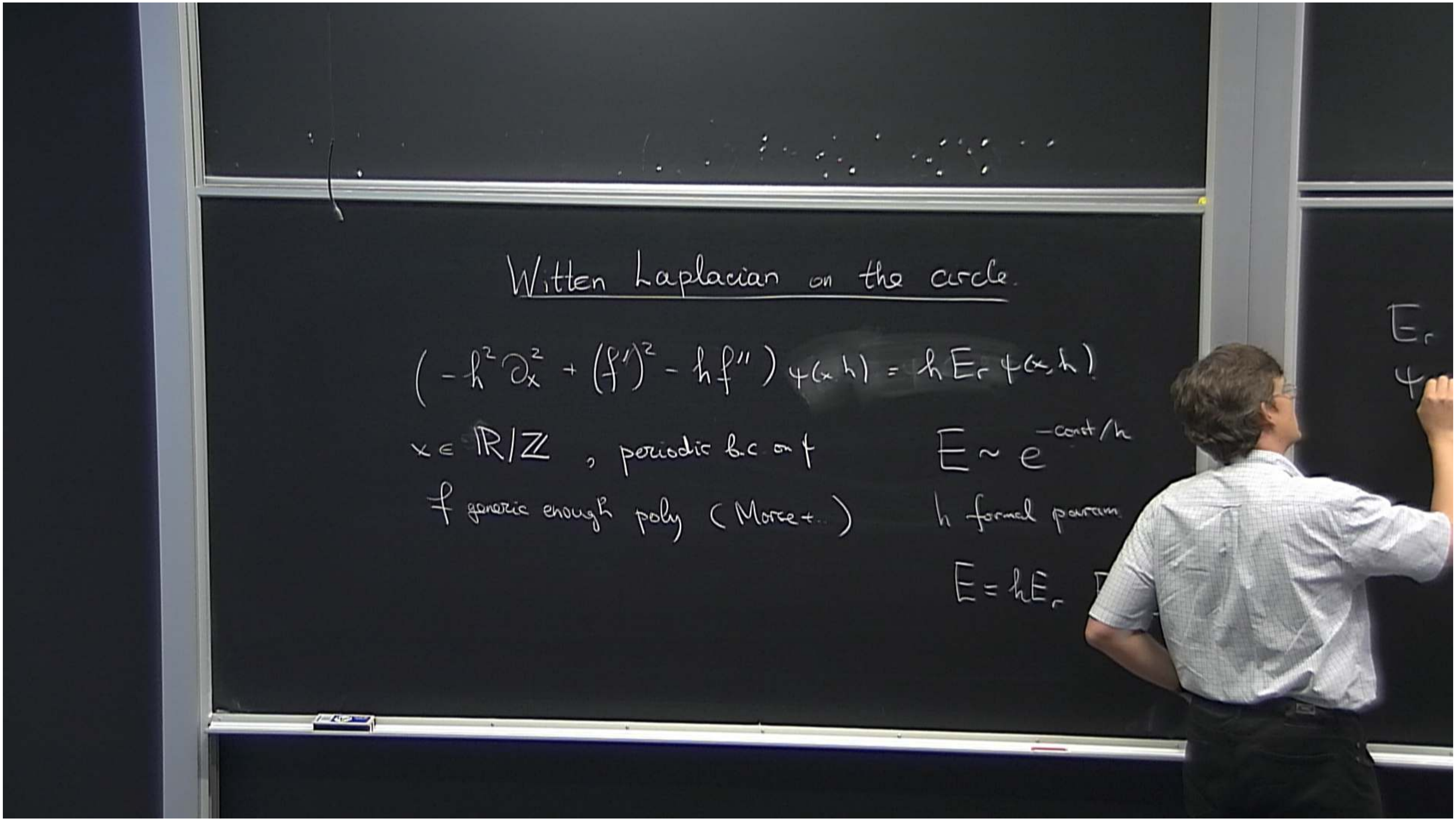
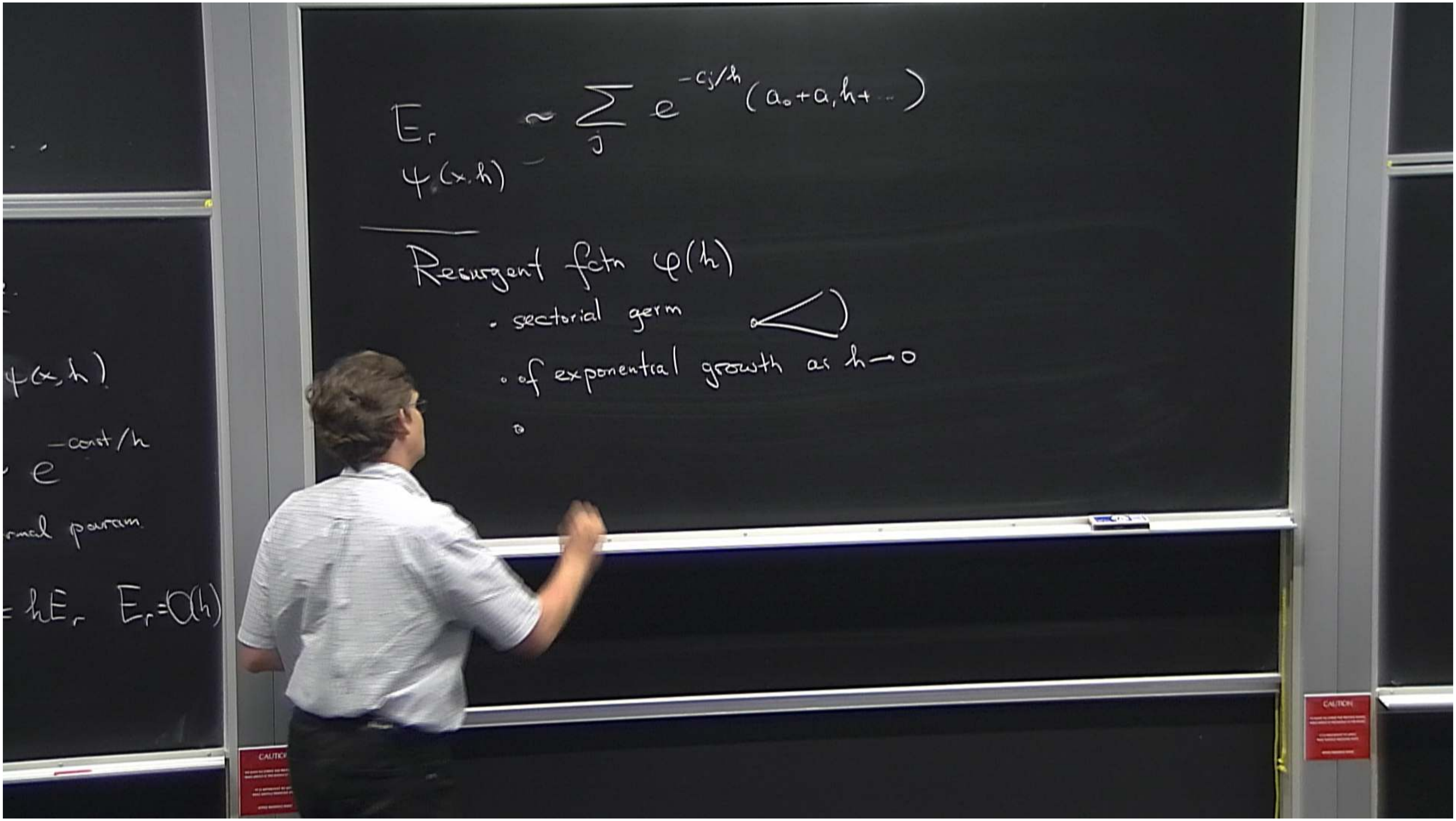
Title: Resurgent analysis and its applications to the Witten Laplacian

