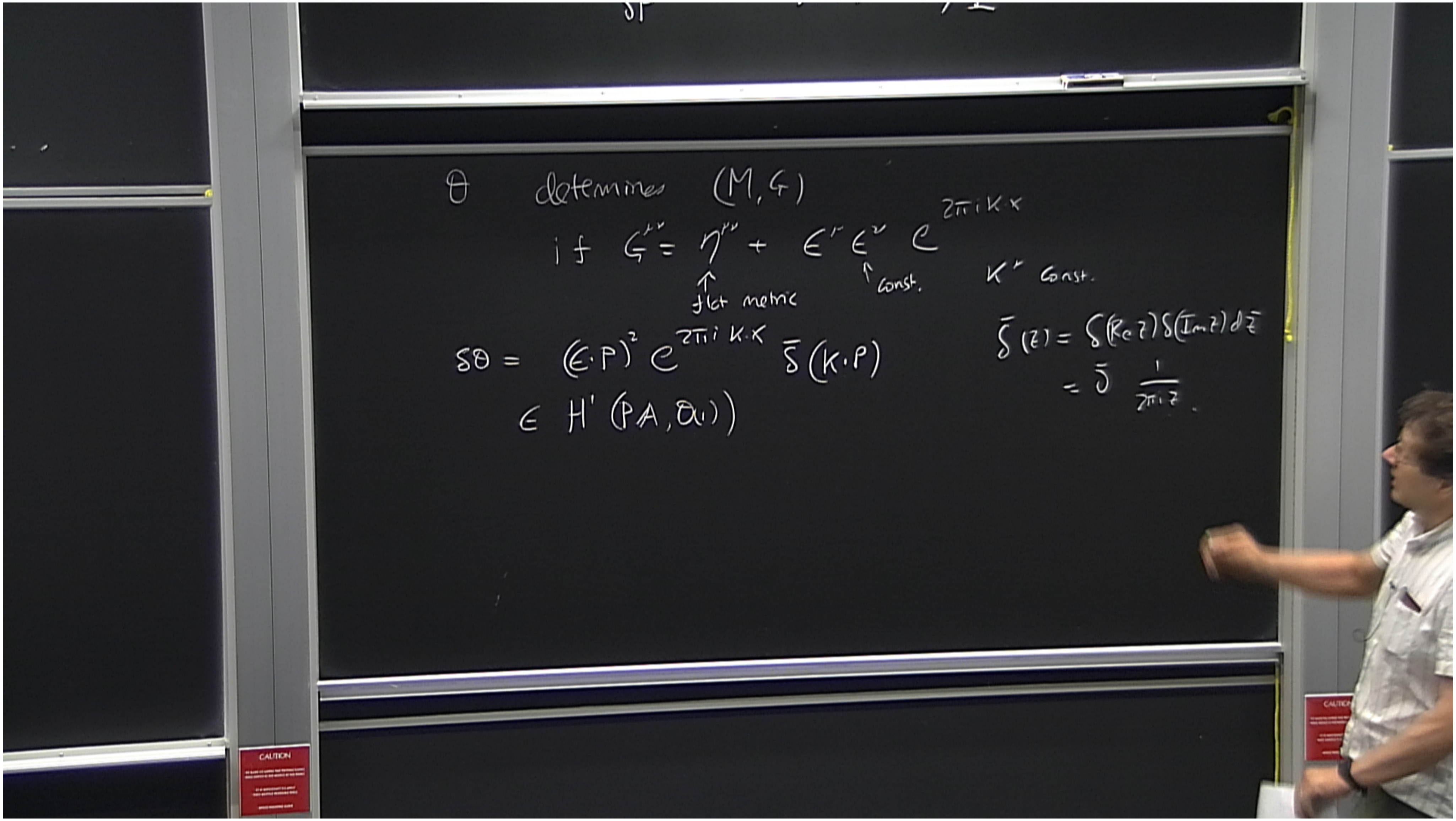
Title: Ambitwistors-strings and amplitudes

Date: May 29, 2015 09:30 AM

URL: http://pirsa.org/15050064

Abstract: These lectures will focus on the geometry of ambitwistor string theories. These are infinite tension analogues of conventional strings and provide the theory that leads to the remarkable formulae for tree amplitudes that have been developed by Cachazo, He and Yuan based on the scattering equations. Although the bosonic ambitwistor string action is expressed in space-time, it will be seen that its target is classically `ambitwistor space', the space of complexified null geodesics in the complexification of a space-time. The lectures will review Ambitwistor constructions from the 70's and 80's that extend the Penrose-Ward twistor constructions for self-dual Yang-Mills and gravitational fields in four dimensions to arbiitrary fields in general dimension. LeBrun showed that the conformal geometry of a space-time is encoded into the complex structure of ambitwistor space. The linearized version encodes linear fields on space-time into sheaf cohomology classes on ambitwistor space. In the case of momentum eigenstates, these give the `scattering equations' that underly the CHY formulae and the ambitwistor string can be used to compute amplitudes via these formulae. If there is time, the lectures will discuss how different matter theories can be obtained, different geometric realizations of ambitwistor space lead to different formulae, the relationship between the asymptotic symmetries of space-time and Weinberg's soft theorems concerning the behaviour of amplitudes when momenta become small, and/or extensions of the ideas to loop amplitudes.


θ determines (M, G)
if $G^{\mu\nu} = \eta^{\mu\nu} + \epsilon^{\mu}\epsilon^{\nu} e^{2\pi i K\cdot X}$
flat metric
const.
K^{μ} const.
$\delta\theta = (\epsilon\cdot P)^2 e^{2\pi i K\cdot X}\, \bar\delta(K\cdot P)$
$\in H^1(PA, \mathcal{O}(1))$
$\bar\delta(z) = \delta(\mathrm{Re}\, z)\,\delta(\mathrm{Im}\, z)\, d\bar z$
$= \bar\partial \frac{1}{2\pi i z}$.

θ determines (M, G)

if $G^{\mu\nu} = \eta^{\mu\nu} + \epsilon^{\mu}\epsilon^{\nu} e^{2\pi i K\cdot X}$

flat metric ($\eta^{\mu\nu}$); ϵ const. K^{μ} const.

$$\delta\theta = (\epsilon\cdot P)^2 e^{2\pi i K\cdot X}\, \bar\delta(K\cdot P)$$

$$\in H^1(\mathbb{P}A, \mathcal{O}(1))$$

$$\bar\delta(z) = \delta(\mathrm{Re}\, z)\,\delta(\mathrm{Im}\, z)\, d\bar z = \bar\partial \frac{1}{2\pi i z}.$$

$$\mathcal{L}_{X_{p^2}} \delta\theta = 2\pi i (K\cdot P)\, \epsilon\cdot P^2 e^{2\pi i K\cdot X}\, \bar\delta(K\cdot P) = 0$$

$$X_{p^2} = P\nabla = P\cdot\frac{\partial}{\partial X}$$

θ determines (M, G)

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flat metric   const. k^{μ} const.

