Title: Ambitwistors-strings and amplitudes

Date: May 28, 2015 09:30 AM

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Abstract: These lectures will focus on the geometry of ambitwistor string theories. These are infinite tension analogues of conventional strings and provide the theory that leads to the remarkable formulae for tree amplitudes that have been developed by Cachazo, He and Yuan based on the scattering equations. Although the bosonic ambitwistor string action is expressed in space-time, it will be seen that its target is classically `ambitwistor space', the space of complexified null geodesics in the complexification of a space-time. The lectures will review Ambitwistor constructions from the 70's and 80's that extend the Penrose-Ward twistor constructions for self-dual Yang-Mills and gravitational fields in four dimensions to arbiitrary fields in general dimension. LeBrun showed that the conformal geometry of a space-time is encoded into the complex structure of ambitwistor space. The linearized version encodes linear fields on space-time into sheaf cohomology classes on ambitwistor space. In the case of momentum eigenstates, these give the `scattering equations' that underly the CHY formulae and the ambitwistor string can be used to compute amplitudes via these formulae. If there is time, the lectures will discuss how different matter theories can be obtained, different geometric realizations of ambitwistor space lead to different formulae, the relationship between the asymptotic symmetries of space-time and Weinberg's soft theorems concerning the behaviour of amplitudes when momenta become small, and/or extensions of the ideas to loop amplitudes.

Ambitwistors &

1311.2564, 1312.3828

1404.6219, 1405.5127, 1406.1462

+ Cachazo, He & Yuan ....

Joint With

Dave Skinner

+ Collaborations with:

Tim Adamo

Eduardo Casali, Yvonne Geyer

A Lipstein, R. Monteiro

1. Ambitwistors from the Last century
2. Ambitwistor Strings
3. Extensions & Applications

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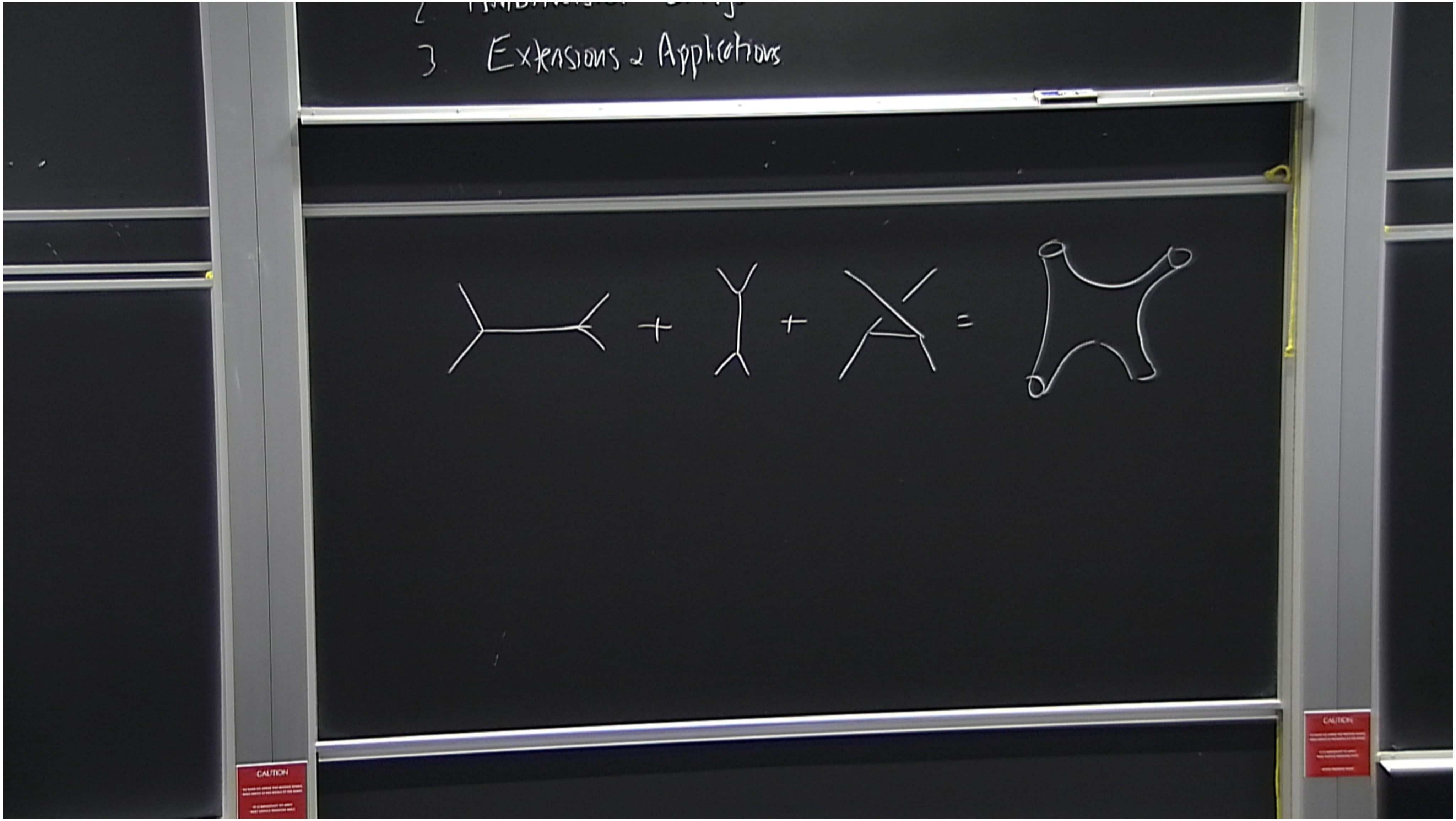
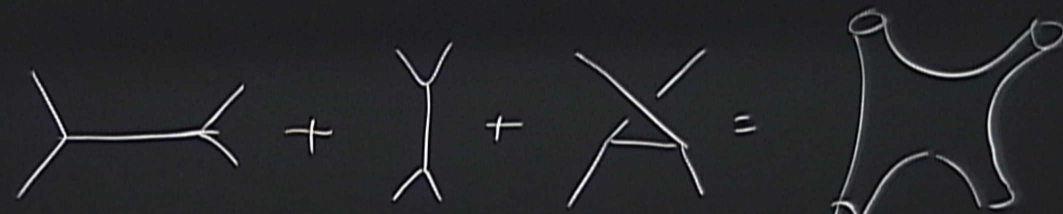

