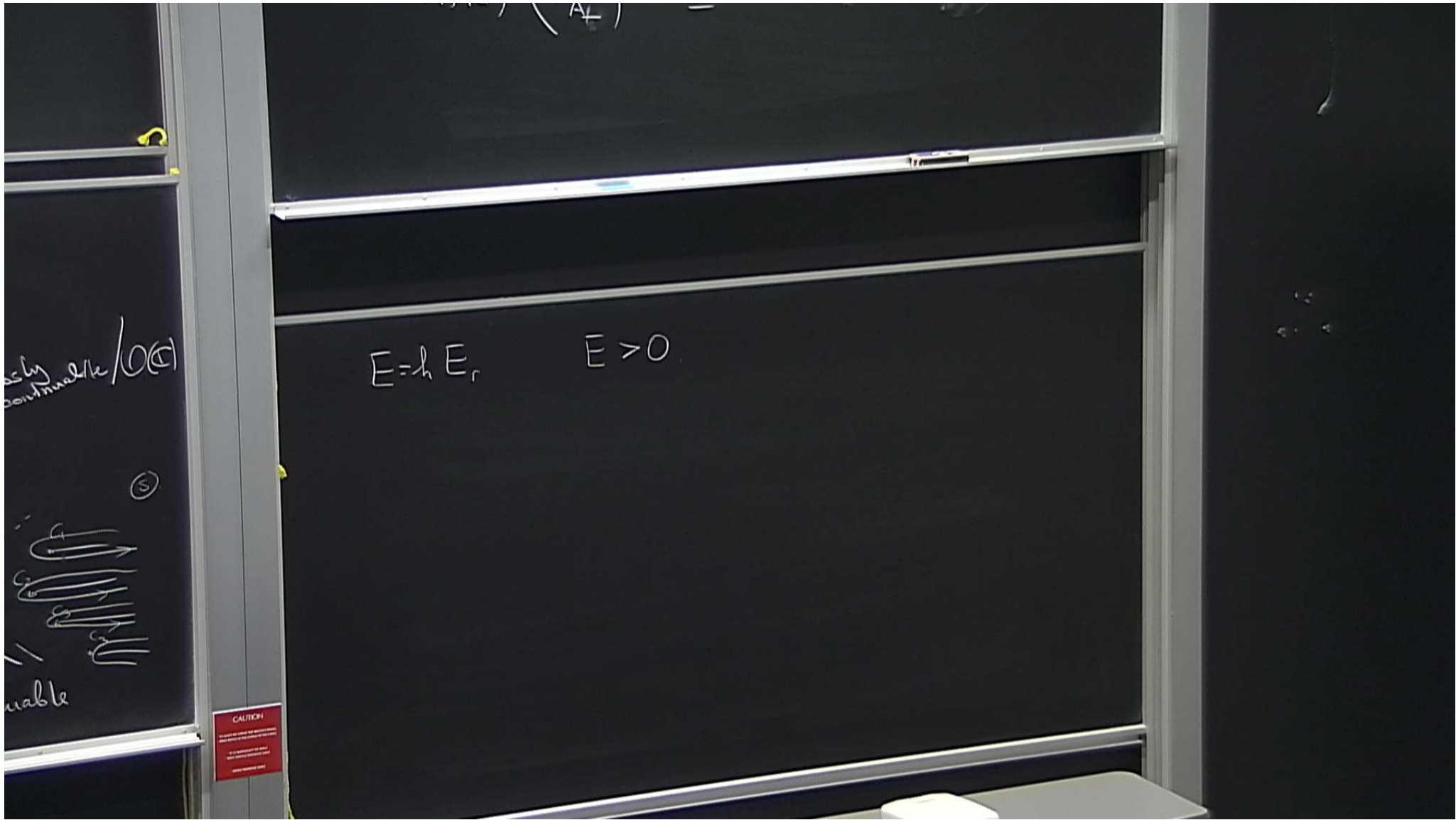
essly  
continua / 100

