Title: Higher-Spin Gravity : One learner's perspective

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Abstract: In these lectures, we will study the bosonic theory of higher-spin gravity in four dimensions. After discussing the reasons for interest in the theory, we will focus on the equations of motion and their content. We will aim to construct the equations from the ground up in a motivated way. The logical order will differ somewhat from standard introductions. As preliminaries, we will discuss the geometry of spinors and twistors in (anti) de Sitter space, along with various viewpoints on free massless fields with arbitrary spin. An ulterior goal of the lectures is to introduce a new version of the theory (arXiv:1502.06685), formulated on a fixed (A)dS background

Let's learn (4d, bosonic) Higher-spin theory!

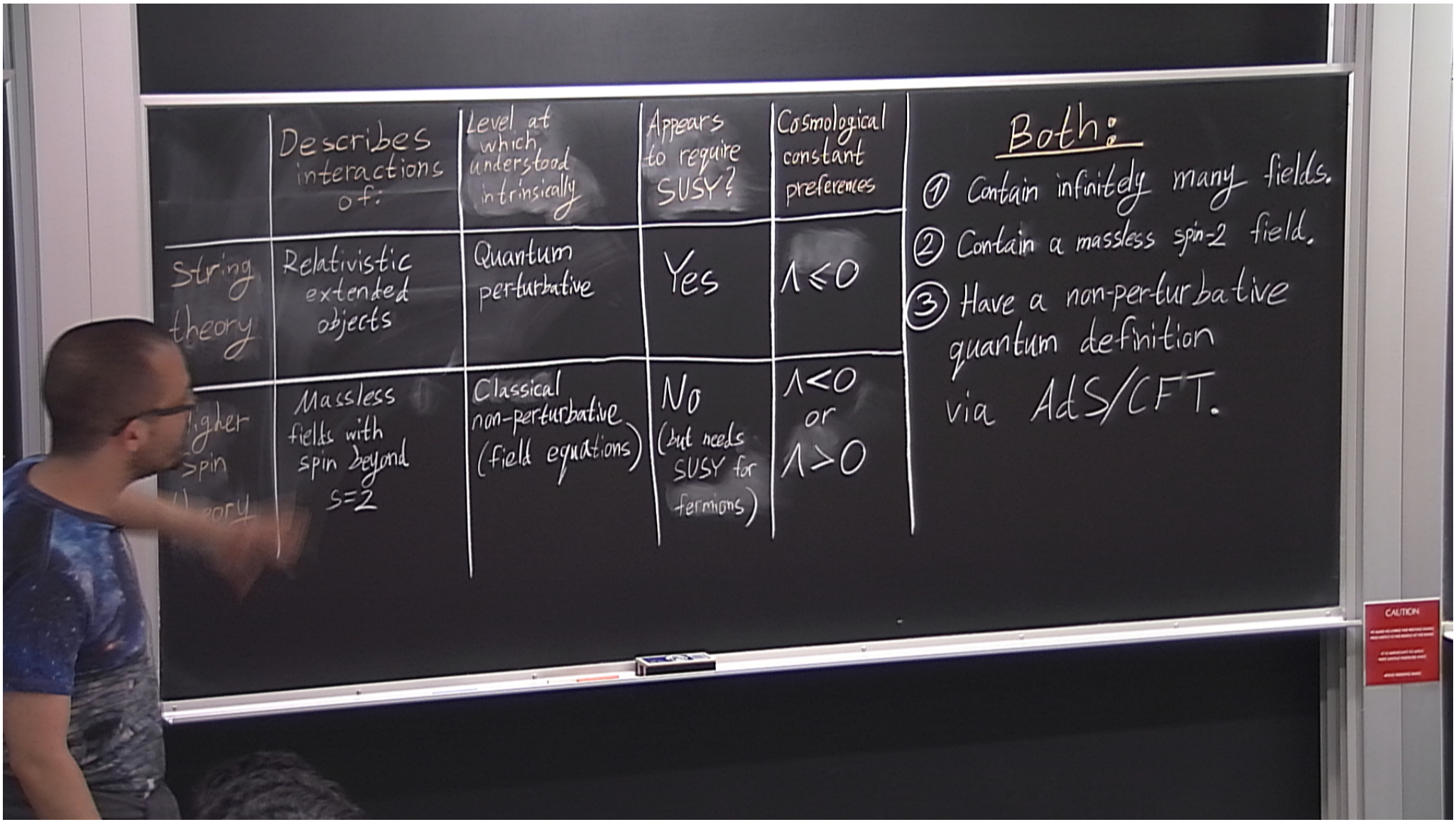
Resources:

1) hep-th/9611024 (Review by Vasiliev)
2) 1401.2975 (Review by Didenko & Skvortsov)
3) Mini-course by Giombi on PIRSA:11030093 (+3 more)

Refs. for Yasha's idiosyncracies:

1) 1312.7842 (Geometry of twistors in (A)dS$_4$)
2) 1502.06685 (Alternative version of HS theory, formulated on non-dynamical (A)dS spacetime)

In equations, will use Lorentz signature & $\Lambda > 0$.