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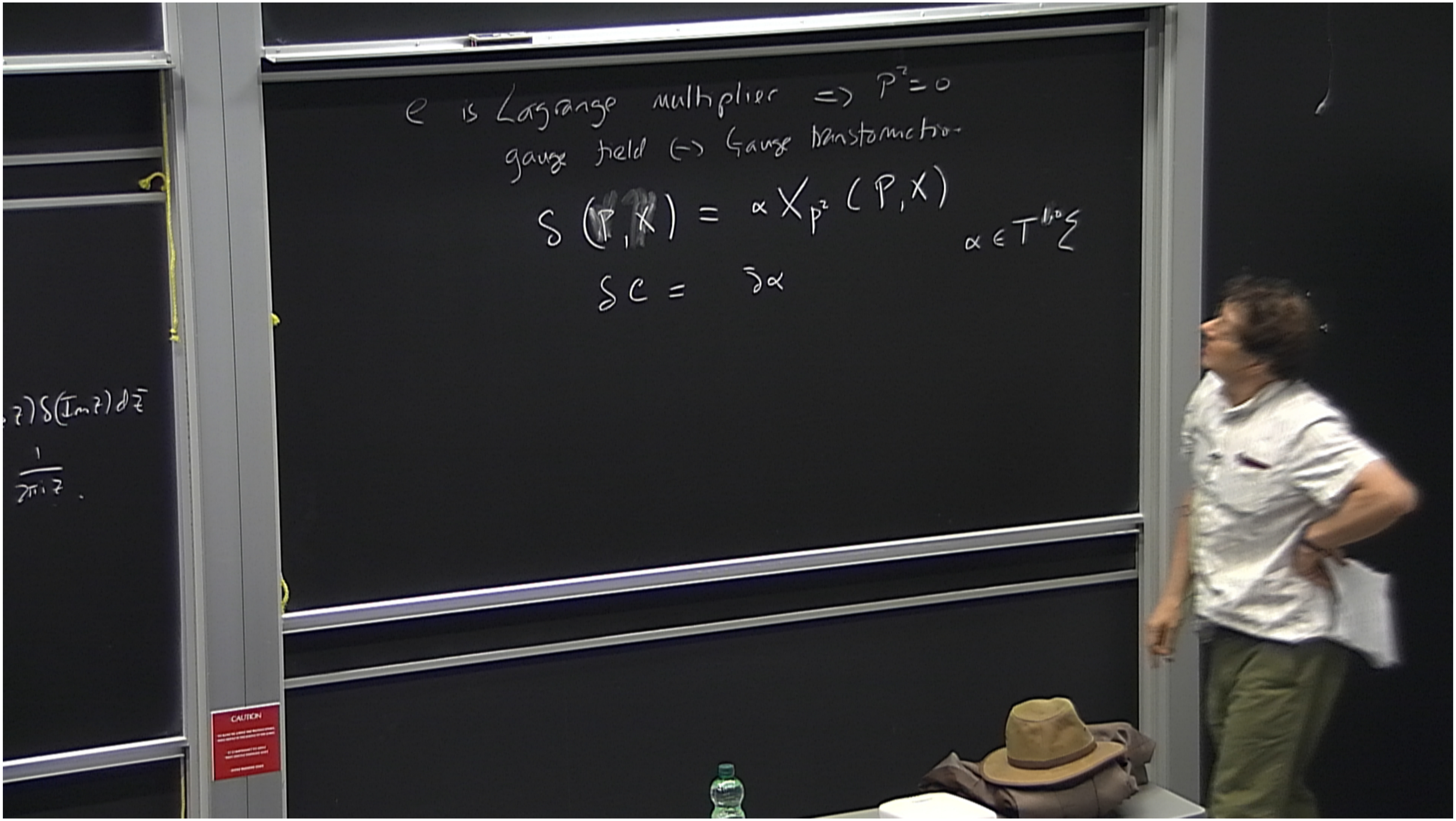
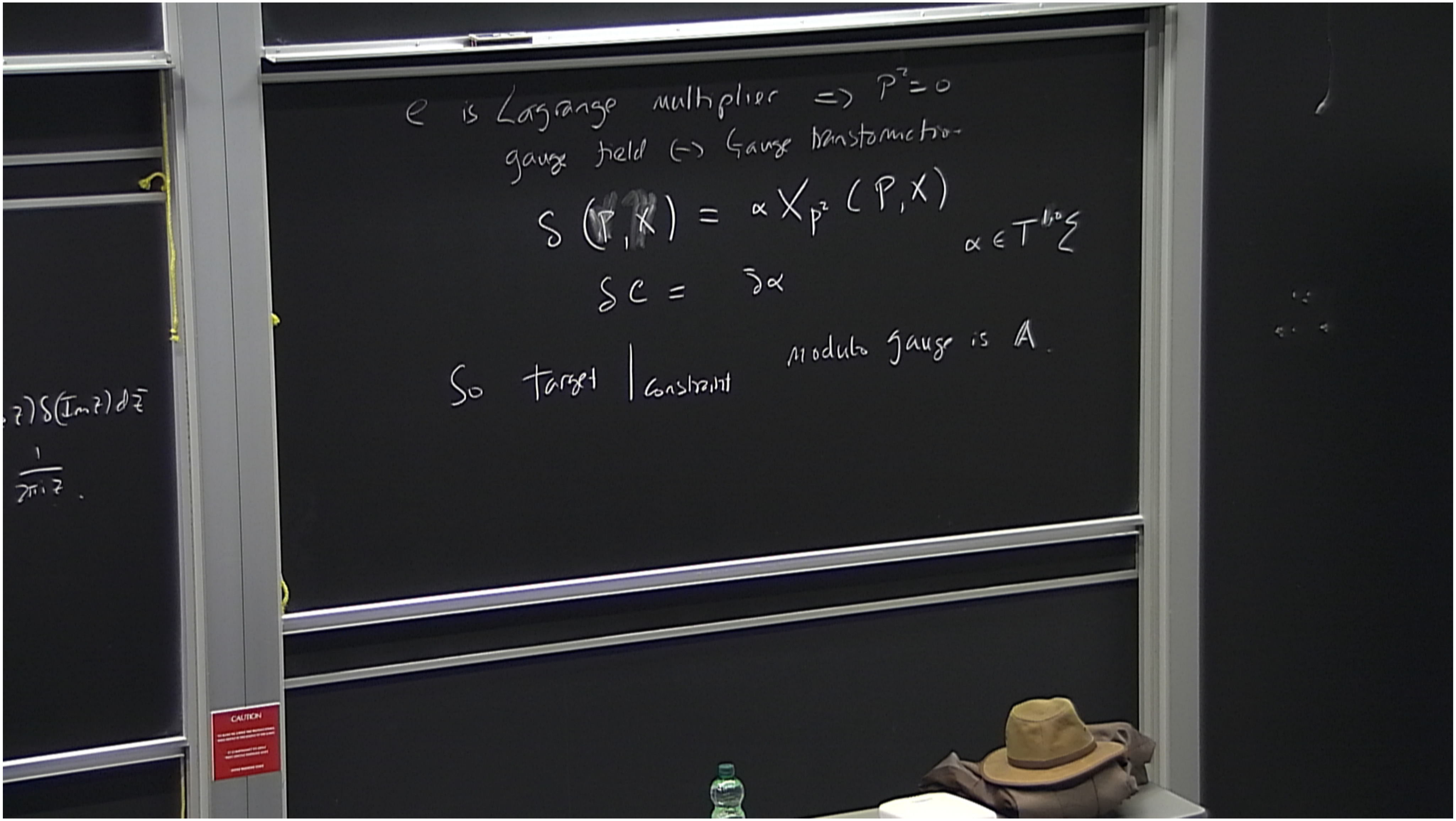
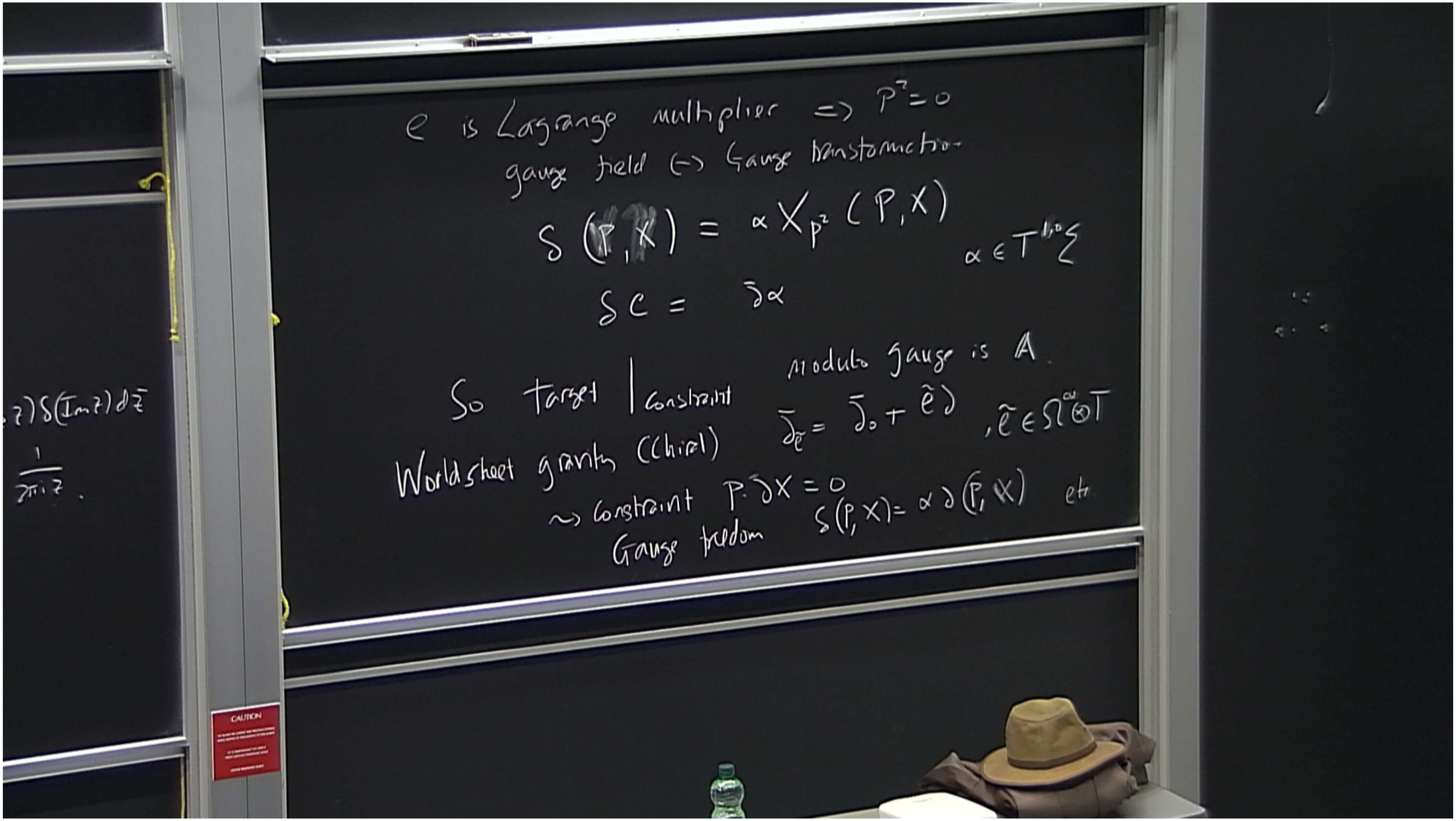

e is Lagrange multiplier => $P^2 = 0$
gauge field <-> Gauge transformation
$\delta(P, X) = \alpha X_{P^2}(P, X)$
$\alpha \in T^{1,0}\Sigma$
$\delta e = \partial\alpha$
$z)\,\delta(\mathrm{Im}\, z)\, d\bar{z}$
$\frac{1}{2\pi i z}$.


e is Lagrange multiplier => P^2 = 0
gauge field <-> Gauge transformation
δ(P, X) = α X_{P^2}(P, X)
α ∈ T^{1,0}Σ
δe = ∂α
So target |constraint modulo gauge is A.
z) δ(Im z) dz̄
1/(2πi z).


e is Lagrange multiplier => P² = 0
gauge field <-> Gauge transformation
δ(P, X) = α X_{P²}(P, X)
α ∈ T^{1,0}Σ
δe = ∂̄α
So target | constraint modulo gauge is A
Worldsheet gravity (chiral)
∂̄_ẽ = ∂̄₀ + ẽ∂
, ẽ ∈ Ω^{0,1} ⊗ T
~> constraint P·∂X = 0
Gauge freedom δ(P, X) = α∂(P, X) etc.

$\ldots z)\,\delta(\mathrm{Im}\, z)\, d\bar z$

$\frac{1}{2\pi i z}$.

e is Lagrange multiplier $\Rightarrow P^2 = 0$

gauge field $\leftrightarrow$ Gauge transformation

$$\delta(P,X) = \alpha X_{P^2}(P,X) \qquad \alpha \in T^{1,0}\Sigma$$

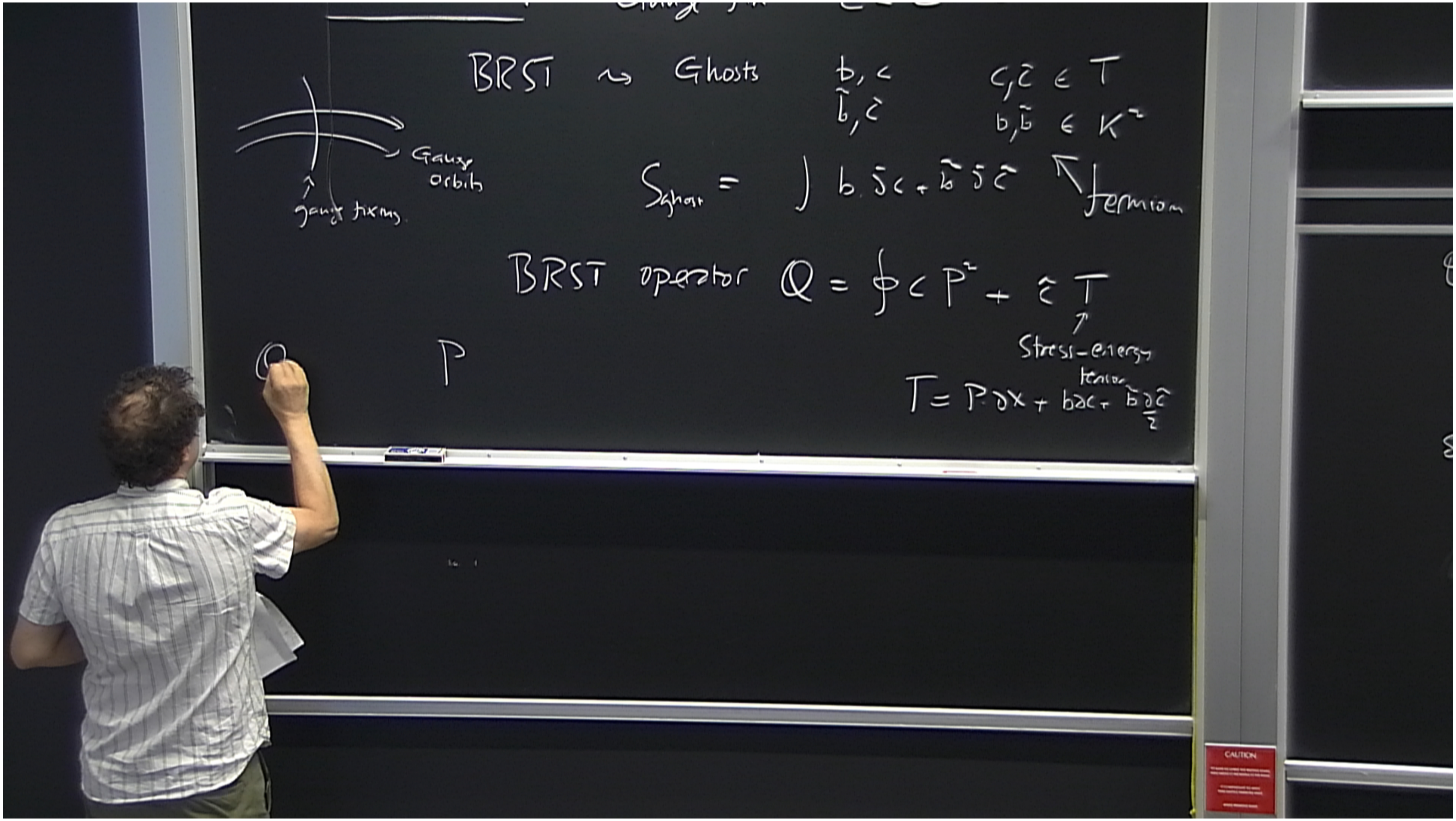
$$\delta e = \bar\partial \alpha$$

So target | constraint modulo gauge is $\mathbb{A}$.

