Title: Scattering Amplitudes and Riemann Surfaces

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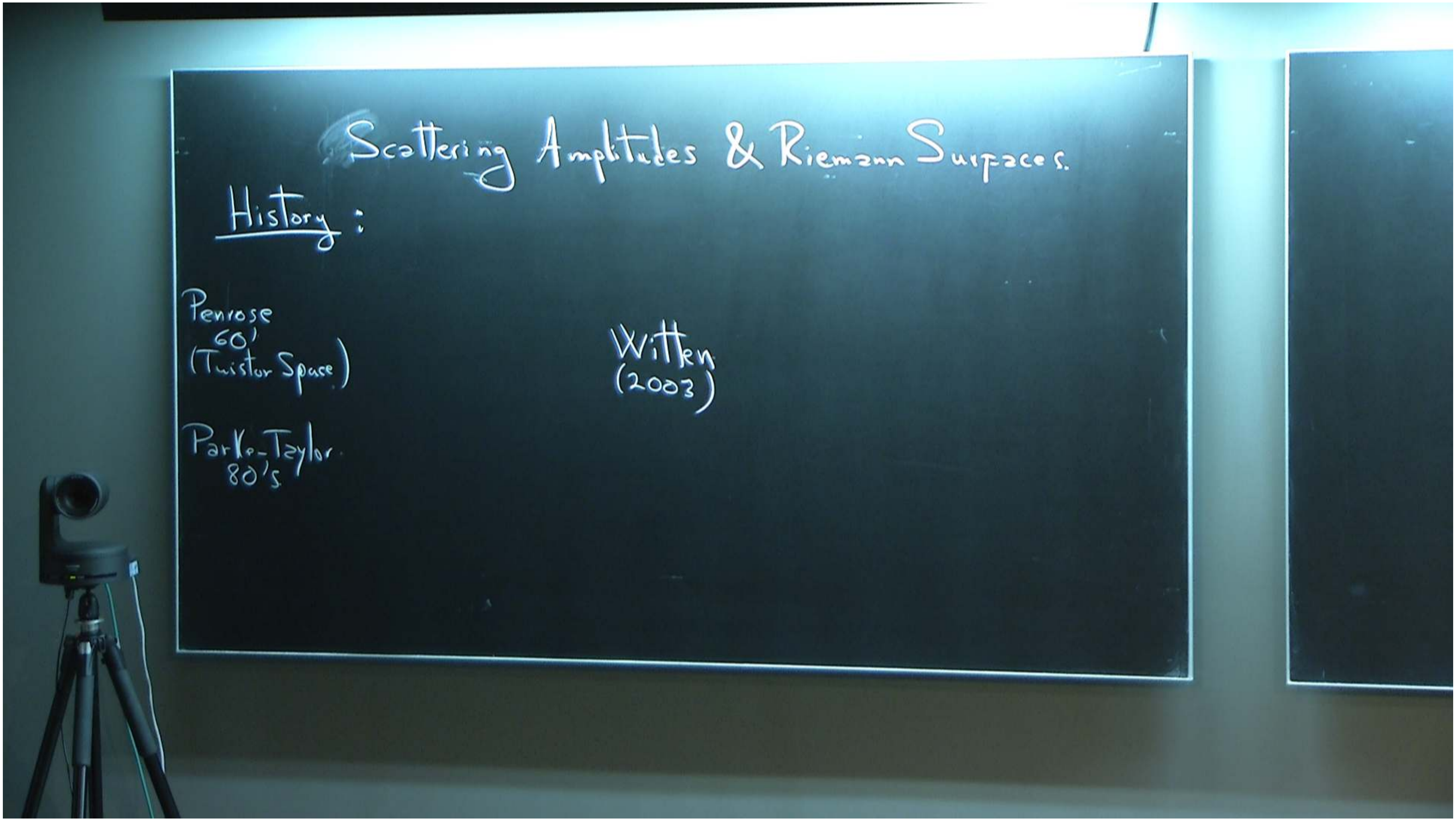
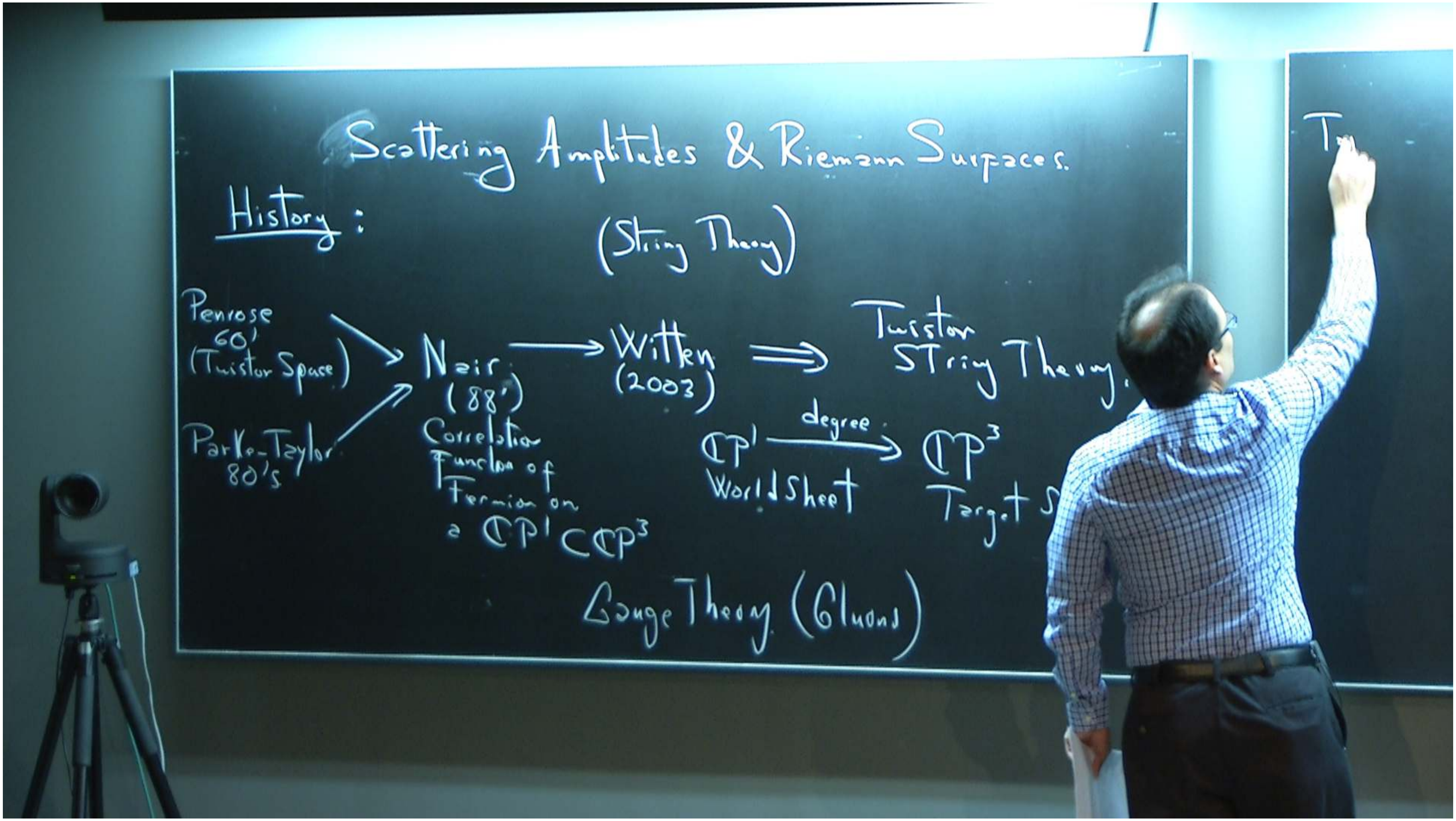
Abstract: In 2003 Witten introduced twistor string theory as a novel description of the scattering matrix of the maximally supersymmetric Yang-Mills theory in four dimensions. In these lectures I will give an introduction to the developments that have led to new formulations, also based on Riemann surfaces, of a large variety of theories, with and without supersymmetry, in arbitrary space-time dimensions.