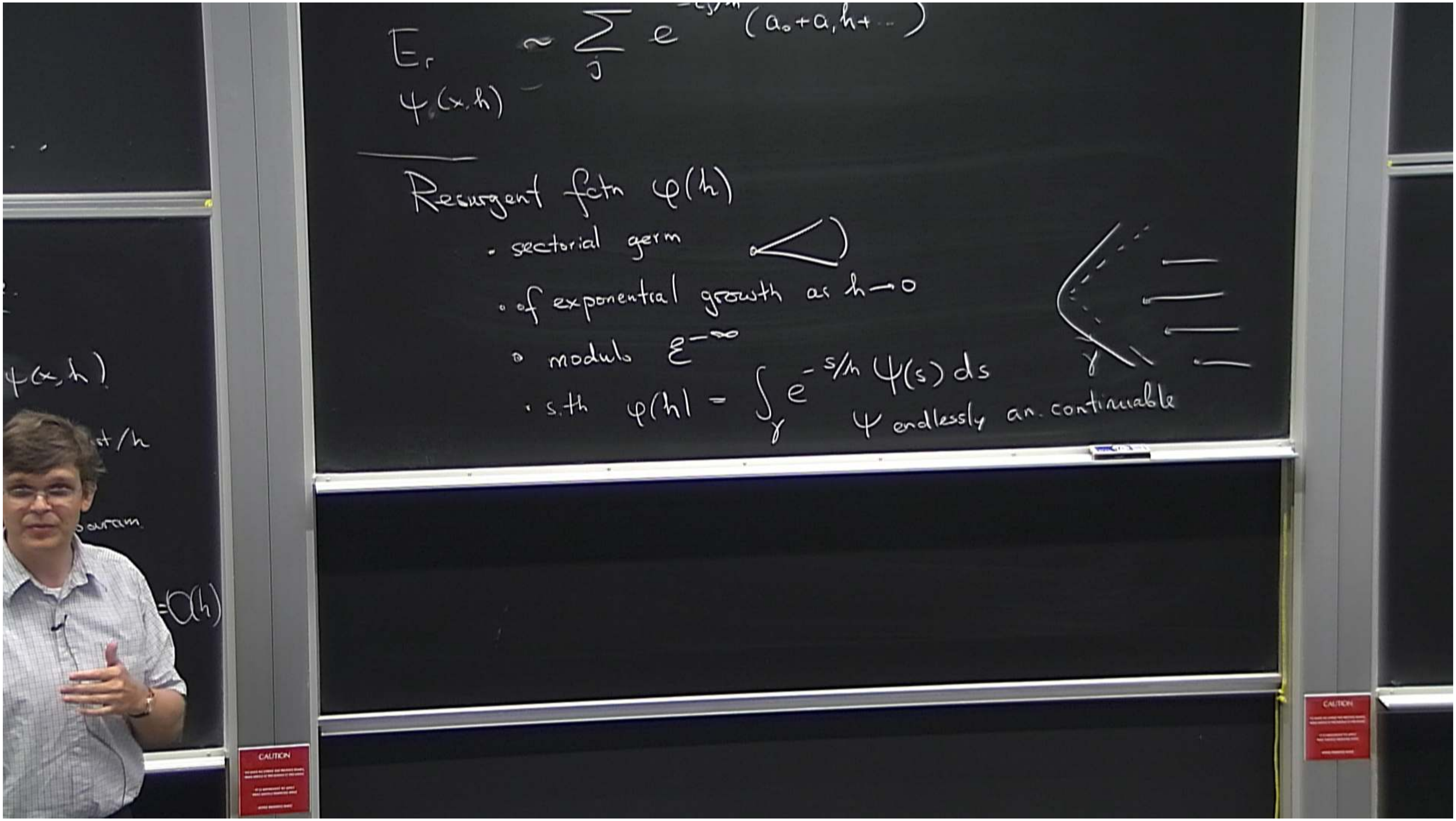
Date: May 29, 2015 11:00 AM

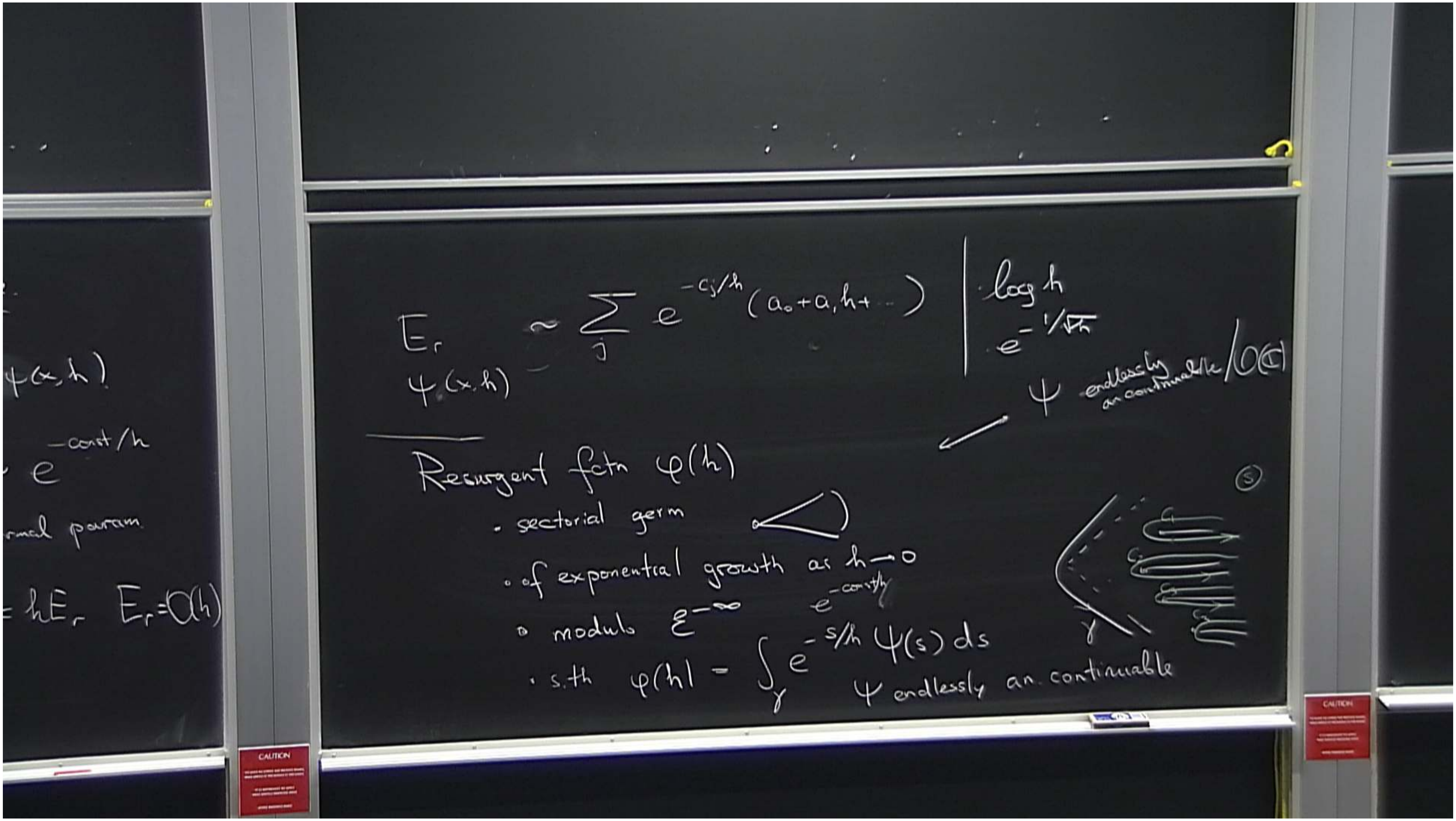
URL: http://pirsa.org/15050065

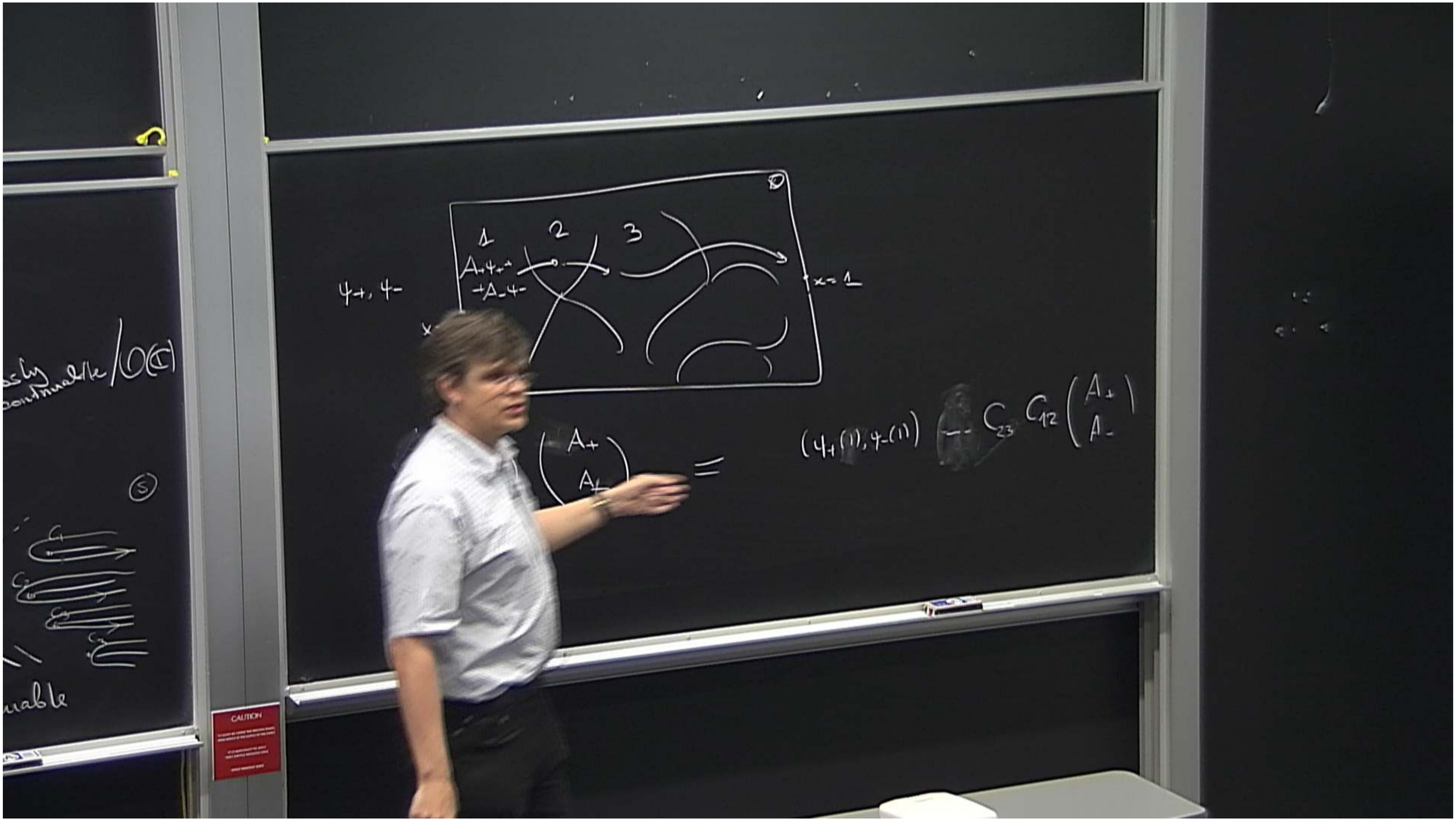
Abstract: The first lecture will be devoted to the review of the classical theory of the Witten Laplacian, the second -- to the concepts of resurgent analysis. The third -- to applications of the resurgent analysis to the Witten Laplacian. Time permitting, we will touch upon some foundational questions of resurgent analysis.