Worldsheet gravity (chiral) $\quad \bar\partial_{\tilde e} = \bar\partial_0 + \tilde e\,\partial \quad , \tilde e \in \Omega^{0,1} \otimes T$

$\rightsquigarrow$ constraint $P \cdot \partial X = 0$

Gauge freedom $\quad \delta(P,X) = \alpha\,\partial(P,X)$ etc.



BRST ⇝ Ghosts
b, c
$\tilde{b}, \tilde{c}$
$c, \tilde{c} \in T$
$b, \tilde{b} \in K^{2}$
Gauge orbits
gauge fixing
$S_{ghost} = \int b\,\bar{\partial} c + \tilde{b}\,\partial \tilde{c}$
fermion
BRST operator $Q = \oint c P^{2} + \tilde{c}\, T$
Stress-energy tensor
$T = P \cdot \partial X + b \partial c + \tilde{b} \partial \tilde{c}$
Q
P

$$T = P\cdot\partial X + b\partial c + \tilde{b}\partial\tilde{c}$$

$$\langle P_\mu(\sigma)\, X^\nu(\sigma')\rangle = \frac{\delta_\mu^\nu}{\sigma-\sigma'}$$

To compute amplitudes: For tree amplitudes

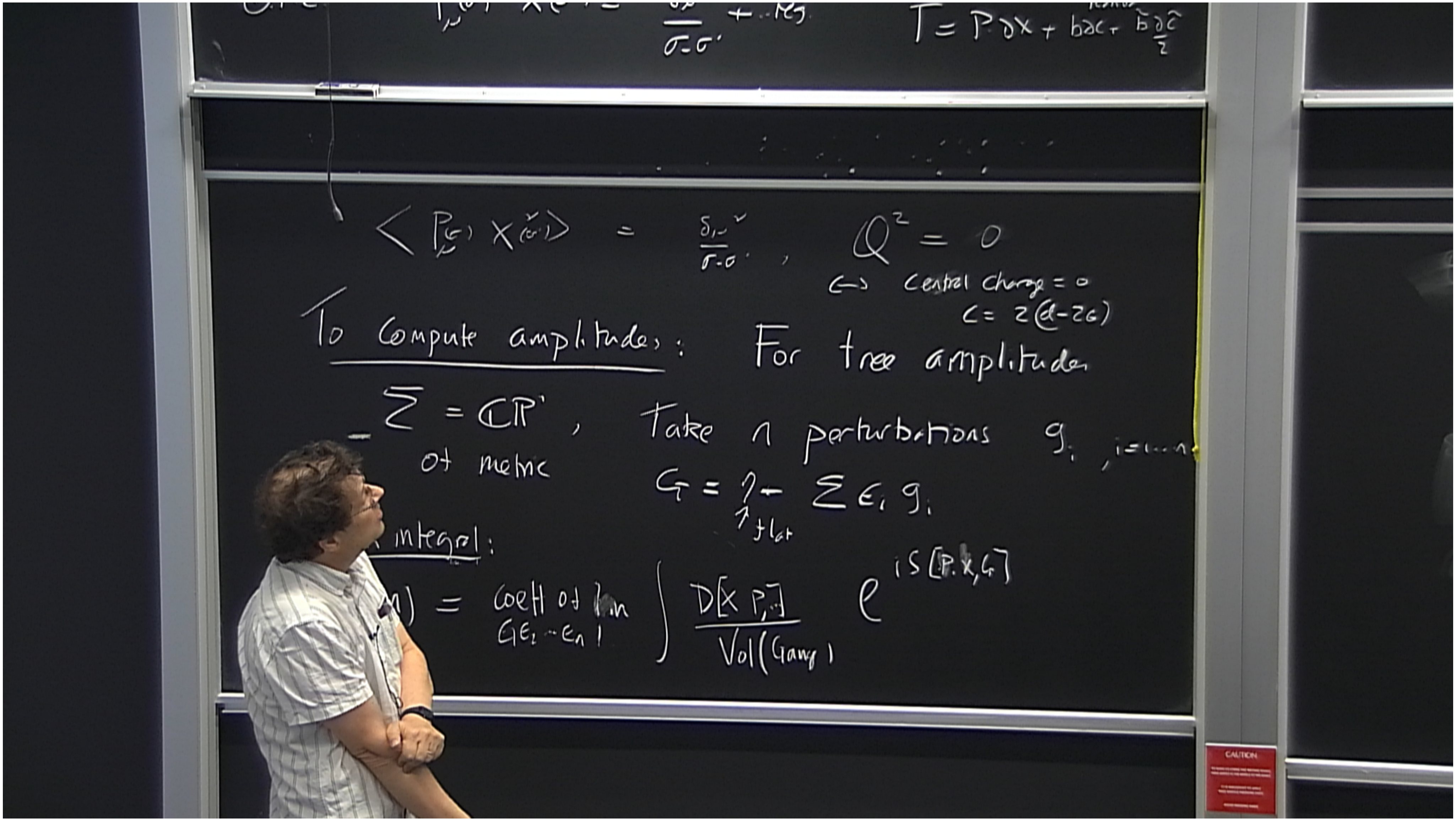
$\Sigma = \mathbb{CP}^1$, Take n perturbations g_i, $i = 1 \dots n$ of metric

$$G = \eta + \sum \epsilon_i g_i$$

(η: flat)

Path integral:

$$M(1,\dots,n) = \text{coeff of } \epsilon_1 \cdots \epsilon_n \text{ in } \int \frac{D[X, P, \dots]}{\mathrm{Vol}(\text{Gauge})}\, e^{iS[P, X, G]}$$


$T = P\partial X + b\partial c + \tilde{b}\partial\tilde{c}$
$\langle P_\mu(\sigma)\, X^\nu(\sigma') \rangle = \frac{\delta_\mu^\nu}{\sigma-\sigma'}$, $Q^2 = 0$
$\Leftrightarrow$ Central charge = 0
$c = 2(d-26)$
To Compute amplitudes:
For tree amplitudes
$\Sigma = \mathbb{CP}^1$,
Take n perturbations g_i , $i=1\ldots n$
of metric
$G = \eta + \sum \epsilon_i g_i$
flat
integral:
$=$ coeff of in $G\epsilon_1 \cdots \epsilon_n$
$\int \frac{D[X\, P, \ldots]}{\mathrm{Vol(Gauge)}}\, e^{iS[P,X,G]}$

$\frac{\delta}{\sigma-\sigma'}$ + ..reg. $T = P\partial X + b\partial c + \tilde{b}\partial\tilde{c}$