Scattering Amplitudes & Riemann Surfaces.
History :
Penrose
60'
(Twistor Space)
Witten
(2003)
Parke-Taylor
80's



Scattering Amplitudes & Riemann Surfaces.
History :
(String Theory)
Penrose
60'
(Twistor Space)
Parke-Taylor
80's
Nair
(88')
Correlation
Function of
Fermion on
a $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Witten
(2003)
Twistor
STring Theory.
$\mathbb{CP}^1 \xrightarrow{\text{degree}} \mathbb{CP}^3$
World Sheet
Gauge Theory (Gluons)

Topological B model $\longrightarrow$ $\mathcal{N}=4$ supersymmetry.

$\mathbb{CP}^1 \xrightarrow{d} \mathbb{CP}^{3|4} \Rightarrow$ Super CY

$\mathcal{M}_{0,n}$ Space-Time is $\mathbb{R}^{3,1}$

• Theory is unique once a Lie Algebra is chosen.

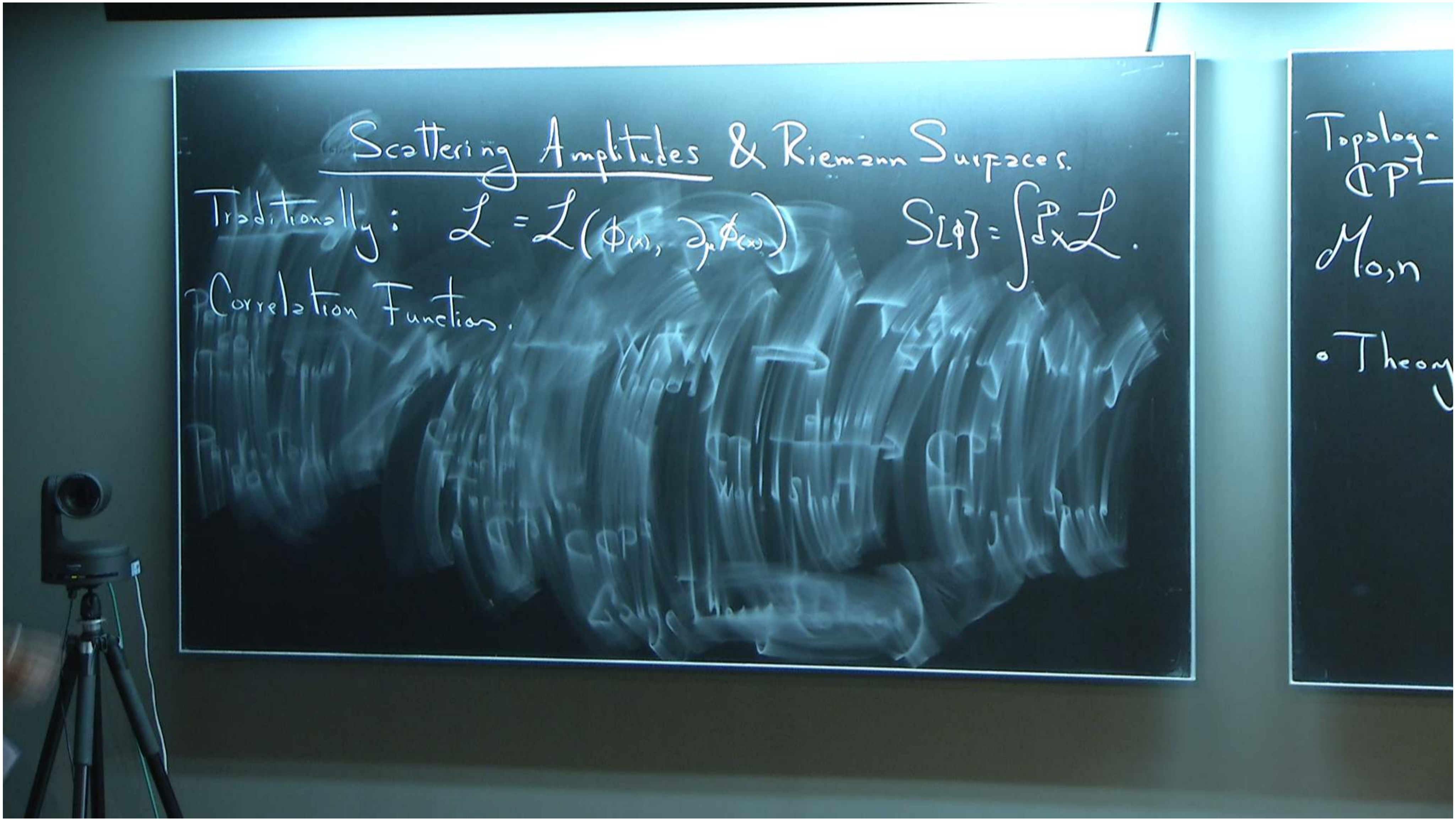
$U(N)$ MSYM $\hookrightarrow$ Yang-Mills.

