

Title: Scattering Amplitudes and Riemann Surfaces

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URL: <http://pirsa.org/15050055>

Abstract: In 2003 Witten introduced twistor string theory as a novel description of the scattering matrix of the maximally supersymmetric Yang-Mills theory in four dimensions. In these lectures I will give an introduction to the developments that have led to new formulations, also based on Riemann surfaces, of a large variety of theories, with and without supersymmetry, in arbitrary space-time dimensions.

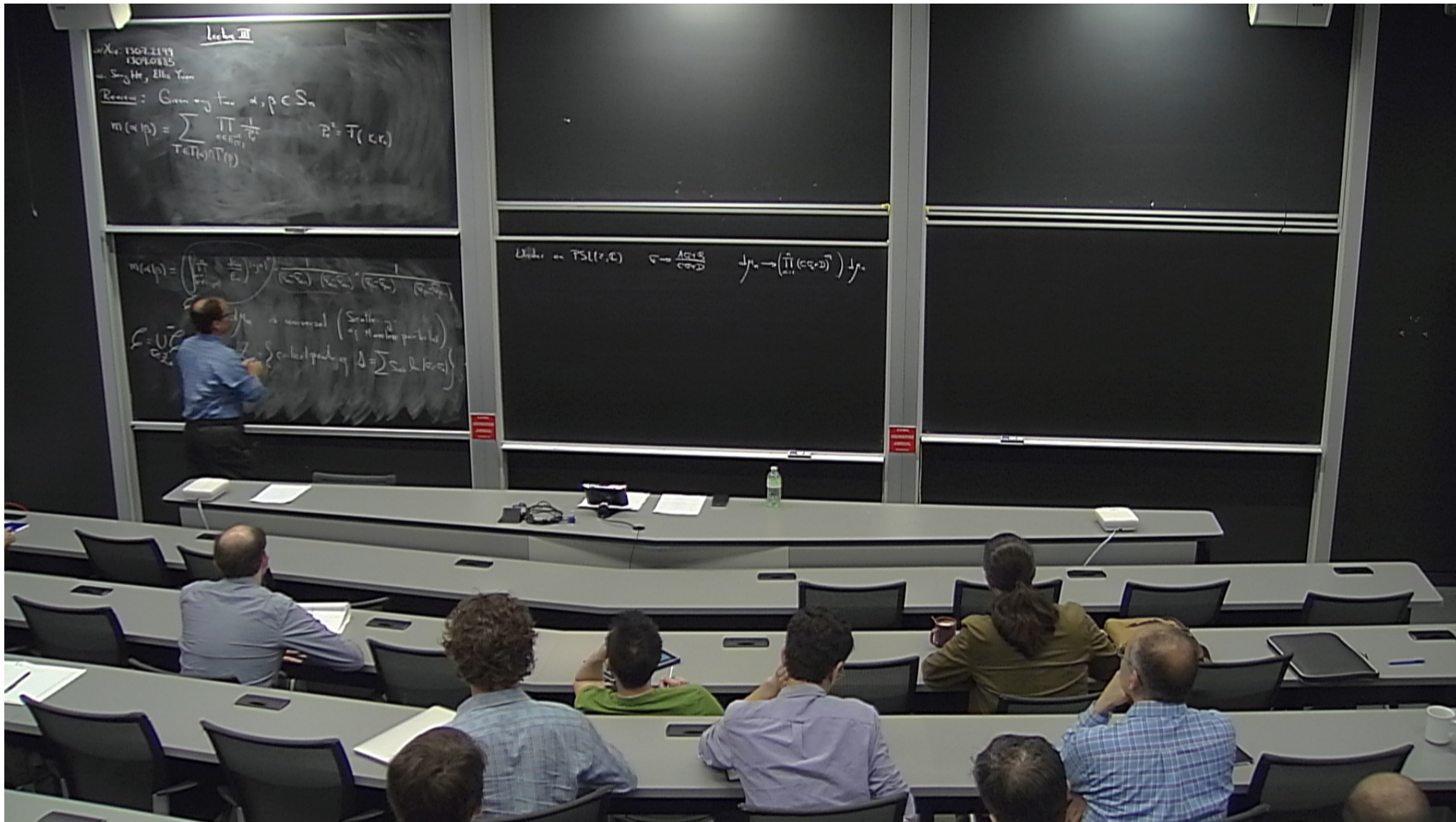
$$T \in T(\alpha) \cap T(\beta)$$

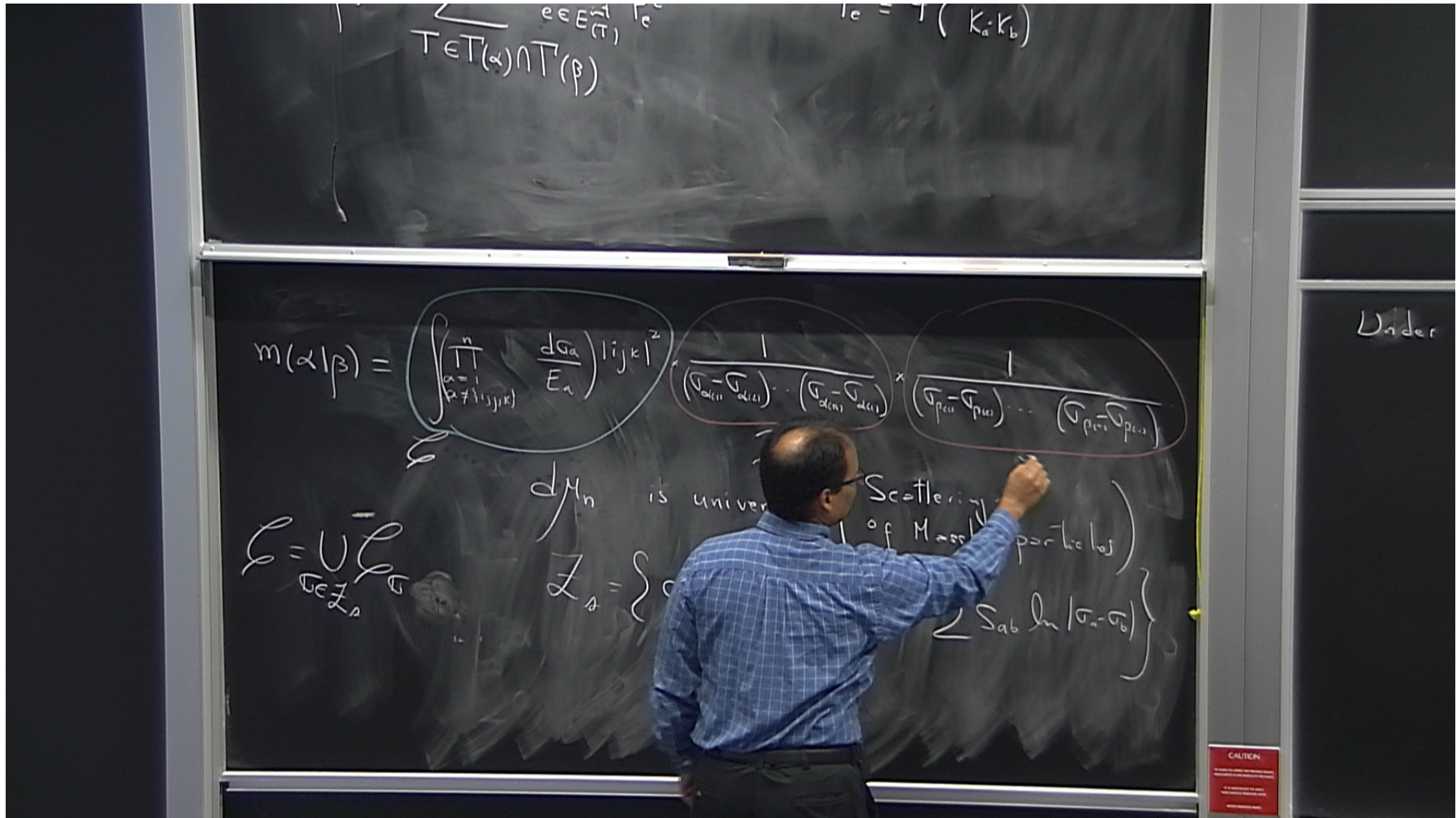
$$m(\alpha|\beta) = \left(\prod_{\substack{\alpha=1 \\ k \neq j, j, k}}^n \frac{d\sigma_\alpha}{E_\alpha} \right) |ijk|^2 \times \frac{1}{(\sigma_{d(1)} - \sigma_{d(2)}) \cdots (\sigma_{d(n)} - \sigma_{d(1)})} \times \frac{1}{(\sigma_{p(1)} - \sigma_{p(2)}) \cdots (\sigma_{p(n)} - \sigma_{p(1)})}$$