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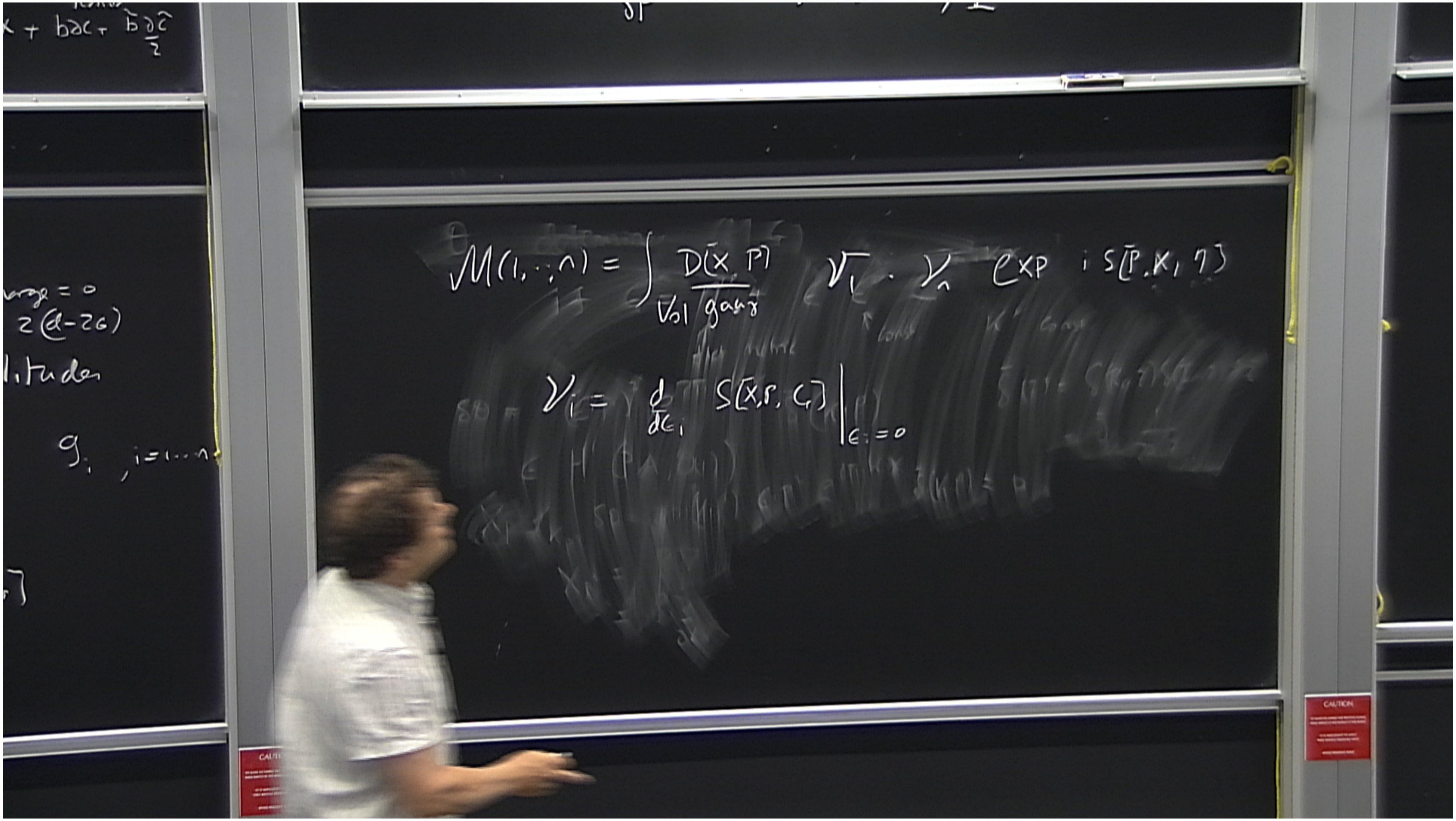
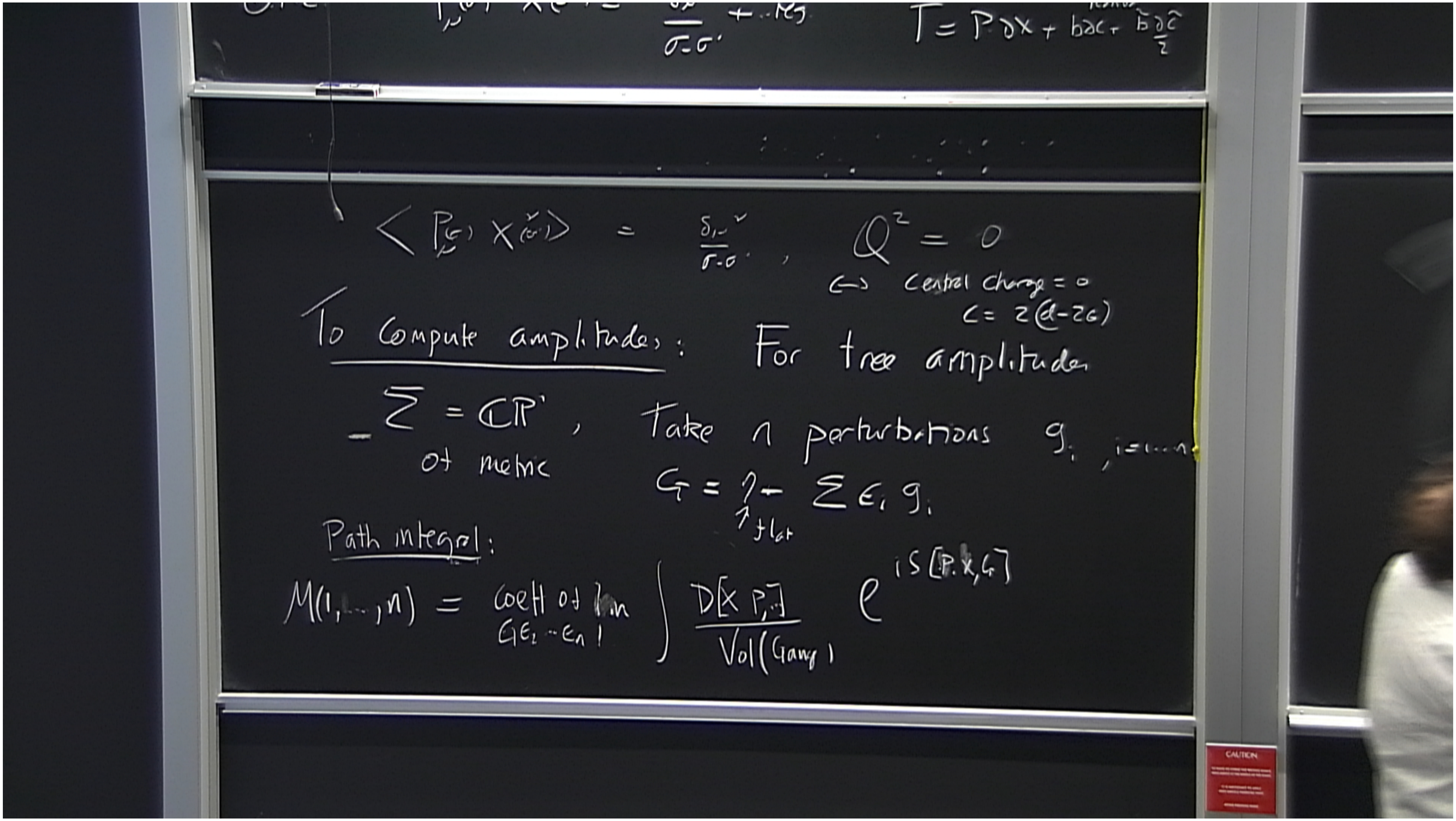