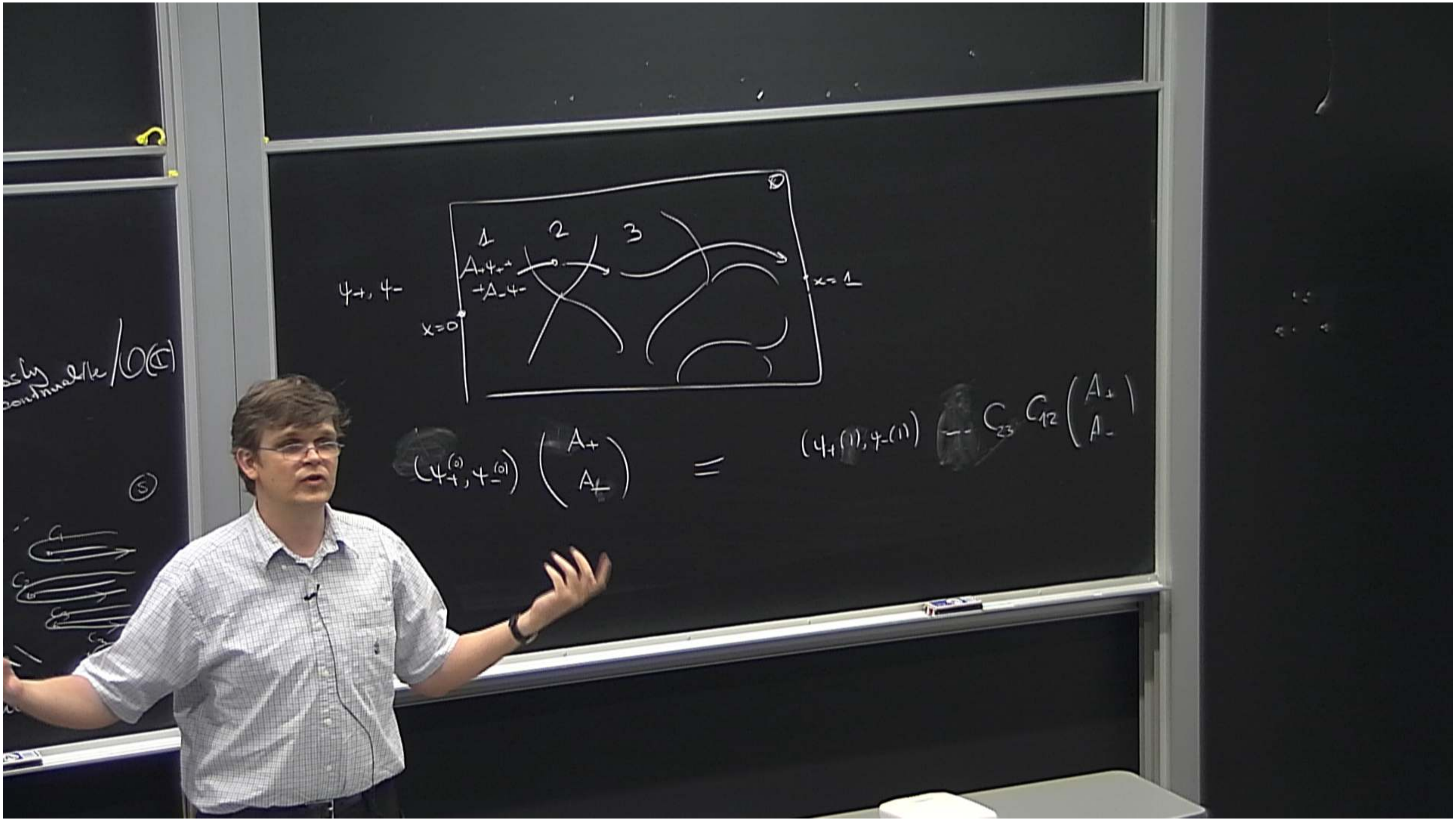

Witten Laplacian on the circle.
$(-\hbar^2\partial_x^2 + (f')^2 - \hbar f'')\,\psi(x,\hbar) = \hbar E_r\,\psi(x,\hbar)$
$x \in \mathbb{R}/\mathbb{Z}$, periodic b.c on f
f generic enough poly (Morse+..)
$E \sim e^{-const/\hbar}$
h formal param
$E = \hbar E_r$
E_r

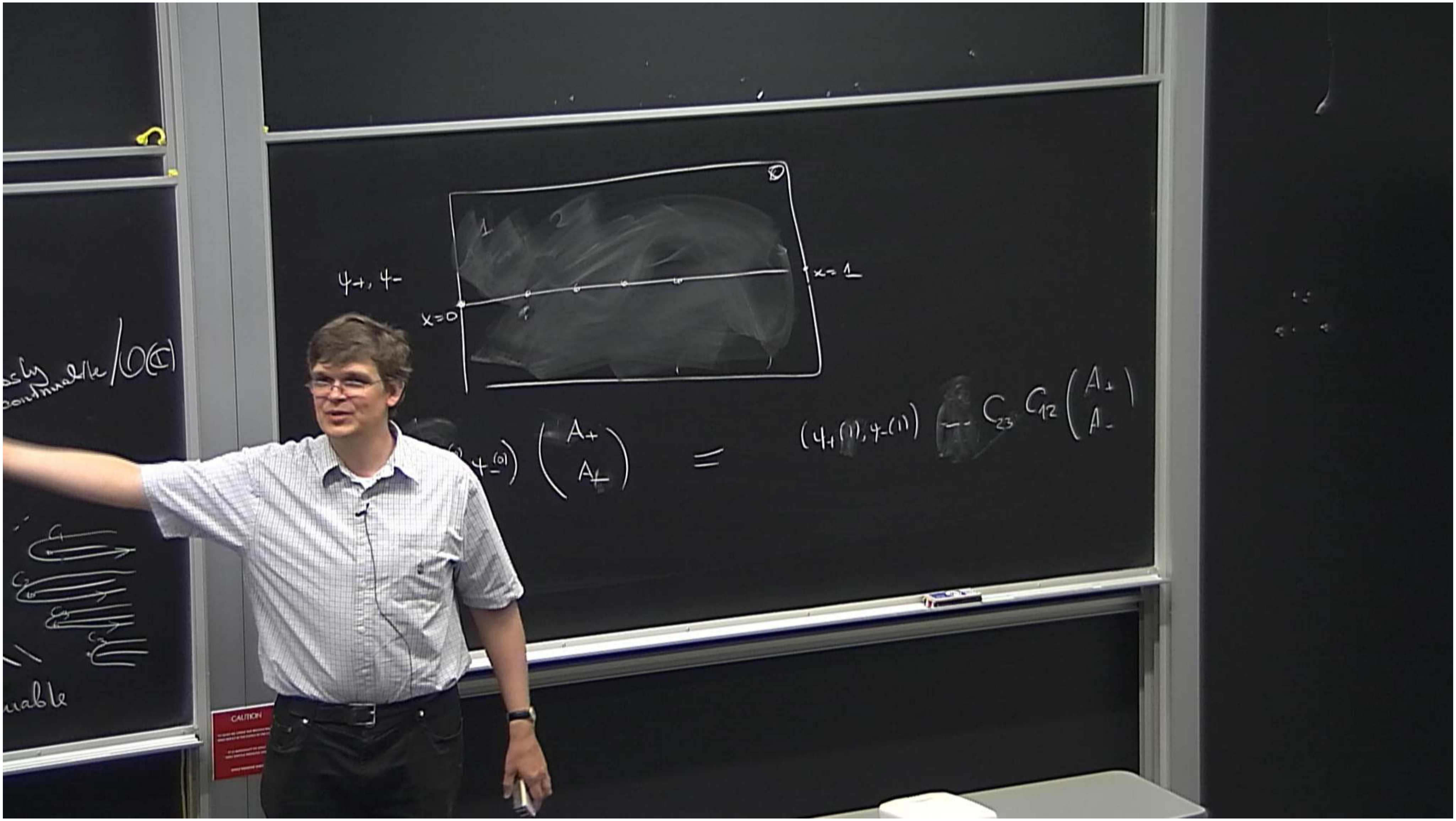

$E_r \sim \sum_j e^{-c_j/h}(a_0 + a_1 h + \dots)$
$\psi(x,h)$
Resurgent fctn $\varphi(h)$
• sectorial germ
• of exponential growth as $h \to 0$
$\psi(x,h)$
$e^{-\text{const}/h}$
formal param
$= hE_r \quad E_r = O(h)$