Describes interactions of:
Level at which understood intrinsically
Appears to require SUSY?
Cosmological constant preferences
String theory
Relativistic extended objects
Quantum perturbative
Yes
Λ≤0
Higher spin theory
Massless fields with spin beyond s=2
Classical non-perturbative (field equations)
No (but needs SUSY for fermions)
Λ<0 or Λ>0
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① Contain infinitely many fields.
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3. Have a non-perturbative quantum definition via $\boxed{\text{AdS/CFT.}}$

(1108.5735)

One massless field of each

$s = (0, 1, 2, 3, 4, \ldots)$

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HS gauge group

Diff.-invariant (diff forms)

(or not)

Single coupling (G_N)

Need $\Lambda \neq 0$, enters $\frac{1}{\sqrt{\Lambda}} \partial_\mu$

At least naively, non-local at Λ scale.

Not those of GR!

① Spontaneous breaking

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Need a good language for all Lorentz reps.

2-spinors! $O(3,1) \approx SL(2,\mathbb{C})$

Two kinds of 2d spinors: ψ^{α}, $\psi^{\dot{\alpha}}$; LH, RH Weyl.

Metric: only antisymm. matrix: $\varepsilon_{\alpha\beta}$, $\varepsilon_{\dot{\alpha}\dot{\beta}}$

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$f_{(\alpha_1 \ldots \alpha_m)(\dot\alpha_1 \ldots \dot\alpha_n)}$: $\left(j_L = \frac{m}{2},\ j_R = \frac{n}{2}\right)$

All the finite-dim reps of Lorentz.

Tensors $\Longleftrightarrow$ Spinors w. even no. of indices.

$\gamma_\mu^{\alpha\dot\alpha}$, $\gamma^{(\alpha\beta)}_{[\mu\nu]}$ (Left), $\gamma^{(\dot\alpha\dot\beta)}_{[\mu\nu]}$ (Right) ; Ex: P_μ, $P_{\alpha\dot\alpha}$

$J_{\mu\nu}$, $(J_{\alpha\beta}, J_{\dot\alpha\dot\beta})$

Field strength of free massless spin-s:

$\varphi_{(\alpha_1 \ldots \alpha_{2s})}$, $\varphi_{(\dot\alpha_1 \ldots \dot\alpha_{2s})}$

$(s, 0)$ $(0, s)$

Spin-1: Maxwell field strength.

Spin-2: Linearized Weyl tensor.

$s>2$: "Generalized" Weyl ".

Field eq: $\nabla_{\beta\dot\beta} \varphi^{\beta}{}_{\alpha_2 \ldots \alpha_{2s}} = 0$

$\nabla_{\beta\dot\beta} \varphi^{\dot\beta}{}_{\dot\alpha_2 \ldots \dot\alpha_{2s}} = 0$.

On-shell independent derivatives:

$$\nabla^{(\dot\alpha_1}_{(\alpha_1} \cdots \nabla^{\dot\alpha_k)}_{\alpha_k} \varphi_{\beta_1 \cdots \beta_{2s})} \sim \varphi^{\dot\alpha_1 \cdots \dot\alpha_k}_{\alpha_1 \cdots \alpha_k \beta_1 \cdots \beta_{2s}}$$

$$\left(s+\frac{k}{2}, \frac{k}{2}\right)$$

$$\left(\frac{k}{2}, s+\frac{k}{2}\right)$$

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$(\Lambda_{dS} = 1)$

Unfolded equations:

Idea: $\nabla_{\beta\dot\beta}\, \varphi_{\alpha_1 \ldots \alpha_{2s}} \sim \varphi_{(\alpha_1 \ldots \alpha_{2s}\beta)\dot\beta}$

Repeat and obtain:

$$\nabla^{\beta}{}_{\dot\beta}\, \varphi^{\alpha_1 \ldots \alpha_m}{}_{\dot\alpha_1 \ldots \dot\alpha_n} = \frac{i}{\sqrt{2}} \Big[\underbrace{(m+1)(n+1)\, \varphi^{\beta\alpha_1 \ldots \alpha_m}{}_{\dot\beta\dot\alpha_1 \ldots \dot\alpha_n}}_{\text{flat-space}} - \underbrace{\varepsilon^{\beta(\alpha_1} \varepsilon_{\dot\beta(\dot\alpha_1}\, \varphi^{\alpha_2 \ldots \alpha_m)}{}_{\dot\alpha_2 \ldots \dot\alpha_n)}}_{\text{Curvature correction in dS.}} \Big]$$