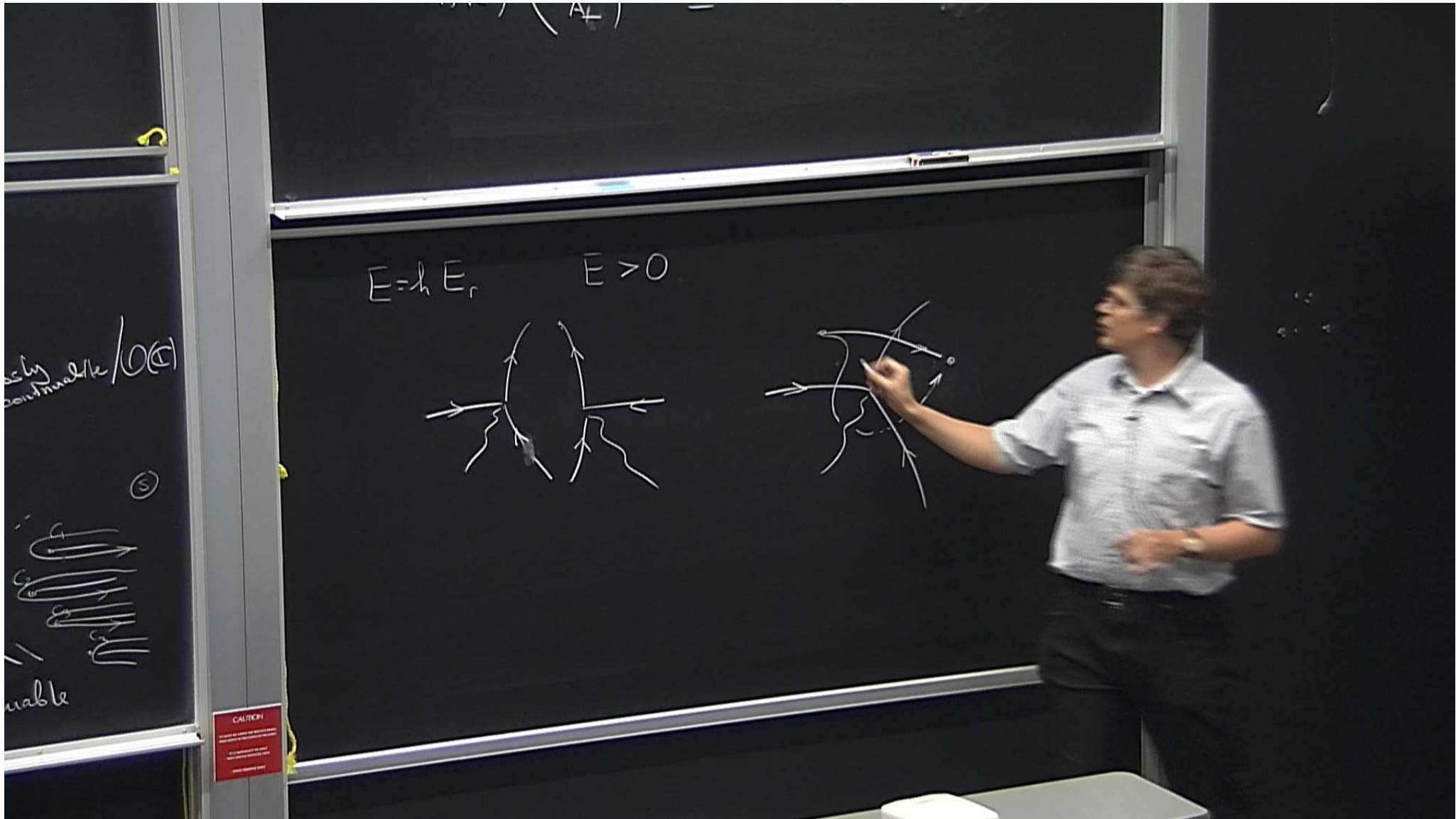


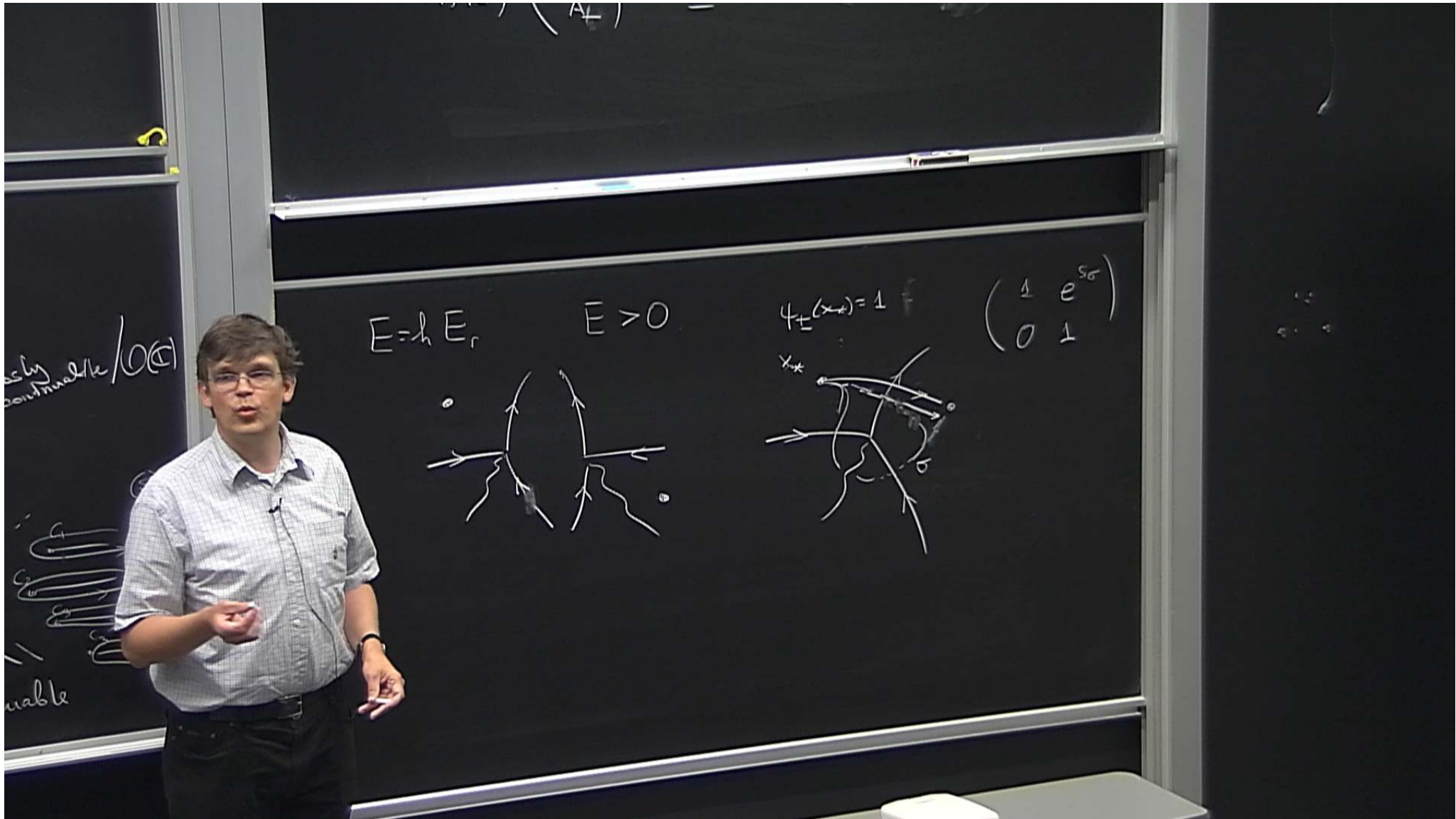