3 Extensions & Applications

3. Extensions & Applications



Witten, Isenberg Yasskin & Green 1978 Yang-Mills

Le Brun 1983 + M 1986, 1996

Gravity

Ambitwistor space $\mathbb{A}$ = Space of complex null geodesic in a complexified space-time

Given analytic real spacetime $(M_{\mathbb{R}}, g_{\mathbb{R}})$

d-dim mfd

metric

extend coords x^μ from $\mathbb{R}^d$ to $\mathbb{C}^d$

Transition functions & metric have finite radius of convergence

extend to some small thickening $\rightsquigarrow$ d-dim $\mathbb{C}$-mfd M

holomorphic metric $g_{\mu\nu}$, $\mu = 1 \ldots d$.

is a ... symplectic manifold.

Let (P_μ, X^μ) be hol. coords on T^*M

On T^*M, Symplectic potential $\Theta = P_\mu dX^\mu$

Symplectic form $\omega = d\Theta$

$= dP_\mu \wedge dX^\mu$

Euler vector field $\underline{Y} = P_\mu \frac{\partial}{\partial P_\mu}$

generates scalings $P \to \alpha P$.

Geodesics are Hamiltonian flow for $P^2 = G^{\mu\nu} P_\mu P_\nu$

generated by Hamiltonian vector field

$$X_{P^2}, \qquad X_{P^2} \lrcorner\, \omega + dP^2 = 0$$

Contraction of vector into form

Integral curves of X_{P^2} = { Geodesic with $\parallel^{l}$ propagated P_μ, $G^{\mu\nu}P_\nu$ tangent to curve }

$$\mathbb{A} = T^*M\big|_{p^2=0} \Big/ \{X_{p^2}\}$$

On $p^2 = 0$: $\quad \mathcal{L}_{X_H}\omega = 0, \quad X_H \lrcorner\, \omega = -dp^2 = 0$