M(1,..,n) = ∫ D[X,P] / Vol gauge V_1 · V_n exp i S[P,X,γ]
V_i = d/dε_i S[X,P,ε_i] |_{ε_i=0}
z(d-26)
g_i , i=1...n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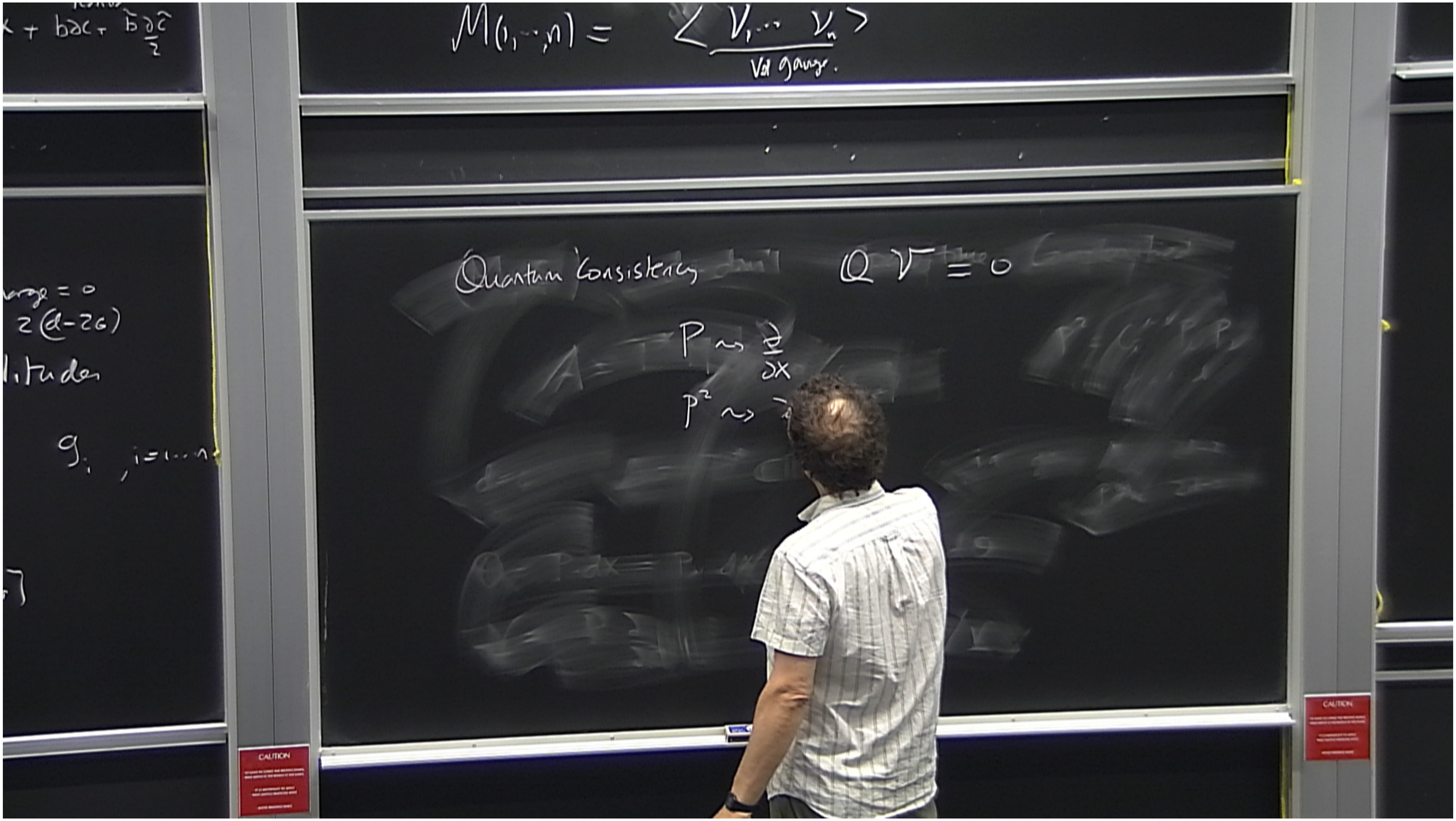
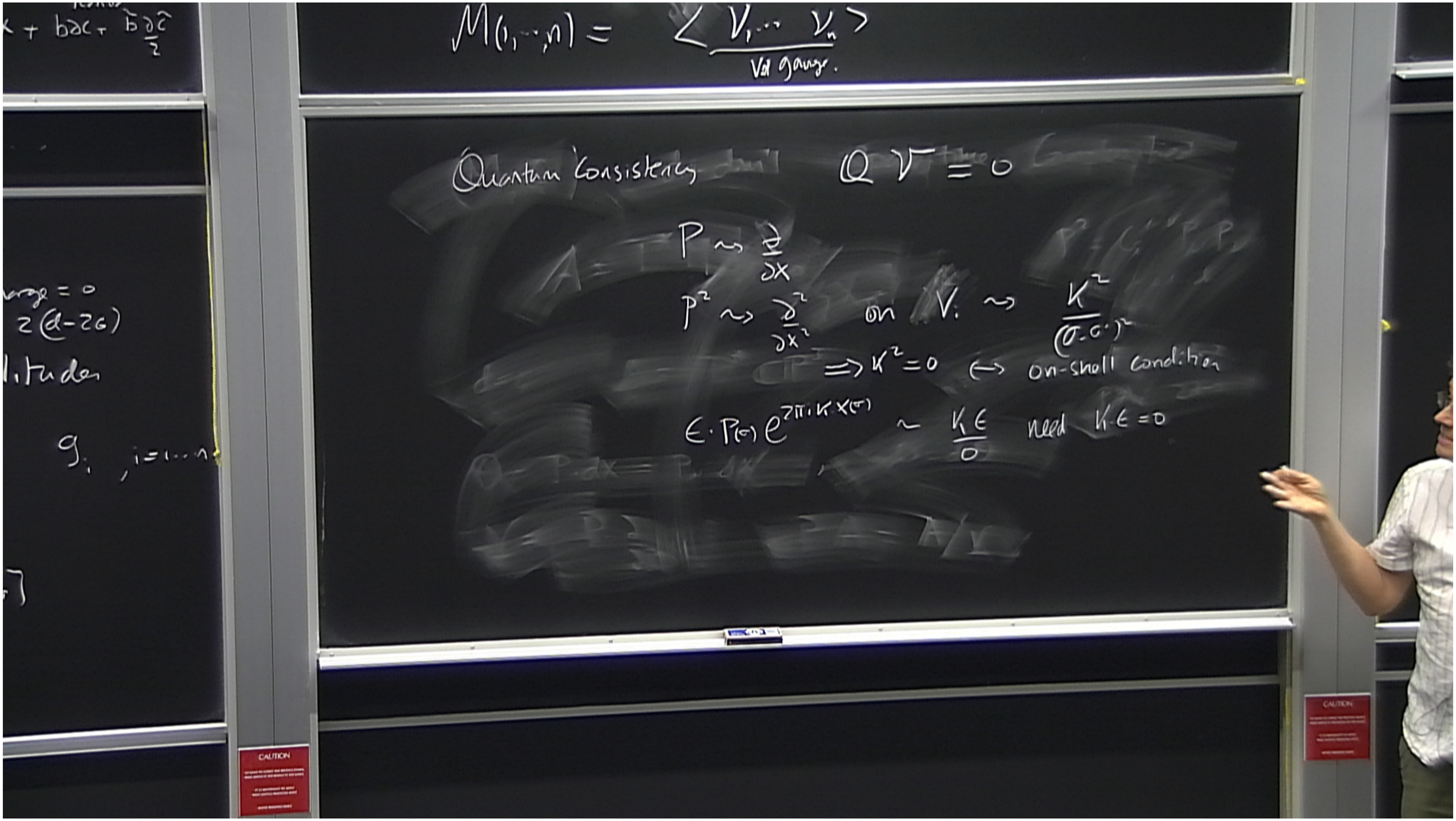
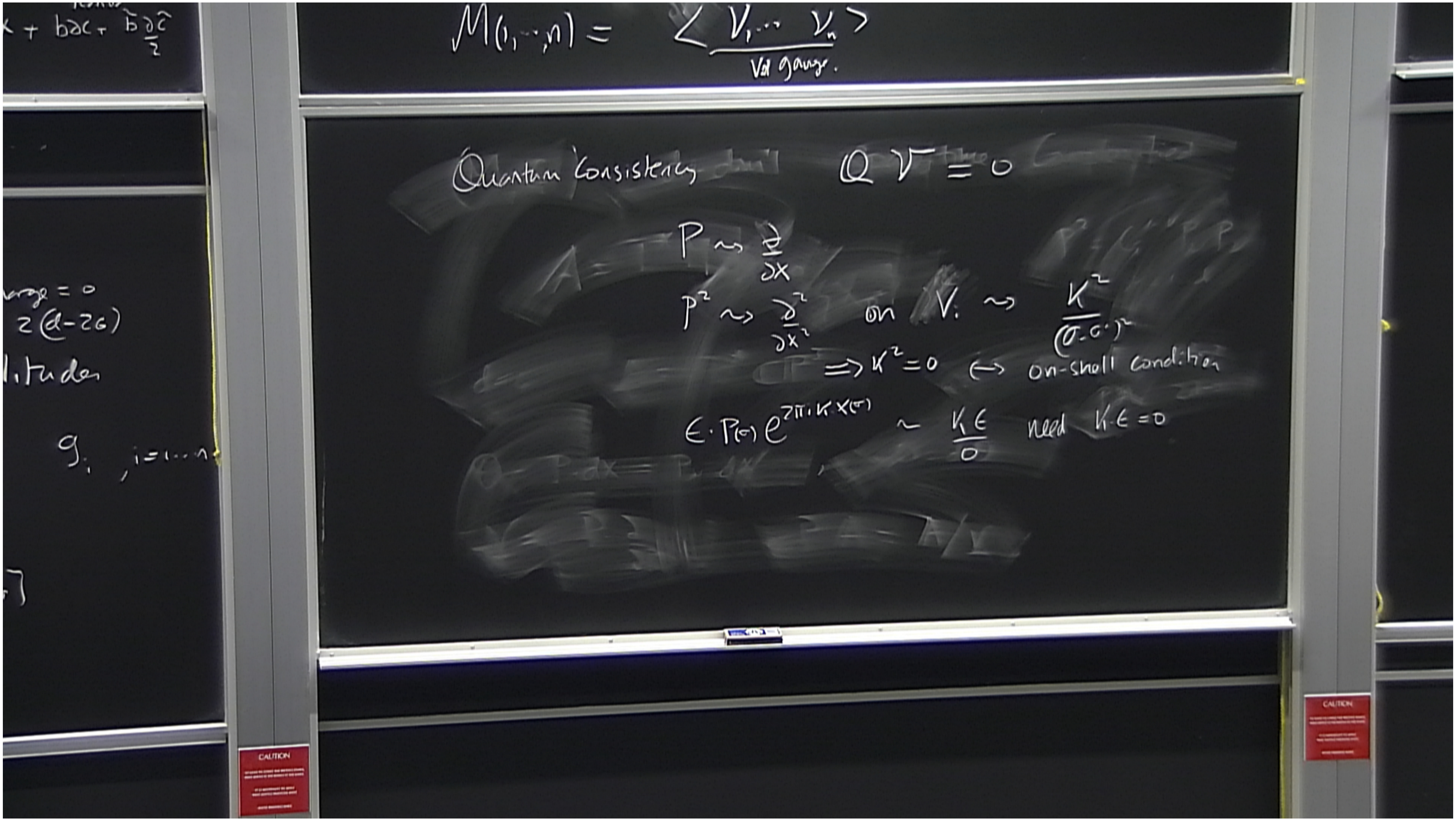
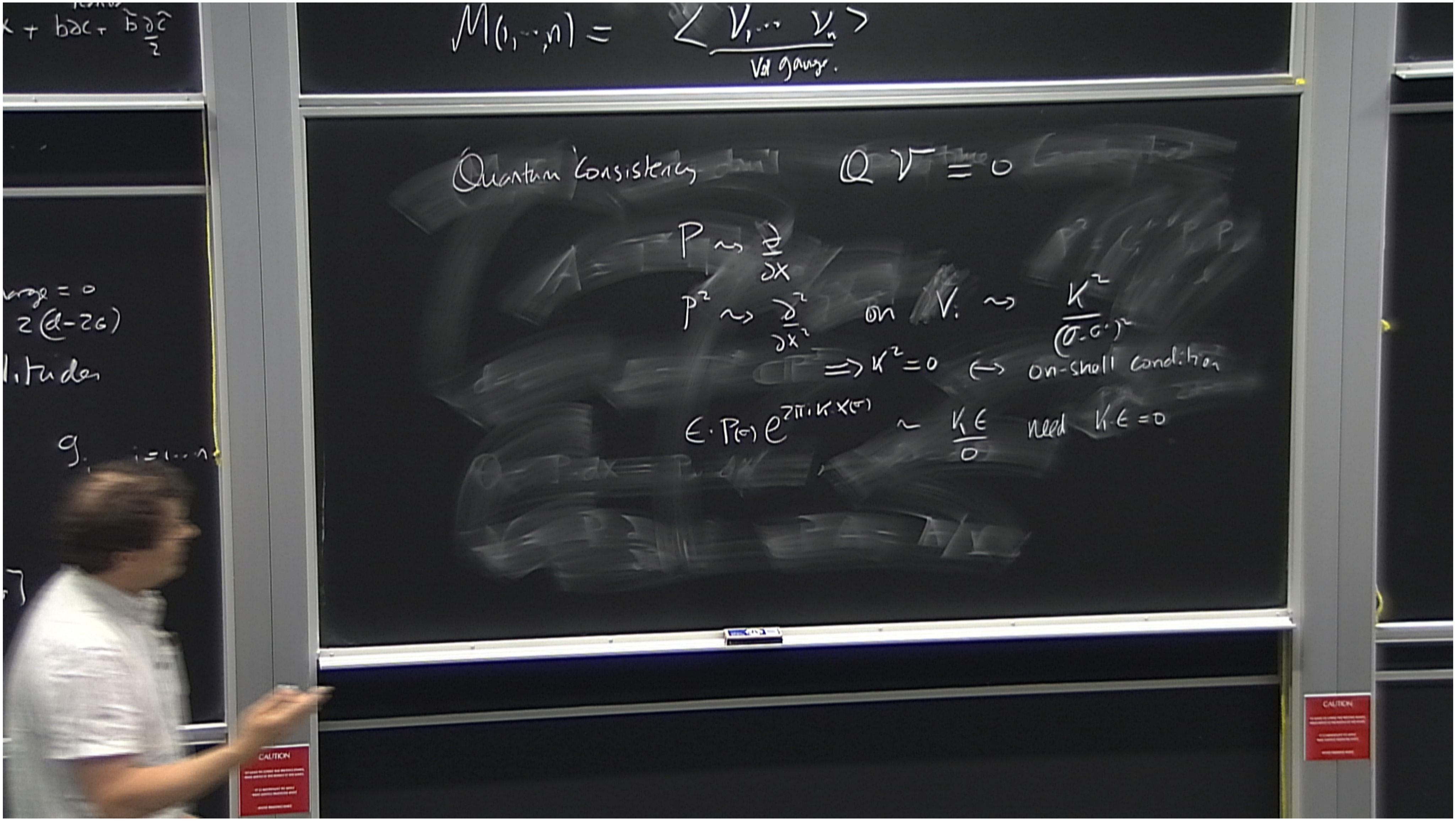

M(1,...,n) = < V_1 ... V_n > / Vol gauge.
Quantum Consistency
Q V = 0
P ~> ∂/∂x
p^2 ~>
z(d-26)
litudes



M(1,…,n) = < V_1 … V_n > / Vol gauge.
Quantum Consistency
QV = 0
P ~ ∂/∂X
P^2 ~ ∂^2/∂X^2 on V_i ~ K^2/(σ-σ_i)^2
⇒ K^2 = 0 ⟷ on-shell condition
ε·P(σ) e^{2πi K·X(σ)} ~ K·ε/0 need K·ε = 0



$M(1,\cdots,n) = \dfrac{\langle V_1 \cdots V_n \rangle}{\text{Vol gauge.}}$
Quantum Consistency $Q V = 0$
$P \rightsquigarrow \frac{\partial}{\partial X}$
$P^2 \rightsquigarrow \frac{\partial^2}{\partial X^2}$ on $V_i \rightsquigarrow \frac{K^2}{(\sigma-\sigma_i)^2}$
$\Rightarrow K^2 = 0 \Longleftrightarrow$ on-shell condition
$\epsilon \cdot P(\sigma)\, e^{2\pi i K \cdot X(\sigma)} \sim \frac{K \cdot \epsilon}{0}$ need $K \cdot \epsilon = 0$



$M(1,\dots,n) = \frac{\langle V_1 \dots V_n \rangle}{\text{Vol gauge}}$
Quantum Consistency
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$z(d-26)$