$d\mathcal{M}_n$ is universal (Scattering of Massless particles)

$\mathcal{L} = \bigcup_{\sigma \in \mathcal{Z}_n} \mathcal{L}_\sigma$

$\mathcal{Z}_n = \{ \text{critical points of } \Delta = \sum S_{ab} \ln |\sigma_a - \sigma_b| \}$





$$T \in T(\alpha) \cap T(\beta)$$

$$T_e = T(k_a, k_b)$$

$$m(\alpha|\beta) = \left(\sum_{\substack{\alpha=1 \\ \alpha \neq i,j,k}}^n \frac{d\sigma_\alpha}{E_\alpha} \right) |ijk|^2 \times \frac{1}{(\sigma_{d(i)} - \sigma_{d(i)}) \cdot (\sigma_{d(n)} - \sigma_{d(i)})} \times \frac{1}{(\sigma_{p(i)} - \sigma_{p(i)}) \cdot (\sigma_{p(n)} - \sigma_{p(i)})}$$

$d\mu_n$ is universal Scattering of Massless particles

$\mathcal{Z}_\alpha = \left\{ \sum_{\sigma \in \mathcal{Z}_\alpha} S_{ab} \ln |\sigma_a - \sigma_b| \right\}$

Impose: $E_a \cdot k_a = 0 \quad \forall a \quad E_a \sim E_a + r_a k_a \quad r_a \in \mathbb{R}$

Claim: Consider the canonical ordering $\alpha = (1, 2, \dots, n)$

$$A(\{k_1, E_1\}, \dots, \{k_n, E_n\}) = \int d^n \frac{1}{(q_1 - q_2)(q_2 - q_3) \dots (q_n - q_1)} \times \underbrace{\tilde{\mathcal{I}}_R^{\text{YM}}(k_a, E_a, q_a)}_{\mathcal{P}_f' \Psi(k_a, E_a, q_a)}$$

- * $\tilde{\mathcal{I}}_R$ must:
- Proper $\text{PSL}(2, \mathbb{C})$ transf. ✓
 - Multilinear in E_a^M ✓
 - Gauge invariant $\Rightarrow \tilde{\mathcal{I}}_R^{\text{YM}} = 0 \quad \checkmark \quad \text{if } E_a^M = k_a^M$
 - ✓ - Function of $k_a \cdot k_b, E_a \cdot k_b, E_a \cdot E_b, q_a - q_b$
 - As simple as possible.

Impose: $E_a \cdot k_a = 0 \quad \forall a \quad E_a \sim E_a + r_a k_a \quad r_a \in \mathbb{R}$

Claim: Consider the canonical ordering $\alpha = (1, 2, \dots, n)$

$$A(\{k_1, E_1\}, \dots, \{k_n, E_n\}) = \int d^n \frac{1}{(G_1 - G_2)(G_2 - G_3) \dots (G_n - G_1)} \times \underbrace{\tilde{\mathcal{L}}_R^{\text{YM}}(k_a^*, E_a^*, G_a)}_{\mathcal{P}_f' \Psi(k_a, E_a, G_a)}$$

- * $\tilde{\mathcal{L}}_R$ must:
- Proper $\text{PSL}(2, \mathbb{C})$ transf. ✓
 - Multilinear in E_a^M ✓
 - Gauge invariant $\Rightarrow \tilde{\mathcal{L}}_R^{\text{YM}} = 0 \quad \checkmark \quad \text{if } E_a^M = k_a^M$
 - ✓ - Function of $k_a \cdot k_b, E_a \cdot k_b, E_a \cdot E_b, G_a - G_b$
 - As simple as possible.