$$\mathcal{L}_{X_H}\theta = 0, \qquad [X_H, \Upsilon] = X_H$$

So $(\theta, \omega, \Upsilon)$ descend to $\mathbb{A}$

$$\omega = d\theta, \qquad \theta = \Upsilon \lrcorner\, \omega$$

$$\mathbb{A}^{2d-2} = T^*M\big|_{p^2=0} \Big/ \{X_{p^2}\}$$

On $p^2 = 0$: $\quad \mathcal{L}_{X_H}\omega = 0 \quad , \quad X_H \lrcorner \omega = -dp^2 = 0$

$$\mathcal{L}_{X_H}\theta = 0 \qquad [X_H, \underline{\Upsilon}] = X_H$$

So $(\theta, \omega, \underline{\Upsilon})$ descend to $\mathbb{A}$

$$\omega = d\theta, \quad \theta = \underline{\Upsilon} \lrcorner \omega$$

holomorphic

$P\mathbb{A}^{2d-3} = \mathbb{A}/\underline{\Upsilon}$, Line bundles $\mathcal{O}(n) \to P\mathbb{A}$

$\mathcal{O}(n) = \{$functions hgs weight n in $P\}$

$\mathbb{A} \longrightarrow P\mathbb{A}$ on $P\mathbb{A}$ $\quad \theta \in \Omega^{1,0} \otimes \mathcal{O}(1)$

$\parallel$

$\mathcal{O}(-1)$

defines a contact structure (holomorphic)

$$\theta \wedge (d\theta)^{d-2} \neq 0.$$

Rmk: · Locally no info in this structure by hol. analogue of Darboux.

$\mathcal{O}(n) = \{$functions hgs weight n in $\mathbb{P}\}$

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extend to ... holomorphic metric ...

$\mathcal{O}(n) = \{$ functions hgs weight n in $P\}$

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Rmk:
- Locally no info in this structure by hol. analogue of Darboux.
- Conformally invariant only requires $[G]$ conformal equ class $G \sim \Omega^2 G$ $\quad \Omega \neq 0$ fn on M.

Thm: [LeBrun 1983]

1. The original $(M,[G])$ can be reconstructed from $\mathbb{C}$-str of $\mathbb{P}\mathbb{A}$

2. Stable under small deformations of $\mathbb{C}$-str of $\mathbb{P}\mathbb{A}$ but must allow $(M,[G],[\nabla])$ null geodesics of a torsion connection $[\nabla]$.

3. $[\nabla]$ is torsion-free if deformation preserves $\exists\ \Theta$.

null geodesics of a torsion connection $[\nabla]$.

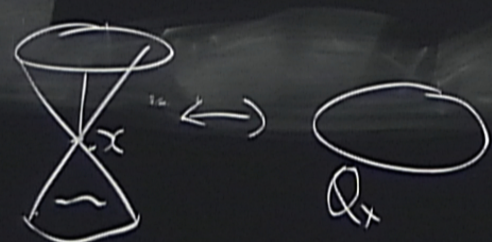
3. $[\nabla]$ is torsion-free iff deformation preserves $\exists\, \Theta$. contact str.

Sketch Proof: Kodaira theory

1. Each $x \in M \longleftrightarrow$ Projective lightcone

$$Q_x = \{P,\ \underbrace{P^2=0}_{}\} \subset PT^*_x M$$

$\uparrow$ compact $\mathbb{C}$-mfd dim $d-2$



Given $Q_x \subset$ ...

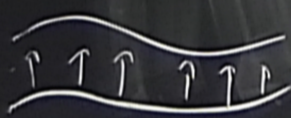
reconstruct $M = \{$ Moduli space of deformations of $Q_x \}$

N_x = Normal bundle of $Q_x \subset \mathbb{P}A$

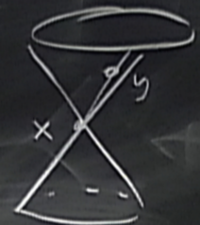
$$\begin{array}{c} N_x \\ \downarrow \\ Q_x \end{array}$$

