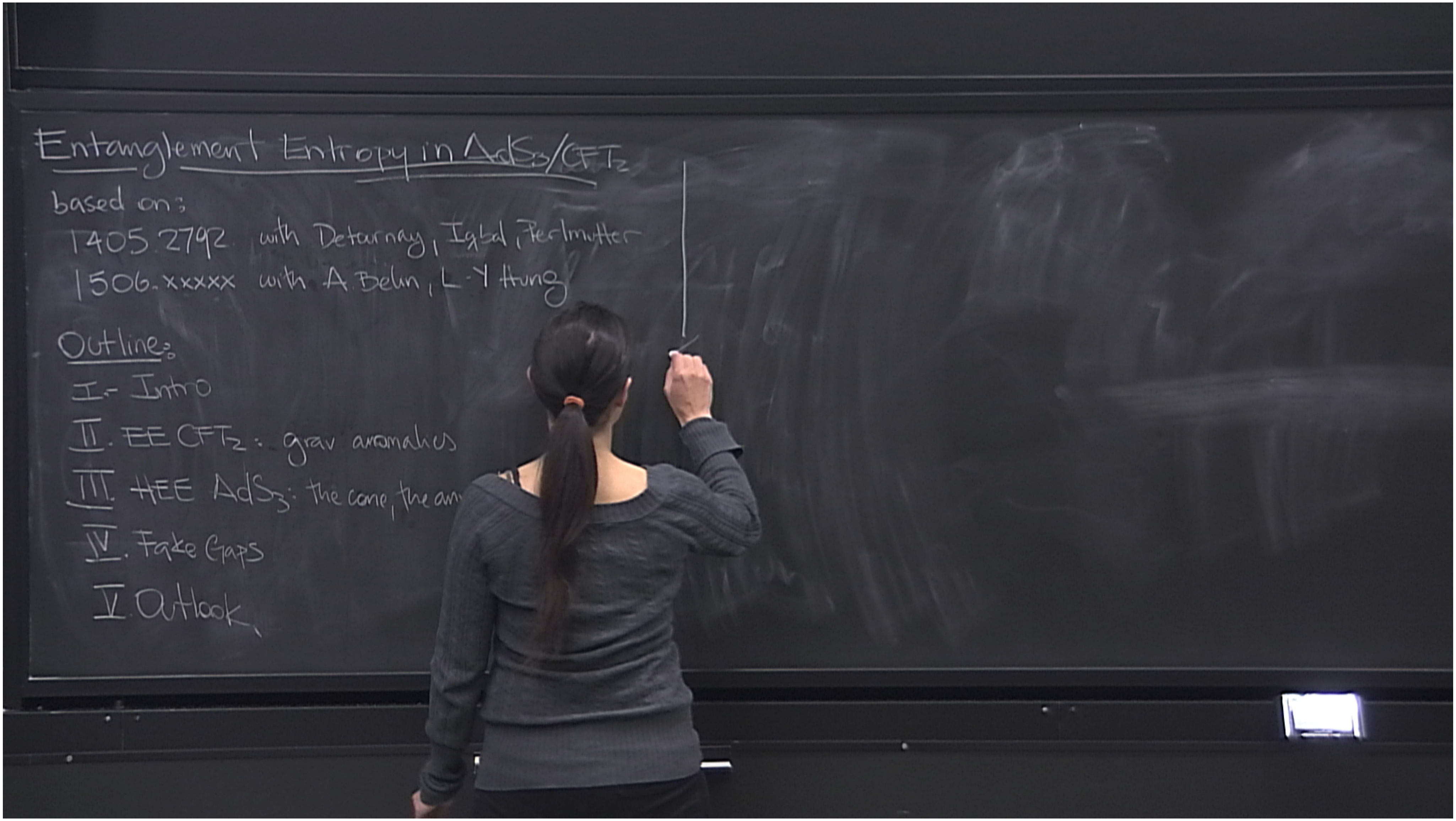
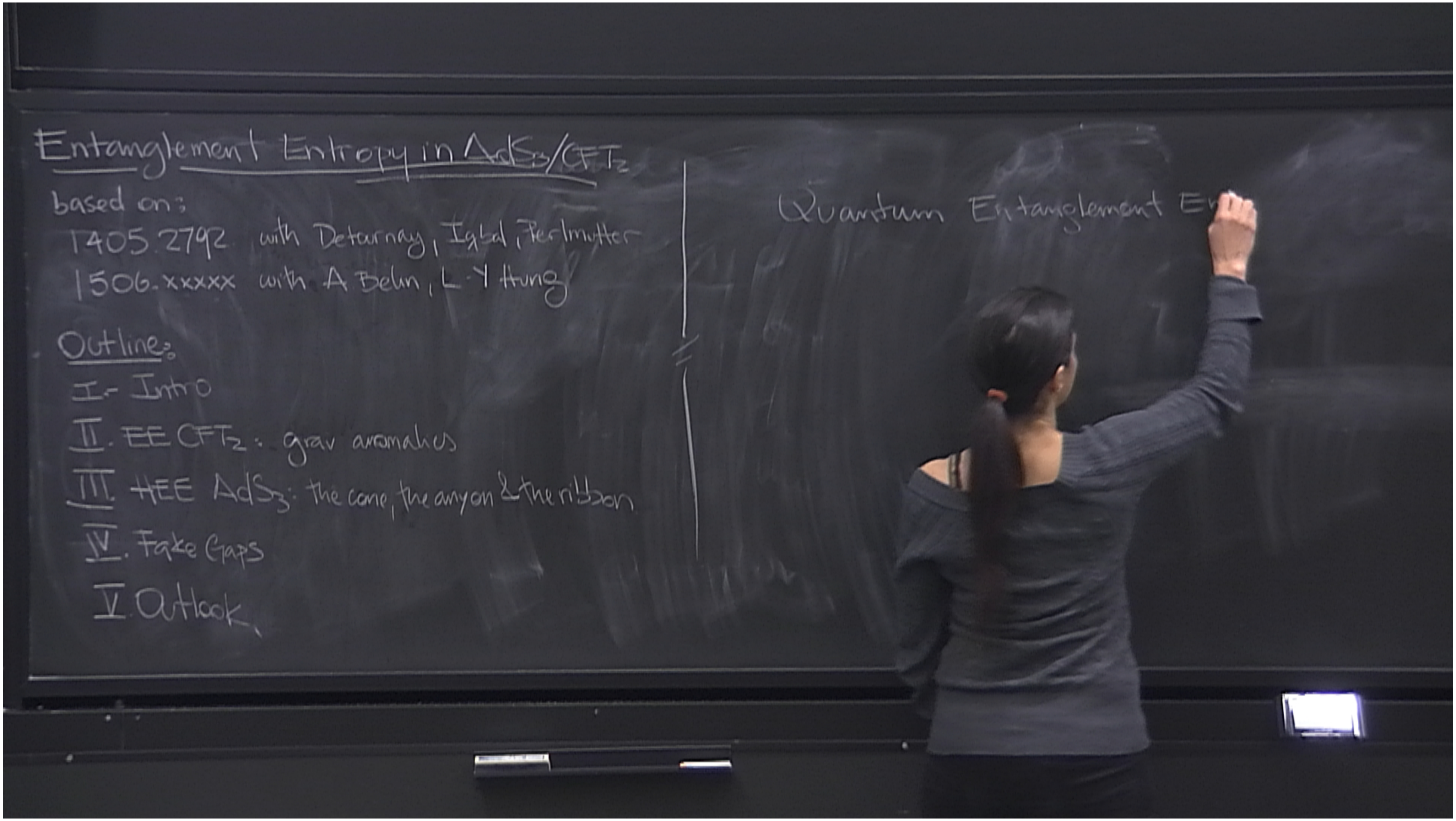
Title: Entanglement Entropy in AdS3/CFT2: anomalies and fake gaps

Date: May 12, 2015 02:00 PM

URL: http://pirsa.org/15050023

Abstract: <p>The concept of quantum entanglement entropy is playing a key role in understanding the mechanism underlying holography. In this talk we will discuss how entanglement can capture non-trivial geometric properties of the bulk spacetime. The goal is to exploit the interplay between anomalies and entanglement entropy, and for concreteness we will focus on AdS3/CFT2. Anomalies play as well a key role in RG flows, and in this context we will see how entanglement, anomalies and geometry conspire to capture dynamically the correct physics.</p>


Entanglement Entropy in AdS3/CFT2
based on:
1405.2792 with Detournay, Iqbal, Perlmutter
1506.xxxxx with A. Belin, L-Y Hung
Outline:
I.- Intro
II. EE CFT2: grav anomalies
III. HEE AdS3: the cone, the
IV. Fake Gaps
V. Outlook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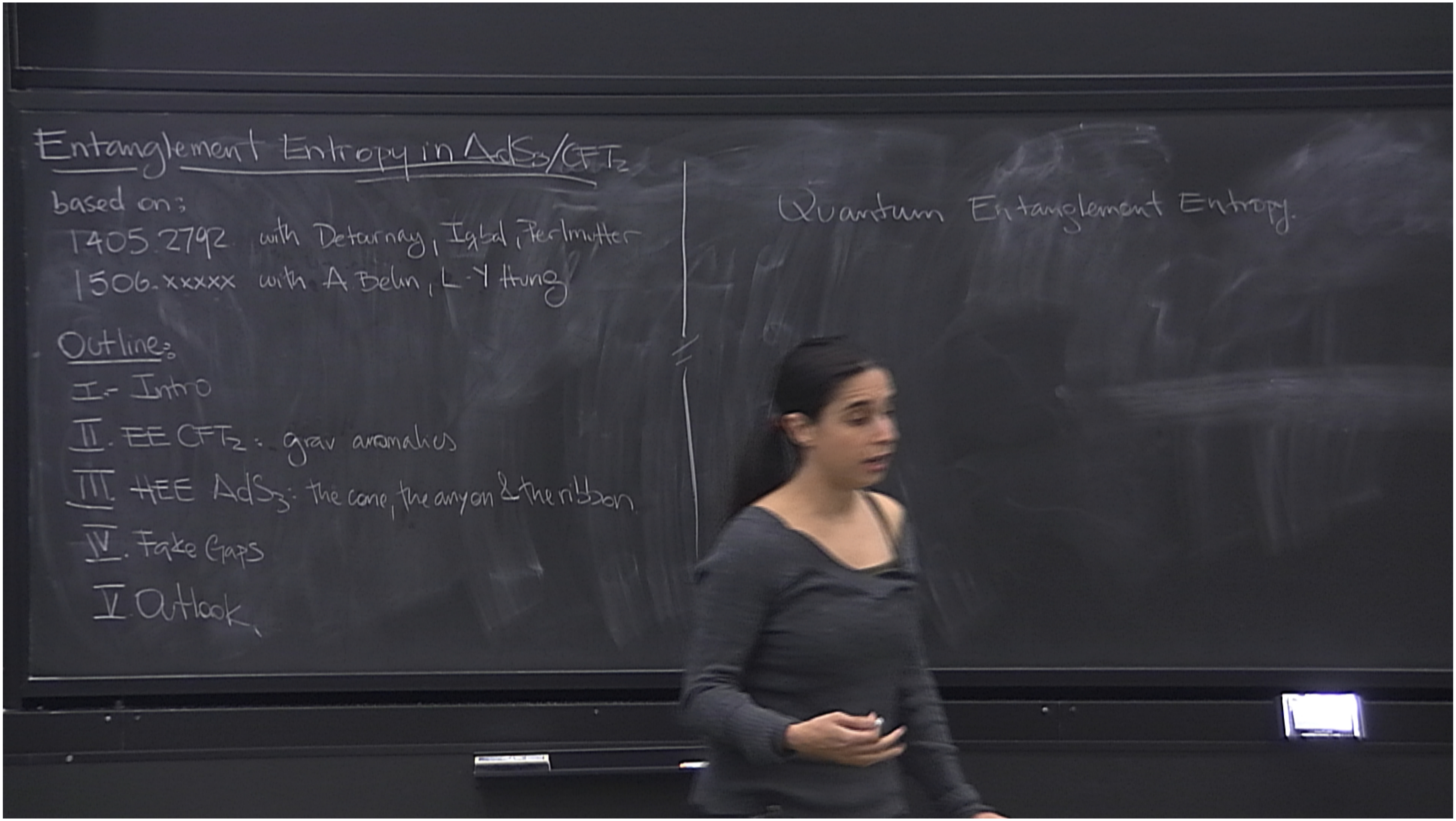
Quantum Entanglement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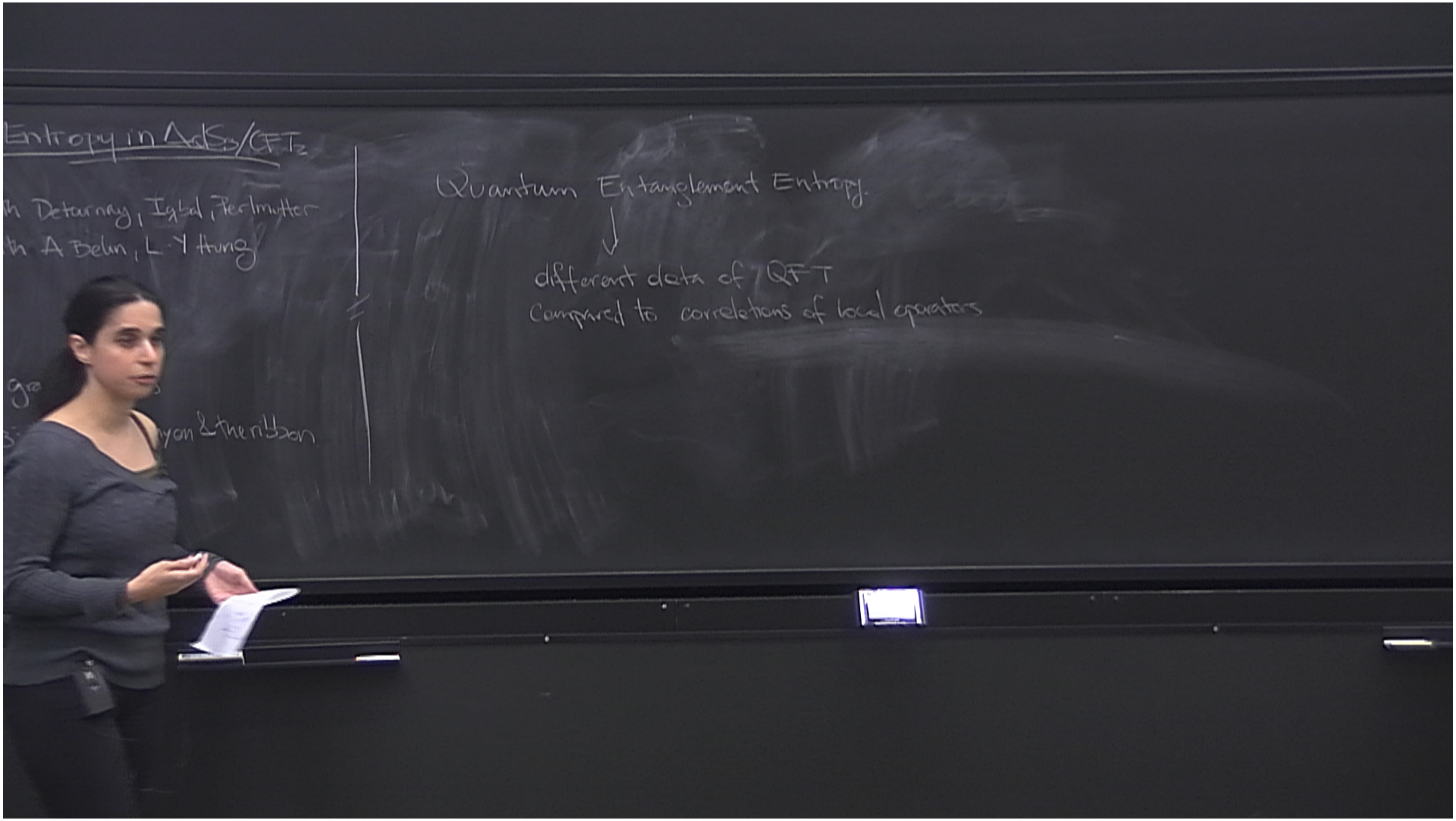
Entropy in AdS3/CFT2
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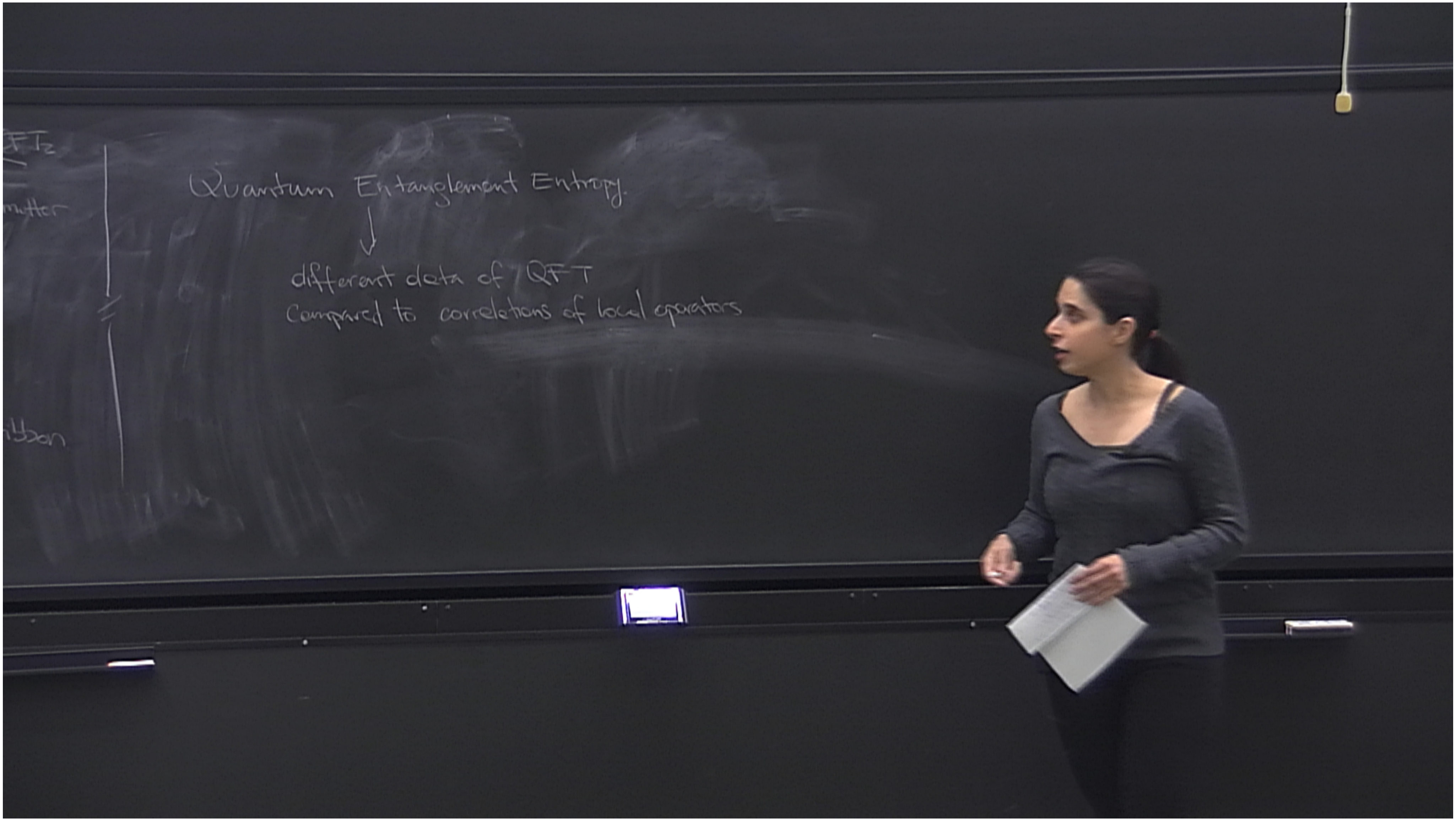
Quantum Entanglement Entropy.
different data of QFT
Compared to correlations of local operators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different data of QFT
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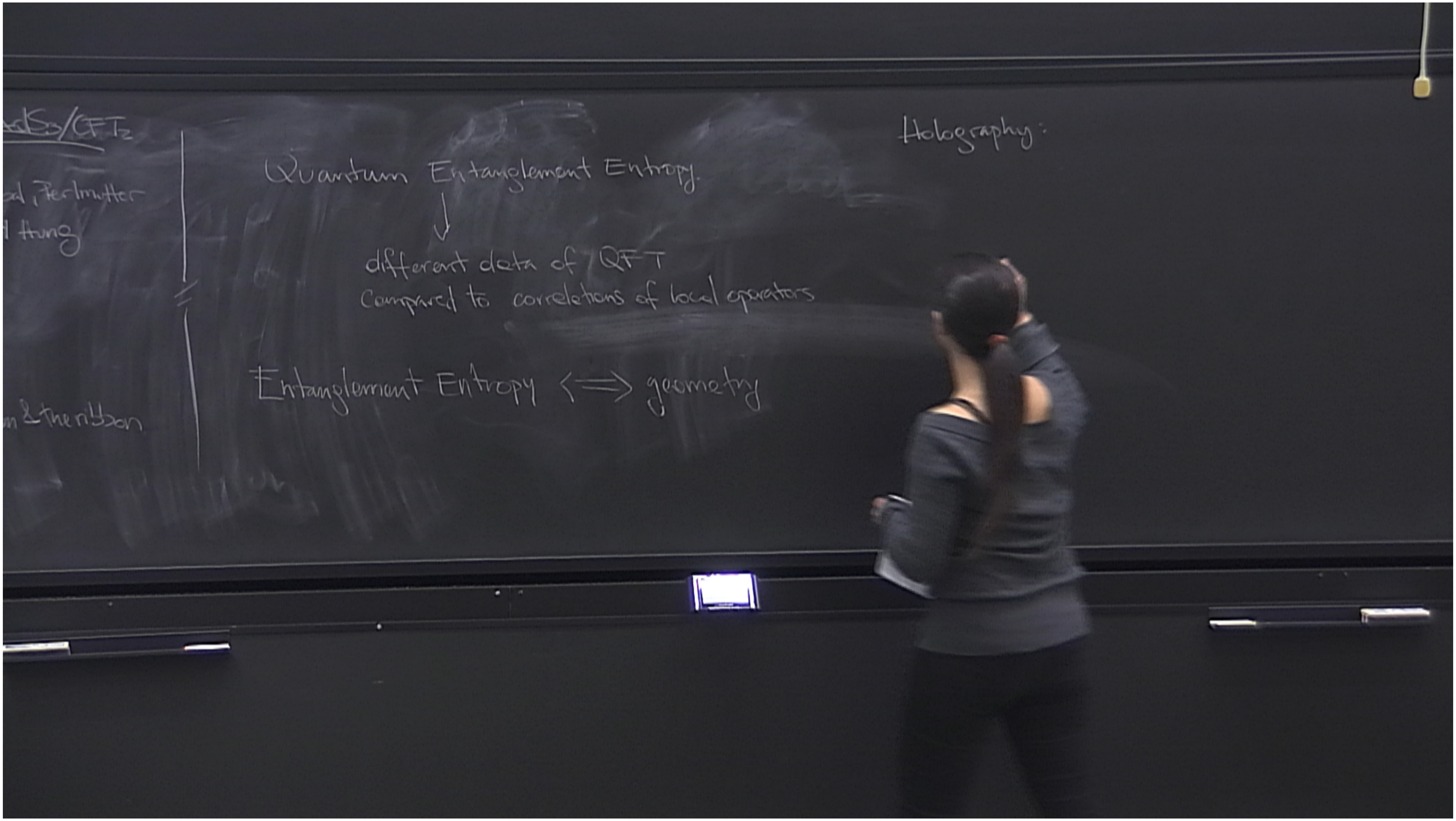
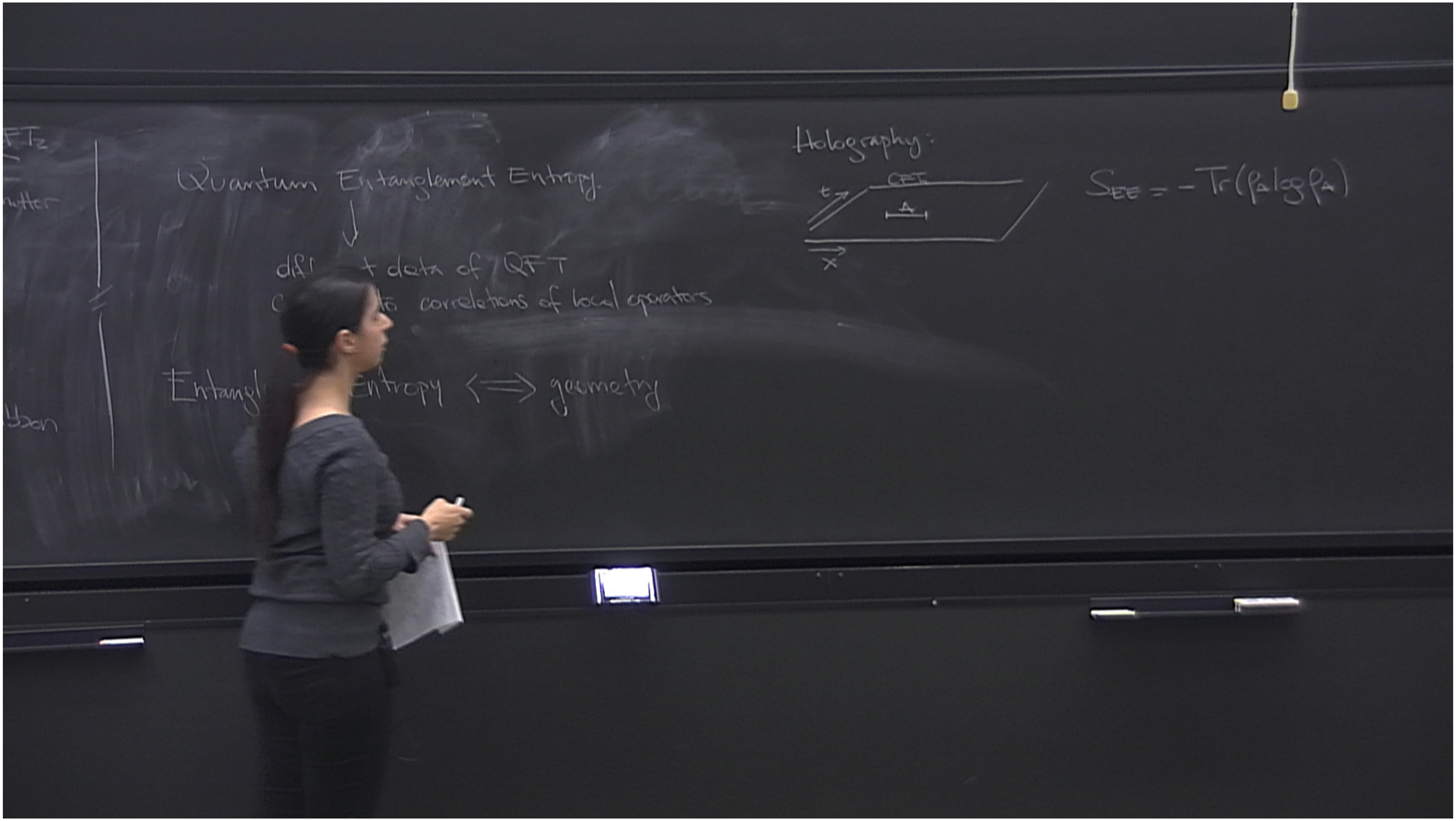
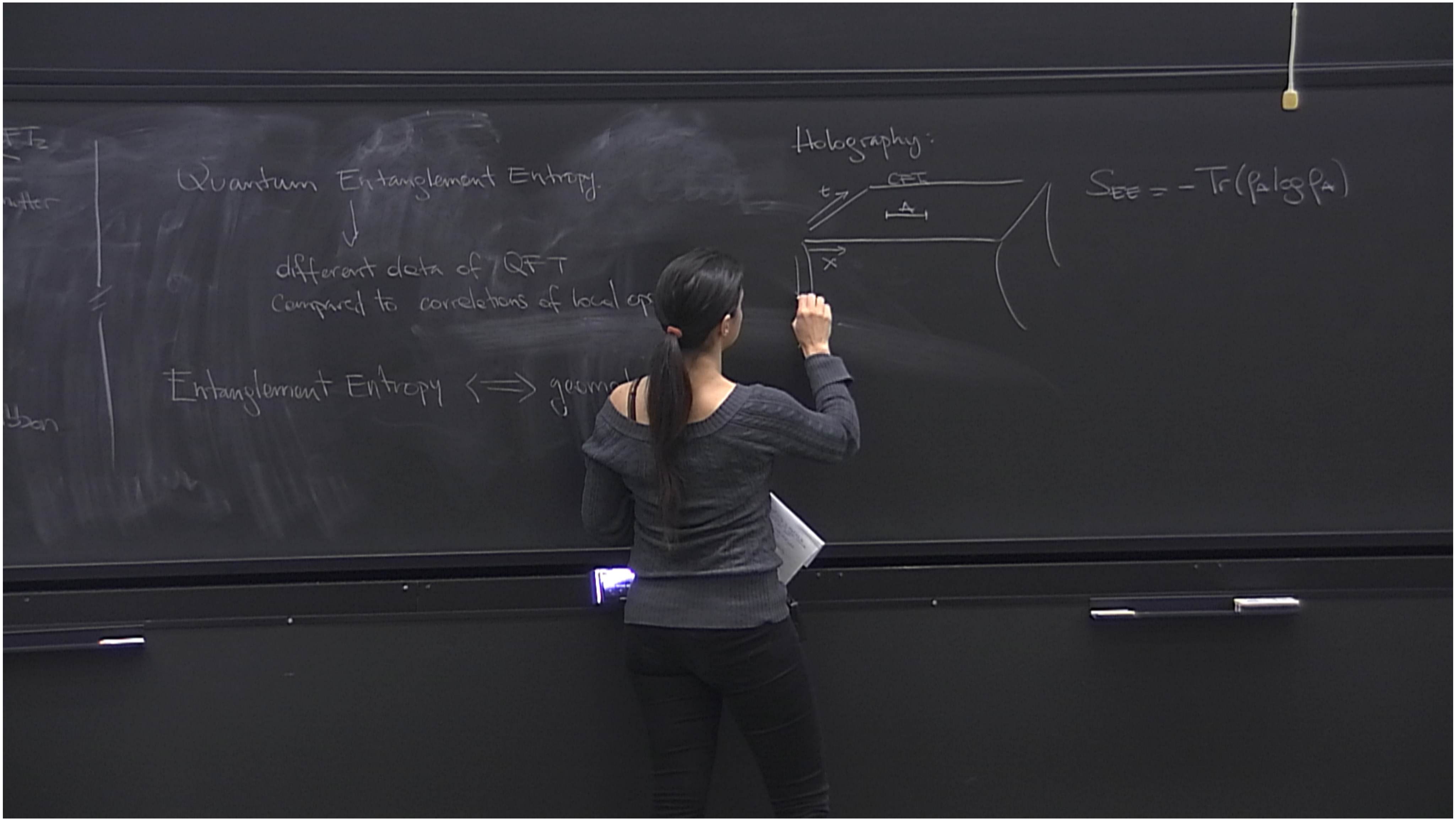