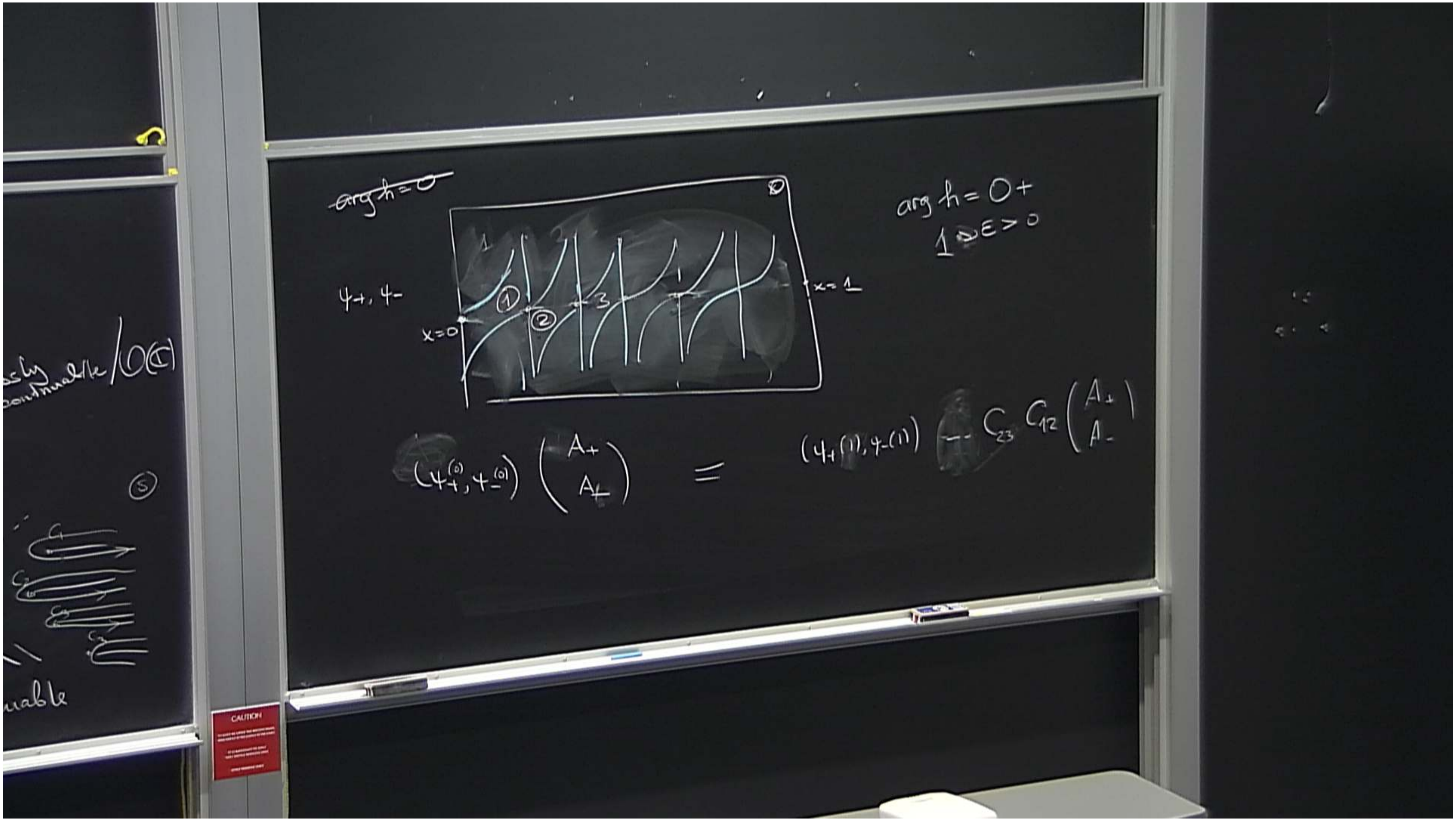

Resurgent fctn φ(h)
· sectorial germ
· of exponential growth as h → 0
· modulo ε^-∞
· s.th φ(h) = ∫γ e^-s/h ψ(s) ds
ψ endlessly an. continuable


$\psi(x,h)$
$e^{-const/h}$
rmal param
$= hE_r \quad E_r = O(h)$
$E_r \quad \psi(x,h) \sim \sum_j e^{-c_j/h}(a_0 + a_1 h + \dots)$
log h
$e^{-1/\sqrt{h}}$
ψ endlessly an. continuable / $\mathcal{O}(\mathbb{C})$
Resurgent fctn φ(h)
· sectorial germ
· of exponential growth as $h \to 0$ $e^{-const/h}$
· modulo $\varepsilon^{-\infty}$
· s.th $\varphi(h) = \int_\gamma e^{-s/h}\,\psi(s)\,ds$
ψ endlessly an. continuable
γ
(S)

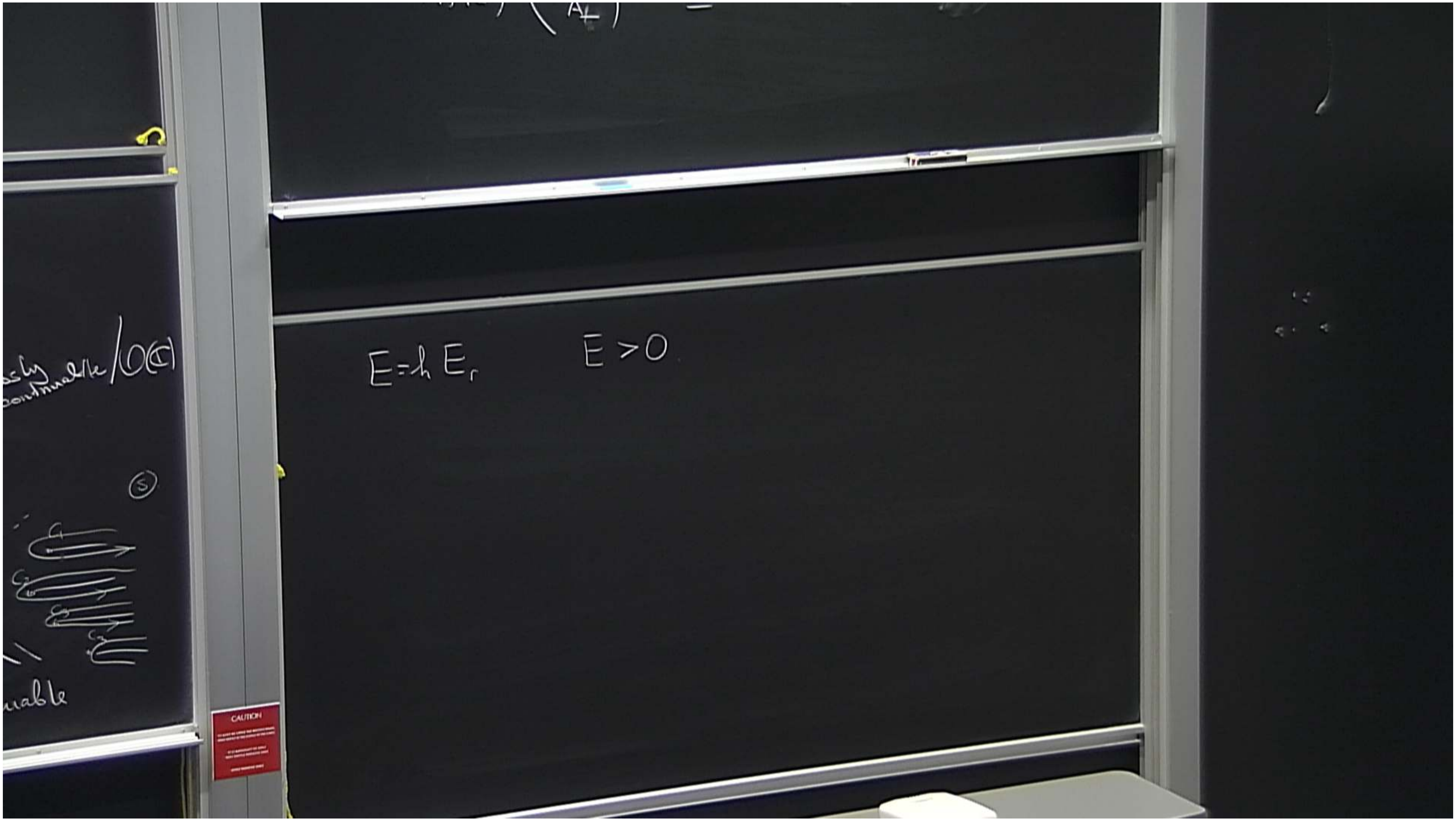

1
2
3
$A_+\psi_+ + A_-\psi_-$
ψ_+, ψ_-
$x=1$
$(\psi_+(1), \psi_-(1))$
$C_{23}\, C_{12} \begin{pmatrix} A_+ \\ A_- \end{pmatrix}$