<u>Kodaira</u>: If $H^1(Q_x, N_x) = 0 = H^1(Q_x, \mathrm{End}\, N_x)$

Then $\exists$ a manifold M of deformations of Q_x



$$T_x M = H^0(Q_x, N_x)$$

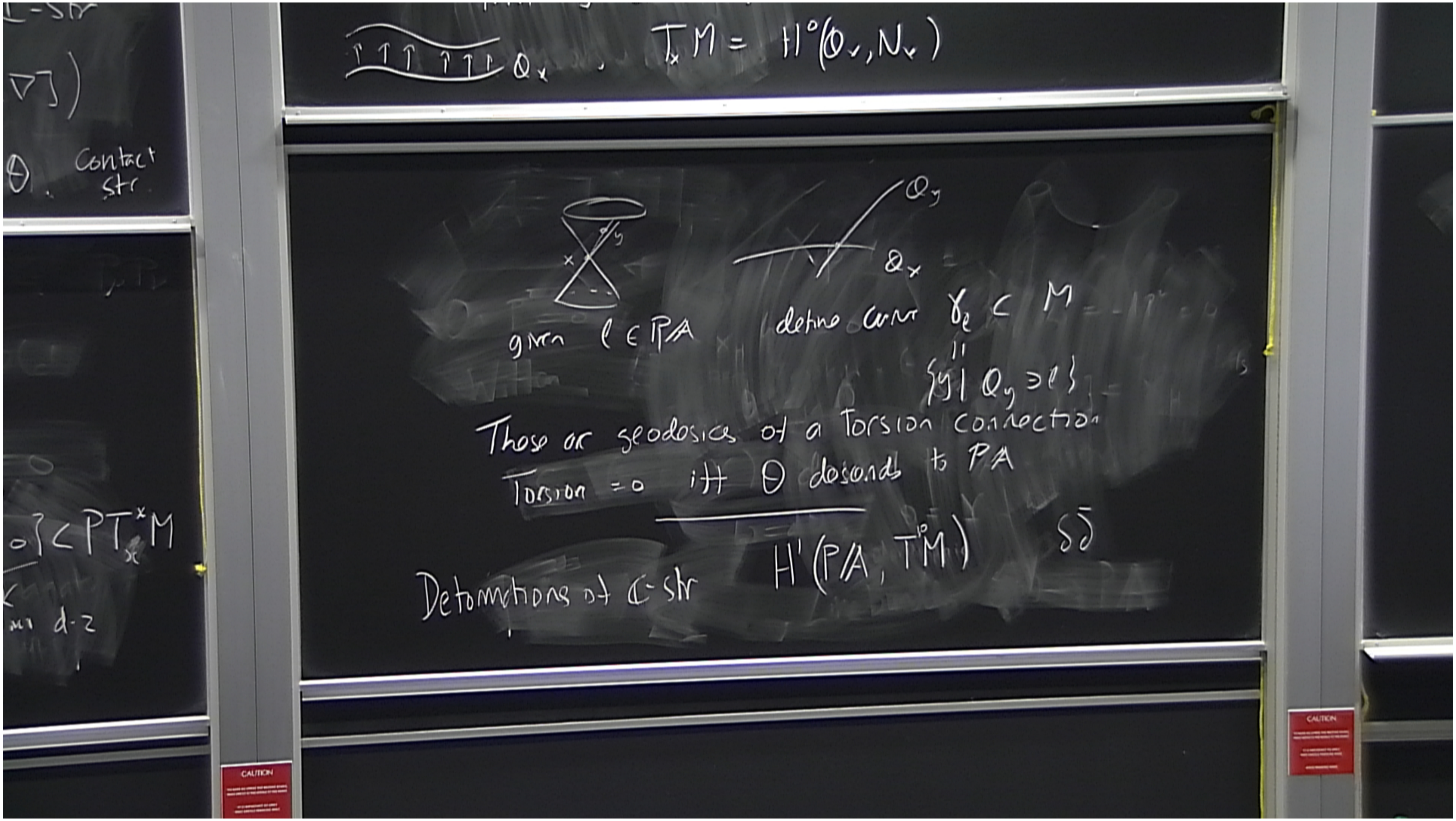


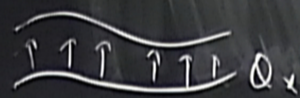
given $\ell \in \mathbb{P}A$ define curve $\gamma_\ell \subset M$

$$\gamma_\ell = \{y \mid Q_y \ni \ell\}$$

Those are geodesics of a Torsion connection

Torsion $= 0$ iff Θ descends to $\mathbb{P}A$


Q_x
T_x M = H^0(Q_x, N_x)
Contact str
Q_y
Q_x
given l ∈ PA
define curve γ_l ⊂ M
= {y | Q_y ∋ l}
Those are geodesics of a Torsion connection
Torsion = 0 iff Θ descends to PA
H^1(PA, T^{1,0}M)
Deformations of C-str
PT*_x M