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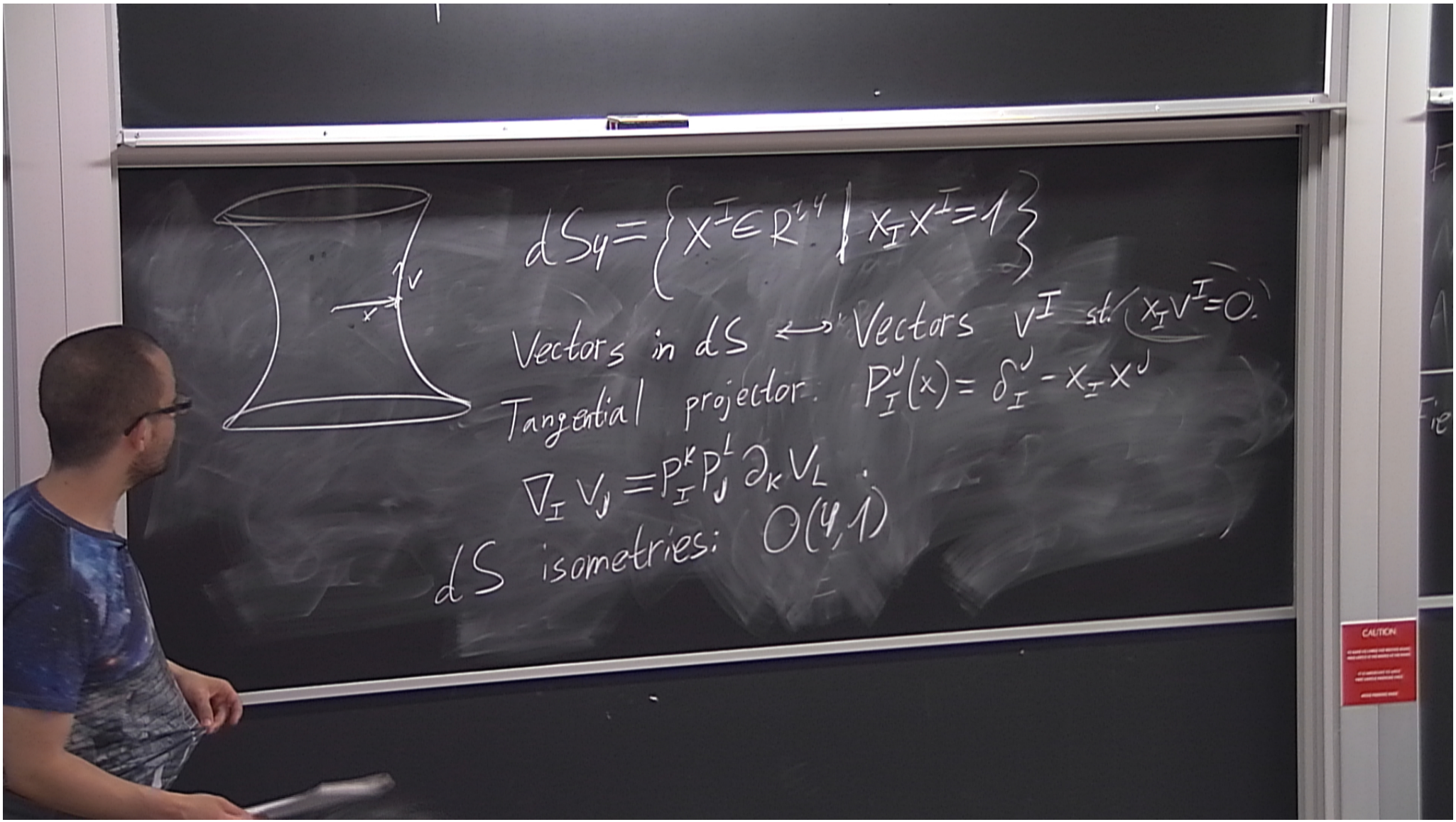
$\varphi_{\alpha\beta}$. $\varphi_{\dot\alpha\dot\beta}$

$\nabla_{\beta\dot\beta} \varphi^{\beta}{}_{\alpha_1\alpha_2\alpha_3}$.

$\nabla_{[\mu} F_{\nu\rho]} = 0$

$\nabla_\mu F^{\mu\nu} = 0$


$dS_4 = \{ X^I \in R^{1,4} \mid X_I X^I = 1 \}$
Vectors in dS $\longleftrightarrow$ Vectors V^I s.t. $X_I V^I = 0$.
Tangential projector: $P_I^J(x) = \delta_I^J - X_I X^J$
$\nabla_I V_J = P_I^K P_J^L \partial_K V_L$
dS isometries: $O(4,1)$
x
v

At a point $x \in dS$: $\quad x^\mu = (0,0,0,0,1)$

$$T_{[IJ]} \longrightarrow T_{[4\mu]} \cdot T_{[\mu\nu]}$$

$$P_\mu \qquad J_{\mu\nu}$$

(But: $[P, P] \sim J$)

$(r_{dS} = 1)$

Spinors of $O(3,1)$ in dS

derive from

spinors of $O(4,1) \approx USp(2,2)$

4-component Dirac spinors ψ^a

$\equiv$

Twistors