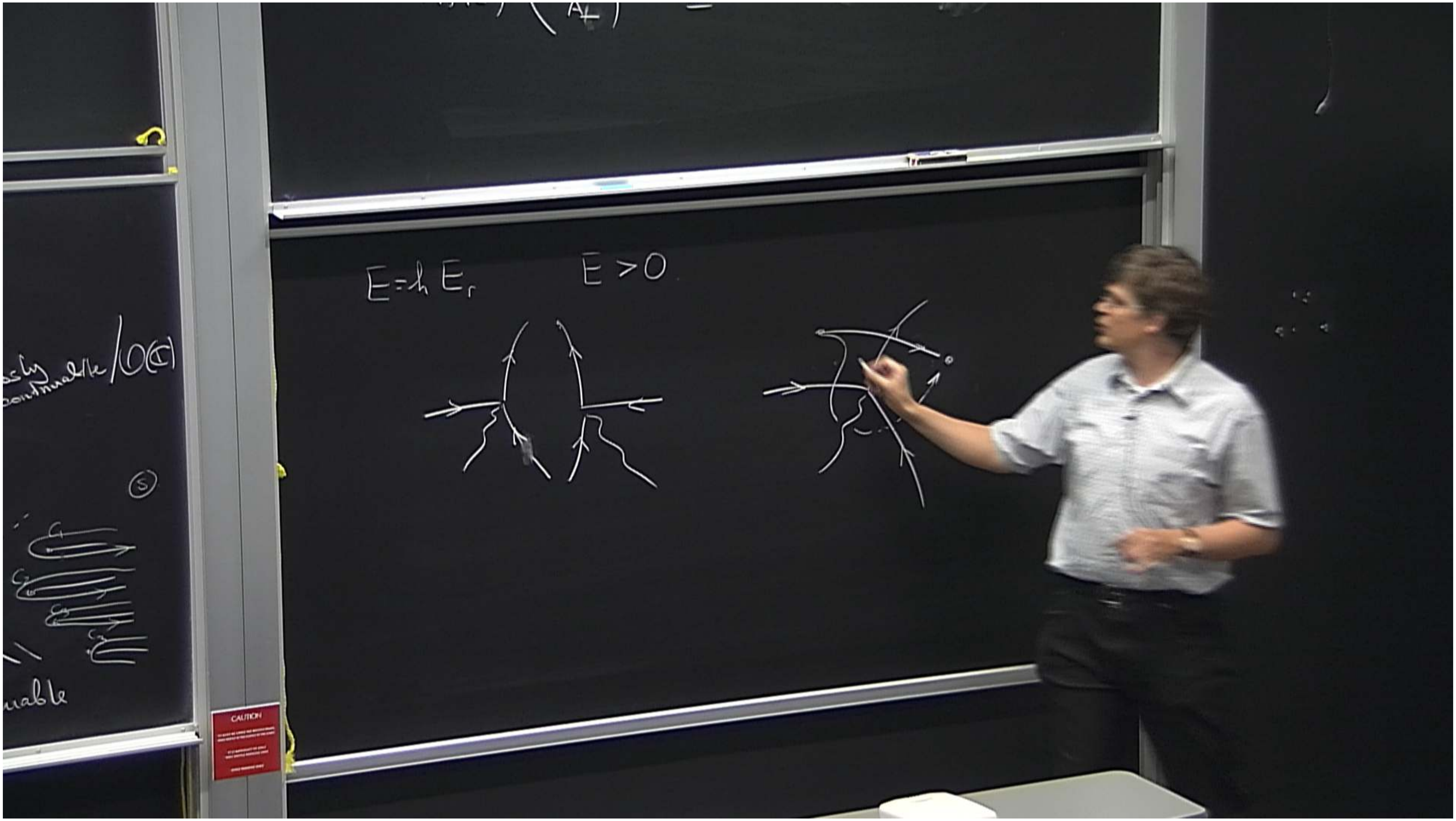

1
2
3
x=0
x=1

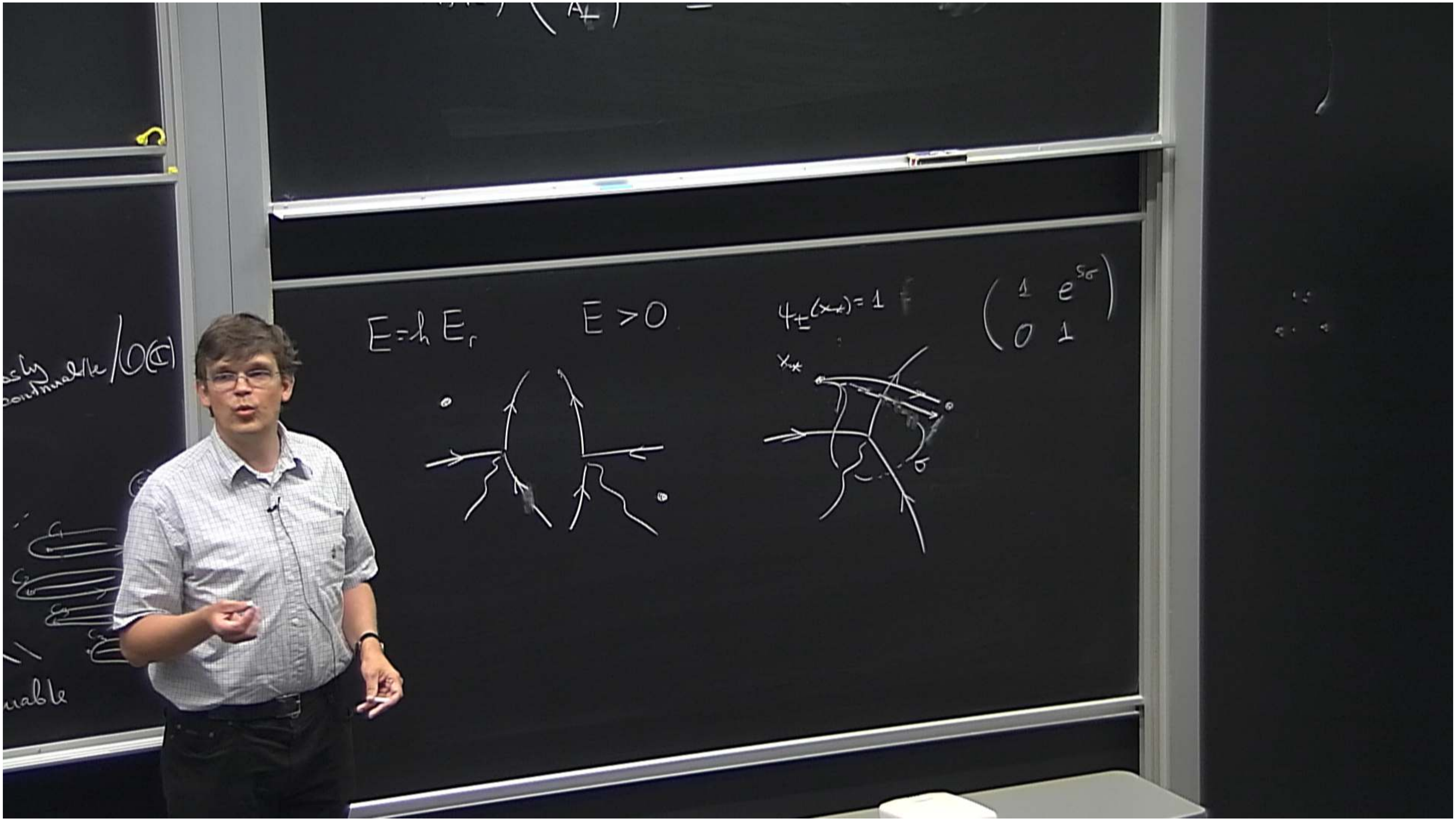


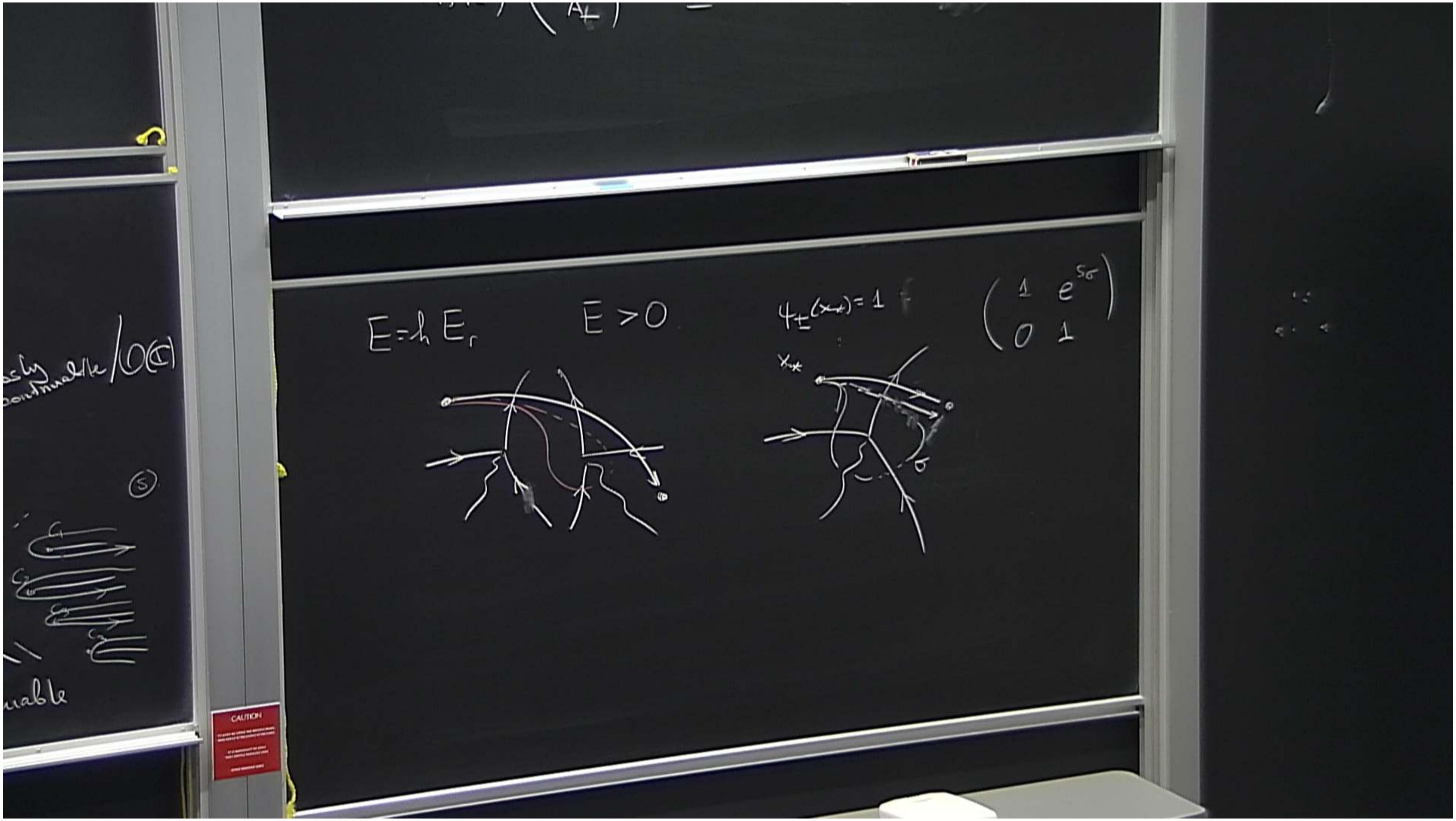

arg ħ = 0+
1 ≫ ε > 0