$+ \, b\partial c + \frac{\tilde{b}\partial \tilde{c}}{2}$

...rge $= 0$

$2(d-26)$

...litudes

$g_i \,, \; i = 1 \ldots n$

$$M(1, \ldots, n) = \frac{\langle V_1 \cdots V_n \rangle}{\text{Vol gauge}}$$

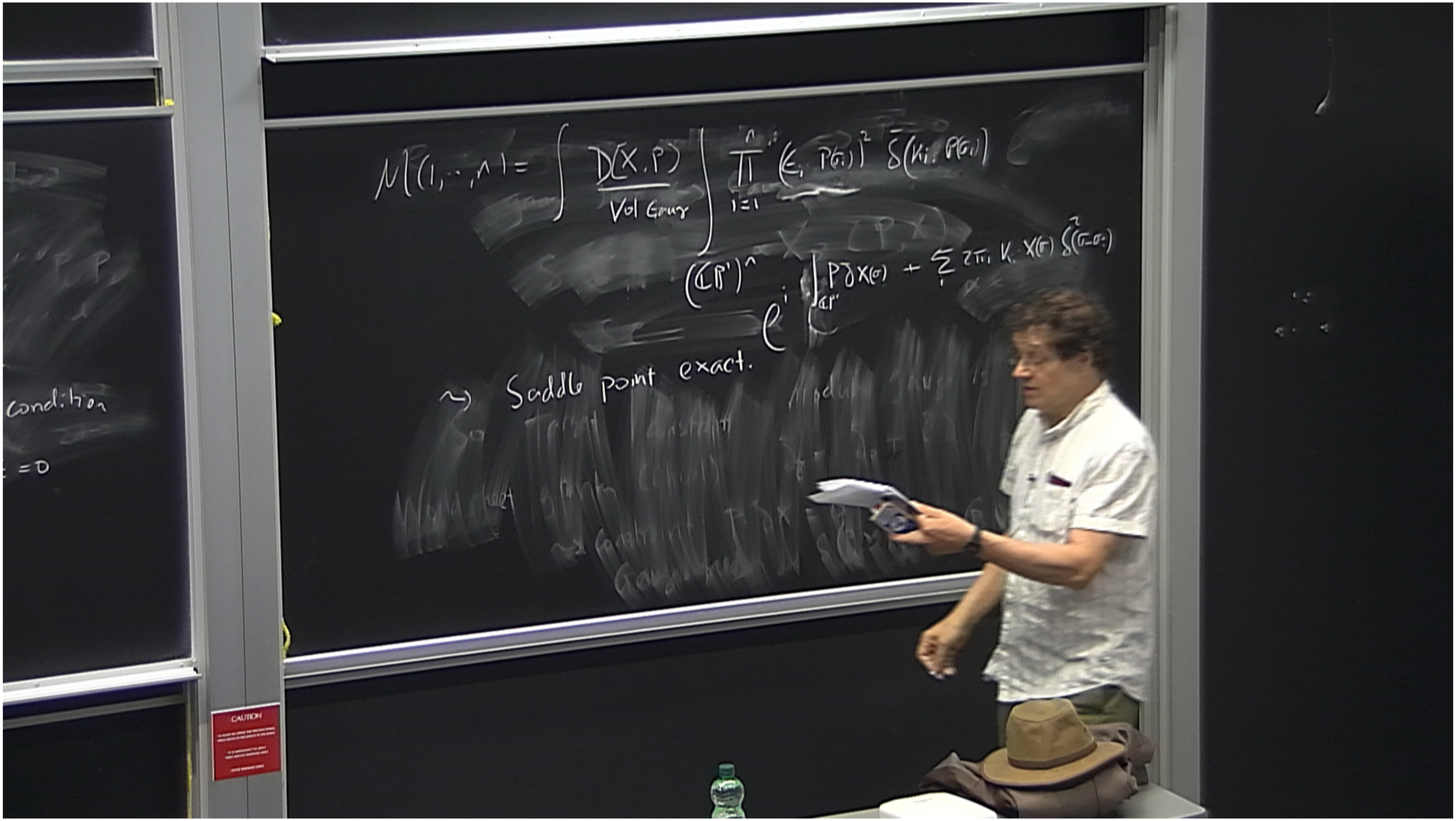
Quantum Consistency $\quad Q\,V = 0$

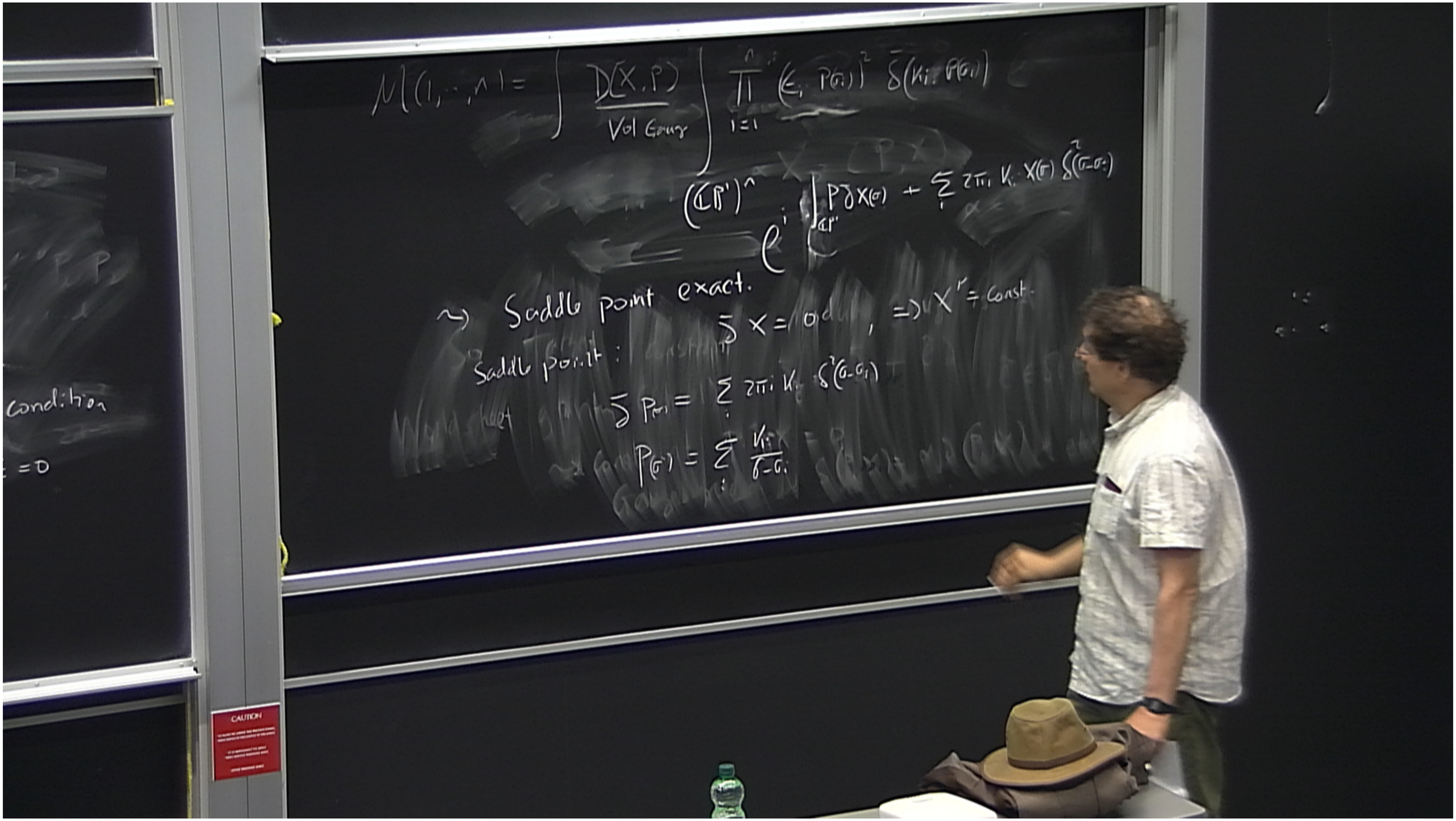
$$P \rightsquigarrow \frac{\partial}{\partial X}$$

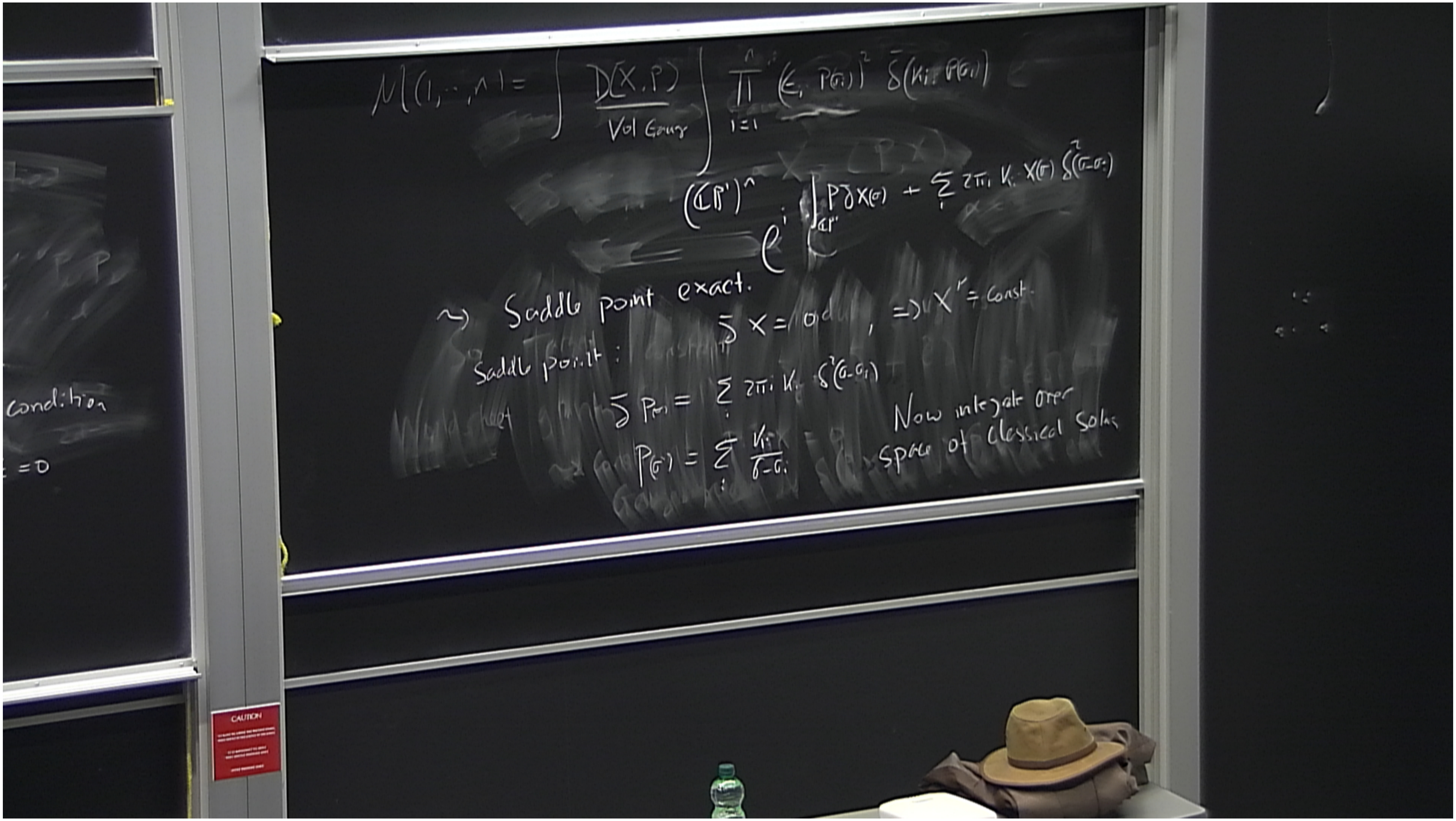
$$P^2 \rightsquigarrow \frac{\partial^2}{\partial X^2} \quad \text{on } V_i \rightsquigarrow \frac{K^2}{(\sigma - \sigma_i)^2}$$