Quantum Entanglement Entropy
different data of QFT
Compared to correlations of local operators
Entanglement Entropy <==> geometry
Holography:


Quantum Entanglement Entropy
data of QFT
correlations of local operators
Entropy <==> geometry
Holography:
CFT
t
A
x
S_EE = - Tr(ρ_A log ρ_A)


Quantum Entanglement Entropy
different data of QFT
Compared to correlations of local op
Entanglement Entropy ⟺
Holography:
CFT
t
x
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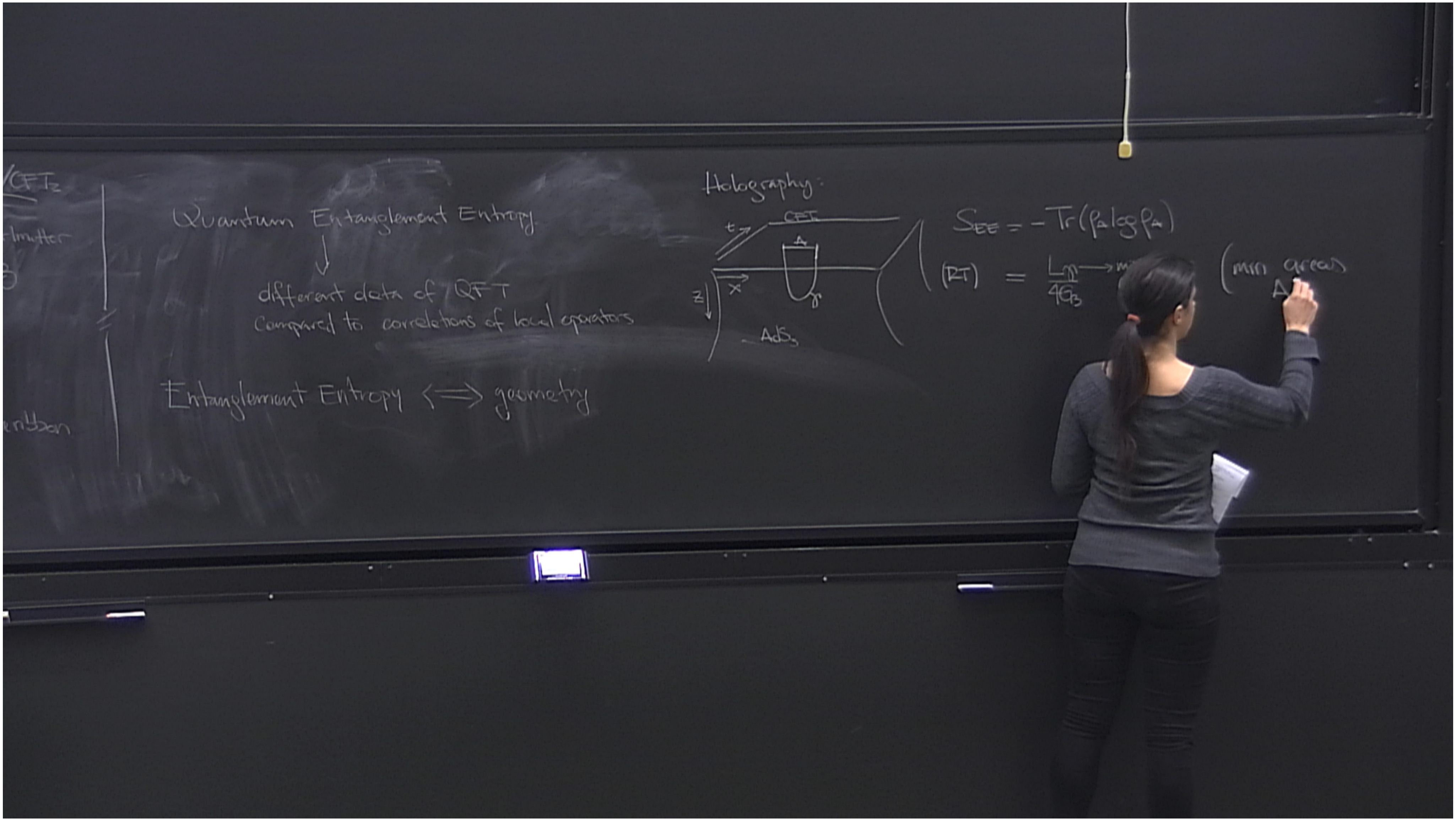
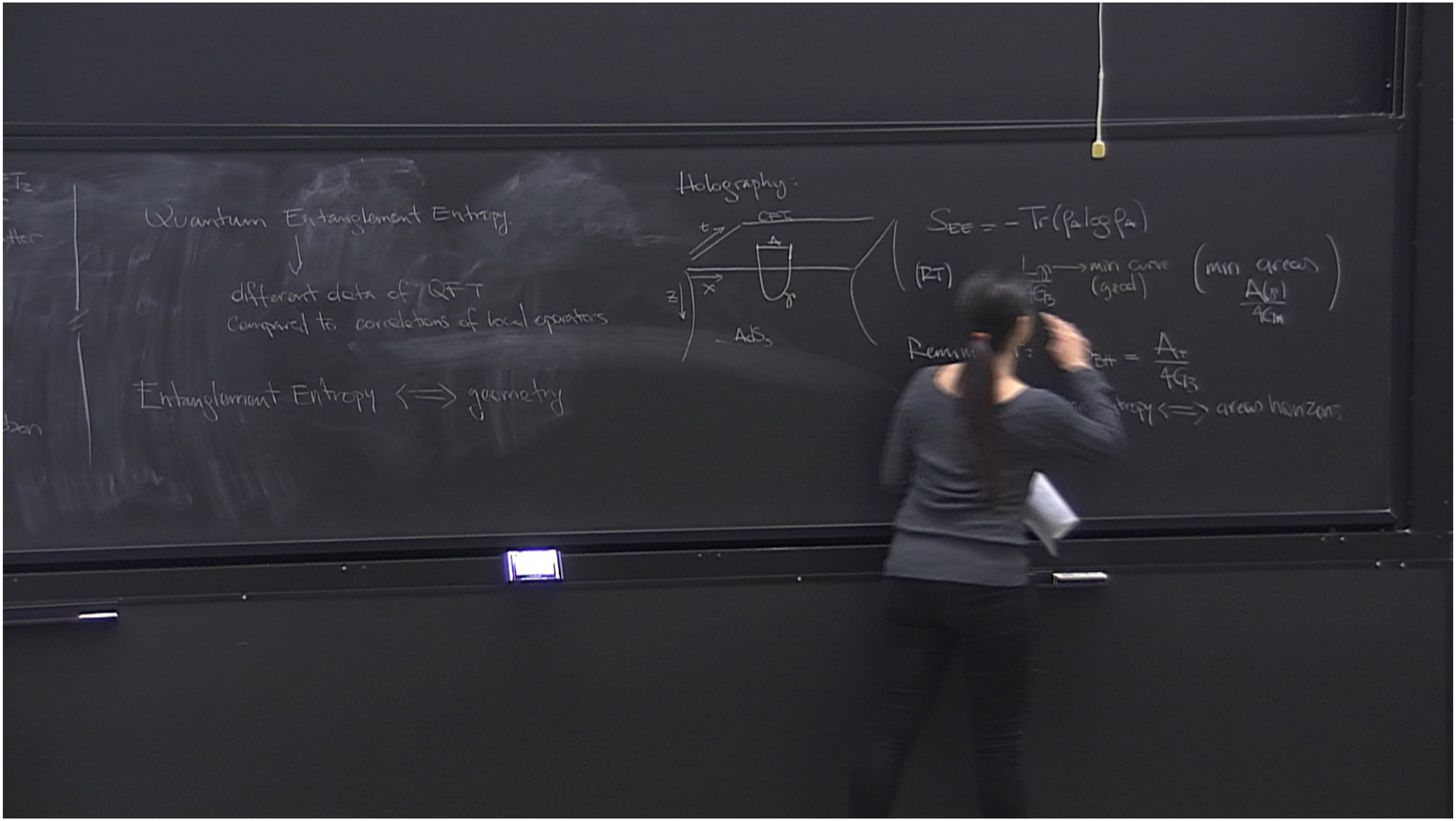
$S_{EE} = -\mathrm{Tr}(\rho_A \log \rho_A)$

Quantum Entanglement Entropy.
different data of QFT
Compared to correlations of loc
Entanglement Entropy <=> geo
Holography:
CFT
t
x
z
A
AdS3
S_EE = -Tr(ρA log ρA)


Holography:
Quantum Entanglement Entropy
different data of QFT
Compared to correlations of local operators
Entanglement Entropy ⟺ geometry
S_EE = −Tr(ρ_A log ρ_A)
(RT) = L_γ / 4G_3
(min areas
CFT
AdS_3


Quantum Entanglement Entropy
different data of QFT
Compared to correlations of local operators
Entanglement Entropy ⟺ geometry
Holography:
CFT
AdS3
S_EE = −Tr(ρ_A log ρ_A)
min curve (geod)
(min areas A(γ)/4G_N)
S_BH = A_r/4G_3