ψ+, ψ-
x=0
x=1

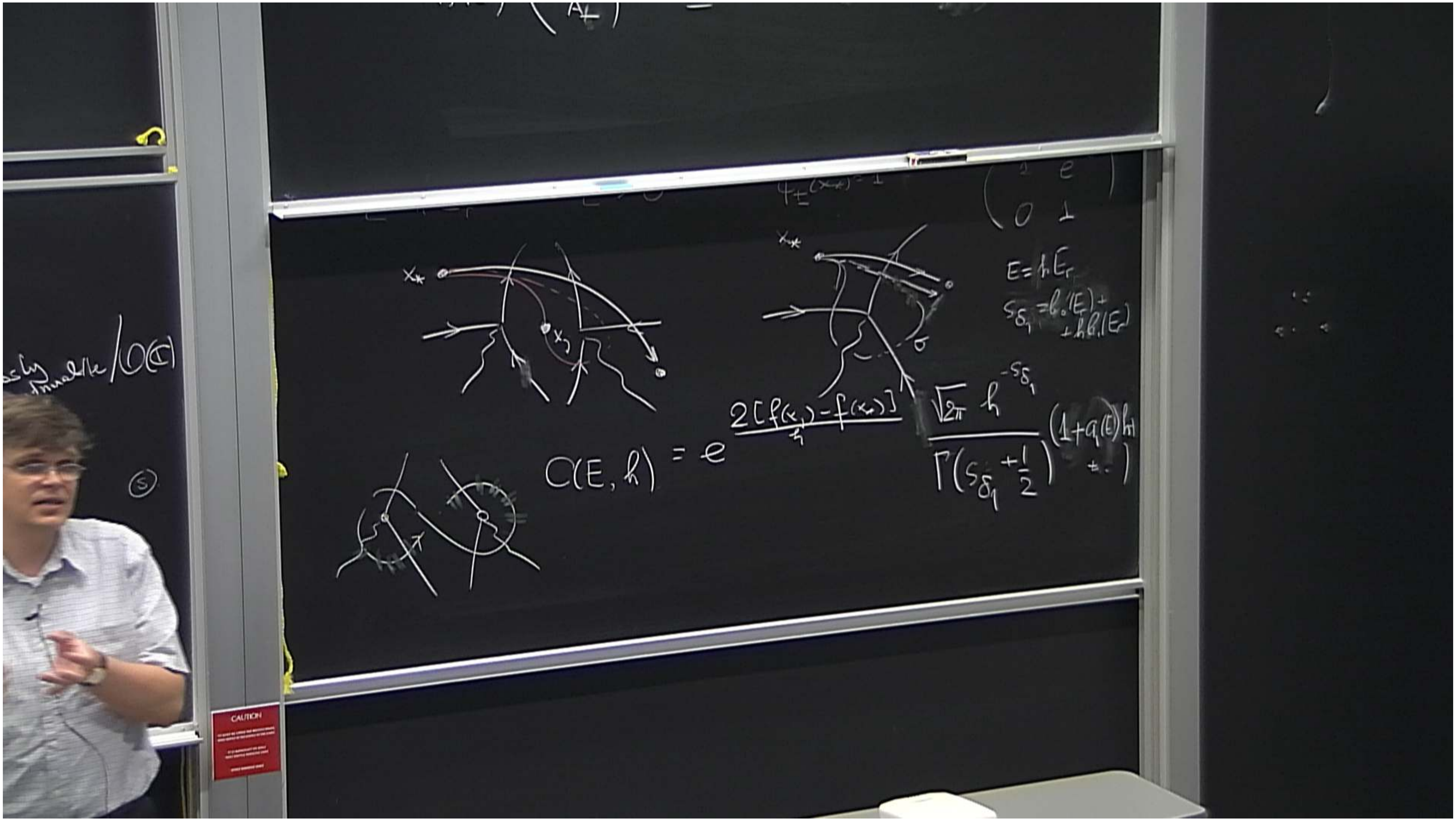

arg ħ = 0+
1 ≫ ε > 0
x=0
x=1
(ψ+(1), ψ−(1))
C23 C12
A+
A−

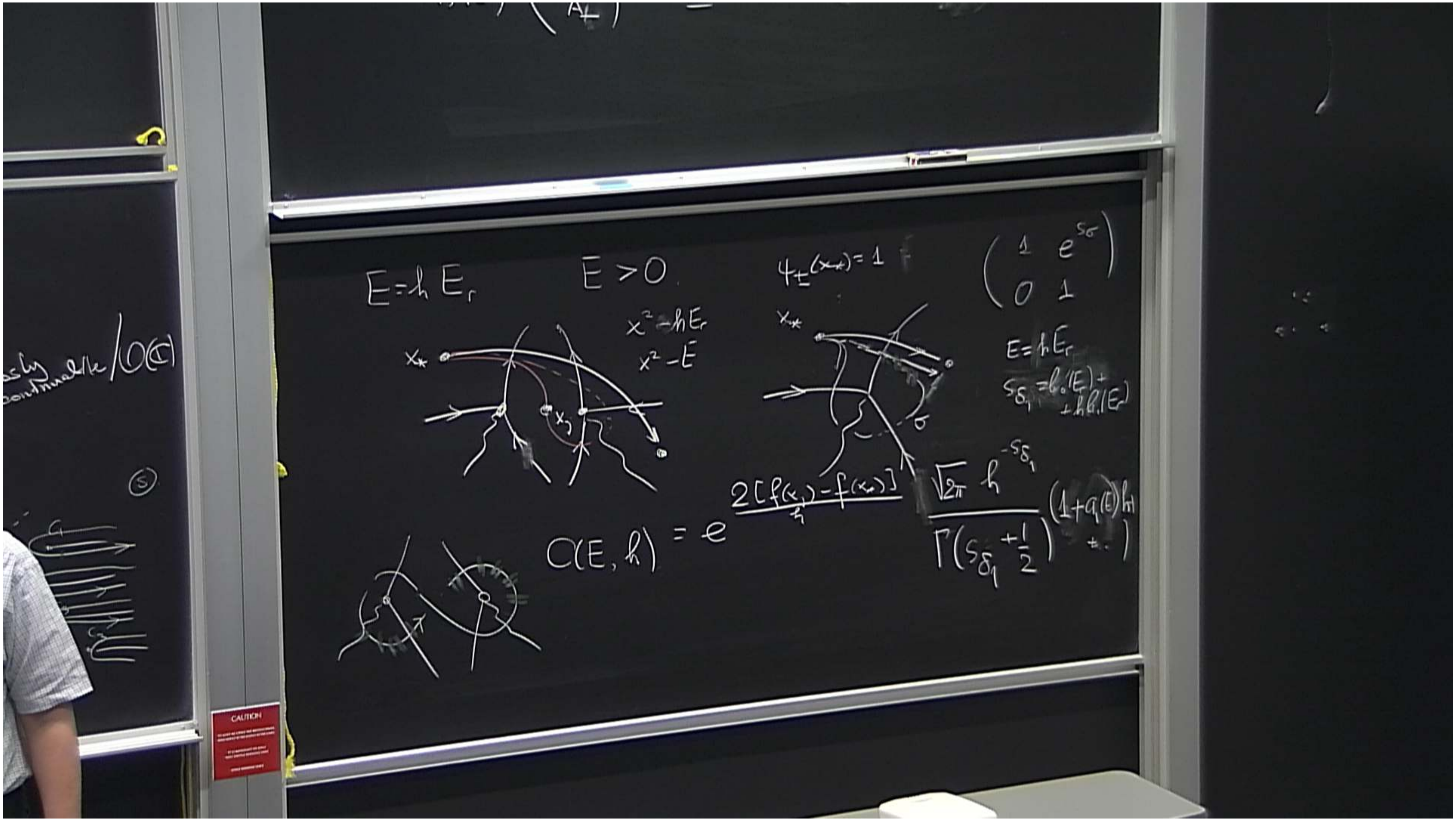

E = ħ E_r