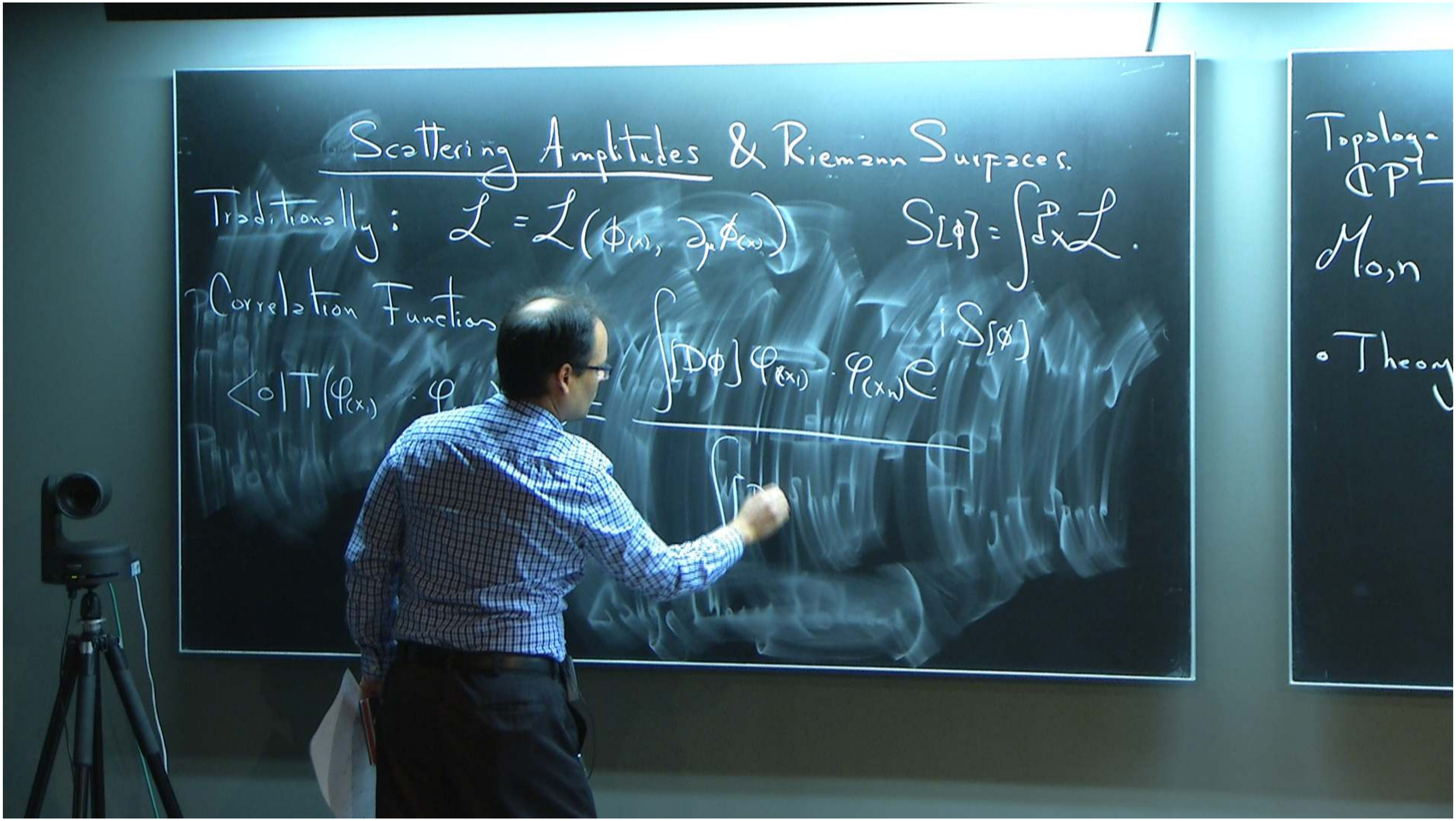
Maldacena's AdS/CFT $\Updownarrow$

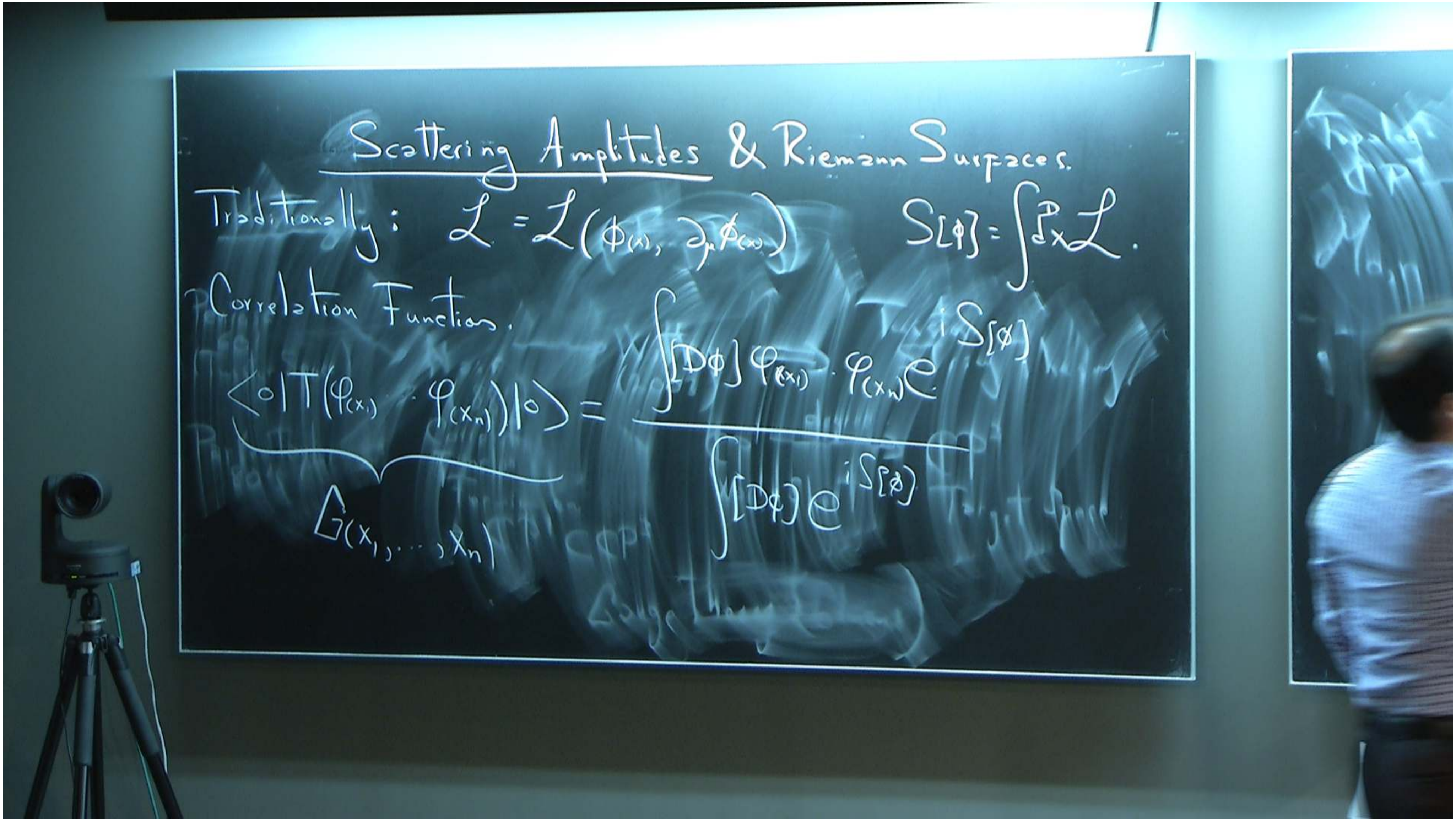
ST. $AdS_5 \times S^5$

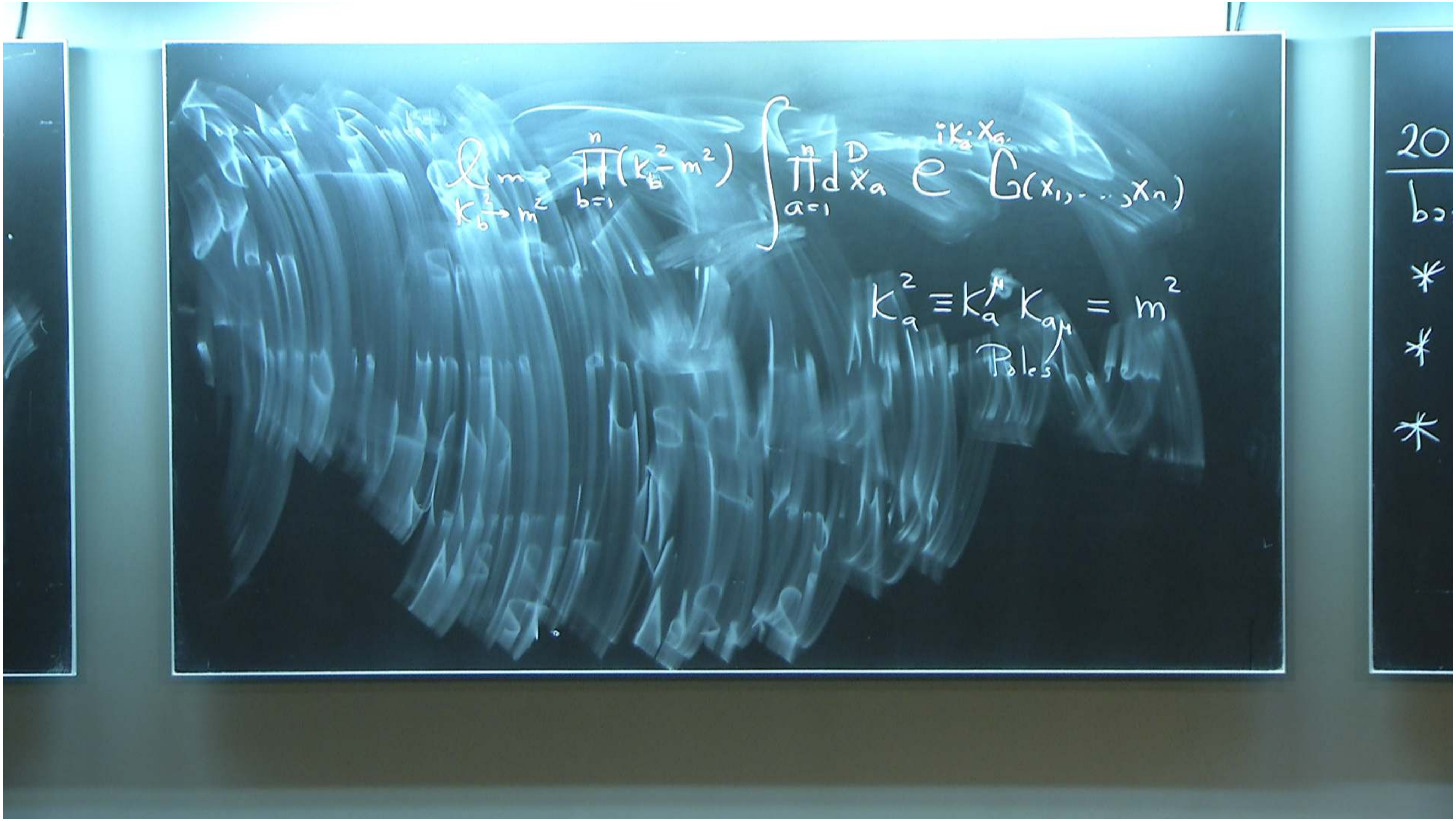
<u>2015</u>: Formulation of QFT scatt. amplitudes
based on $\mathcal{M}_{0,n}$ for theories

* in any space-time dim.
* with or without SUSY
* gluons, gravitons, scalars, Many comb.


Scattering Amplitudes & Riemann Surfaces.
Traditionally: $\mathcal{L} = \mathcal{L}(\phi(x), \partial_\mu \phi(x))$
$S[\phi] = \int d^D x \, \mathcal{L}$.
Correlation Function.
$\mathbb{CP}^1$
$\mathcal{M}_{0,n}$


Scattering Amplitudes & Riemann Surfaces.
Traditionally: $\mathcal{L} = \mathcal{L}(\phi_{(x)}, \partial_\mu \phi_{(x)})$ $S[\phi] = \int d^D x \mathcal{L}$.
Correlation Function
$\langle 0 | T(\varphi_{(x_1)} \cdots \varphi$
$\int [D\phi] \varphi_{(x_1)} \cdots \varphi_{(x_n)} e^{iS[\phi]}$