Quantum Entanglement Entropy
different data of QFT
compared to correlations of local operators
Entanglement Entropy ⟺ geometry
Holography:
CFT
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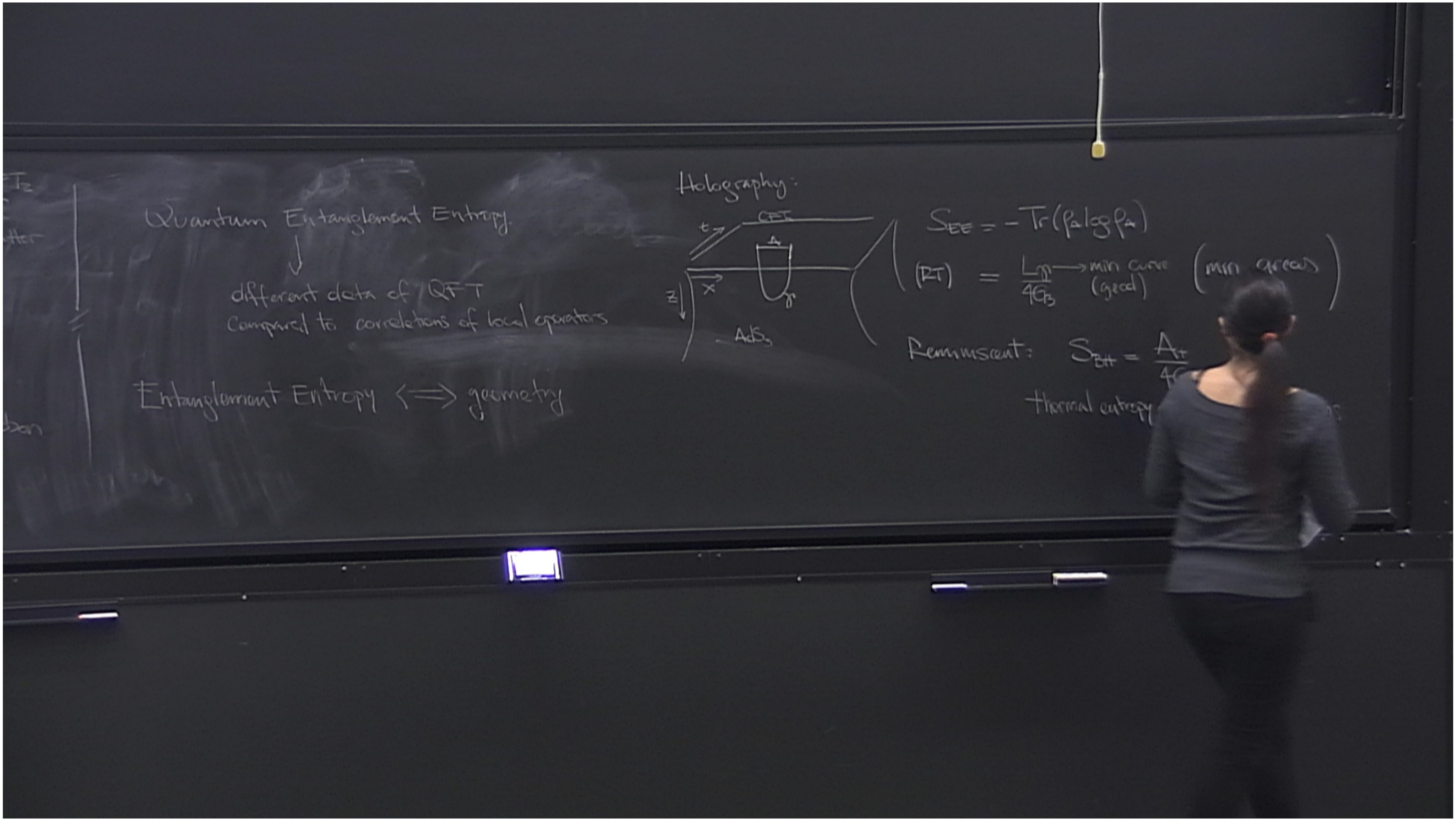
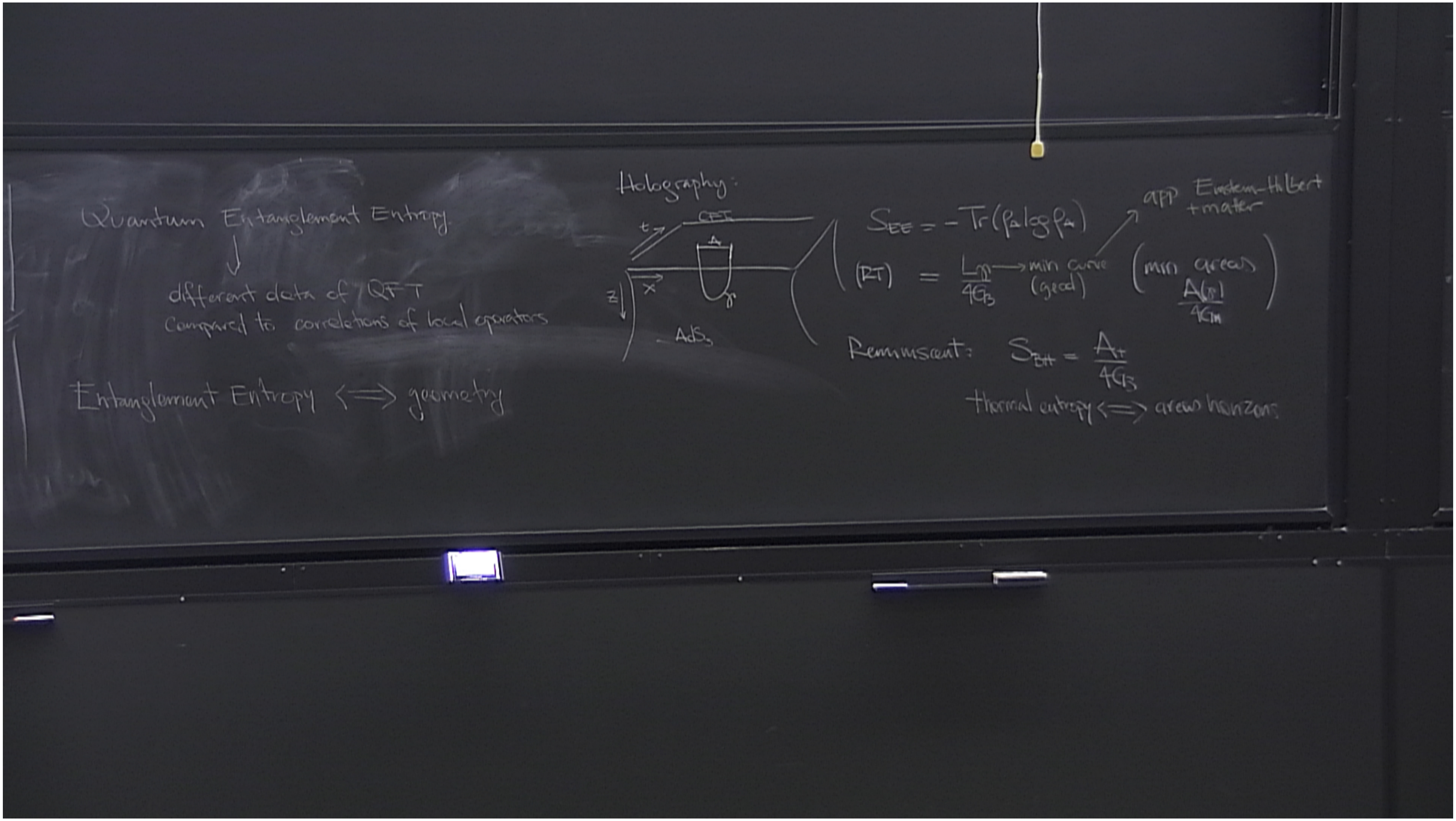
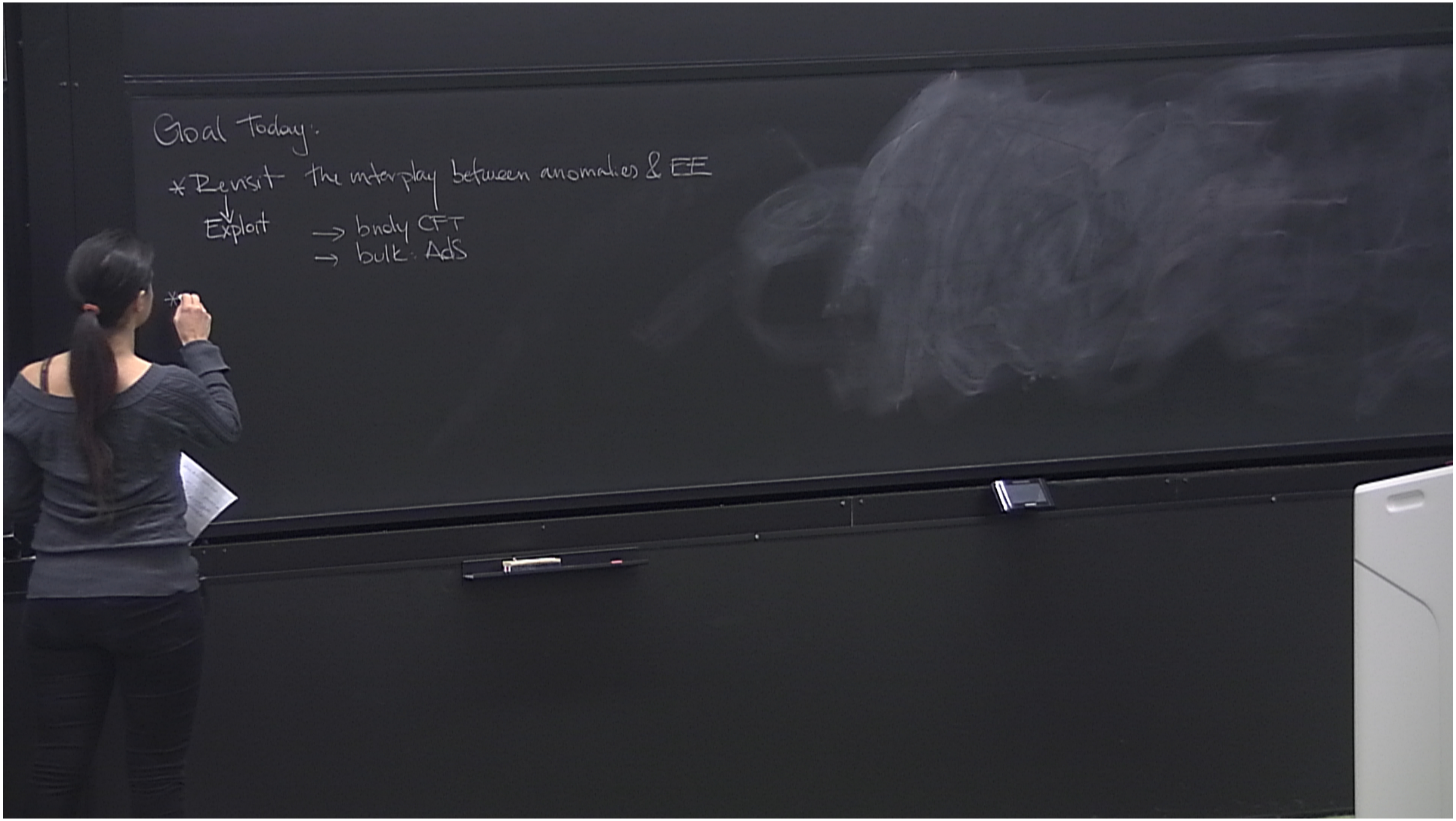
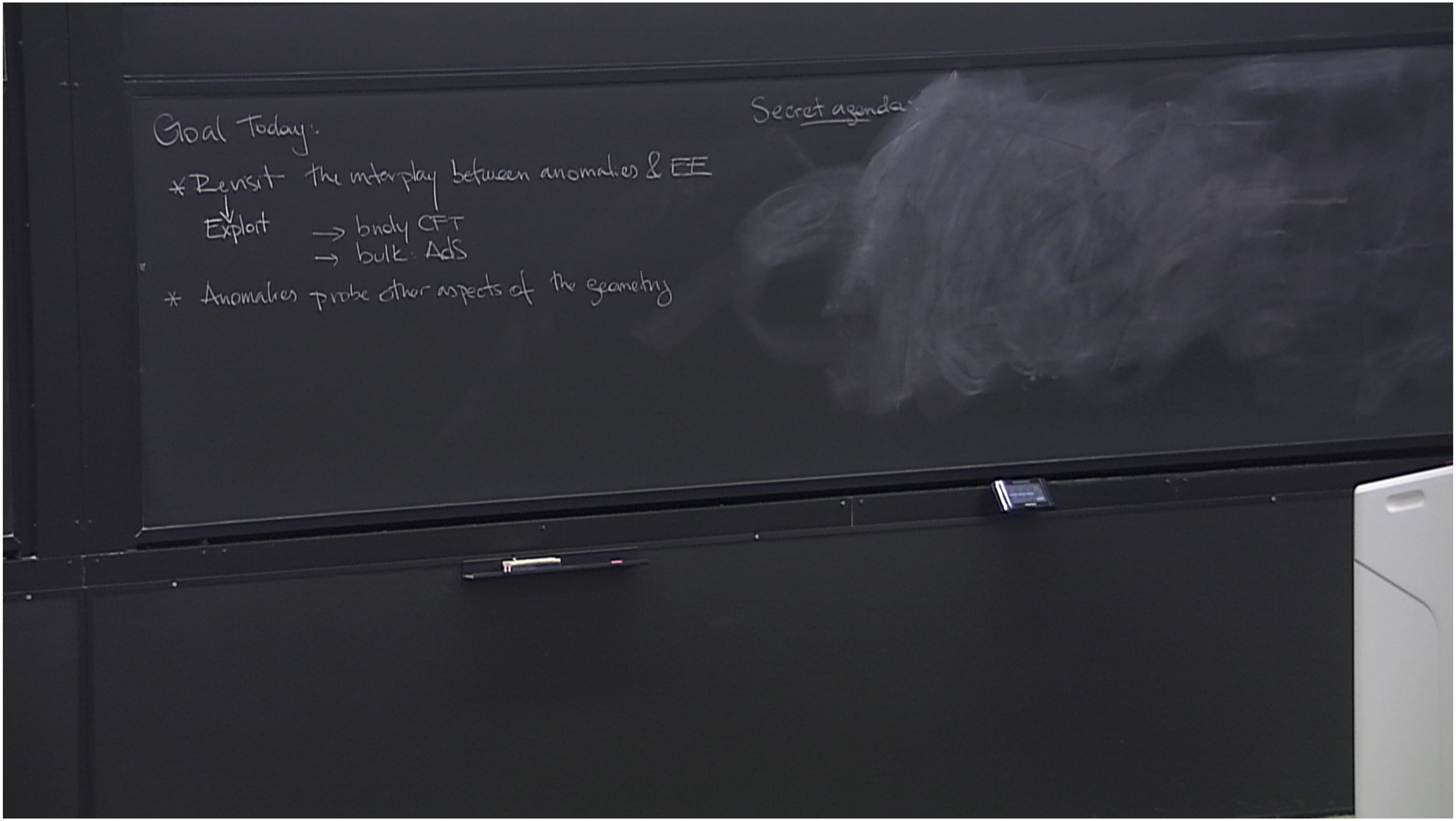
S_EE = −Tr(ρ_A log ρ_A)
(RT)
min curve
(geod)
min areas
Reminder
area horizons


Quantum Entanglement Entropy
different data of QFT
compared to correlations of local operators
Entanglement Entropy ⟺ geometry
Holography:
CFT
AdS3
$S_{EE} = -\mathrm{Tr}(\rho_A \log \rho_A)$
(RT) $= \frac{L_\gamma}{4G_3}$ → min curve (geod)
(min areas)
Reminiscent: $S_{BH} = \frac{A_+}{4G}$
thermal entropy


Quantum Entanglement Entropy
different data of QFT
compared to correlations of local operators
Entanglement Entropy <=> geometry
Holography:
CFT
AdS3
SEE = -Tr(ρA log ρA)
(RT) = Lγ/4G3 → min curve (geod)
(min areas A(γ)/4GN)
app Einstein-Hilbert +matter
Reminiscent: SBH = AH/4G3
thermal entropy <=> area horizons