E > 0
CAUTION

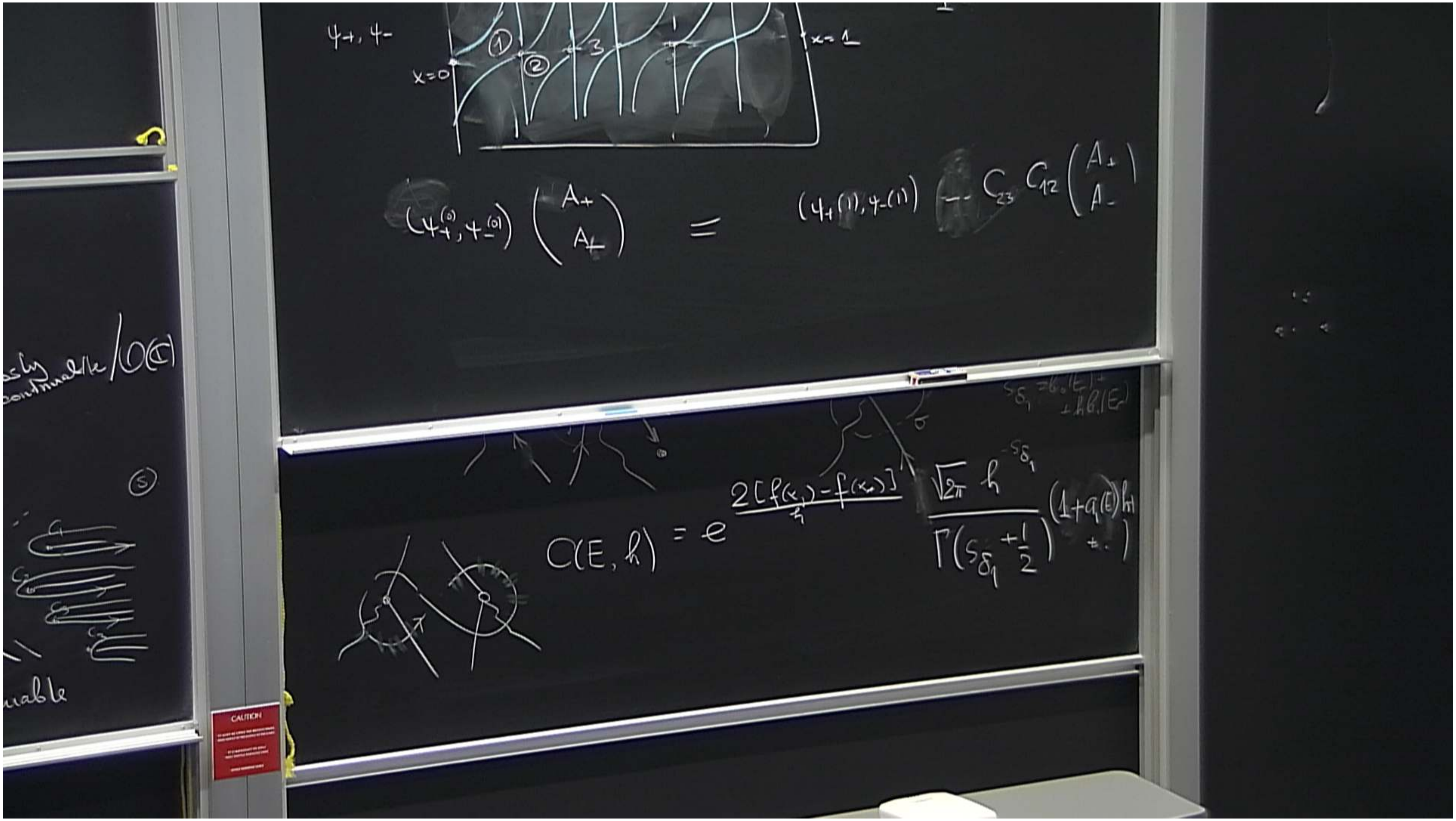

E=ħ E_r
E > 0

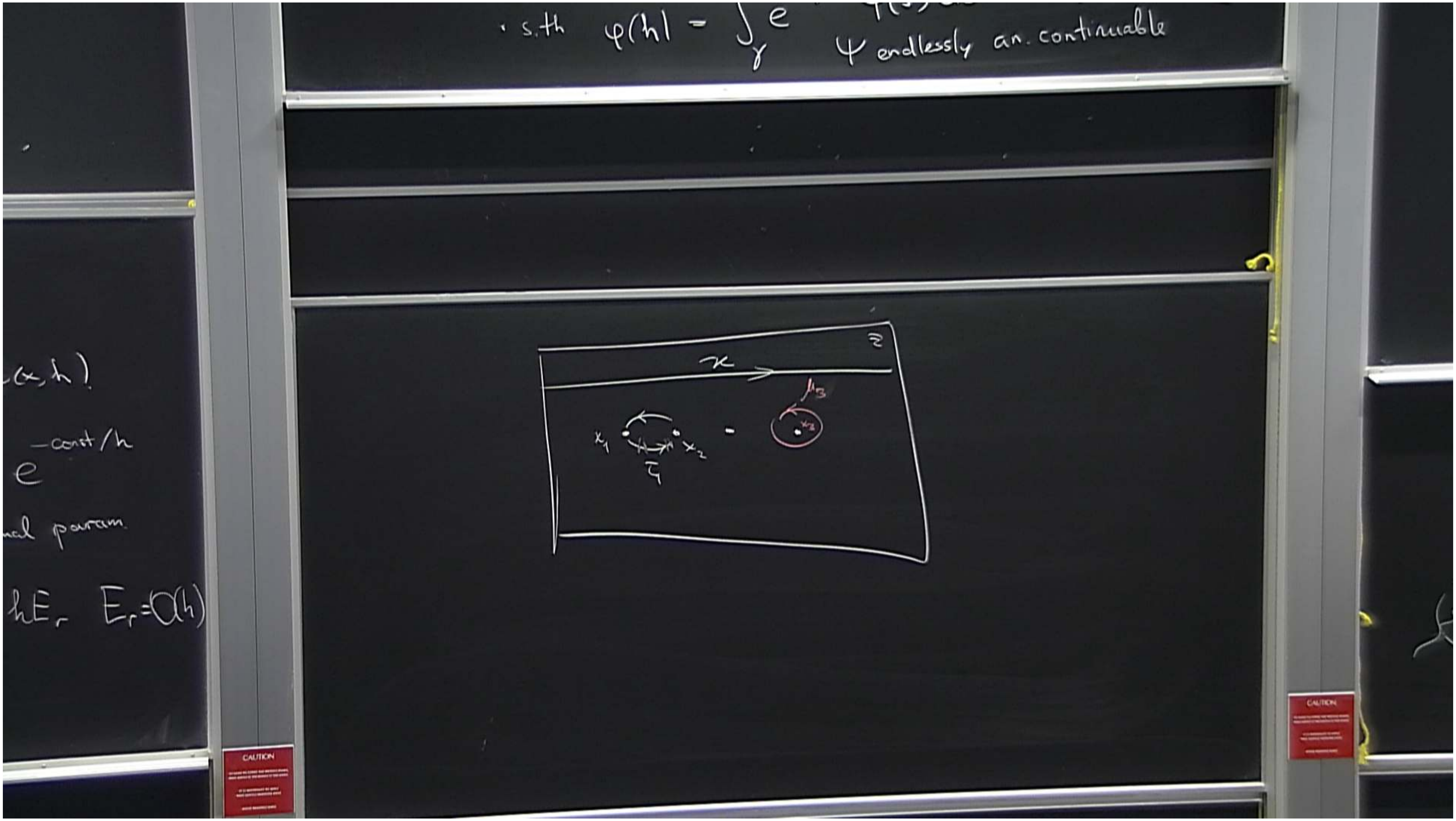

E = ħ E_r
E > 0
ψ±(x*) = 1
x*
σ
(1 e^{S_σ} ; 0 1)