uable

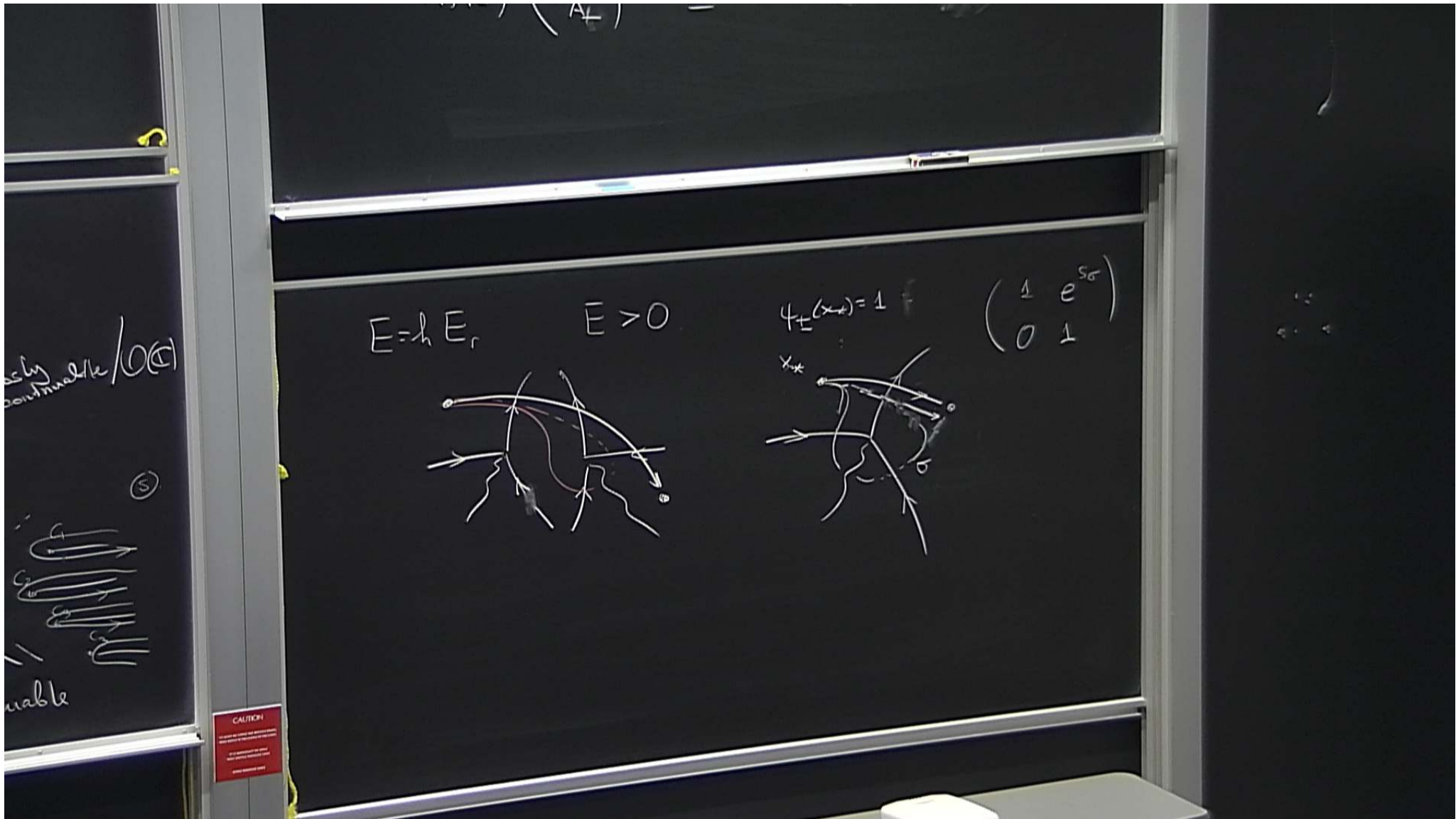
CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD FRAME