Topology
$\mathbb{C}P^1$
$\mathcal{M}_{0,n}$
• Theory


Scattering Amplitudes & Riemann Surfaces.
Traditionally: $\mathcal{L} = \mathcal{L}(\phi(x), \partial_\mu \phi(x))$
$S[\phi] = \int d^Dx \, \mathcal{L}$.
Correlation Function.
$\langle 0 | T(\varphi(x_1) \cdots \varphi(x_n)) | 0 \rangle = \frac{\int [D\phi] \, \varphi(x_1) \cdots \varphi(x_n) \, e^{iS[\phi]}}{\int [D\phi] \, e^{iS[\phi]}}$
$G(x_1, \ldots, x_n)$


$\lim_{K_b^2 \to m^2} \prod_{b=1}^{n} (K_b^2 - m^2) \int \prod_{a=1}^{n} d^D x_a \, e^{iK_a \cdot X_a} G(x_1, \ldots, x_n)$
$K_a^2 \equiv K_a^\mu K_{a\mu} = m^2$
Poles
20

$$\mathcal{A}(k_1^\mu, \dots, k_n^\mu) = \lim_{k_b^2 \to m^2} \prod_{b=1}^{n} (k_b^2 - m^2) \int \prod_{a=1}^{n} d^D x_a \, e^{i k_a \cdot x_a} \, G(x_1, \dots, x_n)$$

S.A. of n particles
with momenta $\{k_1^\mu, \dots, k_n^\mu\}$.