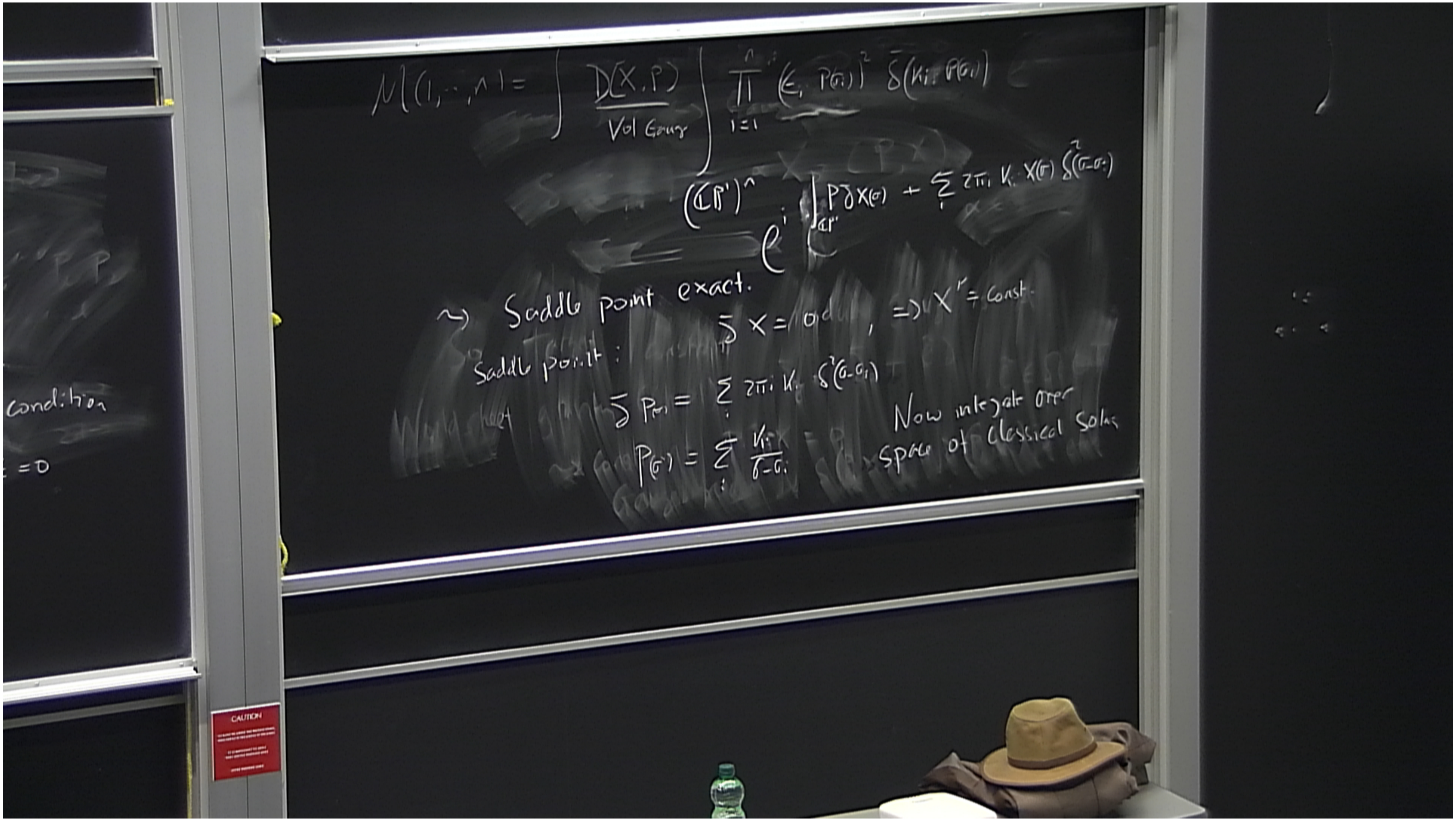
$$\Rightarrow K^2 = 0 \longleftrightarrow \text{on-shell condition}$$

$$\epsilon \cdot P(\sigma)\, e^{2\pi i K \cdot X(\sigma)} \sim \frac{K \cdot \epsilon}{0} \quad \text{need } K \cdot \epsilon = 0$$


Saddle point exact.
condition
CAUTION


Vol Gauge
Saddle point exact.
Saddle point :
Const.
condition
CAUTION


Saddle point exact.
Saddle point :
Now integrate over
space of classical Solns
condition
Vol Gauge


Vol Gauge
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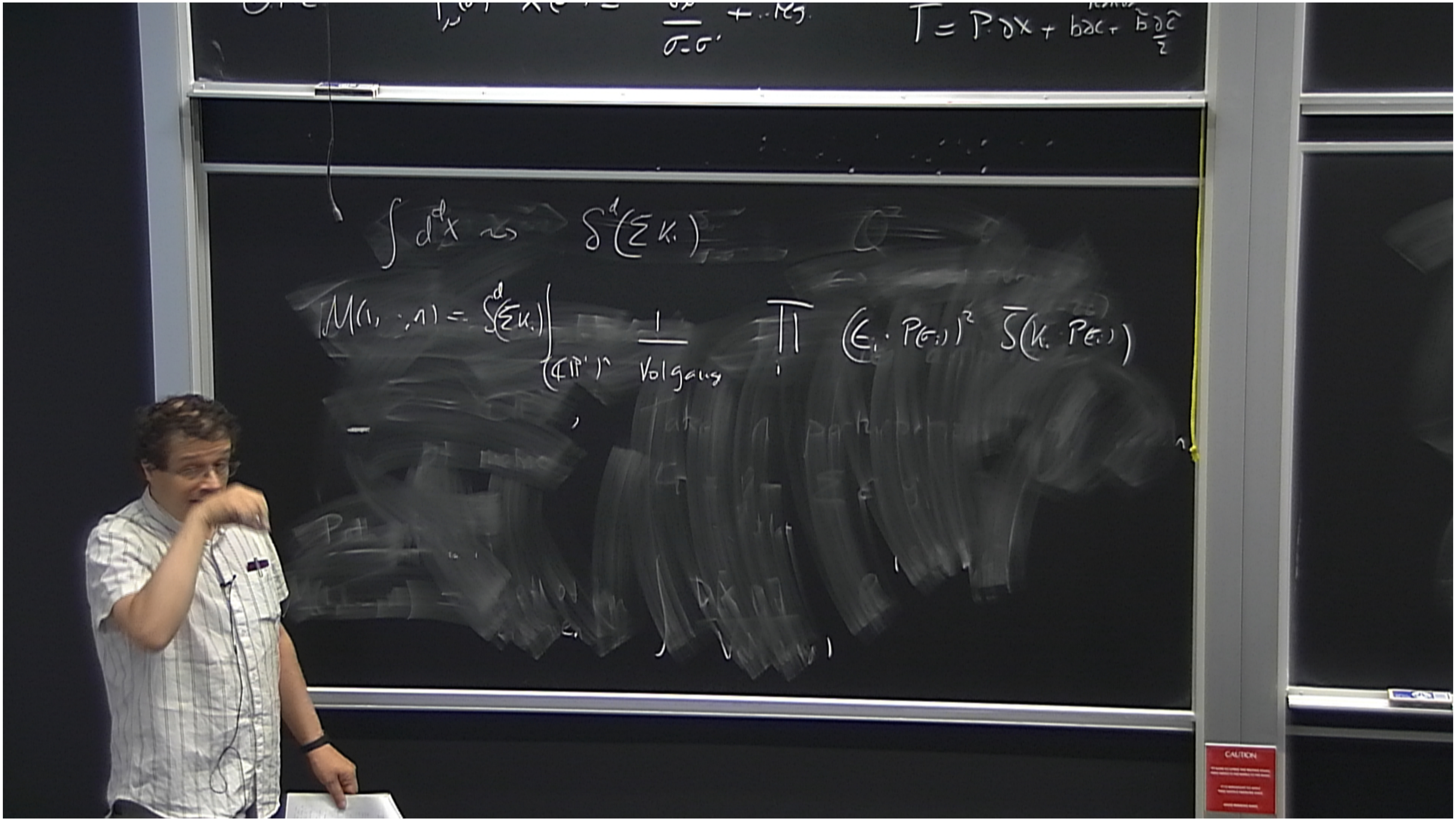
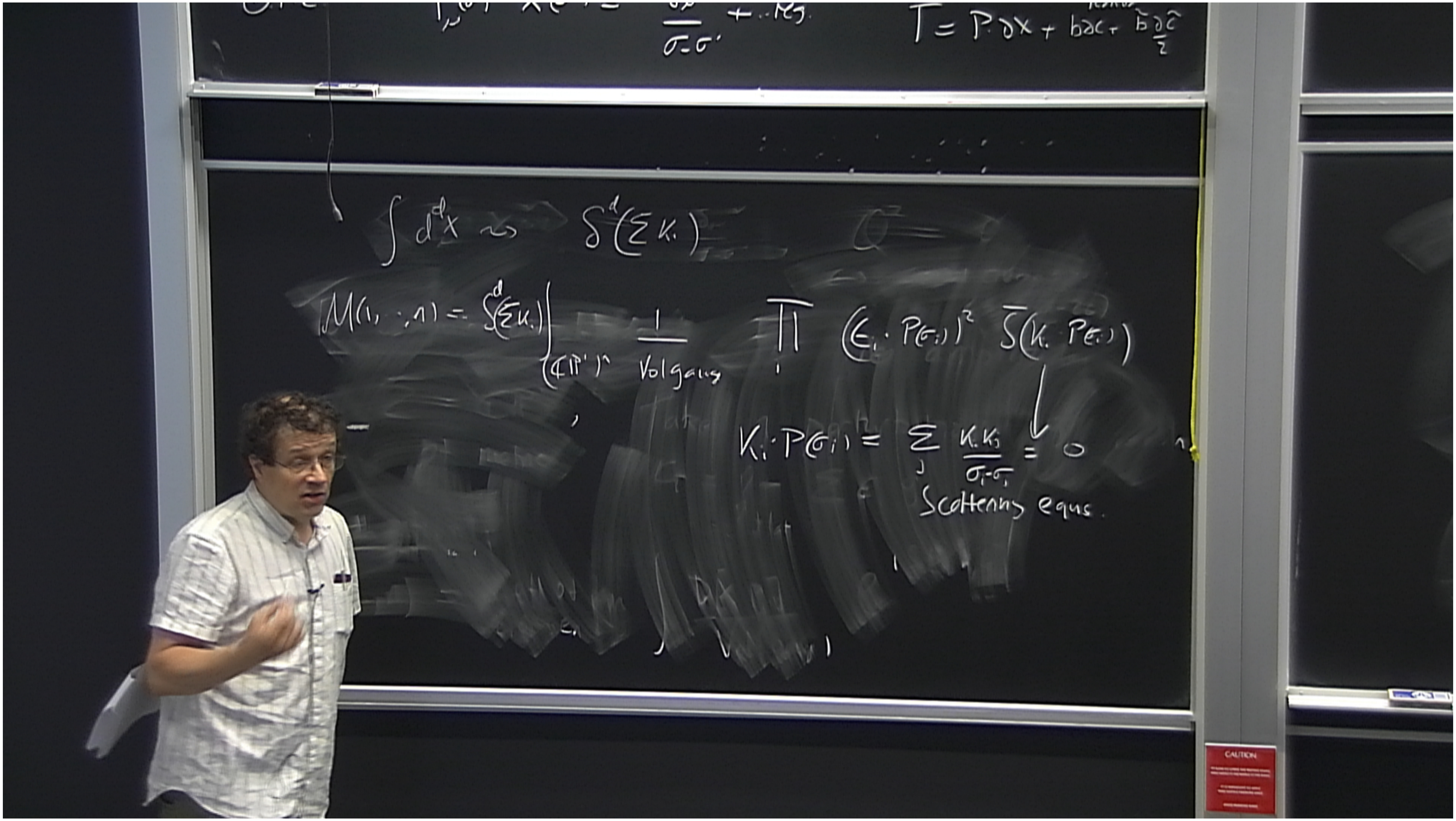
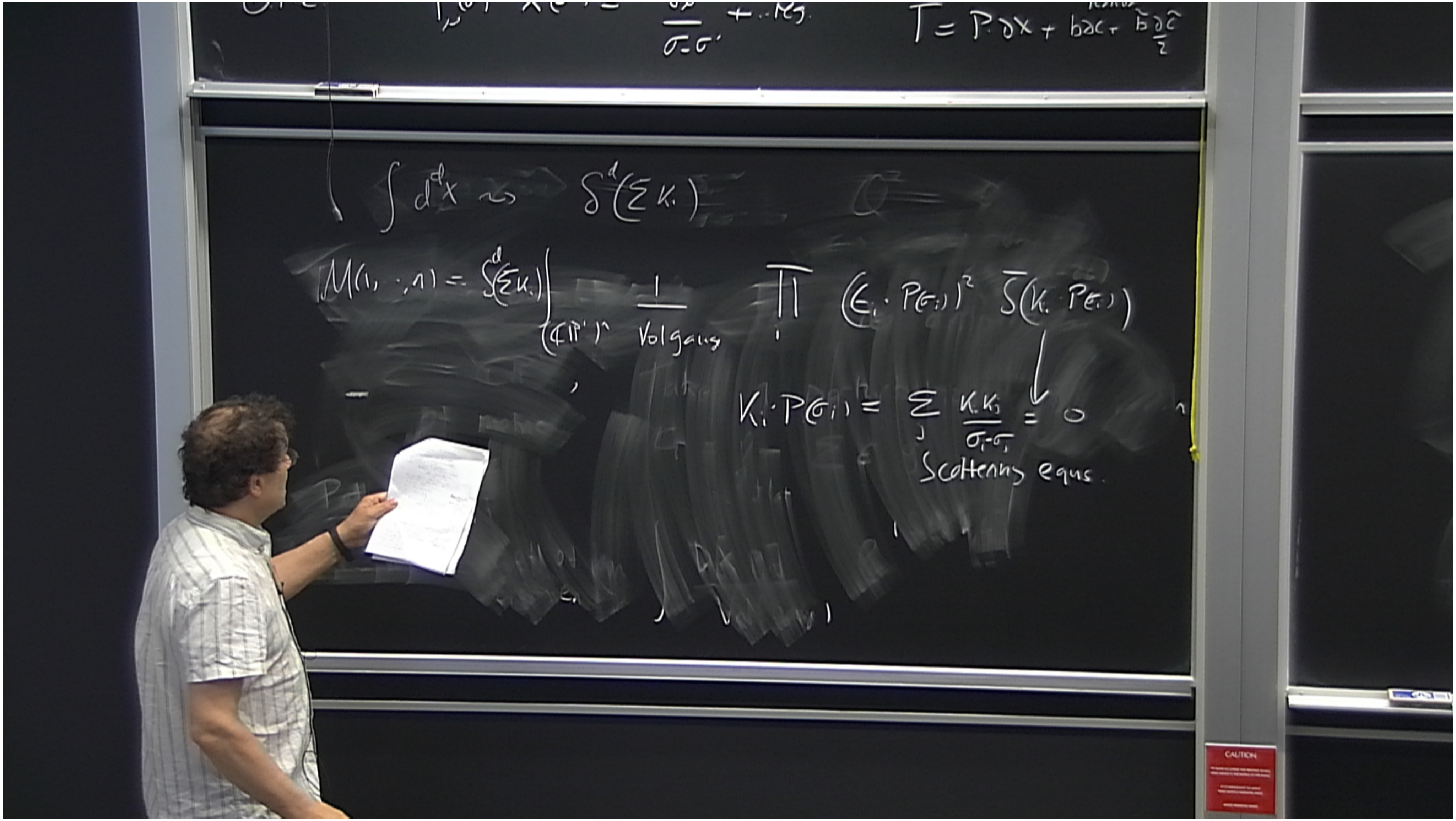