Goal Today:
* Revisit the interplay between anomalies & EE
Exploit
→ bndy CFT
→ bulk: AdS
*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Exploit
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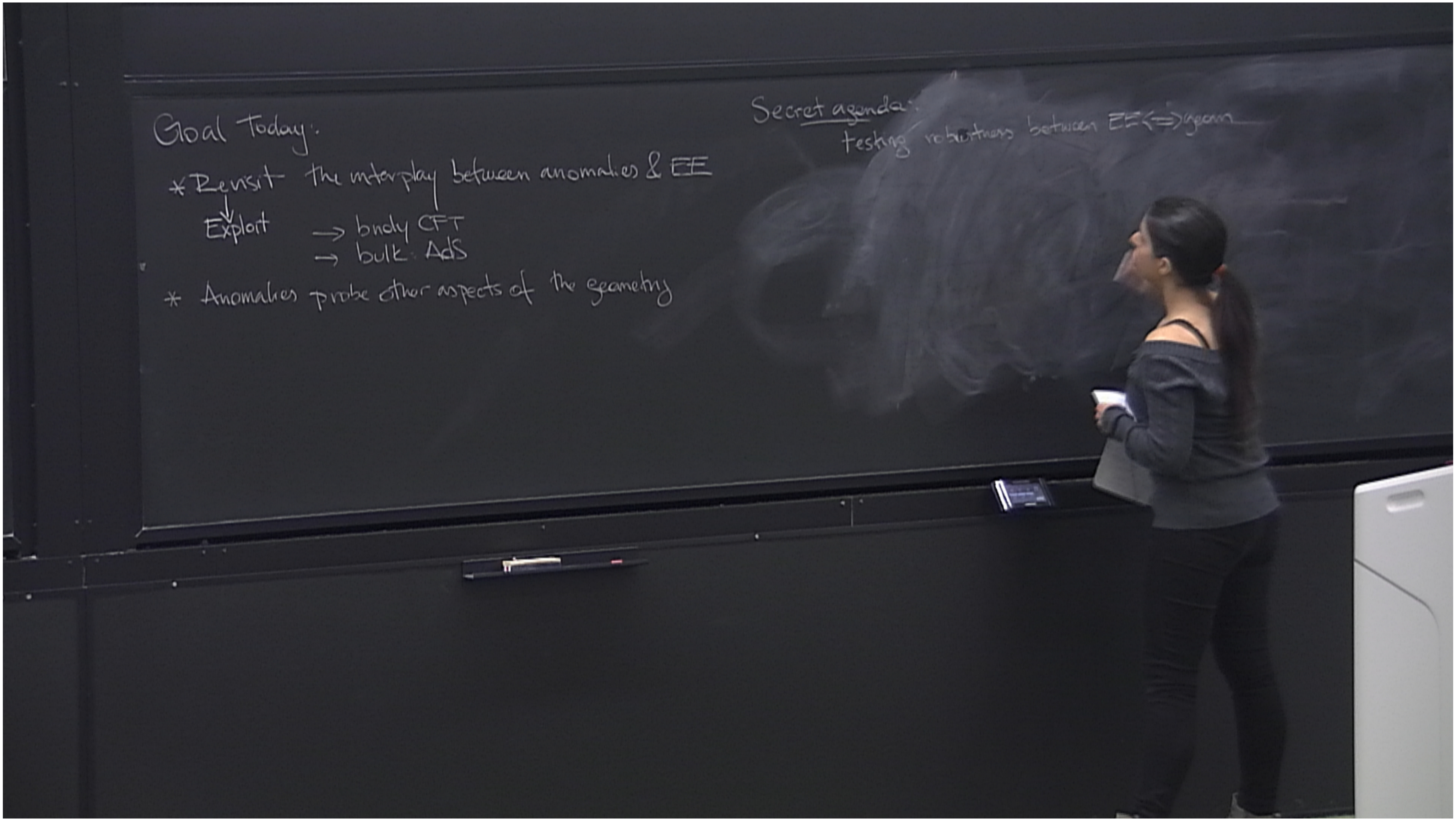
→ bulk: AdS
* Anomalies probe other aspects of the geometry
Secret agenda: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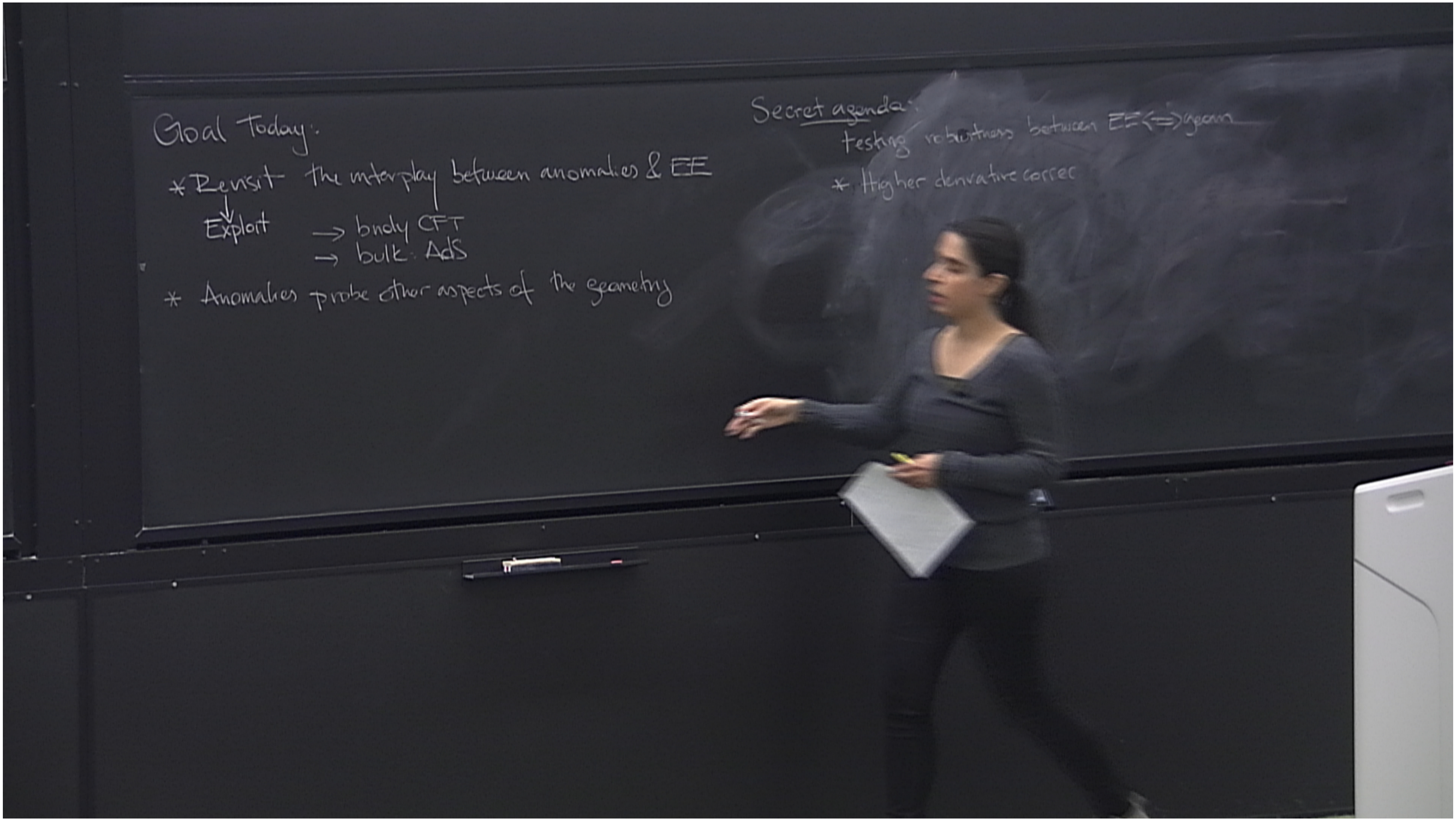
Secret agenda:
testing robustness between EE ⟺ geom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Secret agenda:
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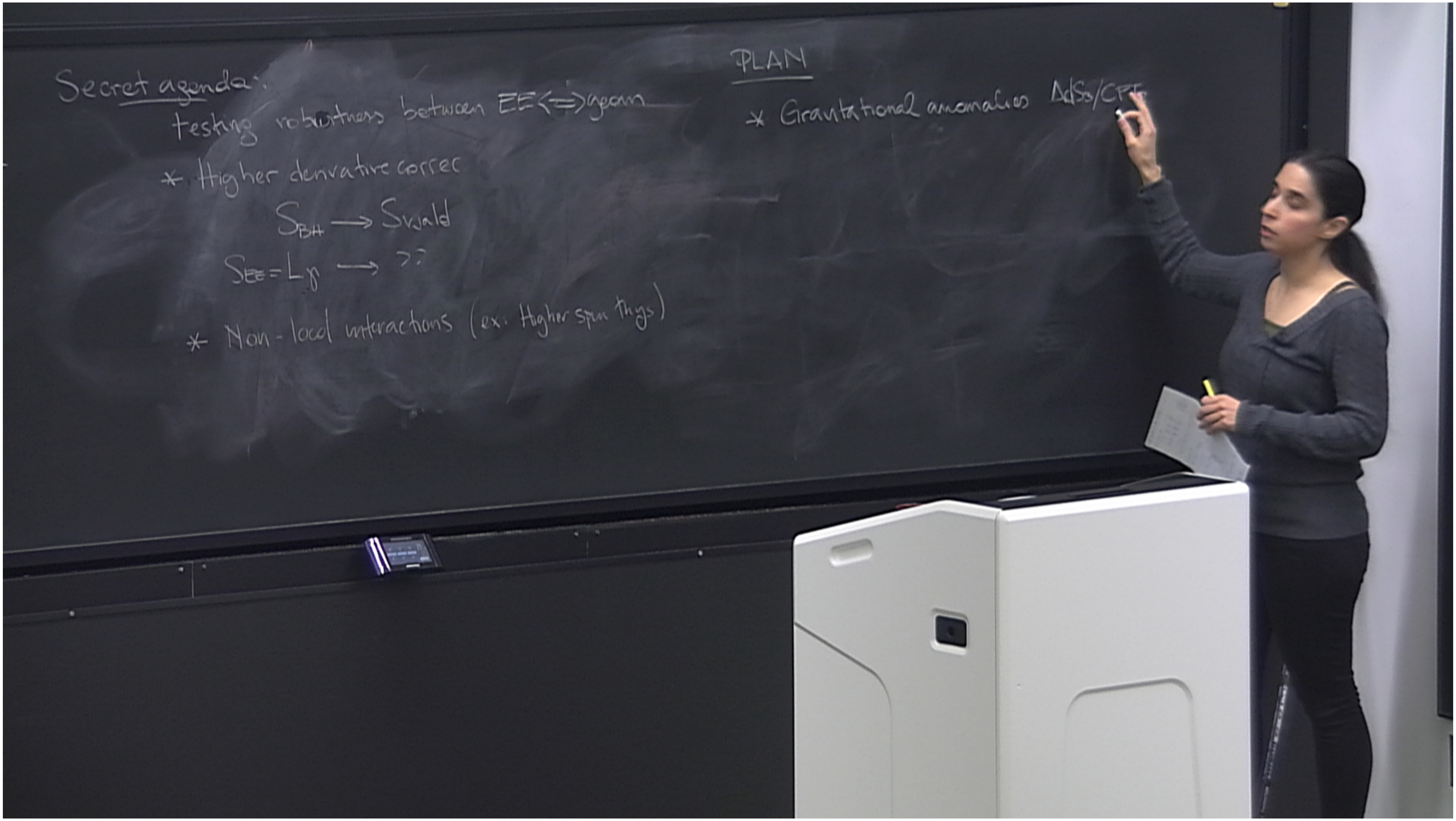
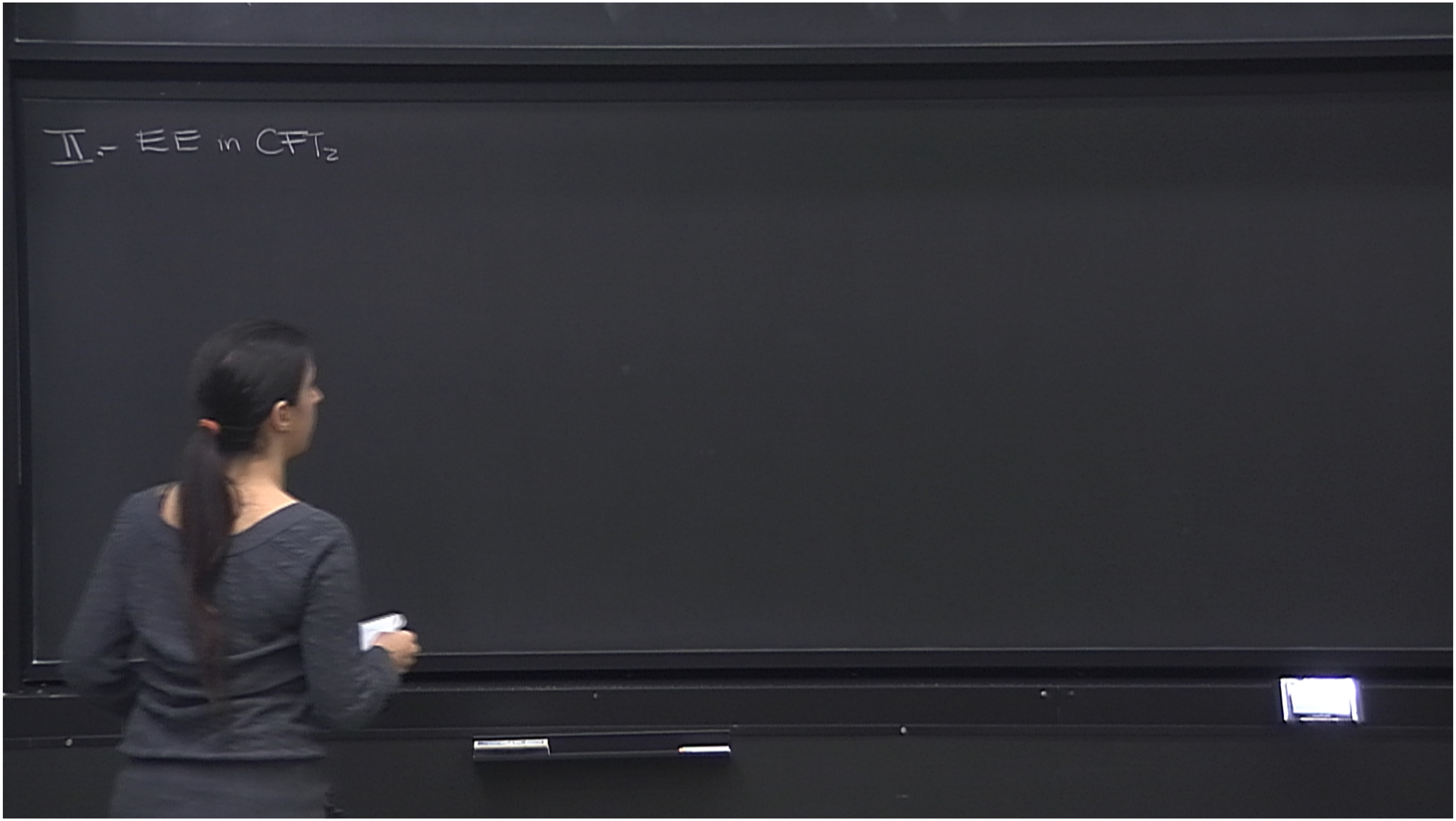
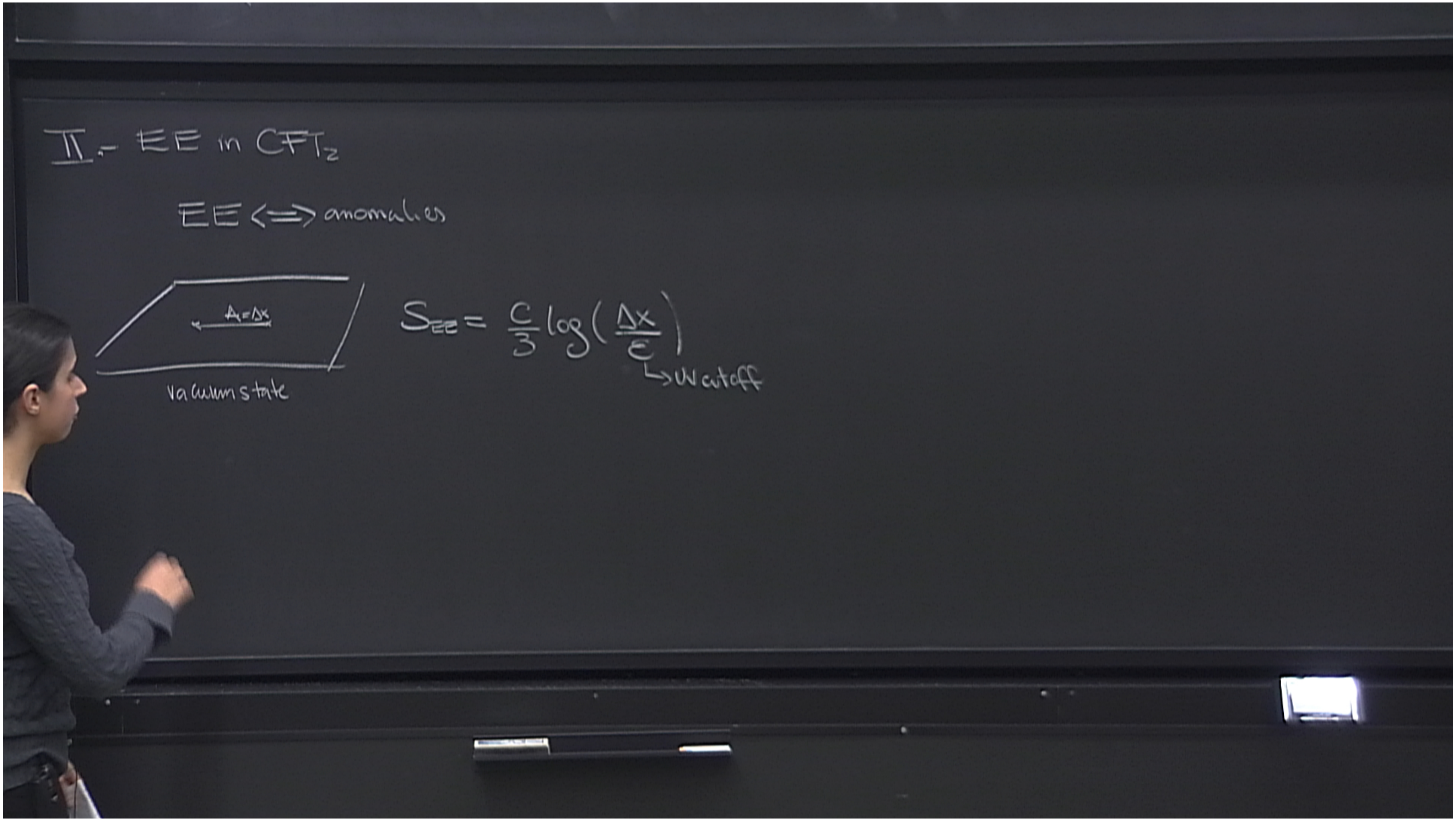
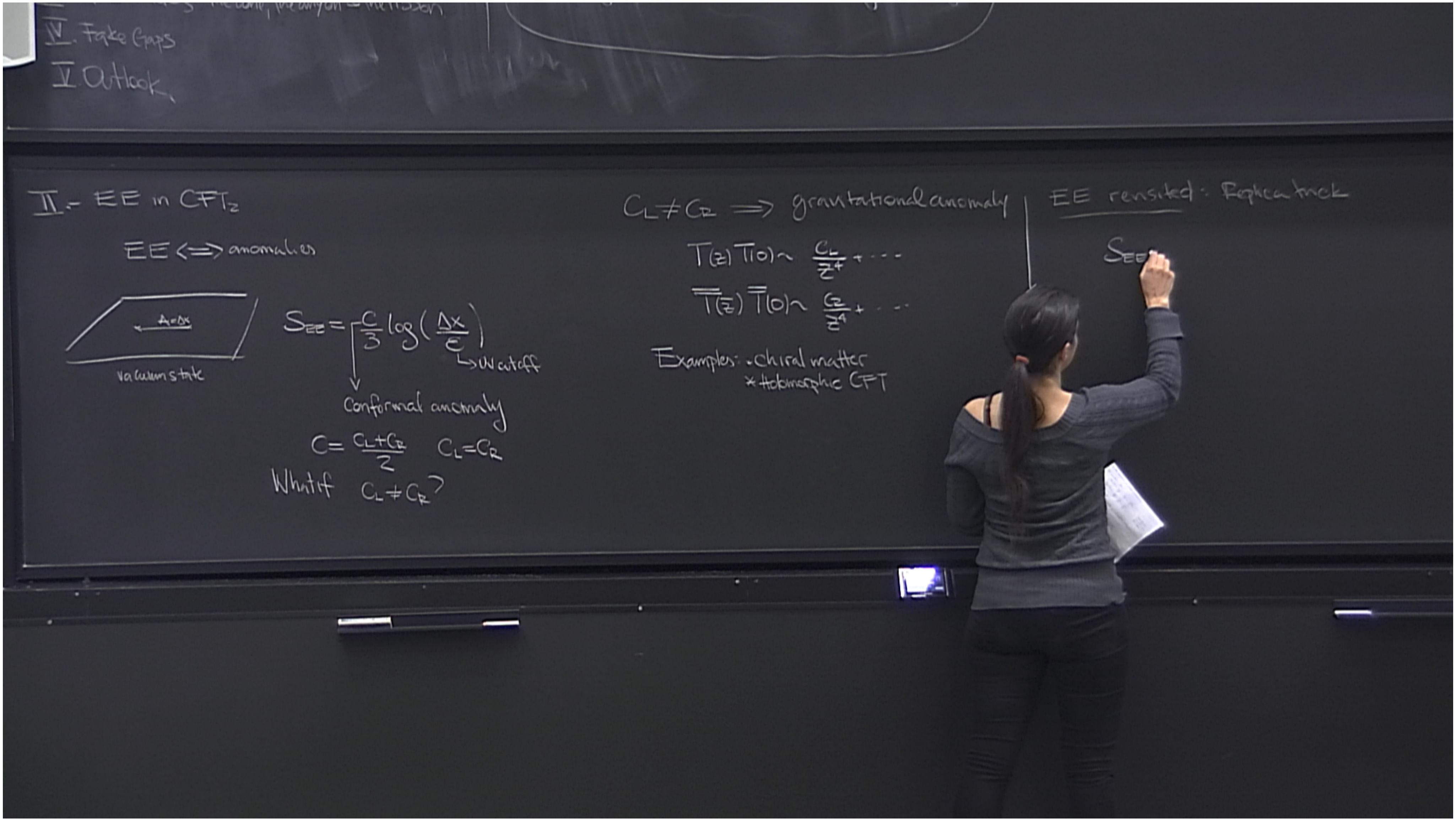
* Higher derivative correc

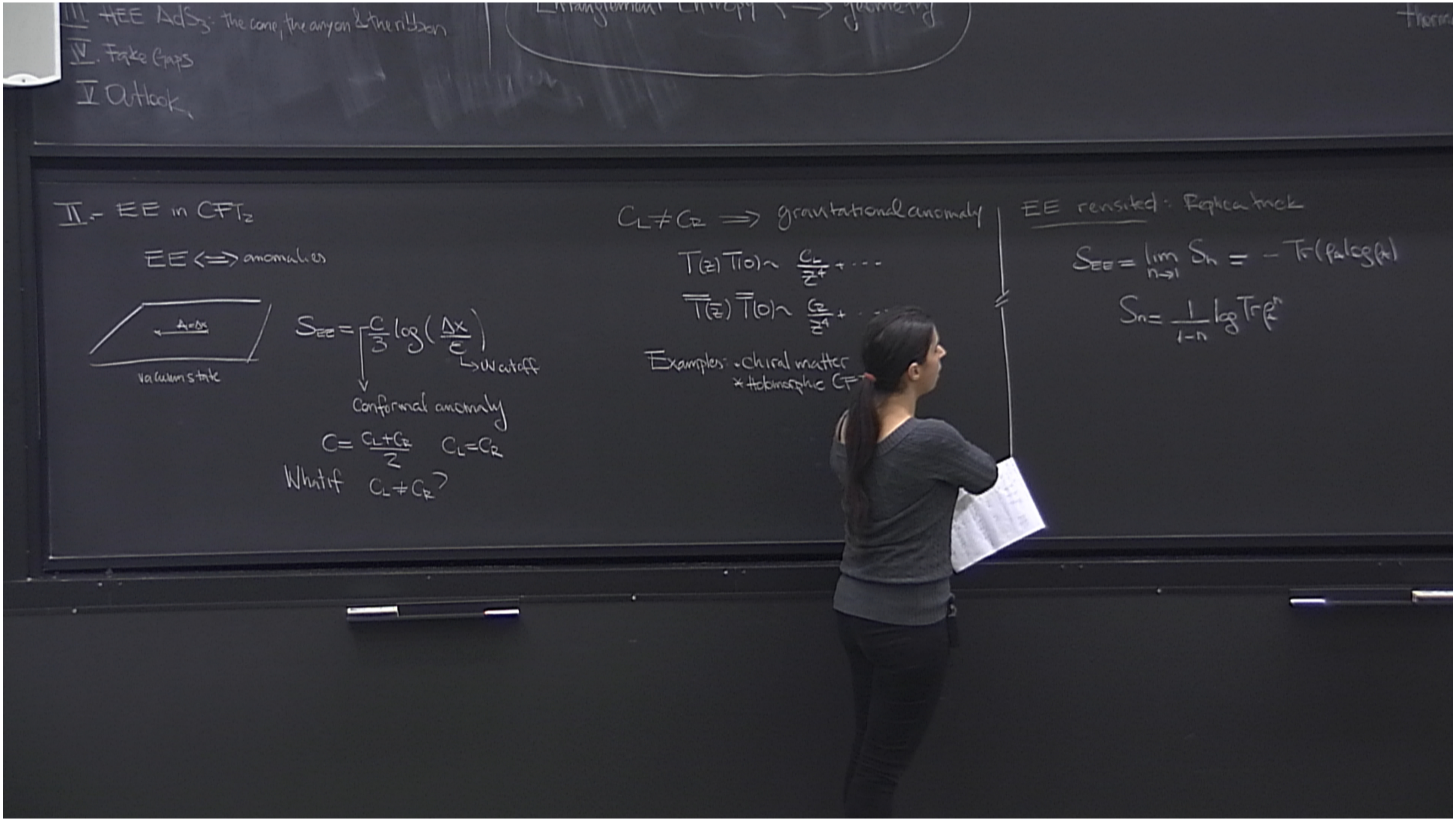

Secret agenda:
testing robustness between EE ⟺ geom
* Higher derivative correc
S_BH → S_Wald
S_EE = L_γ → ??
* Non-local interactions (ex. Higher spin thys)
PLAN
* Gravitational anomalies AdS3/CFT


II.- EE in CFT2


II.- EE in CFT₂
EE ⟺ anomalies
A=Δx
vacuum state
S_EE = c/3 log(Δx/ε)
UV cutoff


II.- EE in CFT2
EE <=> anomalies
A=Δx
vacuum state
S_EE = c/3 log(Δx/ε)
UV cutoff
Conformal anomaly
C = (cL+cR)/2
cL=cR
What if cL ≠ cR?
cL ≠ cR ⇒ gravitational anomaly
T(z)T(0) ~ cL/z^4 + ...