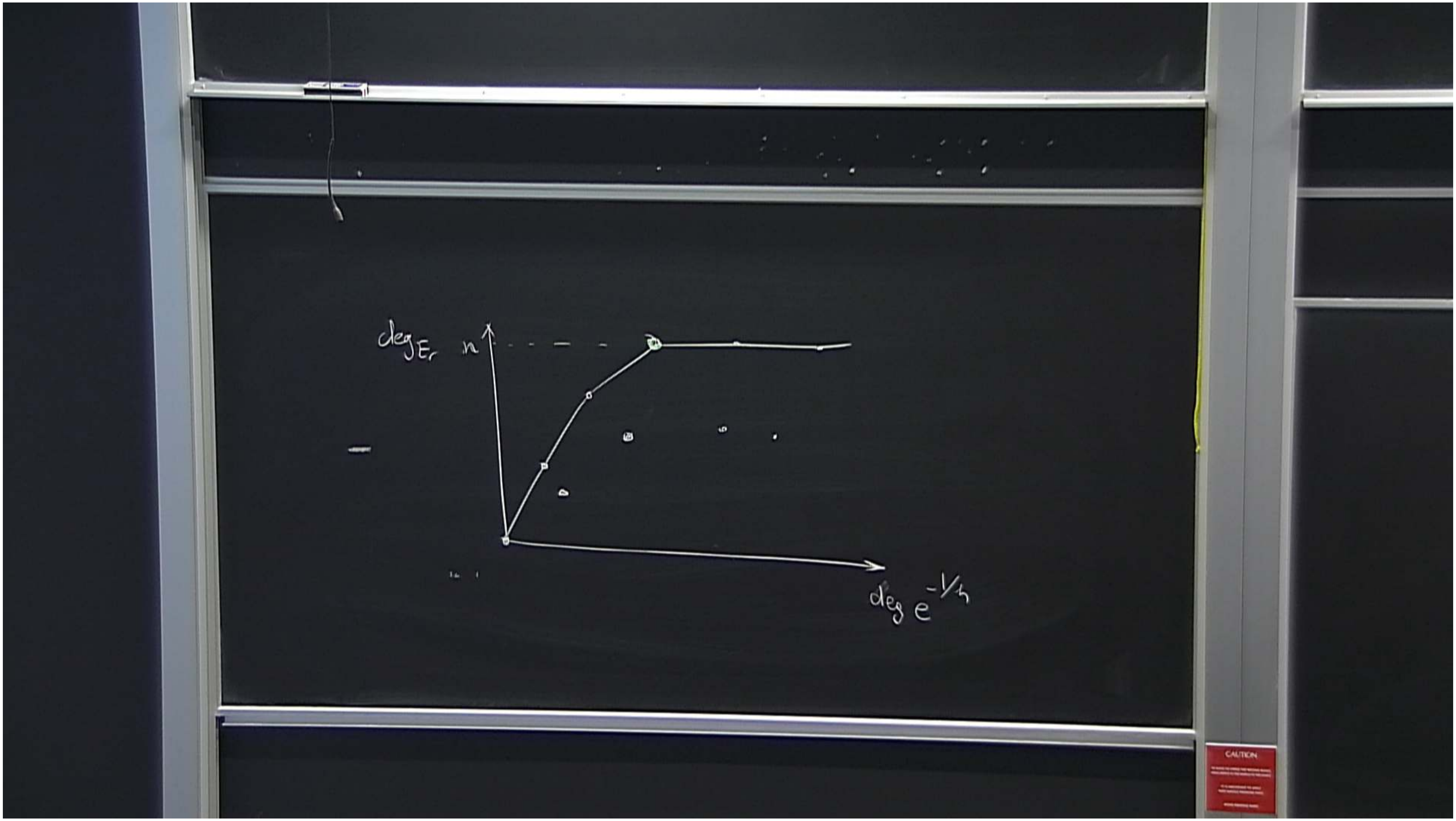

E = ħ E_r
E > 0
ψ±(x*) = 1
x*
σ
(1 e^{sσ}; 0 1)
nable
S



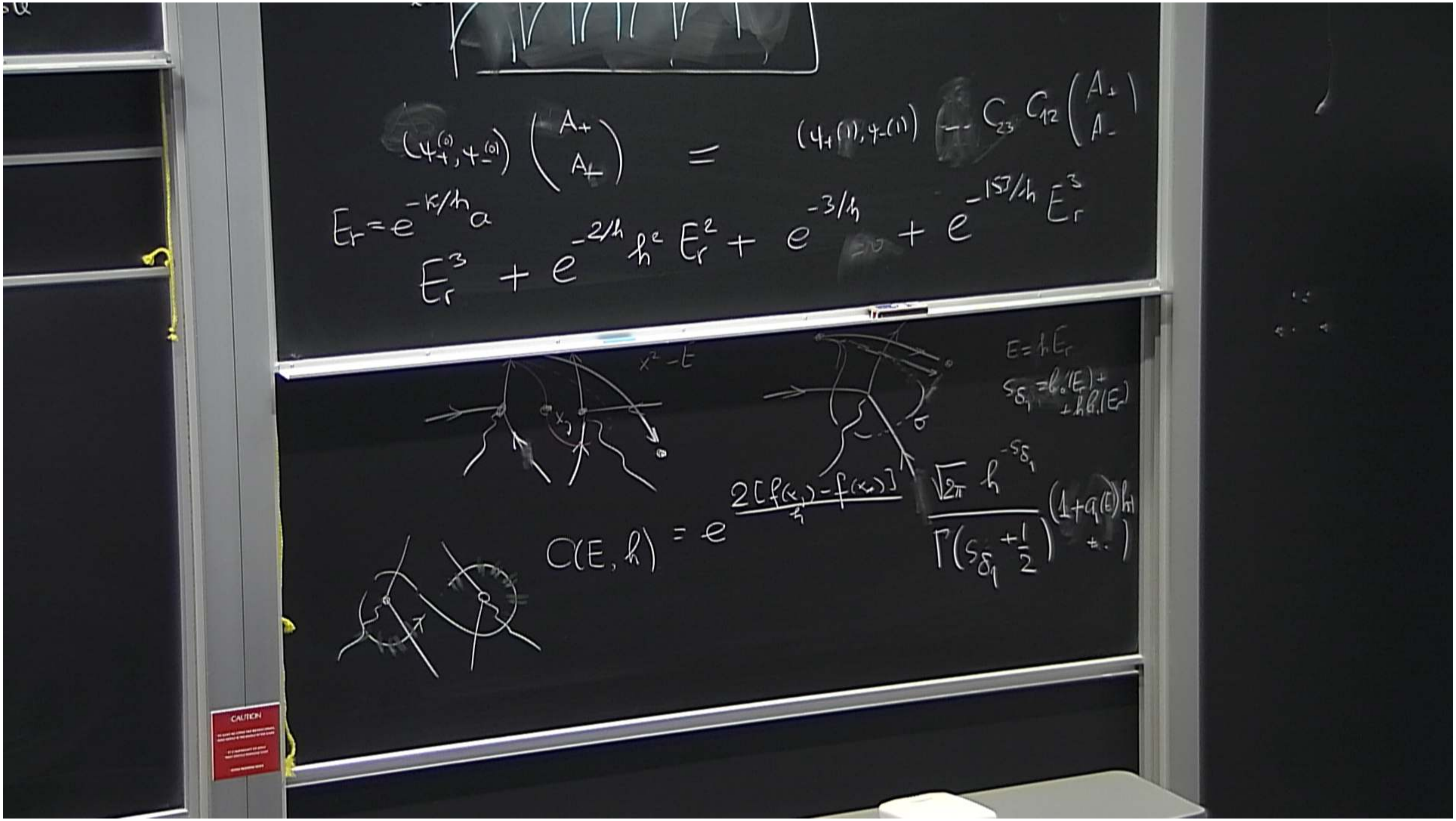

$E = \hbar E_r$
$E > 0$
$x^2 - \hbar E_r$
$x^2 - E$
$C(E, \hbar) = e^{\frac{2[f(x_1) - f(x_*)]}{\hbar}} \frac{\sqrt{2\pi}\, \hbar^{-s_{\delta_1}}}{\Gamma(s_{\delta_1} + \frac{1}{2})} (1 + a_1(E)\hbar + \dots)$
$E = \hbar E_r$
$s_{\delta_1} = b_0(E_r) + \hbar b_1(E_r)$
CAUTION

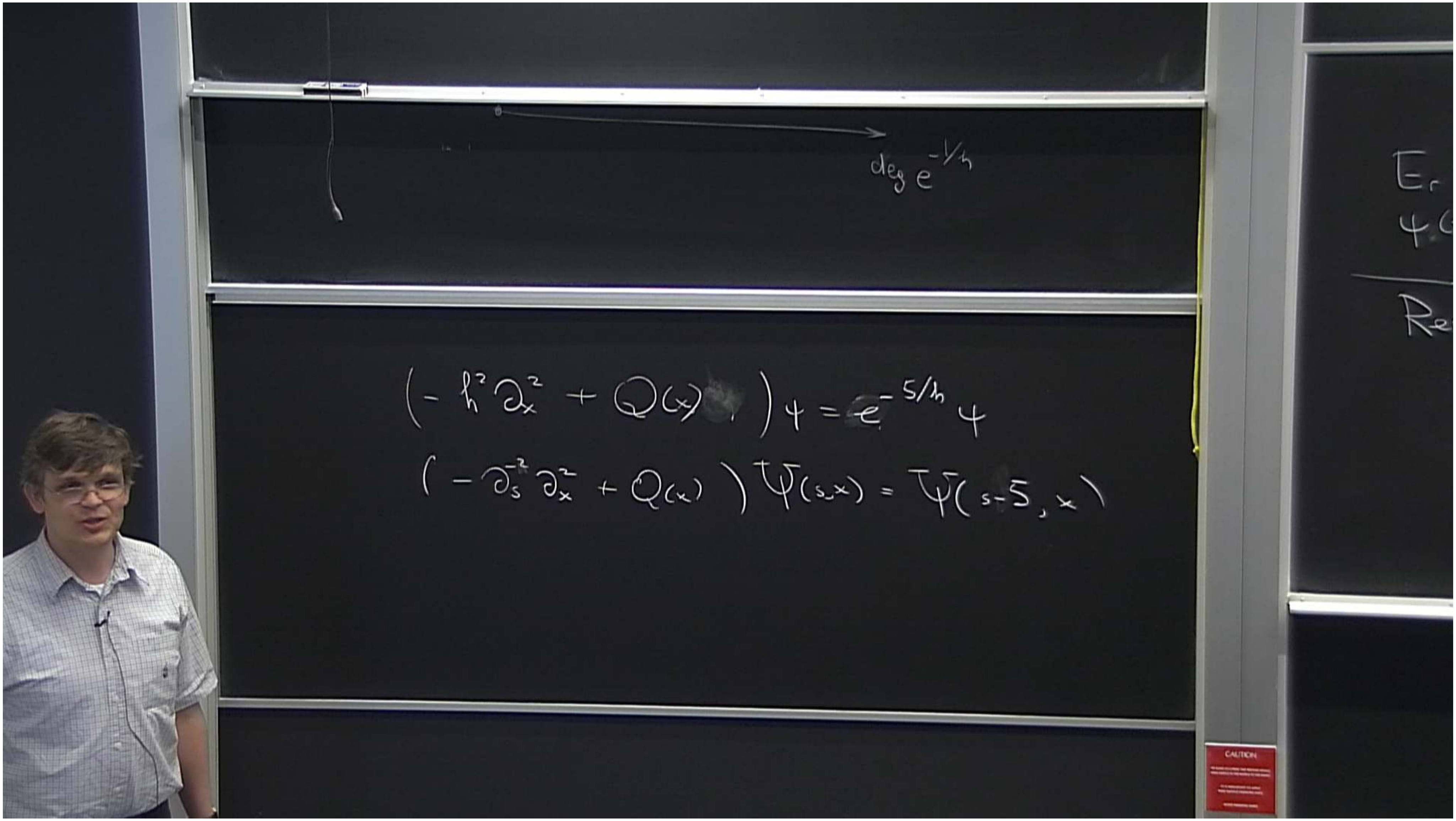

ψ+, ψ-
x=0
x=1
C(E, ħ) = e


s.th
ψ endlessly an. continuable
-const/h
param.
hE_r $E_r = O(h)$
x_1
x_2
x_3



deg$_{E_r}$
n
deg $e^{-1/h}$


(ψ+(0), ψ-(0)) (A+ A+) = (ψ+(1), ψ-(1)) C23 C12 (A+ A-)
Er = e-K/ħ a
E = ħ Er
x² − E
C(E, ħ) = e


deg e^{-1/ħ}
(- ħ² ∂x² + Q(x)) ψ = e^{-S/ħ} ψ
(- ∂s^{-2} ∂x² + Q(x)) Ψ(s,x) = Ψ(s-S, x)