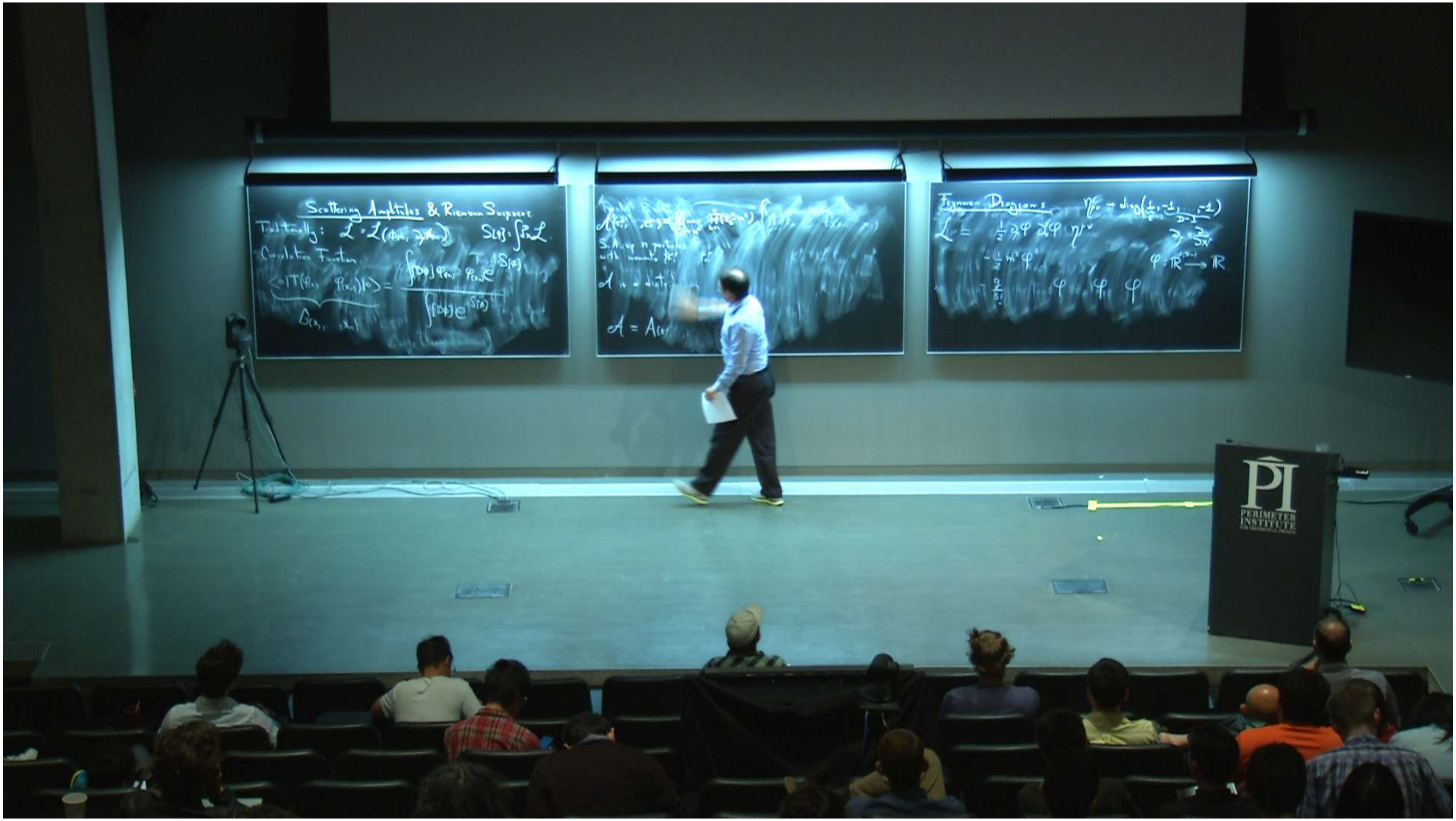
$$k_a^2 \equiv k_a^\mu k_{a\mu} = m^2$$

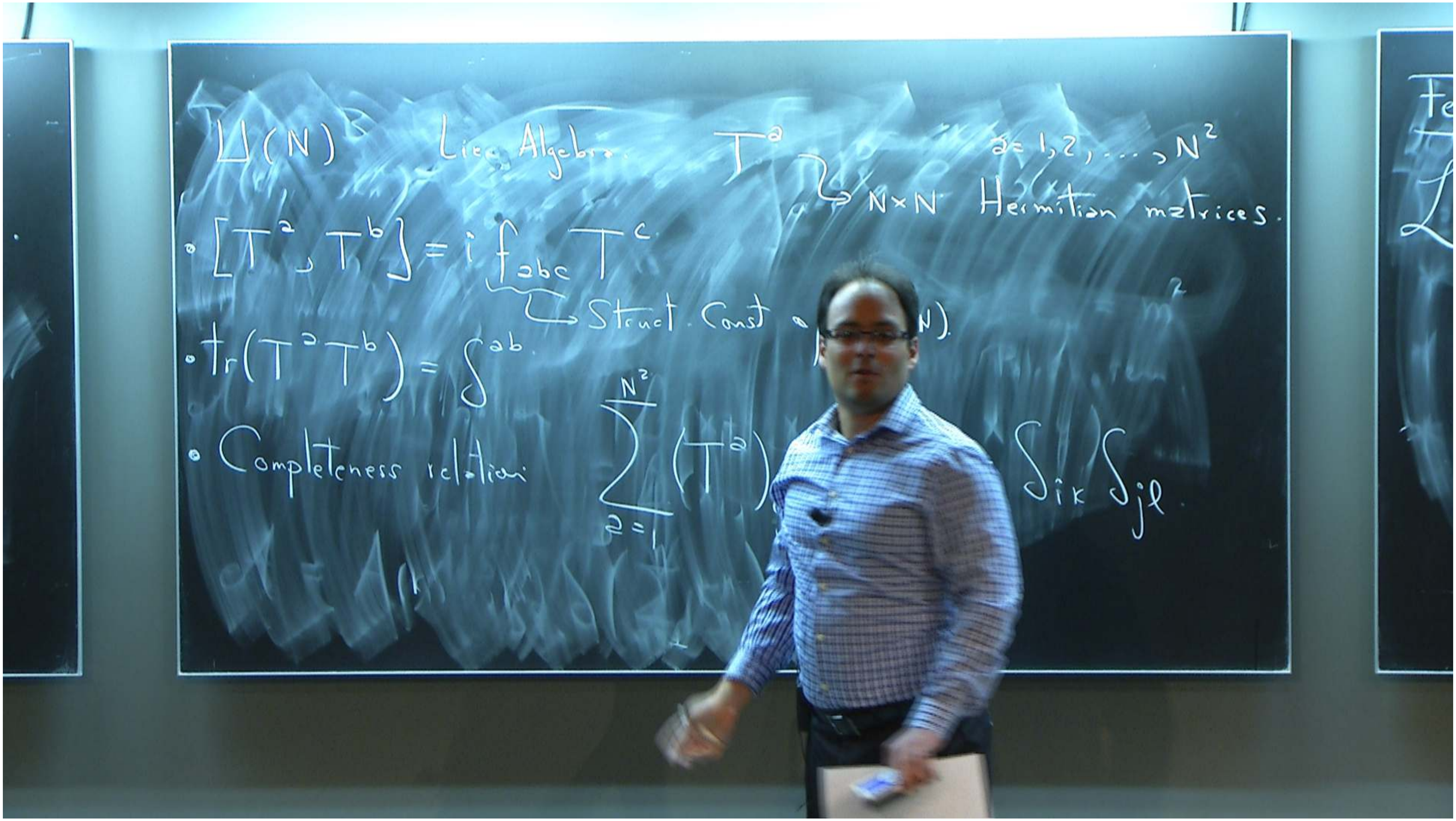
Poles

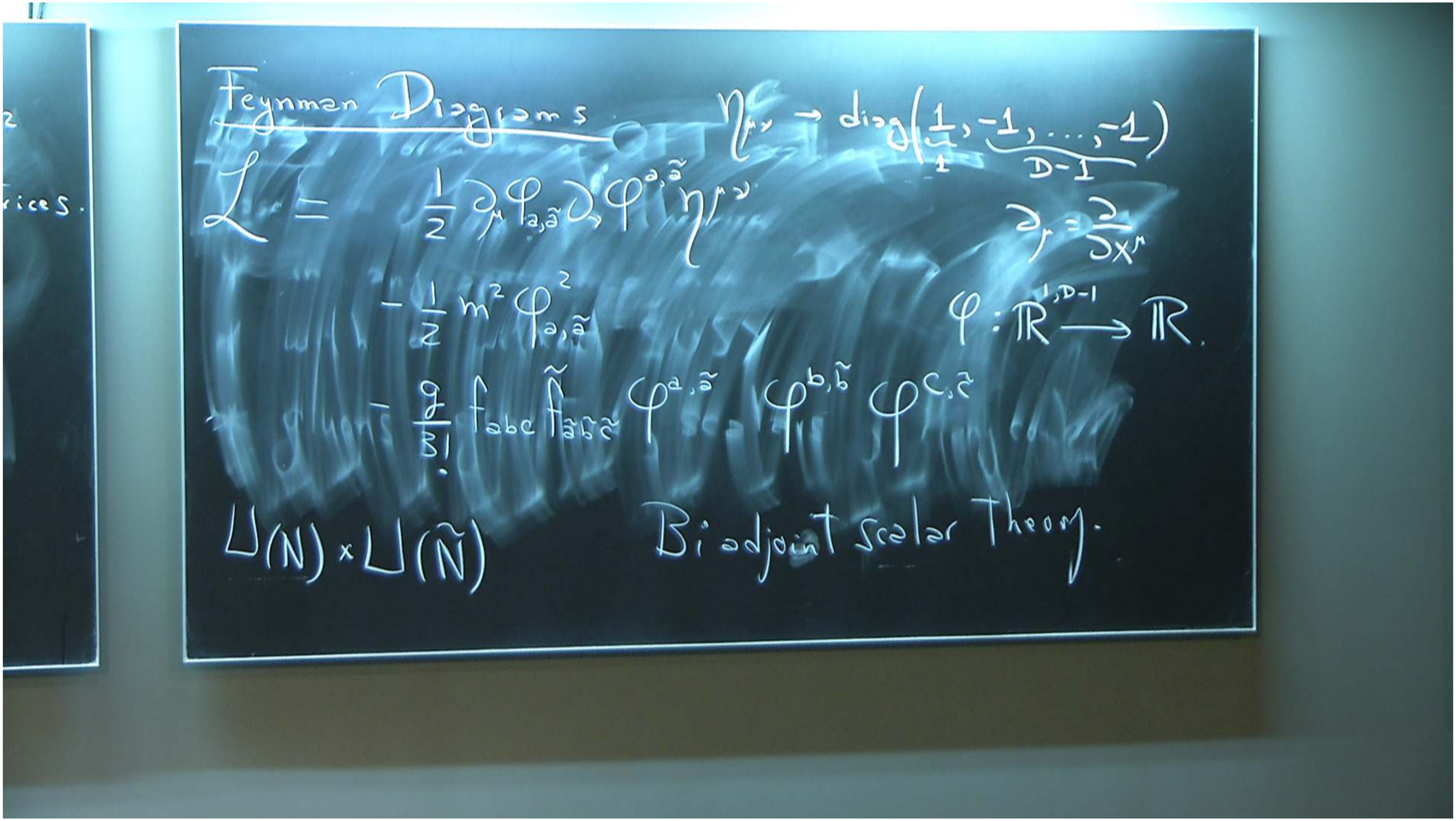