IV. Fake Gaps
V Outlook
II.- EE in CFT2
EE ⟺ anomalies
vacuum state
See = c/3 log(Δx/ε)
UV cutoff
Conformal anomaly
c = (cL+cR)/2   cL=cR
What if cL≠cR?
cL≠cR ⟹ gravitational anomaly
T(z)T(0) ~ cL/z^4 + ...
T̄(z̄)T̄(0) ~ cR/z̄^4 + ...
Examples: ·chiral matter
*Holomorphic CFT
EE revisited: Replica trick
See


III HEE AdS3: the cone, the anyon & the ribbon
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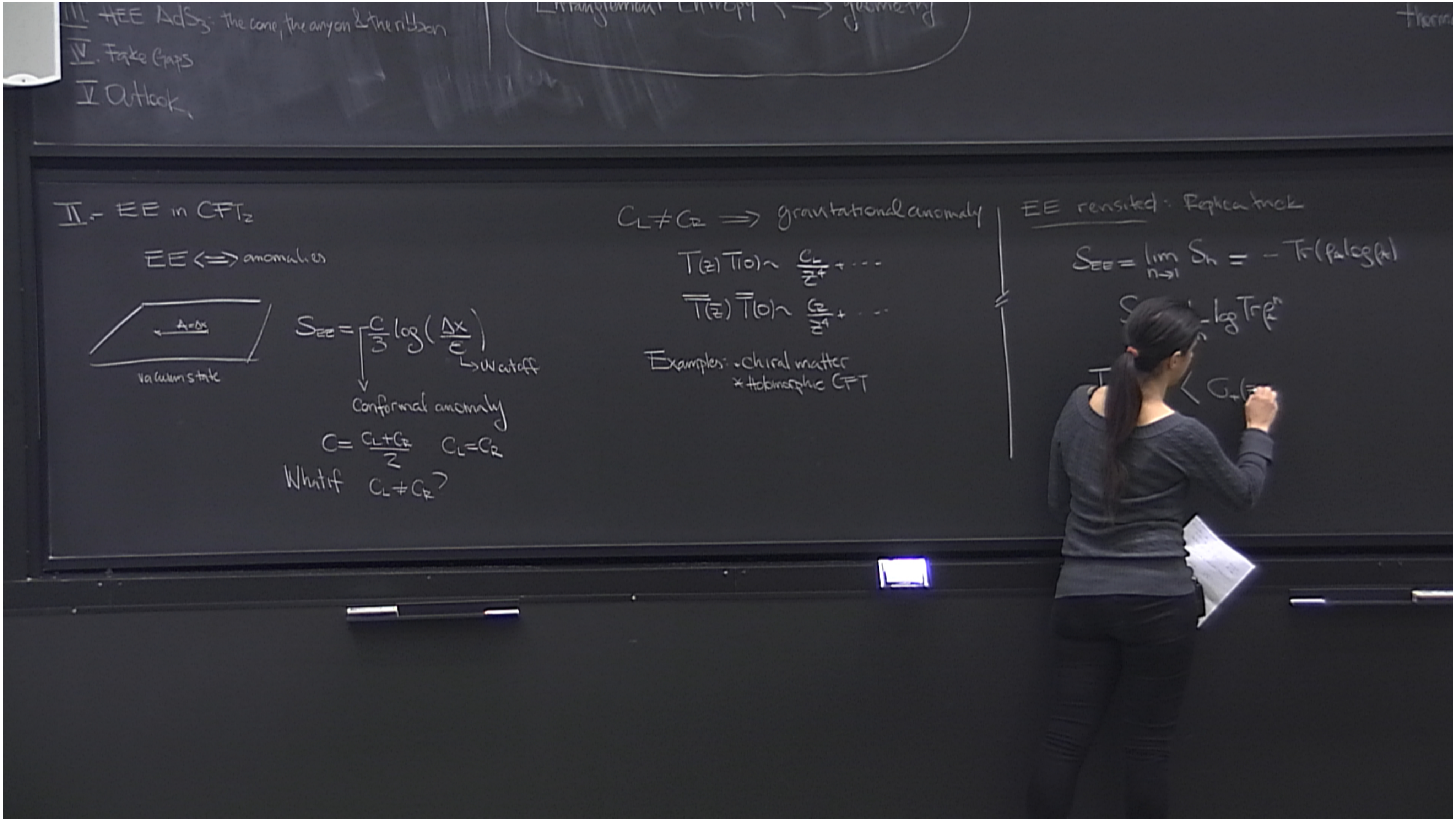
V Outlook
II - EE in CFT2
EE ⟺ anomalies
vacuum state
SEE = c/3 log(Δx/ε)
UV cutoff
Conformal anomaly
C = (CL+CR)/2
CL=CR
What if CL ≠ CR?
CL ≠ CR ⟹ gravitational anomaly
Examples: chiral matter
Holomorphic CFT
EE revisited: Replica trick
SEE = lim n→1 Sn = − Tr(ρA log(ρA))
Sn = 1/(1−n) log Tr ρA^n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S_EE = c/3 log(Δx/ε)
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C = (CL+CR)/2   CL=CR
What if CL ≠ CR?
CL ≠ CR ⟹ gravitational anomaly
Examples: · chiral matter
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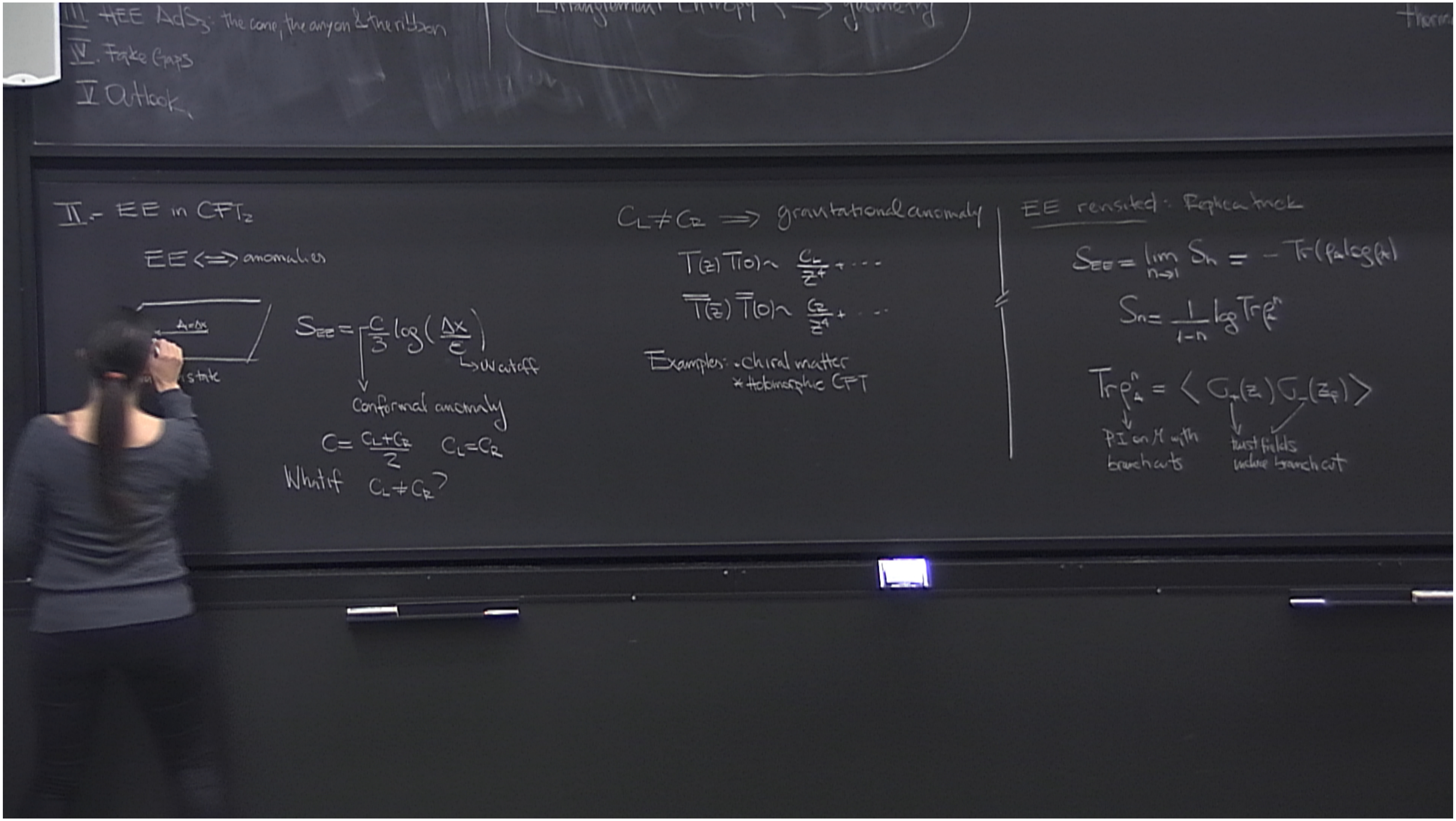
S_EE = lim n→1 S_n = − Tr(ρ_A log ρ_A)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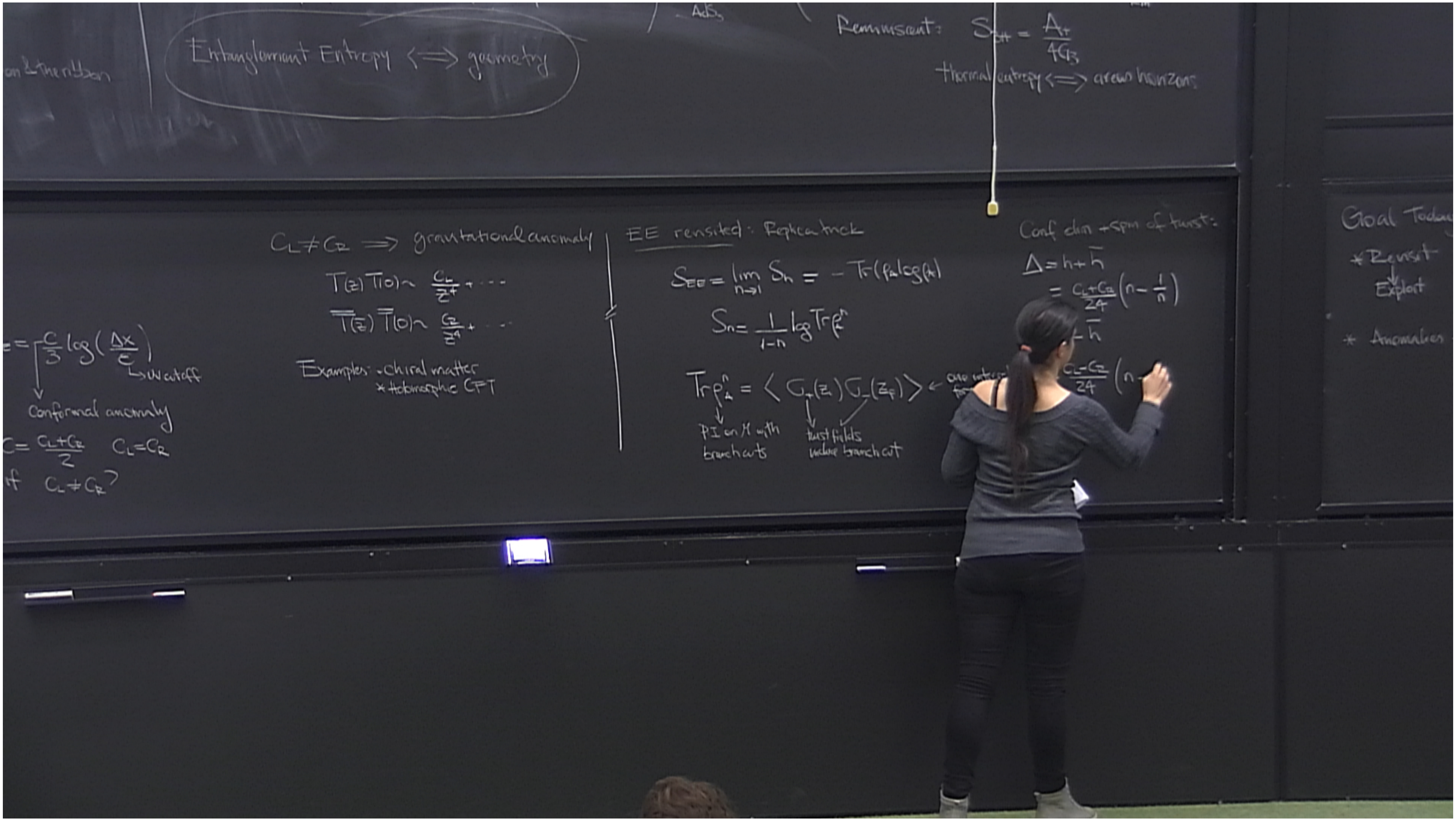
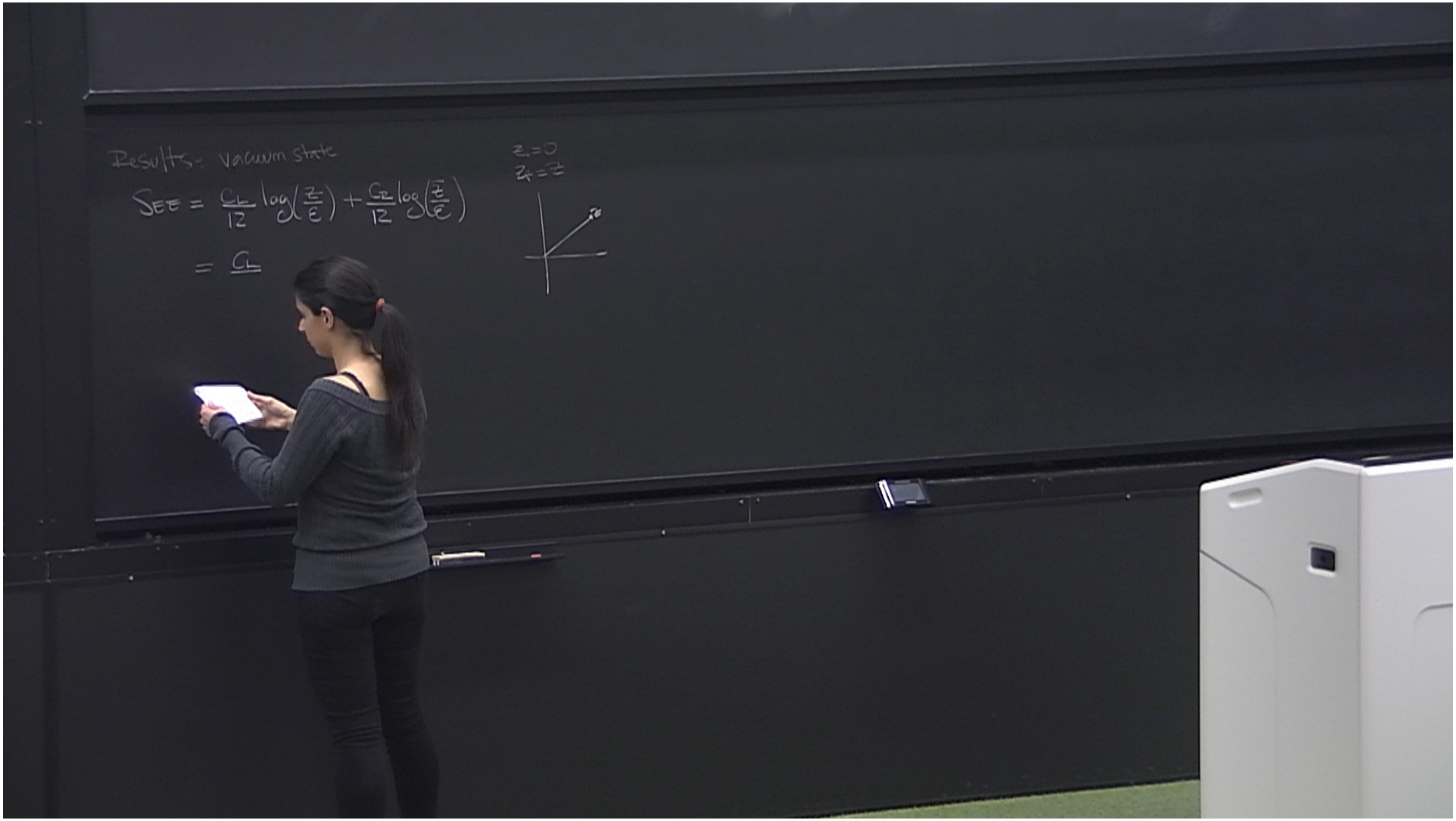
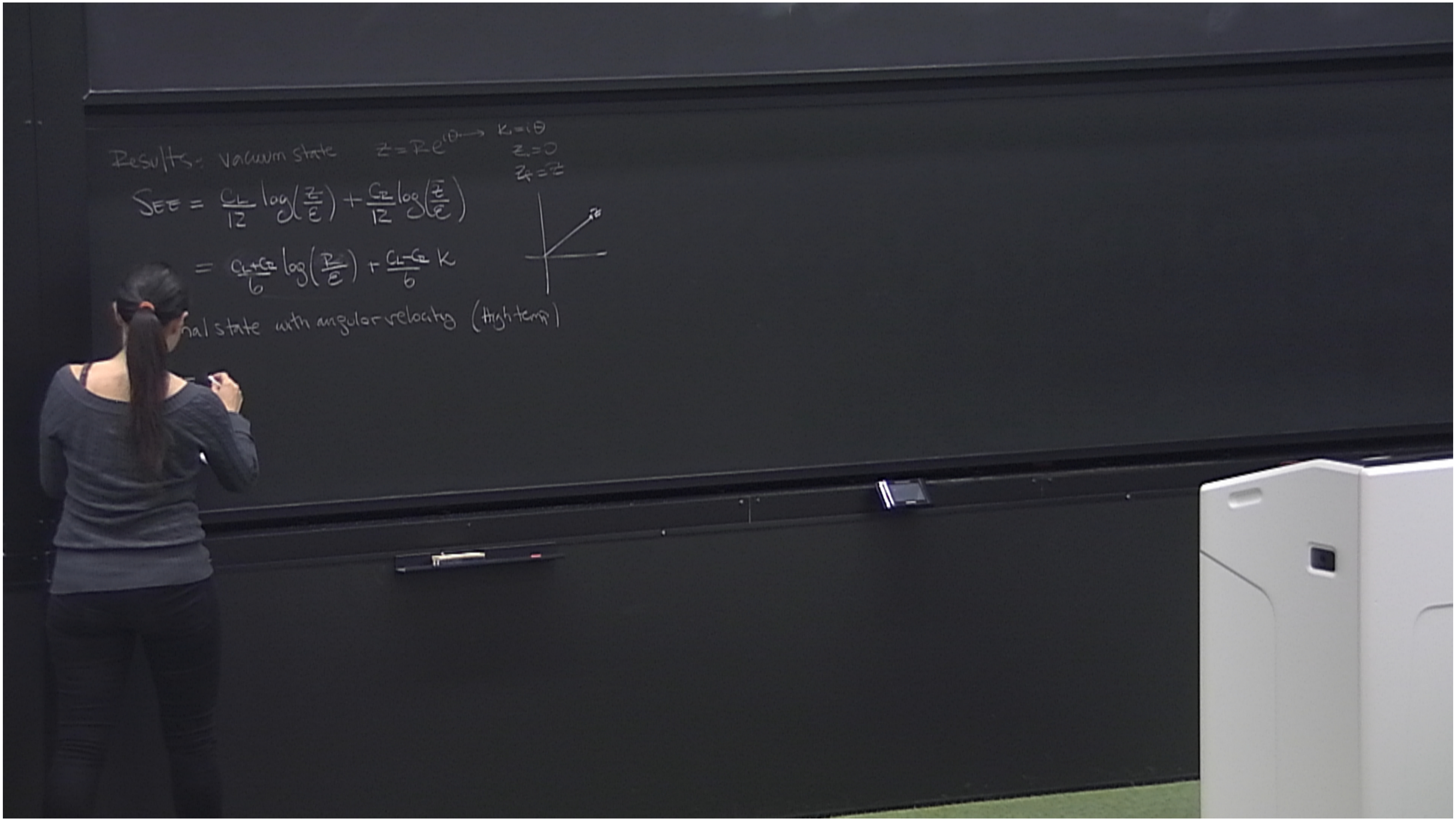
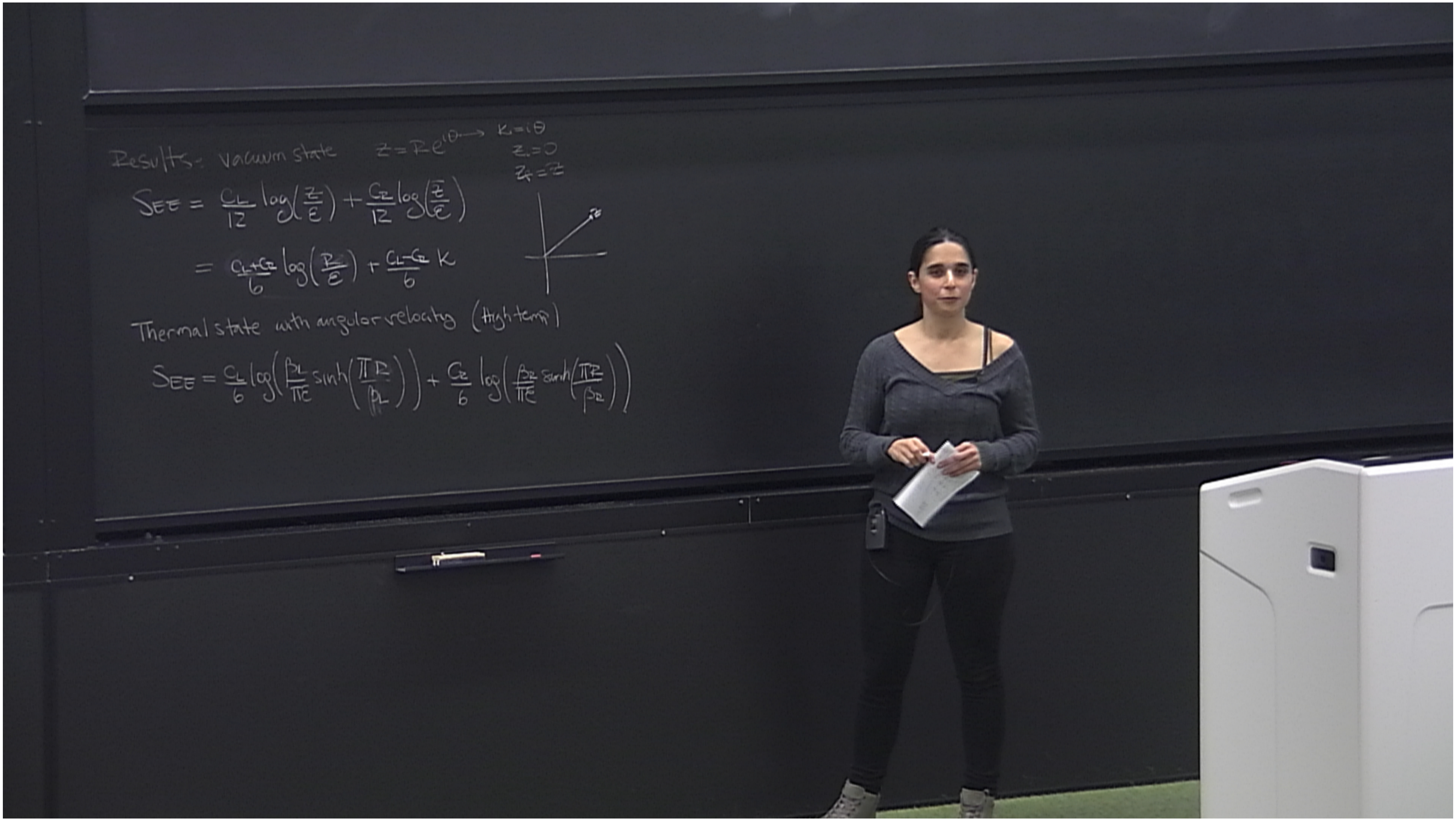
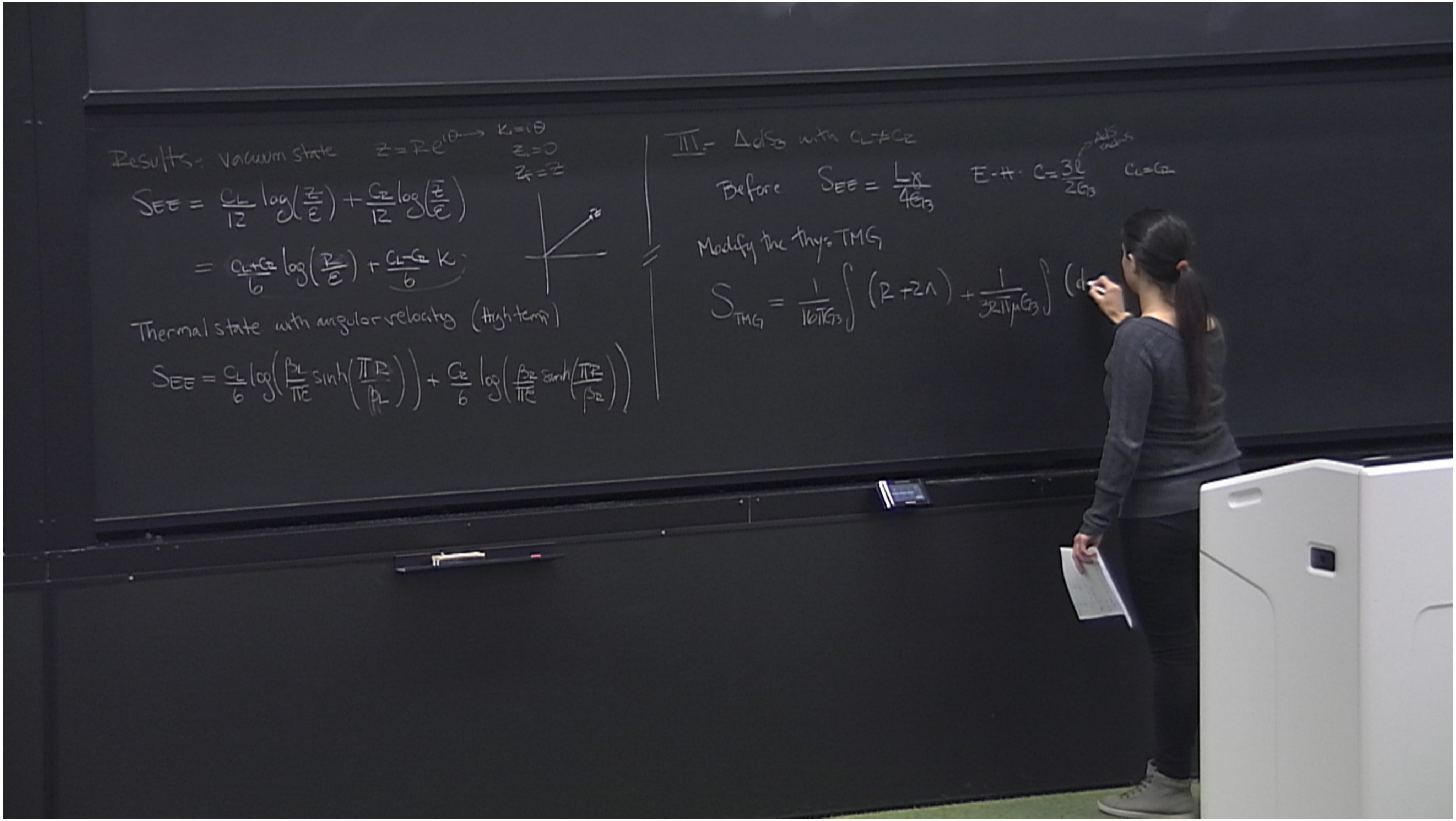
EE <=> anomalies
SEE = c/3 log(Δx/ε)
UV cutoff
Conformal anomaly
C = (CL+CR)/2   CL=CR
What if CL ≠ CR?
CL ≠ CR => gravitational anomaly
Examples: chiral matter
* Holomorphic CFT
EE revisited: Replica trick
SEE = lim n→1 Sn = − Tr(ρA log ρA)
Sn = 1/(1−n) log Tr ρA^n
P.I. on M with branch cuts
twist fields
induce branch cut