Impose: $\epsilon_a \cdot k_a = 0 \quad \forall a \quad \epsilon_a \sim \epsilon_a + r_a k_a \quad r_a \in \mathbb{R}$

Claim: Consider the canonical ordering $\alpha = (1, 2, \dots, n)$

$$A(\{k_1, \epsilon_1\}, \dots, \{k_n, \epsilon_n\}) = \int d^n \frac{1}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)} \times \underbrace{\tilde{\mathcal{I}}_R^{\text{YM}}(k_a, \epsilon_a, \sigma_a)}_{\mathcal{P}_f' \Psi(k_a, \epsilon_a, \sigma_a)}$$

- * $\tilde{\mathcal{I}}_R$ must:
- Proper $\text{PSL}(2, \mathbb{C})$ transf. ✓
 - Multilinear in ϵ_a^M ✓
 - Gauge invariant $\Rightarrow \tilde{\mathcal{I}}_R^{\text{YM}} = 0 \quad \checkmark \quad \text{if } \epsilon_a^M = k_a^M$
 - ✓ - Function of $k_a \cdot k_b, \epsilon_a \cdot k_b, \epsilon_a \cdot \epsilon_b, \sigma_a - \sigma_b$
 - As simple as possible.

CAUTION

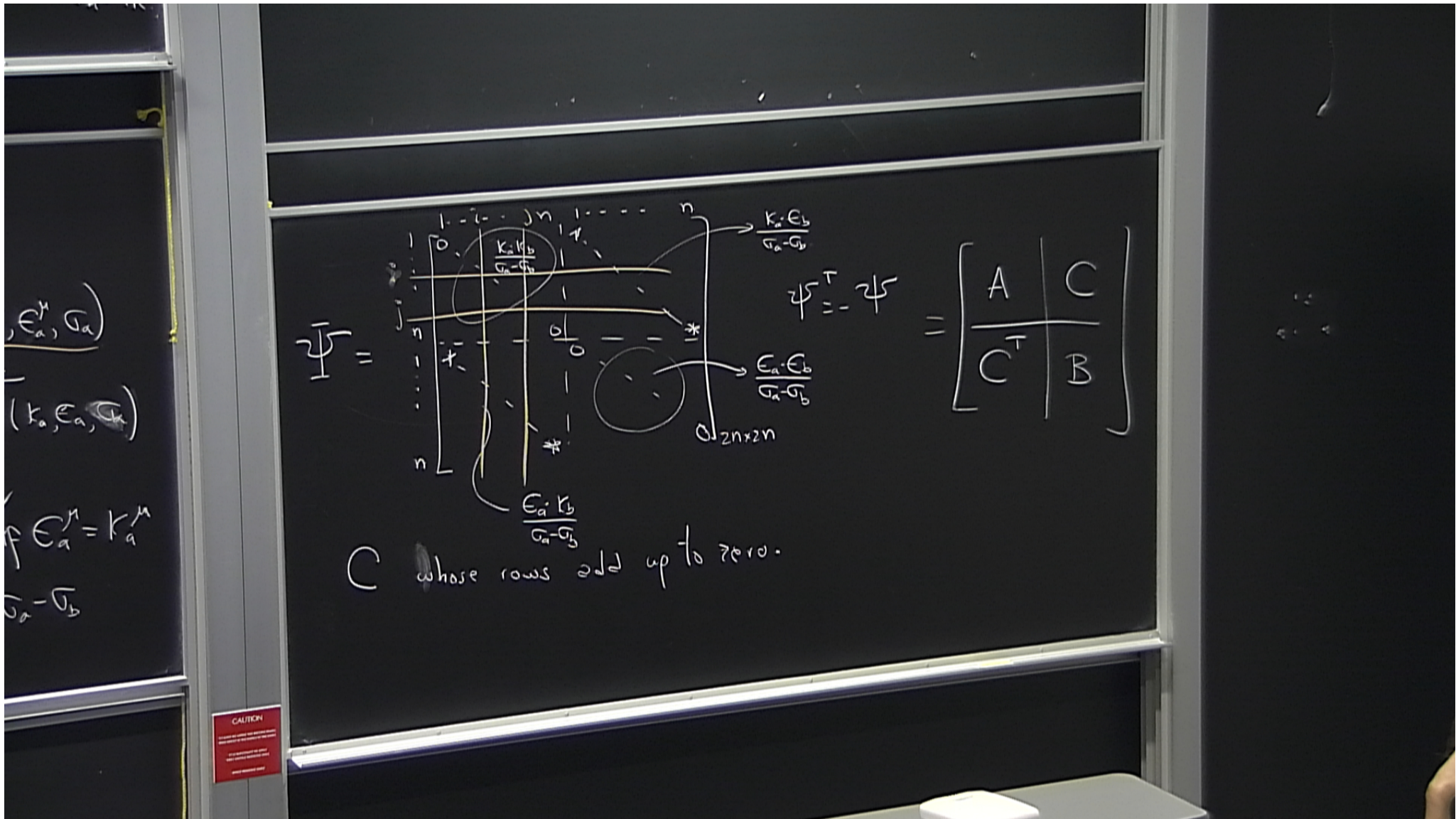
CAUTION

C whose rows add up to zero.