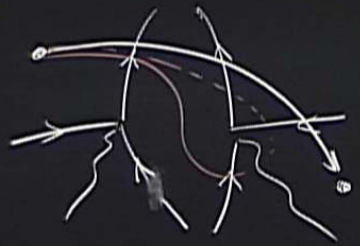








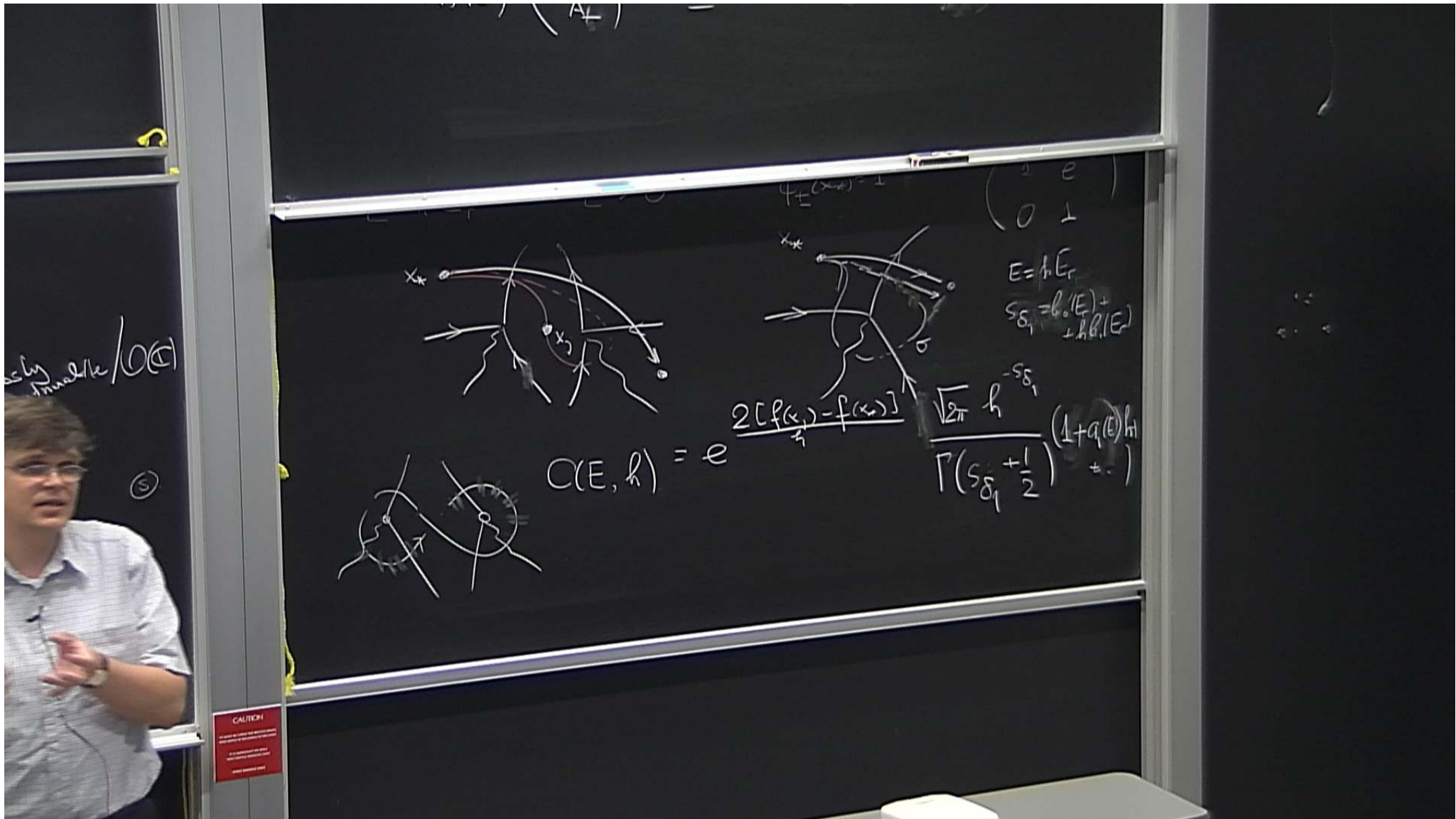
$$E = h E_r \quad E > 0$$



$$\psi_{\pm}(x_{\pm}) = 1 \quad \begin{pmatrix} 1 & e^{s_0} \\ 0 & 1 \end{pmatrix}$$

continua / (0, \infty)  
⑤  
uabile

CAUTION  
ATTENTION  
VERBODEN TOEGANG  
TOEGANG VERBODEN  
ATTENTION  
ATTENTION



continua / 00

(5)