∫ d^d x ⇝ δ^d(Σ k_i)
M(1, …, n) = δ^d(Σ k_i) 1/Vol gauge ∏ (ε_i · P(σ_i))^2 δ̄(k_i · P(σ_i))
T = P·∂X + b∂c + b̃∂c̃/2


Vol gauge
Scattering equs.


T = P·∂X + b∂c + b̃∂c̃
∫ d^d x ⇝ δ^d(Σ k_i)
M(1, …, n) = δ^d(Σ k_i) 1/Vol gauge ∏_i (ϵ_i · P(σ_i))^2 δ̄(k_i · P(σ_i))
k_i · P(σ_i) = Σ_j (k_i·k_j)/(σ_i − σ_j) = 0
Scattering equs.

$$\frac{\ldots}{\sigma-\sigma'} + \ldots \text{Reg.} \qquad T = P\cdot\partial X + b\partial c + \tilde{b}\partial\tilde{c}$$

$$\int d^d x \rightsquigarrow \delta^d\left(\sum k_i\right)$$

$$\mathcal{M}(1,\ldots,n) = \int_{(\mathbb{CP}^1)^n} \delta^d\left(\sum k_i\right) \frac{1}{\text{Vol gauge}} \prod_i \left(\epsilon_i \cdot P(\sigma_i)\right)^2 \bar{\delta}\left(k_i \cdot P(\sigma_i)\right)$$

$$k_i \cdot P(\sigma_i) = \sum_j \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$$

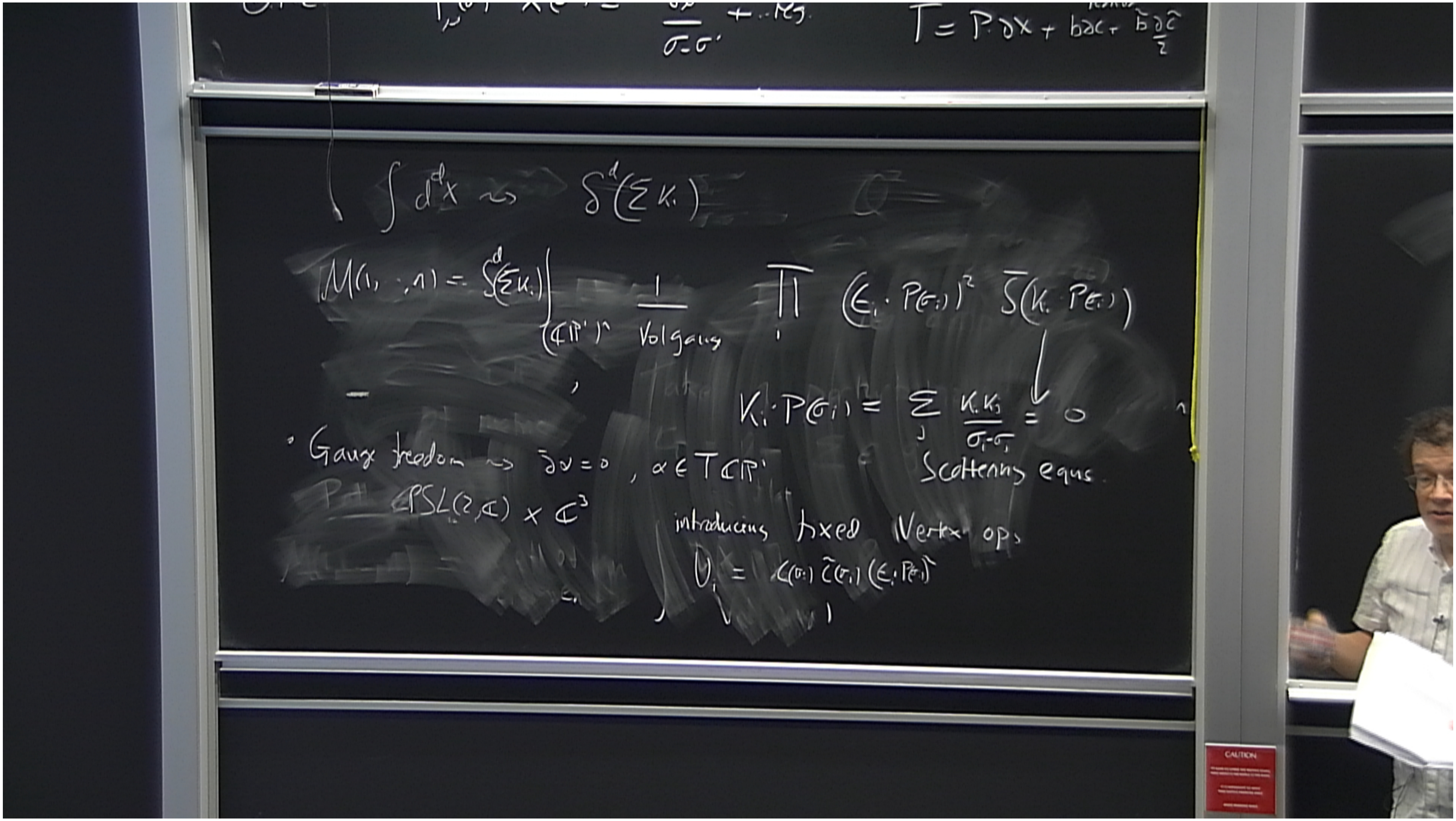
Scattering equs.

- Gauge freedom $\rightsquigarrow \bar{\partial}\alpha = 0$, $\alpha \in T\mathbb{CP}^1$

$PSL(2,\mathbb{C}) \times \mathbb{C}^3$

introducing fixed Vertex ops

$$\mathcal{U}_i = c(\sigma_i)\,\tilde{c}(\sigma_i)$$


$T = P\cdot\partial X + b\partial c + \frac{\tilde b \partial \tilde c}{2}$
$\int d^d x \rightsquigarrow \delta^d(\Sigma k_i)$
$M(1,\dots,n) = \delta^d(\Sigma k_i) \int_{(\mathbb{CP}^1)^n} \frac{1}{\mathrm{Vol\,gauge}} \prod_i (\epsilon_i \cdot P(\sigma_i))^2\, \bar\delta(k_i \cdot P(\sigma_i))$
$k_i \cdot P(\sigma_i) = \sum_j \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$
Scattering equs.
Gauge freedom $\rightsquigarrow \bar\partial \alpha = 0$, $\alpha \in T\mathbb{CP}^1$
$PSL(2,\mathbb{C}) \times \mathbb{C}^3$
introducing fixed vertex ops
$U_i = c(\sigma_i)\,\tilde c(\sigma_i)\,(\epsilon_i \cdot P(\sigma_i))^2$

$$\mathcal{M}(1,\dots,n) = \frac{\langle V_1 \cdots V_n \rangle}{\text{Vol gauge}}.$$

Introduce World sheet matter

$$S_{full} = S_{boson} + S_L + S_R$$

$\sim$ Vertex ops $\quad V = \psi_L \, \psi_R \, e^{2\pi i k\cdot x} \, \bar{\delta}(k\cdot P)$