$\psi =$

$\sum_{b=2}^n \frac{K_b \cdot K_1}{g_b - g_1} = ?$

$\in_1^n = K_1^n$



Klausur 1312-7743

$$\text{PSL}(2, \mathbb{C}) \rightarrow G_1 = -1 \quad G_2 = 1 \quad G_3 = \infty.$$

$$E_a = \sum_{\substack{b=1 \\ b \neq a}}^n \frac{S_{ab}}{G_a - G_b} = \frac{S_{a1}}{G_a - 1} + \frac{S_{a2}}{G_a - 1} + \sum_{b=4}^n \frac{S_{ab}}{G_a - G_b} = 0 \quad \forall a \geq 4$$

Jacobi Polynomials

$$P_m^{(\alpha, \beta)}(x) = B \prod_{i=1}^m (x - x_i) \quad \alpha, \beta \in \mathbb{R}.$$

$$\frac{\left(\frac{x-\beta}{2}\right) + \left(\frac{x+\beta+2}{2}\right)x_i}{(x_i^2 - 1)} + \sum_{\substack{j=1 \\ j \neq i}}^m \frac{1}{x_i - x_j} = 0 \quad \alpha, \beta \geq -1$$

m distinct roots.

$$\text{Impose: } E_a \cdot K_a = 0 \quad \forall a \quad E_a \sim E_a' + r_a K_a \quad r_a \in \mathbb{R}$$

- As simple as possible.

$$S_{ab} = 1 \quad a, b \geq 4 \quad S_{a1} = \frac{(1+\beta)}{2} \quad S_{a2} = \frac{(1+\alpha)}{2}$$

rest are determ. $\sum_{b=1}^n S_{ab} = 0 \quad \forall a$ $m = n-3.$

$$\Rightarrow \{G_4, \dots, G_n\} \rightarrow \int$$

$$\text{Impose: } E_a \cdot K_a = 0 \quad \forall a \quad E_a \sim E_a' + r_a K_a \quad r_a \in \mathbb{R}$$

- As simple as possible.