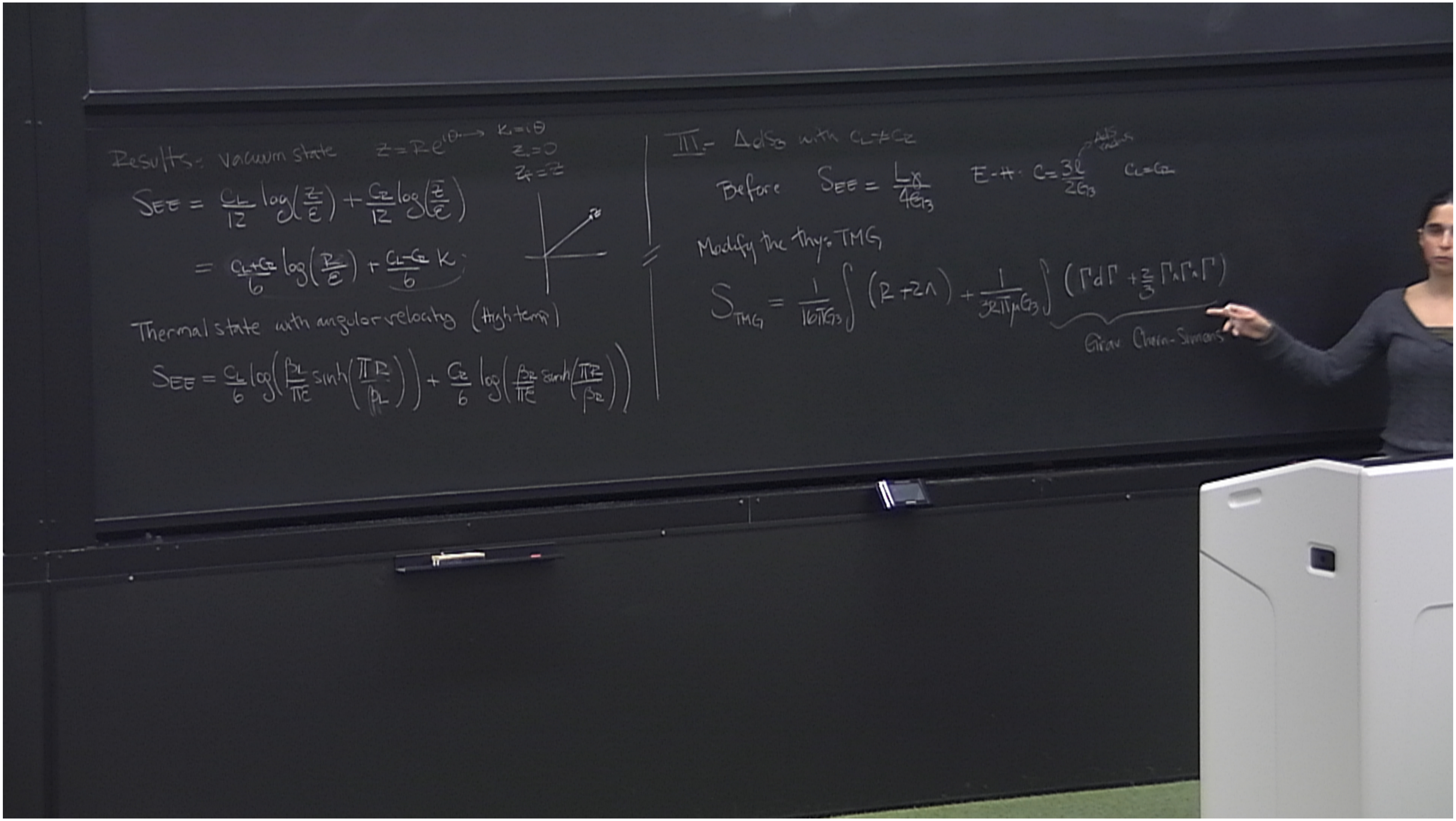

Entanglement Entropy <==> geometry
Reminiscent: S_BH = A_H / 4G_3
thermal entropy <==> area horizon
c_L ≠ c_R ==> gravitational anomaly
T(z) T(0) ~ c_L / z^4 + ...
T̄(z̄) T̄(0) ~ c_R / z̄^4 + ...
Examples: * chiral matter
* Holomorphic CFT
EE revisited: Replica trick
S_EE = lim_{n→1} S_n = − Tr(ρ_A log ρ_A)
S_n = 1/(1−n) log Tr ρ_A^n
Tr ρ_A^n = ⟨ σ_+(z) σ_−(z_f) ⟩
P.I. on M with branch cuts
twist fields induce branch cut
Conf dim + spin of twist:
Δ = h + h̄
= (c_L + c_R)/24 (n − 1/n)
Conformal anomaly
c = (c_L + c_R)/2   c_L = c_R
if c_L ≠ c_R ?
Goal Today
* Revisit
Exploit
* Anomalies