$T_x M = H^0(Q_x, N_x)$

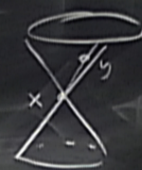
ℂ-str

$[\nabla])$

Θ. Contact str

$=0\} \subset \mathbb{P}T^*_x M$

dim $d-2$



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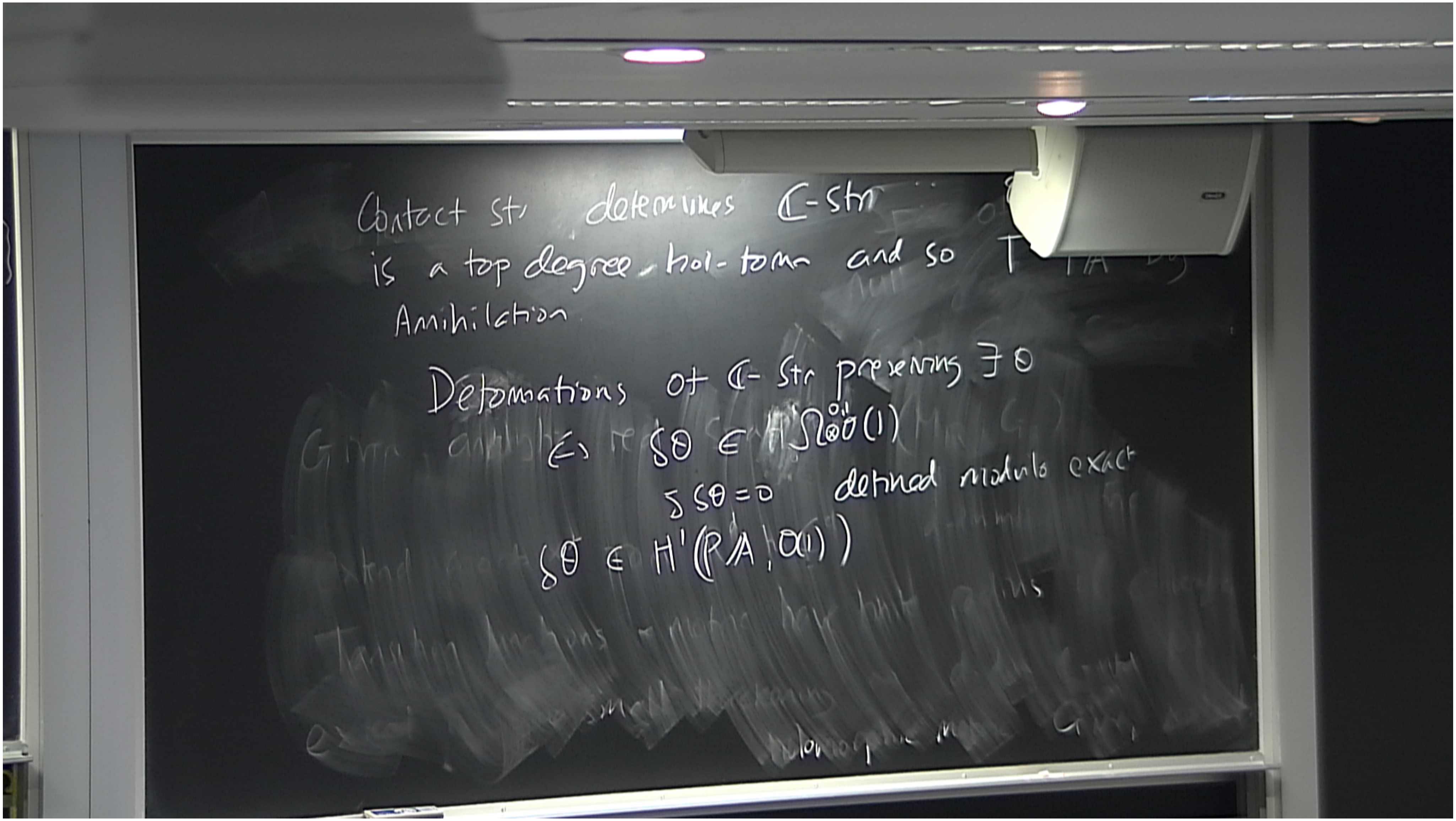
Deformations of ℂ-str $\quad H^1(\mathbb{P}A, T^{1,0}\mathbb{P}A) \quad \delta\bar{\partial}$