$\mathcal{A}$ is a distribution.

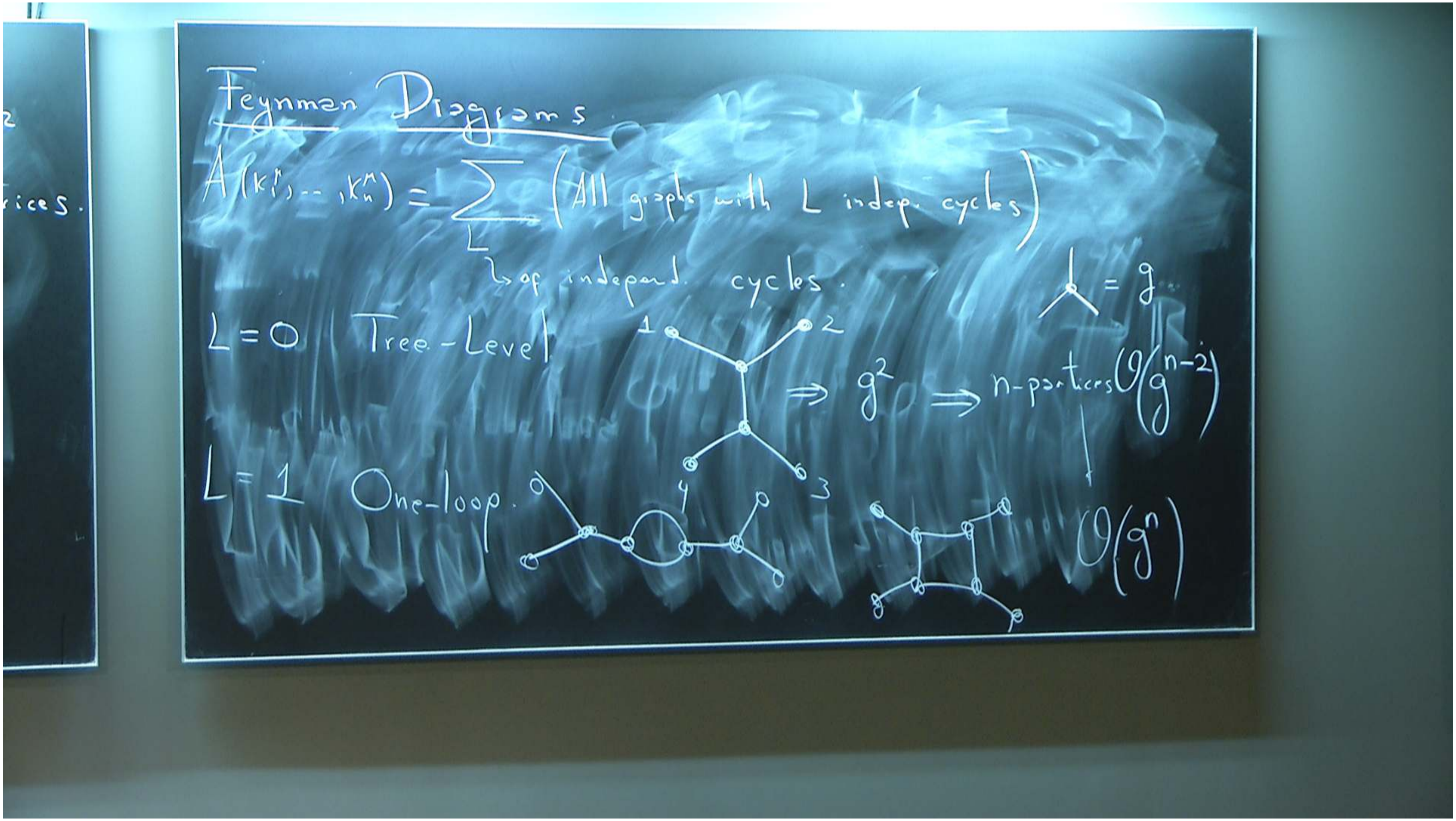
$$G(x_1 + y, x_2 + y, \dots, x_n + y) = G(x_1, \dots, x_n)$$