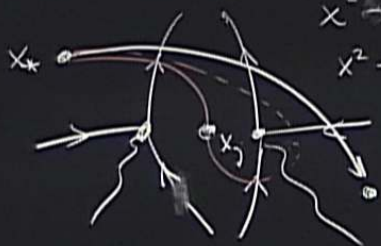


$$E = \hbar E_r$$

$$E > 0$$

$$x^2 = \hbar E_r$$

$$x^2 = E$$



$$\psi_{\pm}(x \pm) = 1$$



$$\begin{pmatrix} 1 & e^{s_0} \\ 0 & 1 \end{pmatrix}$$

$$E = \hbar E_r$$

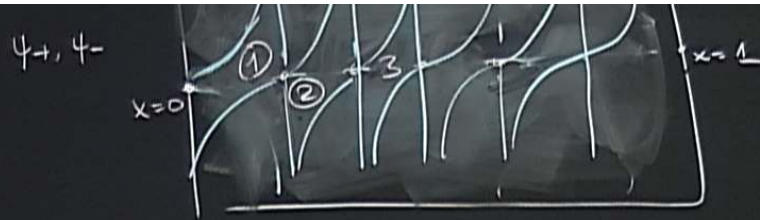
$$s_{\delta_1} = b_0(E) + \frac{1}{\hbar} q(E)$$

$$O(E, \hbar) = e^{\frac{2[f(x_1) - f(x_2)]}{\hbar}}$$

$$\frac{\sqrt{2\pi} \hbar^{-s_{\delta_1}}}{\Gamma(s_{\delta_1} + \frac{1}{2})} (1 + q(E) \hbar)$$



CAUTION



$$(\psi_+^{(0)}, \psi_-^{(0)}) \begin{pmatrix} A_+ \\ A_- \end{pmatrix} = (\psi_+(1), \psi_-(1)) \begin{pmatrix} C_{12} \\ C_{23} \end{pmatrix} \begin{pmatrix} A_+ \\ A_- \end{pmatrix}$$

continuity / O.C.

(5)



uable

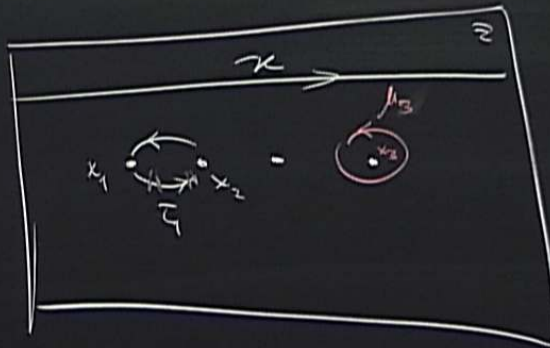
$$C(E, \hbar) = e^{\frac{2[\phi(x_1) - \phi(x_2)]}{\hbar}} \frac{\sqrt{2\pi} \hbar^{-s_{\delta_1}}}{\Gamma(s_{\delta_1} + \frac{1}{2})} (1 + q(E) \hbar^{\pm})$$



CAUTION

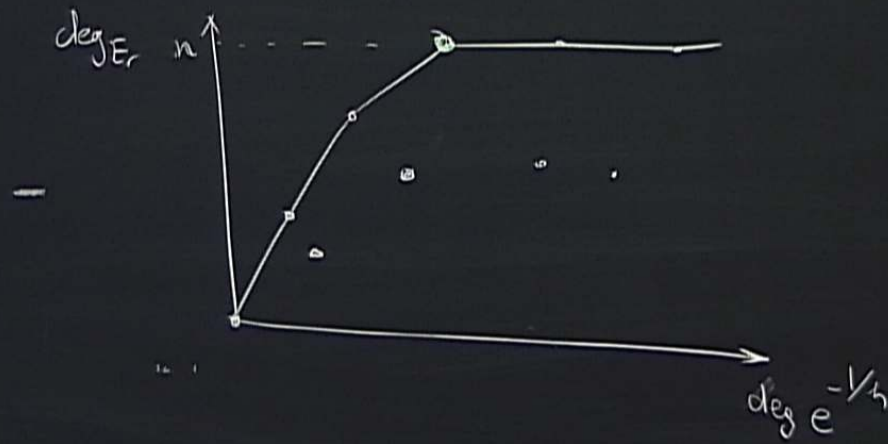
s.t.  $\psi(h) = \int_{\gamma} e^{i\psi} \psi$  endlessly an. continuable