Contact str determines ℂ-str
is a top degree hol-form and so
Annihilation
Deformations of ℂ-str preserving ∃ θ
⟺ δθ ∈ H¹Ω⁰¹⊗O(1)
δδθ = 0 defined modulo exact
δθ ∈ H¹(ℙA, O(1))

$\mathcal{O}(n) = \{\text{functions hgs weight } n \text{ in } \mathbb{P}\}$

<u>Propn</u>: $$H^1(\mathbb{P}A, \mathcal{O}(n)) = \frac{\{\text{Trace-free symmetric tensors } g^{\mu_1 \dots \mu_n}(x) \text{ on } M\}}{\{\nabla^{(\mu_1} f^{\mu_2 \dots \mu_n)}(x)\}}$$

Propn: $H^1(\mathbb{P}A, \mathcal{O}(n)) = \dfrac{\{\text{Trace-free symmetric tensors } g^{\mu_1 \dots \mu_n}(x) \text{ on } M\}}{\{\text{trace-free part of } \nabla^{(\mu_1} f^{\mu_2 \dots \mu_n)}(x)\}}$

$$PT^*M|_{p^2=0} = QM$$

$$\begin{array}{ccc} & QM & \\ \swarrow_{\pi_1, \, P\cdot D} & & \searrow \\ \mathbb{P}A & & M \end{array}$$

$$0 \to \mathcal{O}_{\mathbb{P}A}(n) \xrightarrow{\pi_1^*} \mathcal{O}_{QM}(n) \xrightarrow{P\cdot\nabla} \mathcal{O}_{QM}(n+1) \to 0$$

$\rightsquigarrow$ Long exact sequence $H^0(QM, \mathcal{O}(n)) \xrightarrow{P\cdot D} H^0(QM, \mathcal{O}(n+1)) \xrightarrow{\delta} H^1(\mathbb{P}A, \mathcal{O}(n)) \to 0$

Ex: $g^{\mu_0 \cdots \mu_n}(x) = \epsilon^{\mu_0}\epsilon^{\mu_1}\cdots\epsilon^{\mu_n}\, e^{2\pi i k\cdot x}$

$$g = g^{\mu_0 \cdots \mu_n}(x)\, P_{\mu_0}\cdots P_{\mu_n} = (\epsilon\cdot P)^{n+1}\, e^{2\pi i k\cdot x}$$

Want h in $\Theta(n)$ s.t. $P\cdot\nabla h = g$

$$h = \frac{(\epsilon\cdot P)^{n+1}}{2\pi i (k\cdot P)}\, e^{2\pi i k\cdot x}$$

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$$\delta(g) = V = \bar\partial h = (\epsilon\cdot P)^{n+1} e^{2\pi i k\cdot x} \,\bar\delta(k\cdot P)$$

$$\bar{\delta}(z) = \bar{\partial}\, \frac{1}{2\pi i z} = \delta(\mathrm{Re}\, z)\, \delta(\mathrm{Im}\, z)\, d\bar{z}$$

- $P \cdot \nabla V = 0 \qquad V \in H^1(\mathbb{PA}, \mathcal{O}(n))$
- For $n=2 \qquad g = \delta(P^2) \qquad g = \delta G$
- δ-fn support $\quad \bar{\delta}(k \cdot P) \Rightarrow$ Scattering equ.

$$\underset{\uparrow_{Z \cdot W}}{\mathbb{PA}} \hookrightarrow \underset{Z}{\mathbb{CP}^3} \times \underset{W}{\mathbb{CP}^{3*}}$$