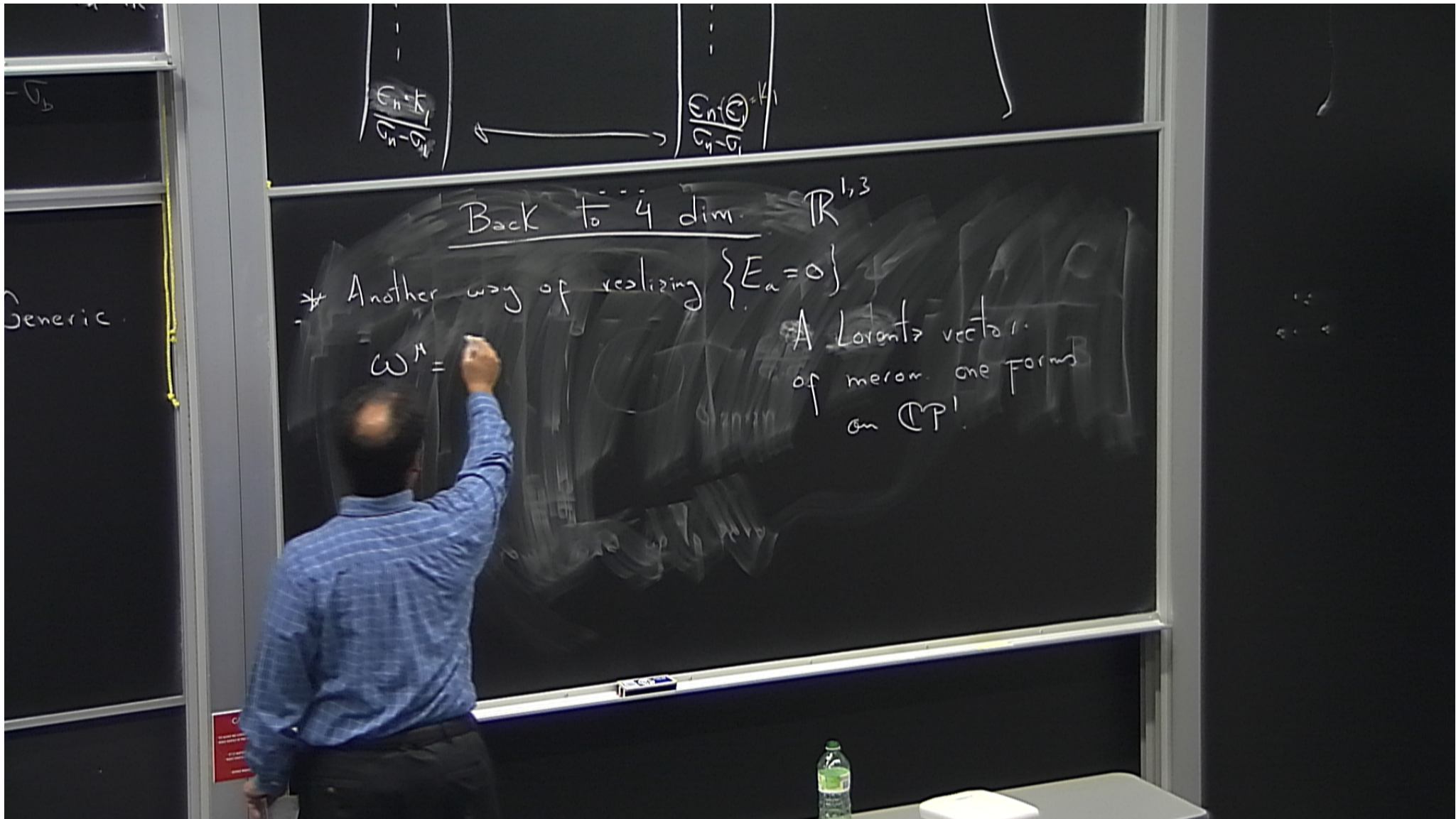
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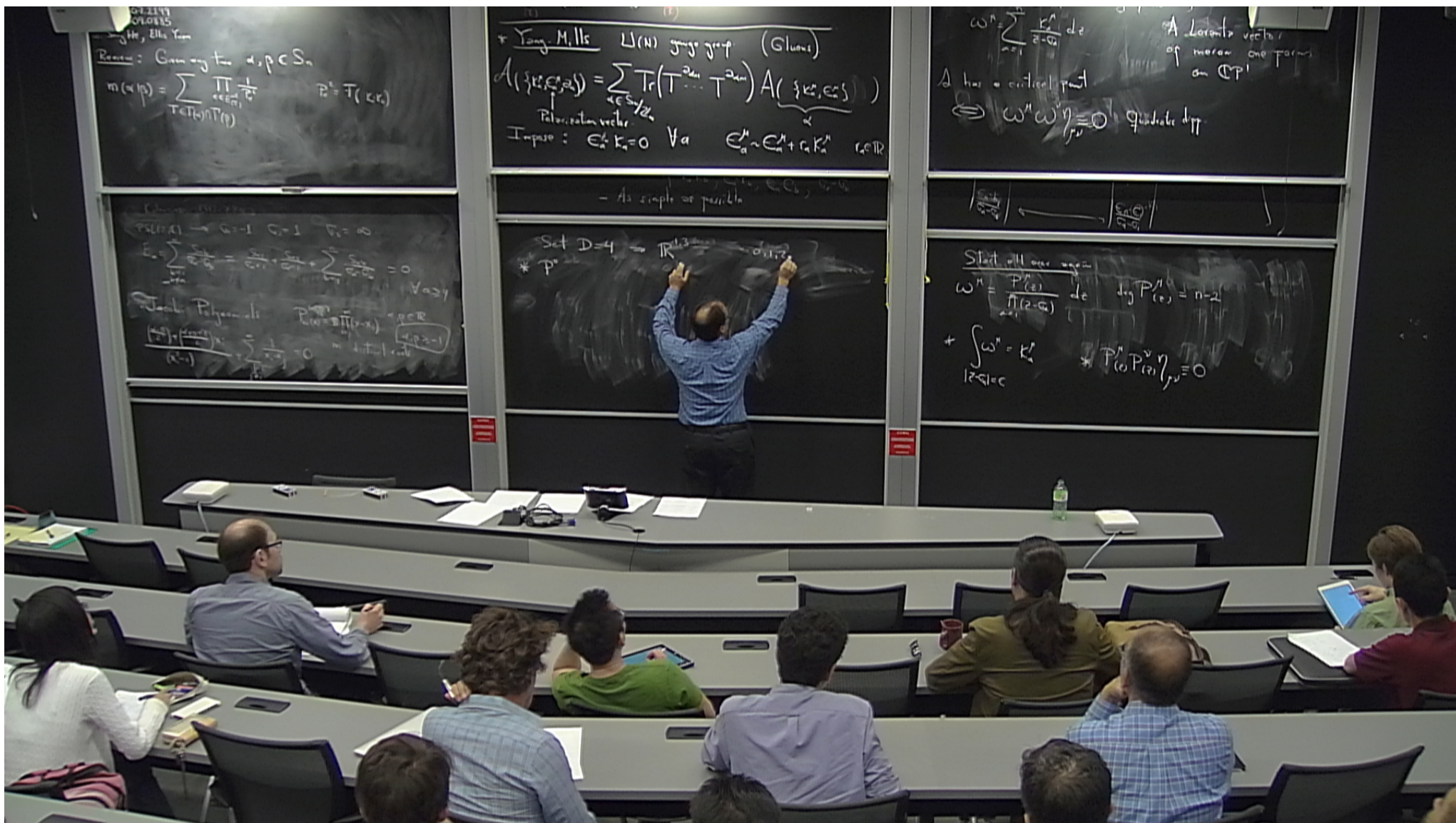
rest are determ. $\sum_{b=1}^n S_{ab} = 0 \quad \forall a$ $m = n-3.$