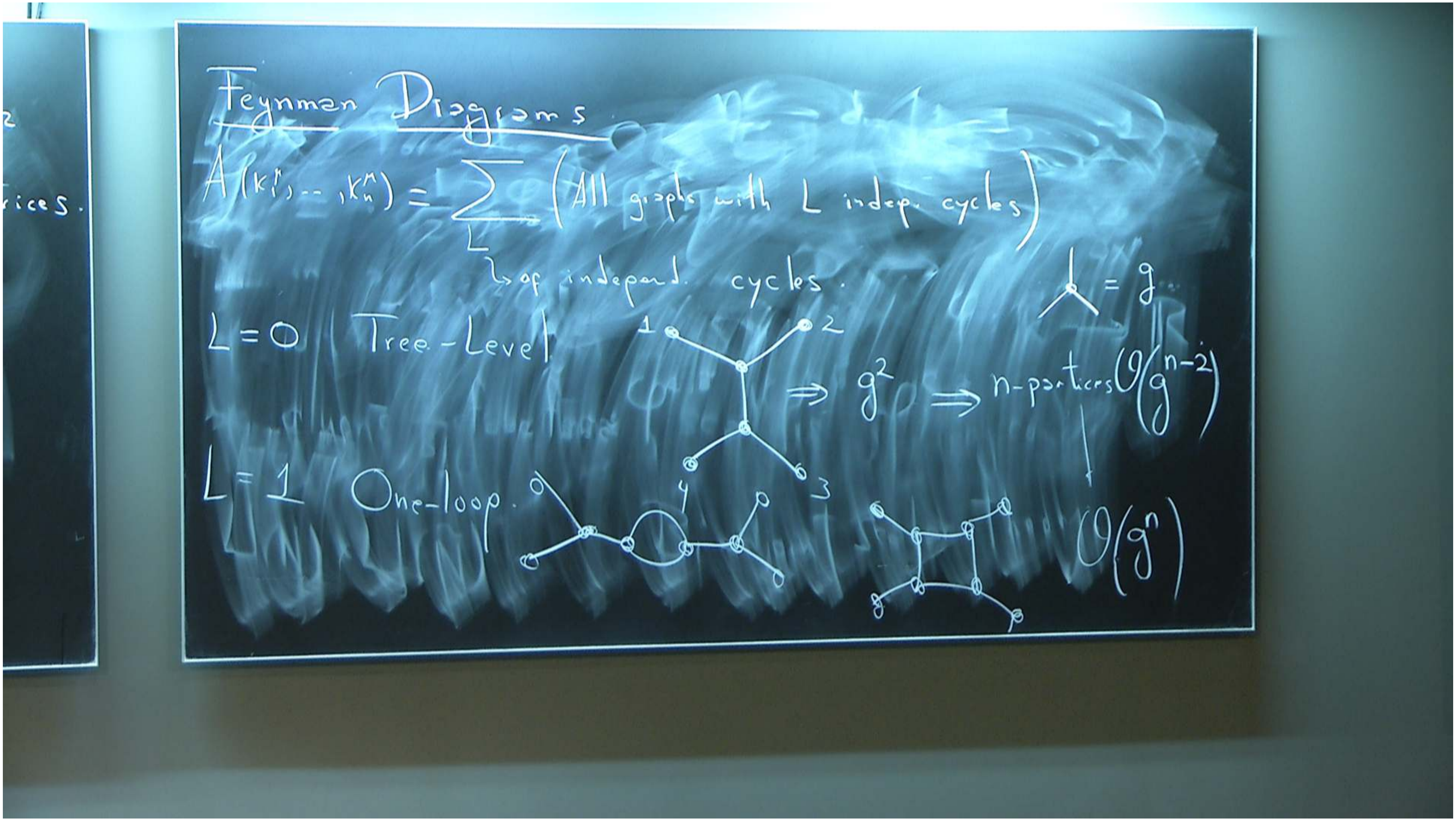
$$\mathcal{A} = A(k_1^\mu, \dots, k_n^\mu) \, \delta^D\!\left(\sum_{a=1}^{n} k_a^\mu\right).$$


Scattering Amplitudes & Riemann Surfaces
Feynman Diagrams
PERIMETER INSTITUTE

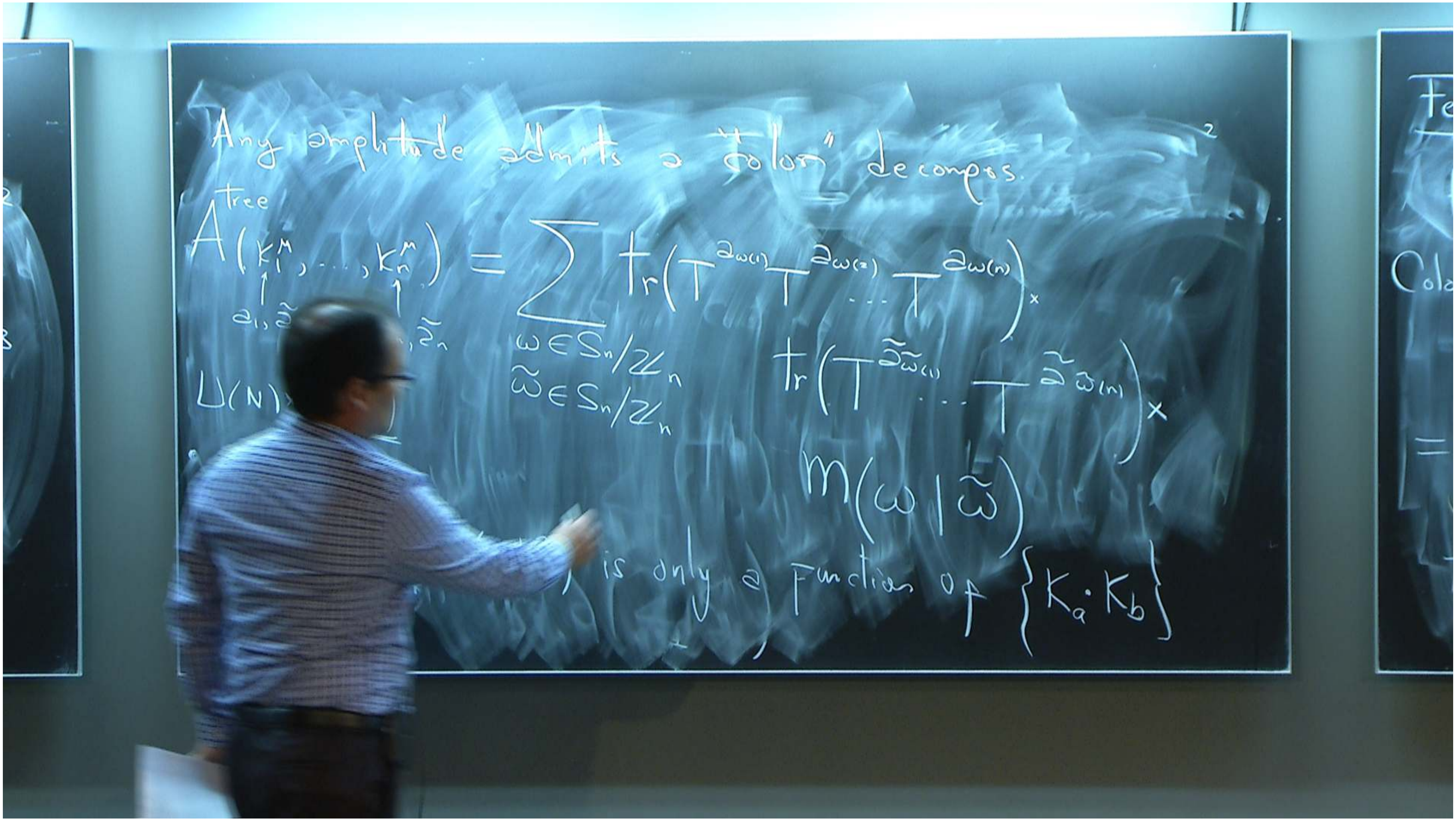

U(N)
Lie Algebra
T^a
a = 1, 2, ..., N^2
N×N Hermitian matrices
[T^a, T^b] = i f_abc T^c
Struct. Const
tr(T^a T^b) = δ^ab
Completeness relation