$\psi(x, h)$   
 $- \text{const}/h$   
 $e$   
 nal param.  
 $hE_r \quad E_r = O(h)$



CAUTION  
 ATTENTION  
 ATTENTION

CAUTION  
 ATTENTION  
 ATTENTION

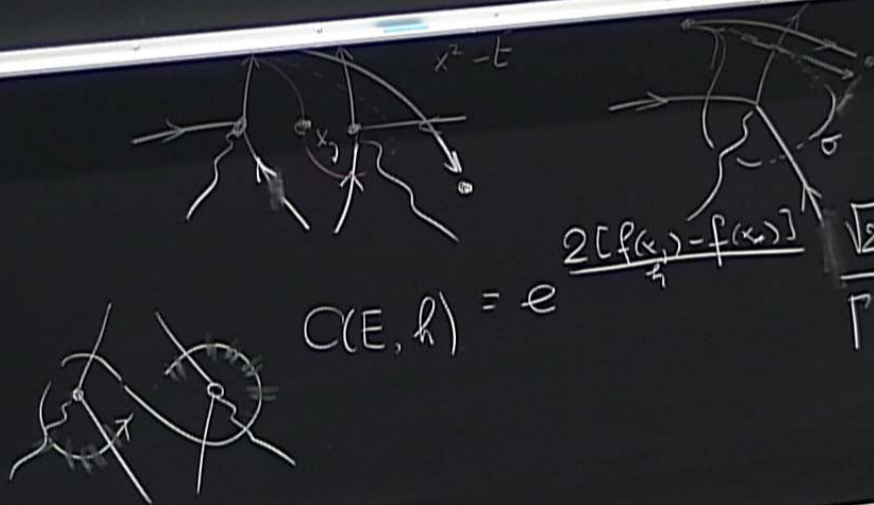


CAUTION  
 DO NOT TOUCH THE SURFACE OF THE BOARD  
 IT IS HEATED BY THE LIGHTS AND MAY BE HOT TO TOUCH  
 PLEASE BE CAREFUL

$$(\psi_{+}^{(0)}, \psi_{-}^{(0)}) \begin{pmatrix} A_{+} \\ A_{-} \end{pmatrix} = (\psi_{+}^{(1)}, \psi_{-}^{(1)}) \dots C_{23} C_{12} \begin{pmatrix} A_{+} \\ A_{-} \end{pmatrix}$$

$$E_r = e^{-k/\hbar} a$$

$$E_r^3 + e^{-2/\hbar} \hbar^2 E_r^2 + e^{-3/\hbar} + e^{-15/\hbar} E_r^3$$



$$C(E, \hbar) = e^{\frac{2[f(x_1) - f(x_2)]}{\hbar}} \frac{\sqrt{2\pi} \hbar^{-s_{\delta_1}}}{\Gamma(s_{\delta_1} + \frac{1}{2})} (1 + q(E) \hbar) \dots$$

$E = \hbar E_r$   
 $s_{\delta_1} = \frac{6.0(E)}{\hbar} + \frac{1}{\hbar} \ln(E)$

CAUTION  
 Do not touch the blackboard  
 as it is very hot and may cause  
 burns. Please use the eraser  
 provided for cleaning.

deg  $e^{-V/h}$

$$(-\hbar^2 \partial_x^2 + Q(x)) \psi = e^{-5/h} \psi$$

$$(-\partial_s^2 \partial_x^2 + Q(x)) \tilde{\psi}(s, x) = \tilde{\psi}(s-5, x)$$