$$\Rightarrow \{G_4, \dots, G_n\} \rightarrow \{x_4, \dots, x_{n-3}\}$$

Any such map provides solution.

There solution





$$\text{Impose: } \epsilon_a \cdot k_a = 0 \quad \forall a \quad \epsilon_a \sim \epsilon_a + r_a k_a \quad r_a \in \mathbb{R}$$

$\epsilon_a \cdot k_b, \epsilon_a \cdot \epsilon_b, \epsilon_a - \epsilon_b$
 - As simple as possible.

$$\text{Set } D=4 \Rightarrow \mathbb{R}^{1,3} = \{0,1,2,3\}$$

$$* (P^0)^2 - (P^1)^2 - (P^2)^2 - (P^3)^2 = 0$$

$$(P^0)^2 - (P^3)^2 = (P^1)^2 + (P^2)^2$$

$$\forall a \geq 4$$

Impose : $E_a \cdot k_a = 0 \quad \forall a \quad E_a \sim E_a + r_a k_a \quad r_a \in \mathbb{R}$

- As simple as possible.

Set $D=4 \Rightarrow \mathbb{R}^{1,3} = \{0,1,2,3\}$

$$* (P^0)^2 - (P^1)^2 - (P^2)^2 - (P^3)^2 = 0$$

$$(P^0)^2 - (P^3)^2 = (P^1)^2 + (P^2)^2$$

$$(P^0 - P^3)(P^0 + P^3) = (P^1 + iP^2)(P^1 - iP^2)$$

$$\text{Impose: } \epsilon_a \cdot k_a = 0 \quad \forall a \quad \epsilon_a \sim \epsilon_a + r_a k_a \quad r_a \in \mathbb{R}$$

$\epsilon_a \cdot k_b, \epsilon_a \cdot \epsilon_b, \epsilon_a - \epsilon_b$
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$$\text{Set } D=4 \Rightarrow \mathbb{R}^{1,3} \quad \{0,1,2,3\}$$

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$$(P^0)^2 - (P^3)^2 = (P^1)^2 + (P^2)^2$$

$$\underbrace{(P^0 - P^3)}_{n-2} \underbrace{(P^0 + P^3)}_{n-2} = \underbrace{(P^1 + iP^2)}_{n-2} \underbrace{(P^1 - iP^2)}_{n-2}$$

$$\lambda_{\perp(z)} \lambda$$

$$\lambda_1(z)$$

Polyn. eq. split into branches.

$$\deg \lambda_1(z) \in \{0, 1, 2, \dots, n-3, n-2\}$$

→ Inconsistent
w/ Δ