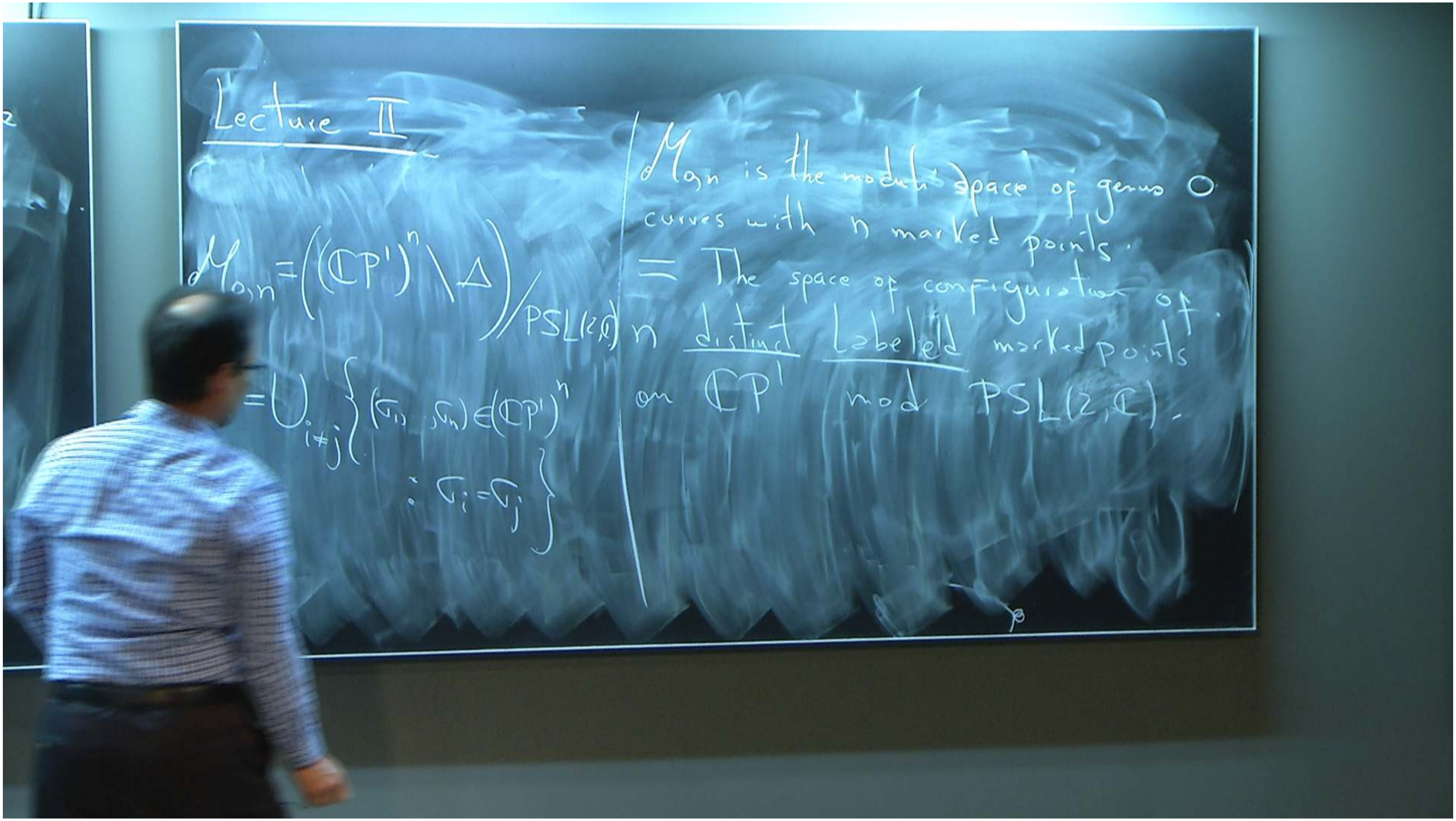

Feynman Diagrams
$\eta_{\mu\nu} \to \mathrm{diag}(1, \underbrace{-1, \ldots, -1}_{D-1})$
$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi_{a,\tilde{a}} \partial_\nu \varphi^{a,\tilde{a}} \eta^{\mu\nu}$
$\partial_\mu = \frac{\partial}{\partial x^\mu}$
$- \frac{1}{2} m^2 \varphi_{a,\tilde{a}}^2$
$\varphi : \mathbb{R}^{1,D-1} \longrightarrow \mathbb{R}.$
$- \frac{g}{3!} f_{abc} \tilde{f}_{\tilde{a}\tilde{b}\tilde{c}} \varphi^{a,\tilde{a}} \varphi^{b,\tilde{b}} \varphi^{c,\tilde{c}}$
$U(N) \times U(\tilde{N})$
Biadjoint scalar Theory.


Feynman Diagrams
A(k_1^μ, ..., k_n^μ) = Σ_L (All graphs with L indep. cycles)
↳ of independ. cycles.
= g
L = 0 Tree-Level
1
2
3
4
⇒ g² ⇒ n-particles O(g^(n-2))
L = 1 One-loop.
O(g^n)



Feynman Diagrams
$A(k_1^\mu, \ldots, k_n^\mu) = \sum_L$ (All graphs with L indep. cycles)
↳ of independ. cycles.
= g
L = 0 Tree-Level
1
2
3
4
⇒ g^2 ⇒ n-particles $\mathcal{O}(g^{n-2})$
L = 1 One-loop.
$\mathcal{O}(g^n)$


Any amplitude admits a "color" decompos.
$A^{tree}(k_1^\mu, \ldots, k_n^\mu) = \sum tr(T^{a_{\omega(1)}} T^{a_{\omega(2)}} \cdots T^{a_{\omega(n)}}) \times$
$\omega \in S_n/\mathbb{Z}_n$
$\tilde{\omega} \in S_n/\mathbb{Z}_n$
$tr(T^{\tilde{a}_{\tilde{\omega}(1)}} \cdots T^{\tilde{a}_{\tilde{\omega}(n)}}) \times$
$U(N)$
$m(\omega|\tilde{\omega})$
is only a function of $\{K_a \cdot K_b\}$


Lecture II
$\mathcal{M}_{0,n} = \left((\mathbb{CP}^1)^n \setminus \Delta \right) / PSL(2,\mathbb{C})$
$= \bigcup_{i \neq j} \{ (\sigma_1, \ldots, \sigma_n) \in (\mathbb{CP}^1)^n : \sigma_i = \sigma_j \}$
$\mathcal{M}_{0,n}$ is the moduli space of genus 0 curves with n marked points.
= The space of configuration of n distinct labeled marked points on $\mathbb{CP}^1$ mod $PSL(2,\mathbb{C})$.