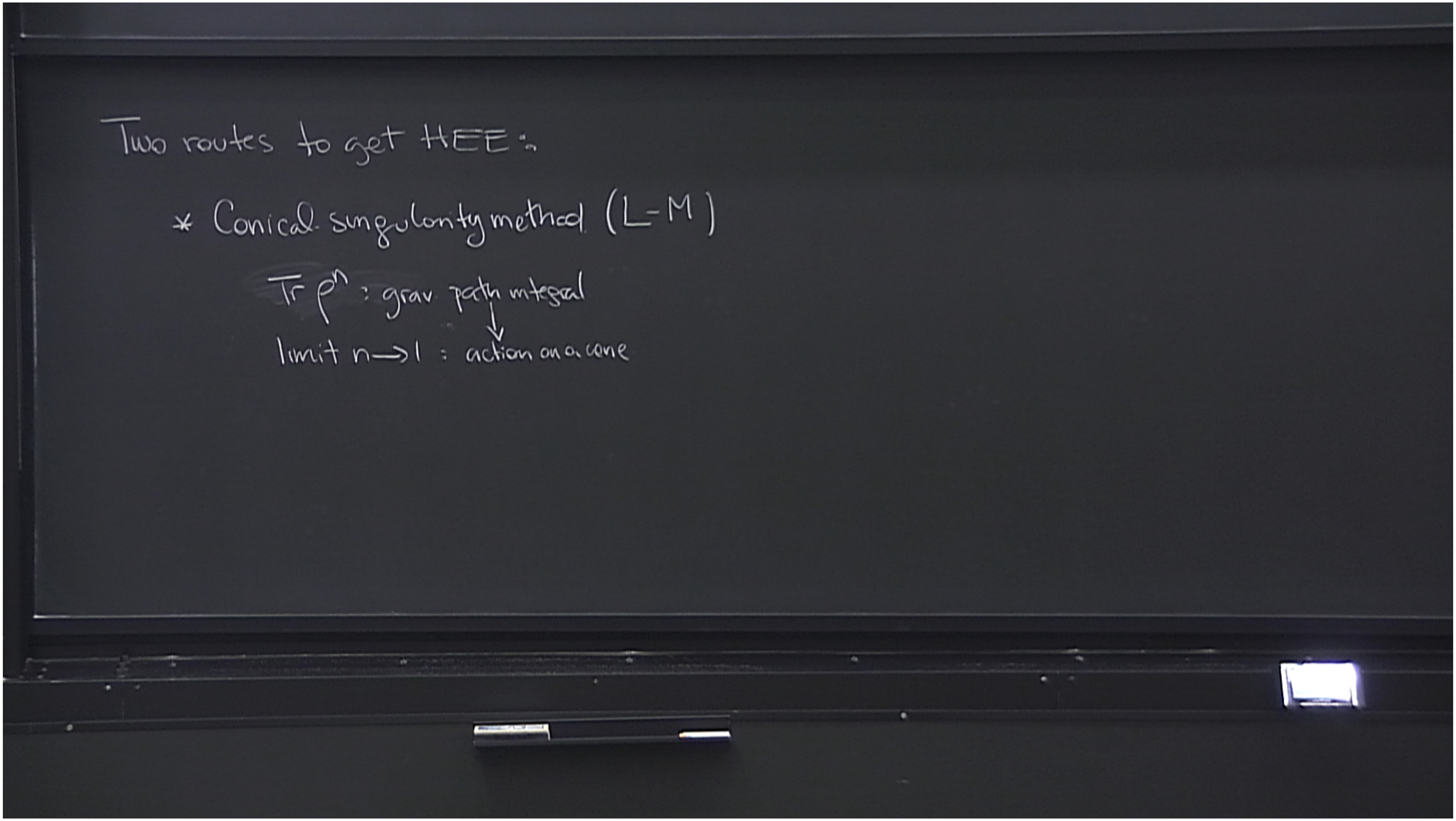

Results: vacuum state
z1 = 0
z2 = z
SEE = cL/12 log(z/ε) + cR/12 log(z̄/ε)
= cL

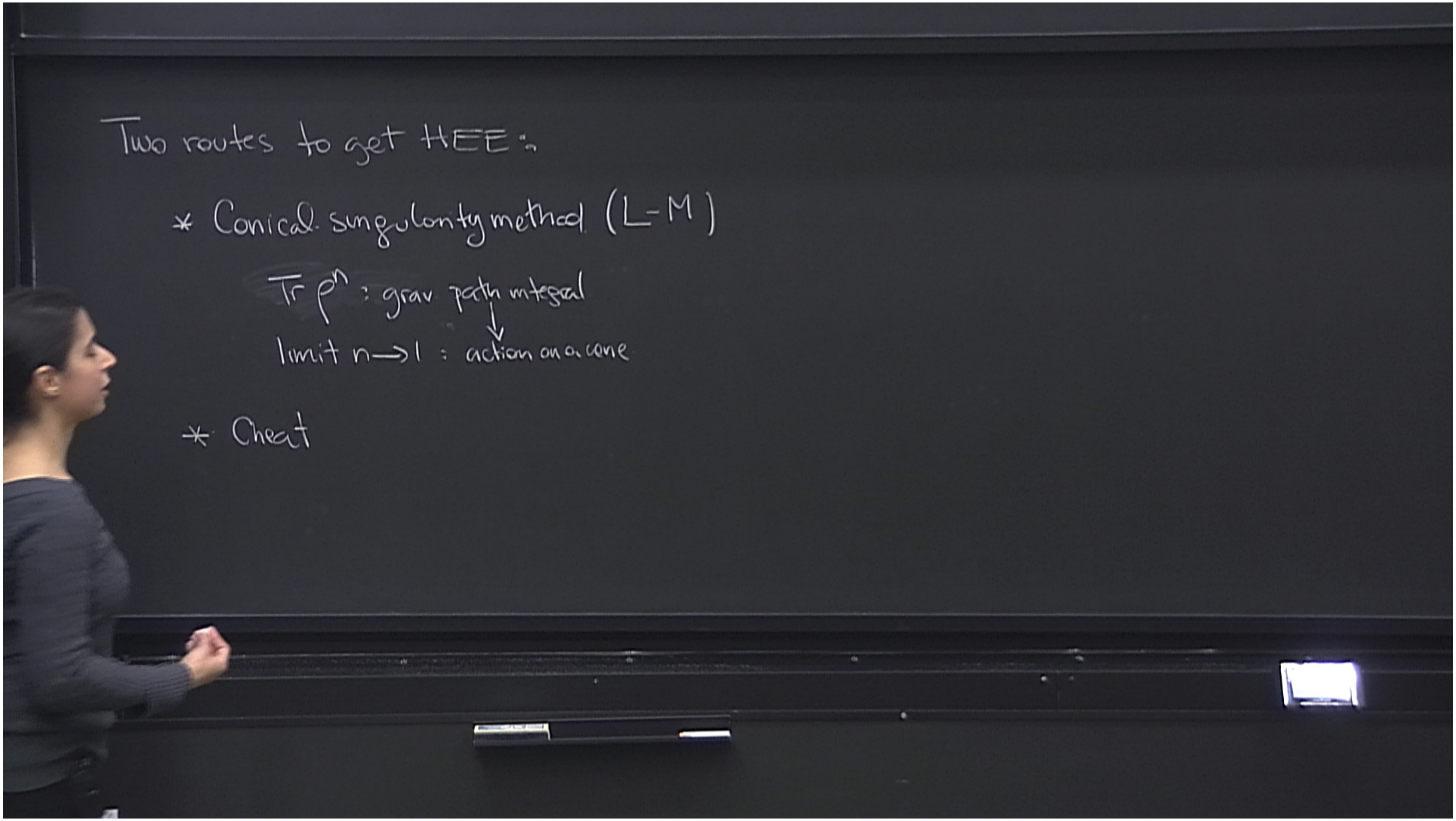

Results: vacuum state
z = Re^{iθ} → κ = iθ
z_1 = 0
z_2 = z
S_EE = c_L/12 log(z/ε) + c_R/12 log(z̄/ε)
= (c_L+c_R)/6 log(R/ε) + (c_L−c_R)/6 κ
nal state with angular velocity (high temp)

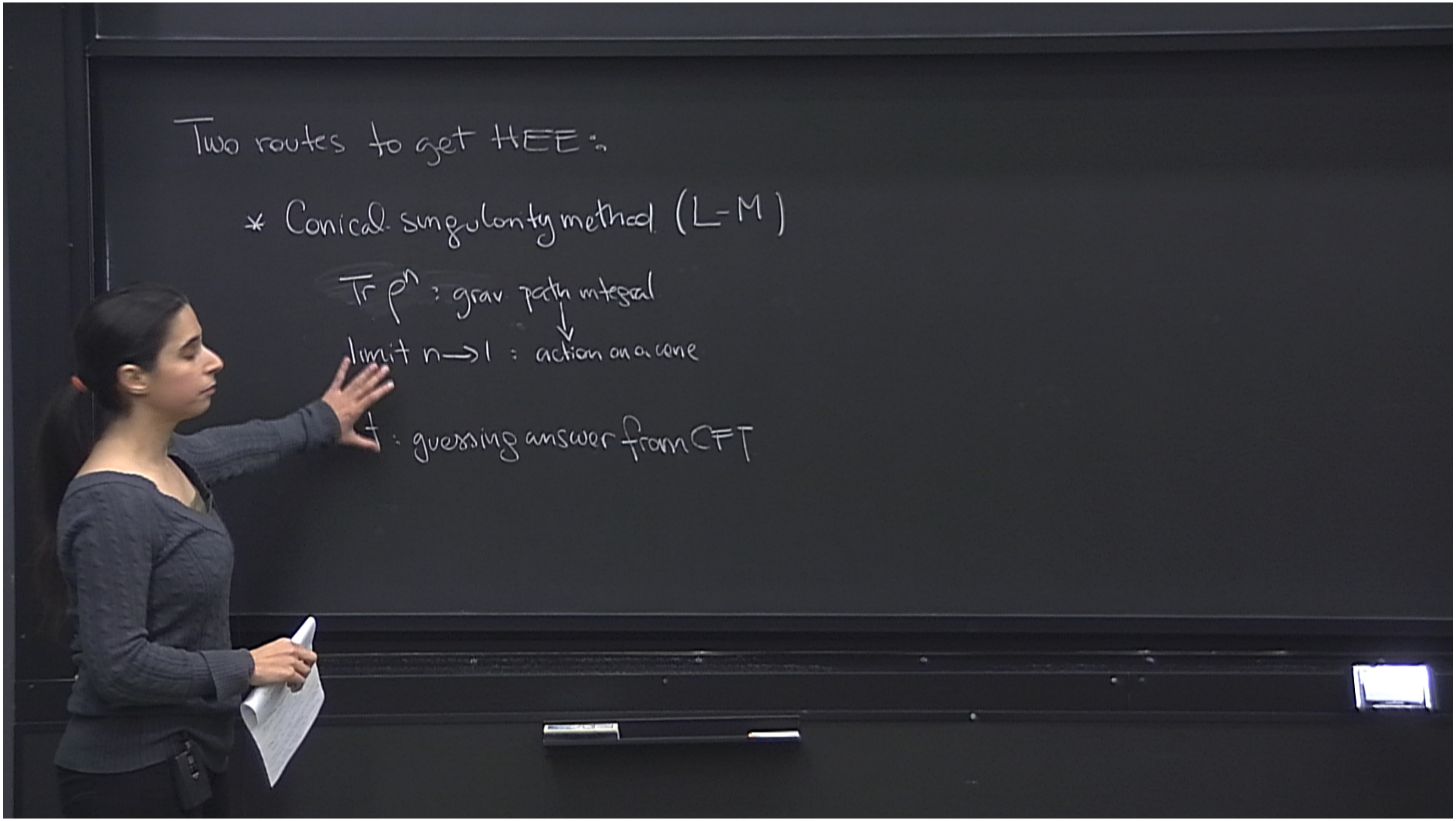

Results: vacuum state z = Re^{iθ} → K = iθ
z₋ = 0
z₊ = z
S_EE = c_L/12 log(z/ε) + c_R/12 log(z̄/ε)
= (c_L+c_R)/6 log(R/ε) + (c_L−c_R)/6 K
Thermal state with angular velocity (high temp)
S_EE = c_L/6 log(β_L/πε sinh(πR/β_L)) + c_R/6 log(β_R/πε sinh(πR/β_R))

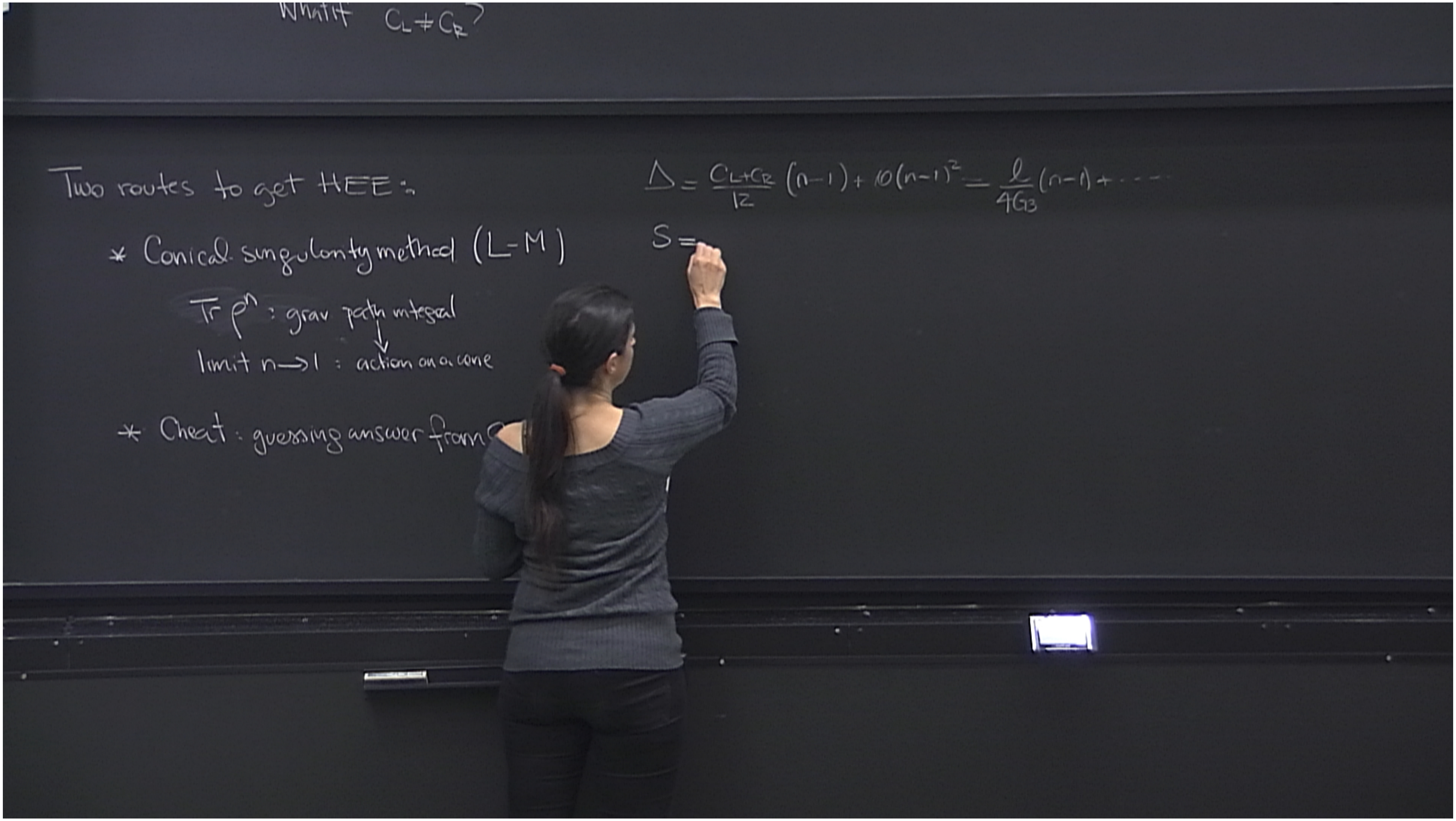

Results: vacuum state
$z = Re^{i\theta} \to \kappa = i\theta$
$z_1 = 0$
$z_2 = z$
$S_{EE} = \frac{c_L}{12}\log\left(\frac{z}{\epsilon}\right) + \frac{c_R}{12}\log\left(\frac{\bar{z}}{\epsilon}\right)$
$= \frac{c_L+c_R}{6}\log\left(\frac{R}{\epsilon}\right) + \frac{c_L-c_R}{6}\kappa$
Thermal state with angular velocity (High temp)
$S_{EE} = \frac{c_L}{6}\log\left(\frac{\beta_L}{\pi\epsilon}\sinh\left(\frac{\pi R}{\beta_L}\right)\right) + \frac{c_R}{6}\log\left(\frac{\beta_R}{\pi\epsilon}\sinh\left(\frac{\pi R}{\beta_R}\right)\right)$
III - AdS_3 with $c_L \neq c_R$
Before $S_{EE} = \frac{L_\gamma}{4G_3}$
E-H: $c = \frac{3\ell}{2G_3}$
$c_L = c_R$
Modify the thy: TMG
$S_{TMG} = \frac{1}{16\pi G_3}\int (R + 2\Lambda) + \frac{1}{32\pi\mu G_3}\int (\ldots$

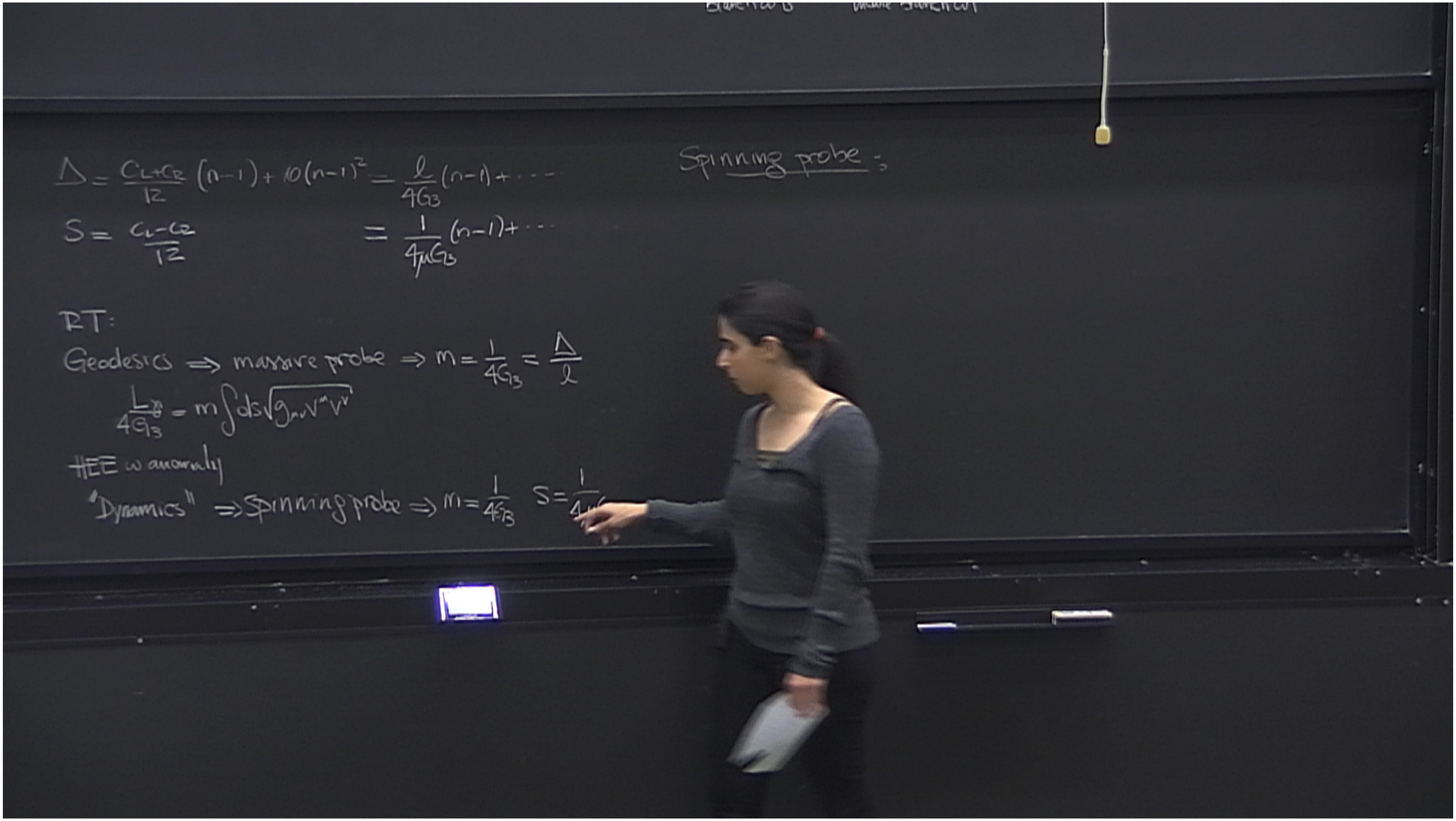

Results: vacuum state
z = Re^{iθ} → K = iθ
z_L = 0
z_R = z
S_EE = c_L/12 log(z/ε) + c_R/12 log(z̄/ε)
= (c_L+c_R)/6 log(R/ε) + (c_L−c_R)/6 K
Thermal state with angular velocity (high temp)
S_EE = c_L/6 log(β_L/πε sinh(πR/β_L)) + c_R/6 log(β_R/πε sinh(πR/β_R))
III - AdS_3 with c_L ≠ c_R
Before S_EE = Lγ/4G_3
E-H: c = 3ℓ/2G_3
AdS radius
c_L = c_R
Modify the thy: TMG
S_TMG = 1/16πG_3 ∫ (R + 2Λ) + 1/32πμG_3 ∫ (ΓdΓ + 2/3 Γ∧Γ∧Γ)
Grav Chern-Simons

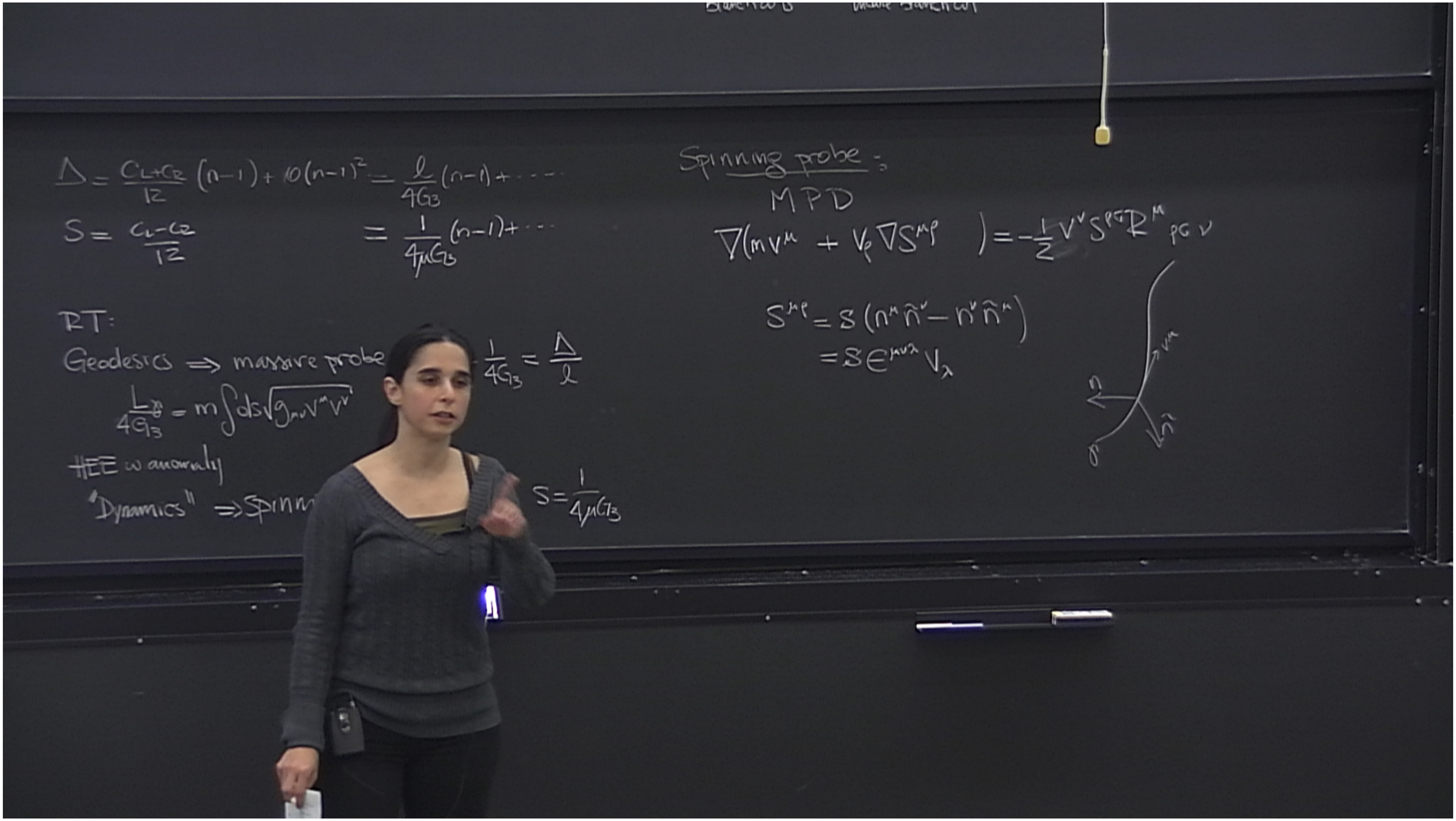

Two routes to get HEE:
* Conical singularity method (L-M)
Tr ρ^n : grav path integral
limit n → 1 : action on a cone


Two routes to get HEE :
* Conical singularity method (L-M)
Tr ρ^n : grav path integral
limit n → 1 : action on a cone
* Cheat


Two routes to get HEE :
* Conical singularity method (L-M)
Tr ρ^n : grav path integral
limit n→1 : action on a cone
: guessing answer from CFT


What if C_L ≠ C_R ?
Two routes to get HEE :
* Conical singularity method (L-M)
Tr ρ^n : grav path integral
limit n→1 : action on a cone
* Cheat : guessing answer from
Δ = (C_L+C_R)/12 (n−1) + O(n−1)^2 = ℓ/4G_3 (n−1) + ...
S =


Δ = (cL+cR)/12 (n−1) + O(n−1)^2 = ℓ/4G3 (n−1) + ...
S = (cL−cR)/12 = 1/4μG3 (n−1) + ...
RT:
Geodesics ⇒ massive probe ⇒ m = 1/4G3 = Δ/ℓ
L/4G3 = m∫ds√(gμν v^μ v^ν)
HEE w anomaly
"Dynamics" ⇒ Spinning probe ⇒ m = 1/4G3 S = 1/4μG3
Spinning probe:


Δ = (cL+cR)/12 (n−1) + 10(n−1)² = ℓ/4G3 (n−1) + . . .
S = (cL−cR)/12 = 1/4μG3 (n−1) + . . .
RT:
Geodesics ⇒ massive probe
1/4G3 = Δ/ℓ
L/4G3 = m∫ds√(gμν v^μ v^ν)
HEE w anomaly
"Dynamics" ⇒ Spinn
S = 1/4μG3
Spinning probe:
MPD
∇(m v^μ + vρ ∇S^μρ) = −½ v^ν S^ρσ R^μ ρσν
S^μρ = s(n^μ ñ^ρ − n^ρ ñ^μ)
= s ε^μρλ vλ
v^μ
n
ñ
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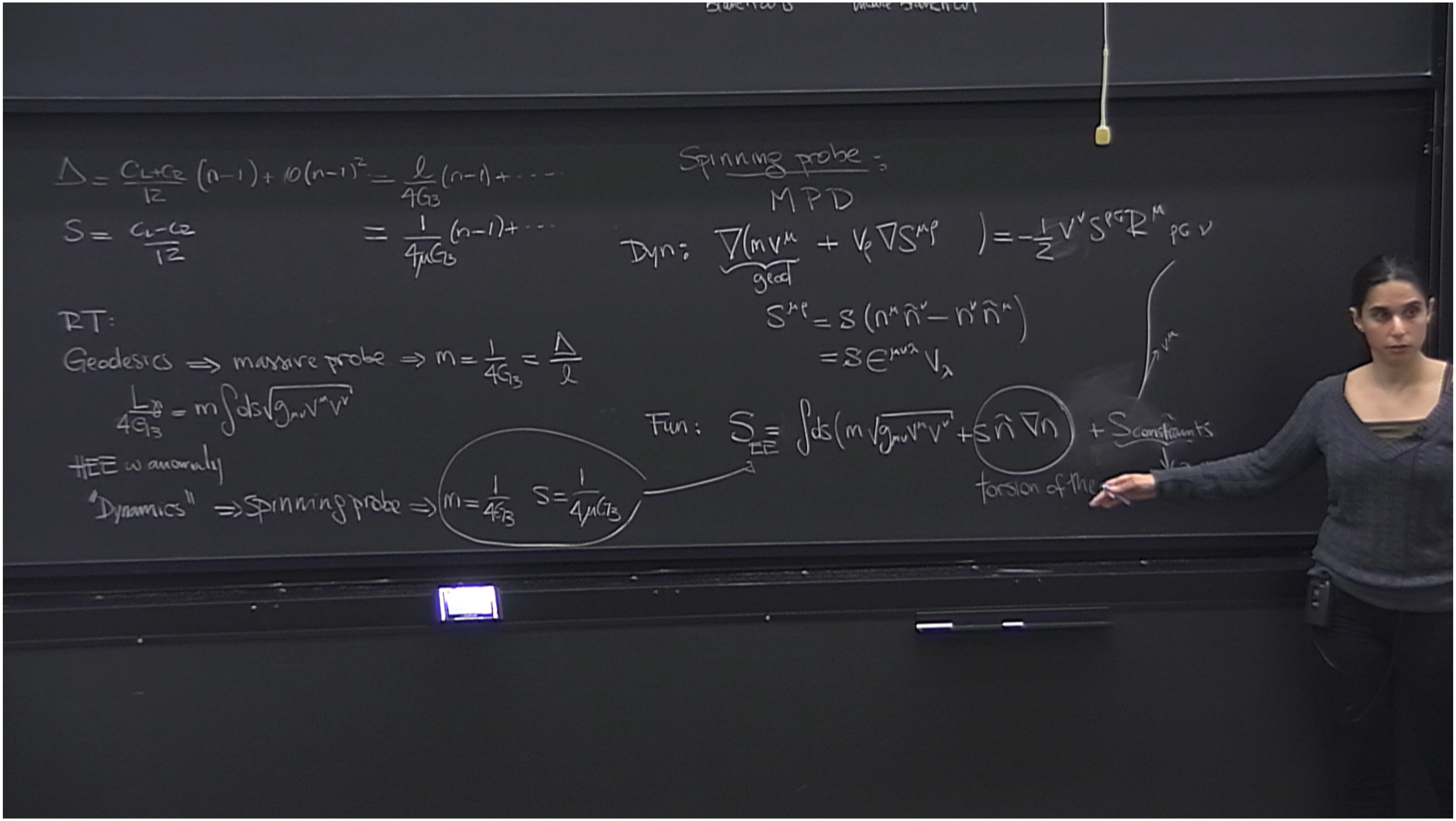
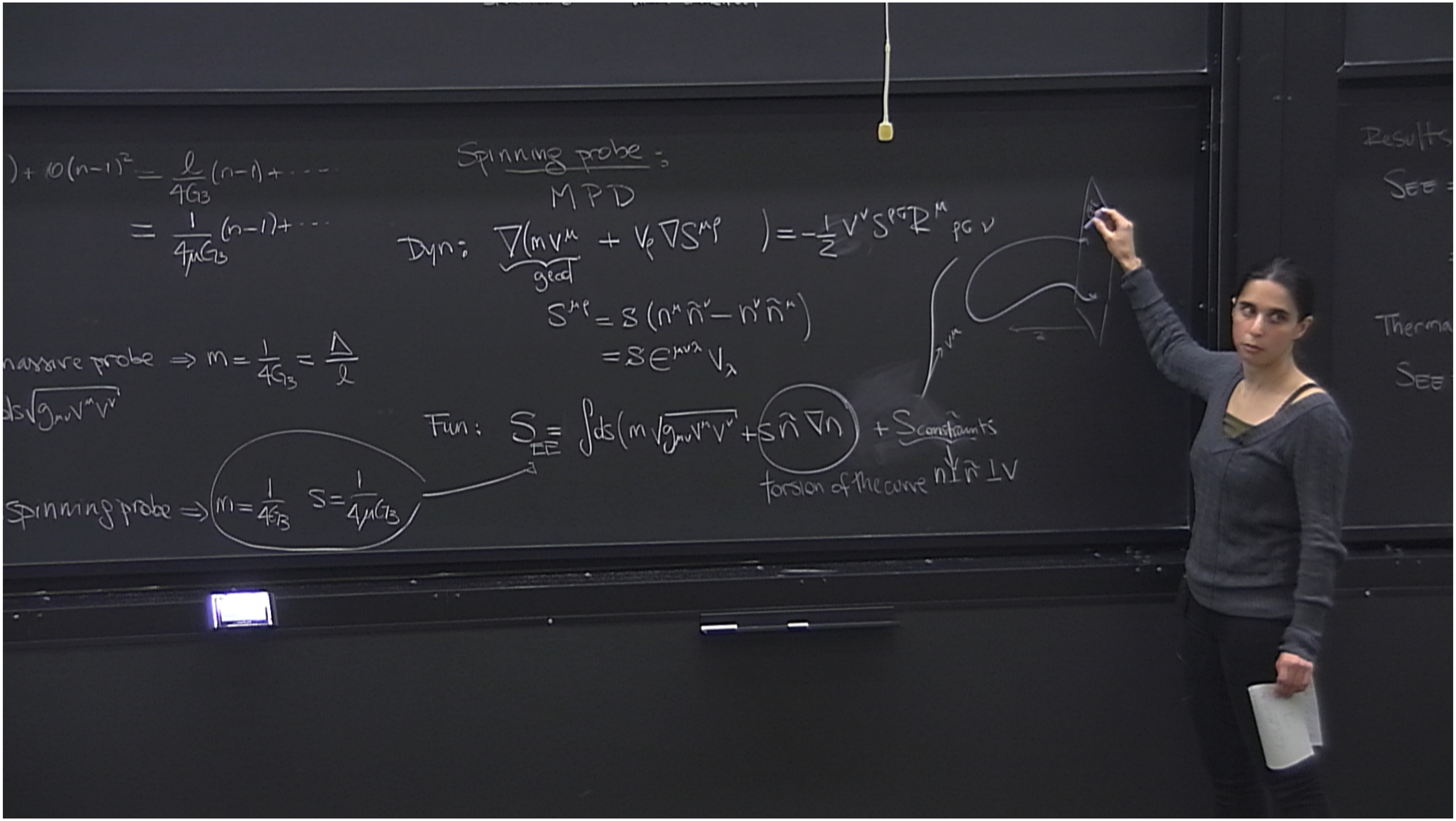
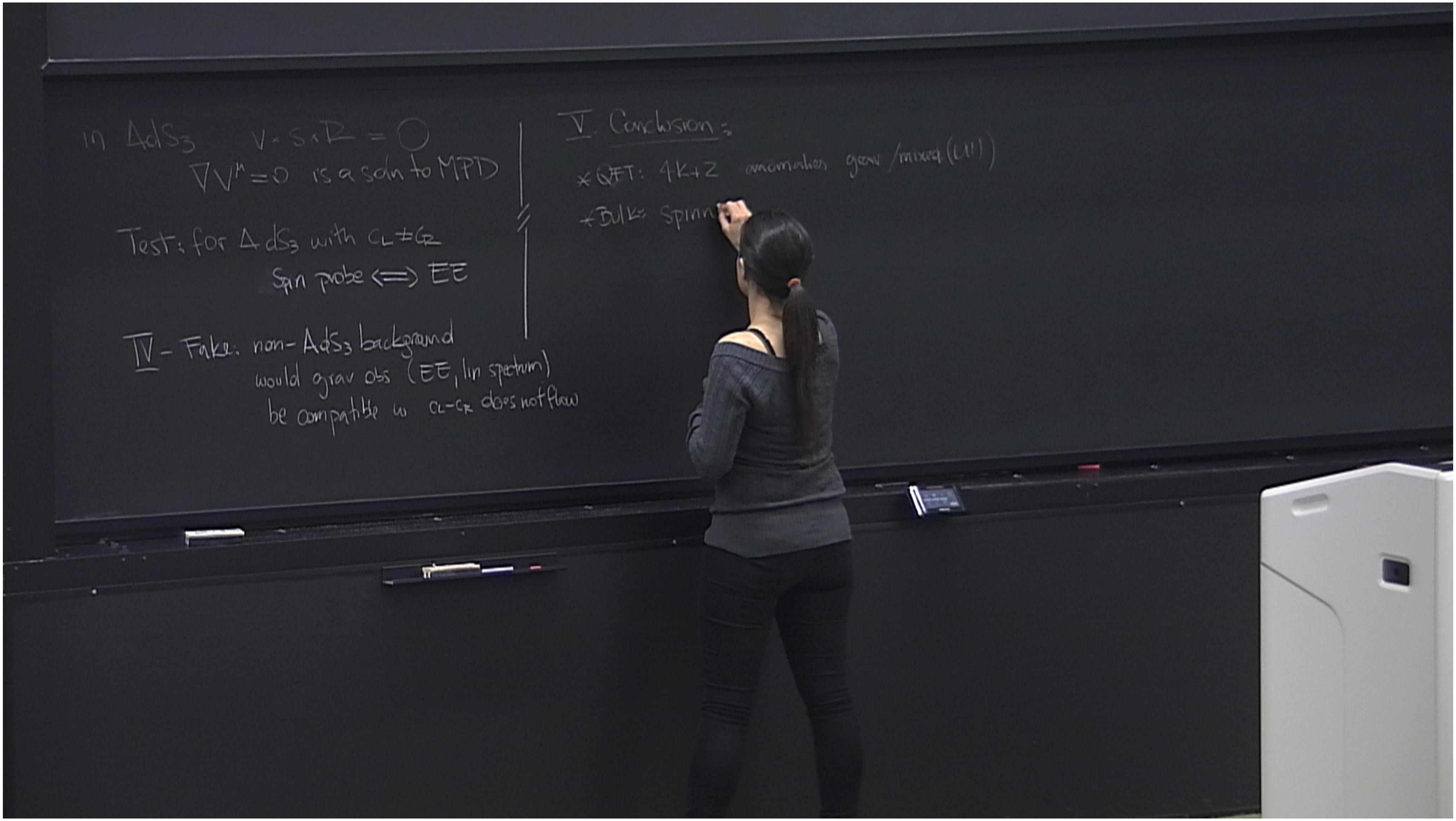
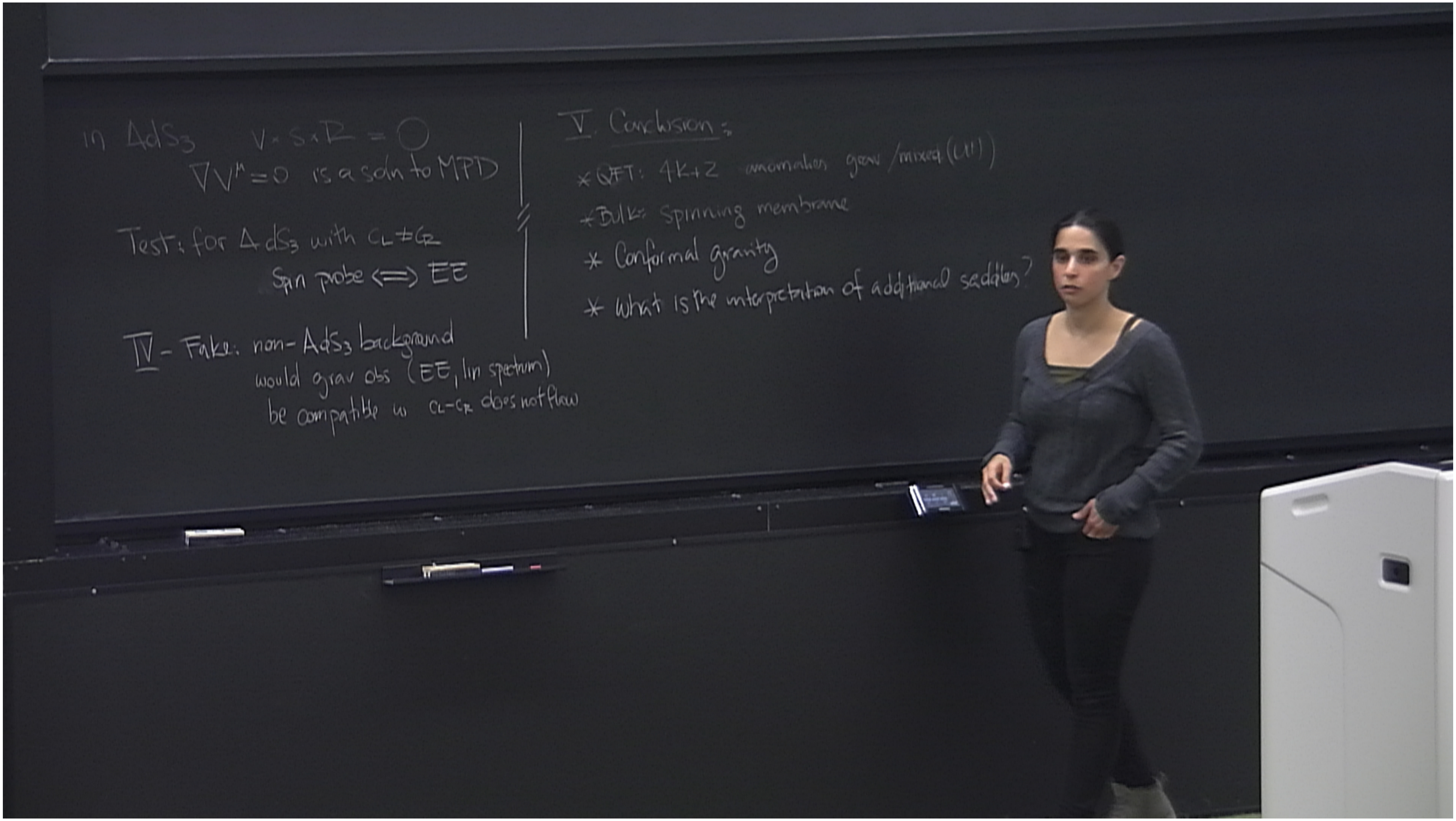


$\Delta = \frac{c_L+c_R}{12}(n-1) + 10(n-1)^2 = \frac{\ell}{4G_3}(n-1) + \dots$
$S = \frac{c_L-c_R}{12} = \frac{1}{4\mu G_3}(n-1) + \dots$
RT:
Geodesics ⇒ massive probe ⇒ $m = \frac{1}{4G_3} = \frac{\Delta}{\ell}$
$\frac{L}{4G_3} = m\int ds\sqrt{g_{\mu\nu}v^\mu v^\nu}$
HEE w anomaly
"Dynamics" ⇒ Spinning probe ⇒ $m = \frac{1}{4G_3}$ $s = \frac{1}{4\mu G_3}$
Spinning probe:
MPD
Dyn: $\nabla(m v^\mu + v_\rho \nabla S^{\mu\rho}) = -\frac{1}{2} v^\nu S^{\rho\sigma} R^\mu{}_{\rho\sigma\nu}$
geod
$S^{\mu\rho} = s(n^\mu \tilde{n}^\nu - n^\nu \tilde{n}^\mu)$
$= s\,\epsilon^{\mu\nu\lambda} v_\lambda$
v^μ
Fun: $S_{EE} = \int ds\left(m\sqrt{g_{\mu\nu}v^\mu v^\nu} + s\,\tilde{n}\nabla n\right) + S_{constraints}$
torsion of the


Spinning probe :
MPD
Dyn:
geod
torsion of the curve
massive probe
spinning probe
Results
Therma


in AdS3
∇V = 0 is a soln to MPD
Test: for AdS3 with cL ≠ cR
Spin probe ⟺ EE
IV - Fake: non-AdS3 background
would grav obs (EE, lin spectrum)
be compatible w cL-cR does not flow
V. Conclusion:
* QFT: 4K+Z anomalies grav/mixed (U(1))
* Bulk: spinn


in AdS3 V·S×R = 0
∇V^μ = 0 is a soln to MPD
Test: for AdS3 with cL ≠ cR
Spin probe ⟺ EE
IV - Fake: non-AdS3 background
would grav obs (EE, lin spectrum)
be compatible w cL-cR does not flow
V. Conclusion:
* QFT: 4k+2 anomalies grav / mixed (U(1))
* Bulk: spinning membrane
* Conformal gravity
* What is the interpretation of additional saddles?