Title: Quantum phenomena modelled by interactions between many classical worlds

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Abstract: <p>One necessity to avoid the measurement problem in quantum mechanics is a clear ontology. Such an ontology is for instance provided by Bohmian mechanics. In the non-relativistic regime, Bohmian mechanics is a theory about particles whose motion is governed by a velocity field. The latter is generated by a wave</p>

<p>function solving the SchrÃ¶dinger equation. In view of Feynman's criticism towards classical field theory one may wonder whether such a complex object as the wave function is needed to account for quantum phenomena. After all the value of the velocity field, i.e., of the wave function, is only needed in the vicinity of the configuration of the particle positions, however, it is defined everywhere in configuration space, even in places where the configuration might never roam. In a joint work with M. Hall and H. Wiseman we were able to formulate an approach to quantum mechanics that is capable to describe typical quantum phenomena like interference without a wave function, having only particles. This approach comes at the cost of introducing many classical worlds, hence the name Many-Interacting-Worlds approach (MIW). In MIW the force on each particle is given by 1) Newton's force describing the interaction within each world and 2) an additional force term describing an interaction between the worlds. Similar approaches have been suggested by B. Poirier and C. Sebens. I will give an overview on MIW, discuss the nature of its equations of motion, and its empirical import.</p>

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07 May 2015 14:09

Quantum Mechanics and the Wave Function

Perimeter Institute, Waterloo, 2015

Dirk - André Deckert
Interaction between Light and Matter Group
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Outline

- The practitioners' perspective on quantum theory
- A successful ontology for quantum theory
- Quantum theory without a wave function?

Without doubt, quantum mechanics is one of the fundamental column in physics:

General Relativity	large scales?	quantum gravity?
Classical Mechanics	intermediate scales?	macroscopic interference?
Quantum Theory	small scales?	scattering regime?

But when should one theory be preferred over another?

Practitioner: "Quantum theoretic description is good when it gives good predictions."

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What is quantum theory about? What is it that it is so good predicting in?

Defining equation of motion in quantum theory:

$$i\partial_t \Psi_t = H \Psi_t$$

What is the entity Ψ_t that moves?

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"Why not look it up in a good book?", J.S. Bell, Against Measurement (1990)

Postulates of Quantum Theory:

1. The state of a quantum system is represented by the wave function ψ or ket $|\psi\rangle$.
2. Observables are represented by Hermitian operators A that act on kets.
3. The only possible outcome of a measurement is the eigenvalue of the operator $A|\psi_n\rangle = a_n|\psi_n\rangle$.
4. The probability of measuring a_n is $\mathbb{P}(a_n) = |\langle\psi_n|\psi\rangle|^2$.
5. After a measurement yielding a_n the new state is a normalized projection $|\langle\psi|P_n|\psi\rangle|^{-1/2} P_n|\psi\rangle$, $P_n = |\psi_n\rangle\langle\psi_n|$.
6. The time evolution of the state is given by $i\partial_t|\psi\rangle = H|\psi\rangle$.

"For those who are not shocked when they first come across quantum theory cannot possibly have understood it."

N. Bohr, quoted by W. Heisenberg in
Physics and Beyond (1971)

1. A <u>system</u> of what? Particles, Fields? What is their state?
2. <u>Observables</u>? A thing that humans can experience? Only humans?
3. <u>Measurement</u>? Do I have to look at the pointer in a lab, or is the pointer being there, without me looking, enough? Is there a one-to-one map between Hermitian operators and experiments? No!
4. <u>Probability</u>? Is there something intrinsically stochastic? Or is this only the effective description of a deterministic, even more fundamental theory? Or are all possibilities realized, we just don't know in which world we live?
5. When is <u>after</u> a measurement? When I look at the pointer? "Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a PhD?", (J.S. Bell, 1990)
6. That the time evolution of Ψ is ruled by the Schrödinger evolution contradicts projection postulate 5. Schrödinger's cat...

"I think I can safely say that nobody understands quantum mechanics."

R.P. Feynman, The character of a physical law (1965)

So what is quantum theory about?

"I like to think that the moon is there even when I am not looking at it."

A. Einstein

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One version quantum theory with a simple and precise ontology is Bohmian mechanics:

Ontology: There are N matter points, they have spatial relations w.r.t. each other, and those change.

Mathematics:
$$\frac{d}{dt}\begin{pmatrix} Q_t \\ \Psi_t \end{pmatrix} = \begin{pmatrix} v^{\Psi_t}(Q_t) \\ -iH\Psi_t \end{pmatrix}$$

$Q_t = (q_{1,t}, \dots, q_{N,t})$ encodes the spatial relations of the N matter points, $Q_t \in \mathbb{R}^{3N}$

$v^{\Psi_t} = \frac{j^{\Psi_t}}{\rho^{\Psi_t}}$ is the velocity field defined by the current j_t and density ρ_t generated by Ψ_t

Given suff. regular initial data (Q_0, Ψ_0) there is a

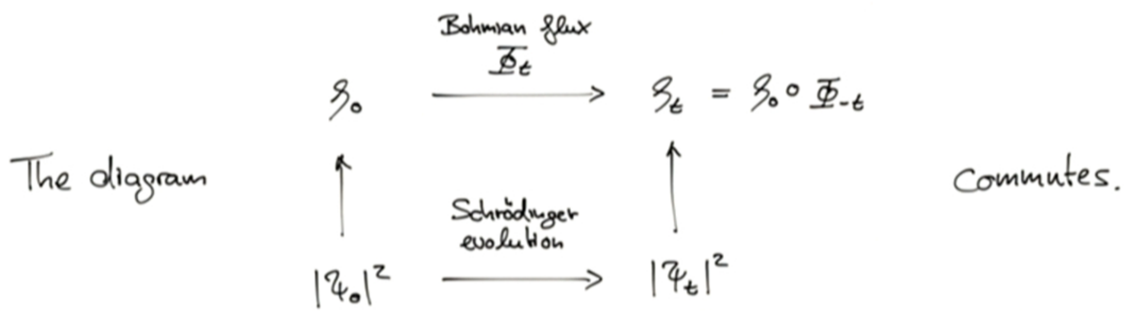
If Ψ_0 is given, one can carry out Boltzmann's program as in statistical mechanics:

Replace the unknown initial data by random variables distributed by
a measure on configuration space $\mathbb{R}^{3N}$ given by the principle of stationarity:

Typicality does not change in time.

The resulting probabilistic predictions then capture what is to expect in the typical universes.

In Bohmian mechanics this measure is given by a feature called "equivariance":



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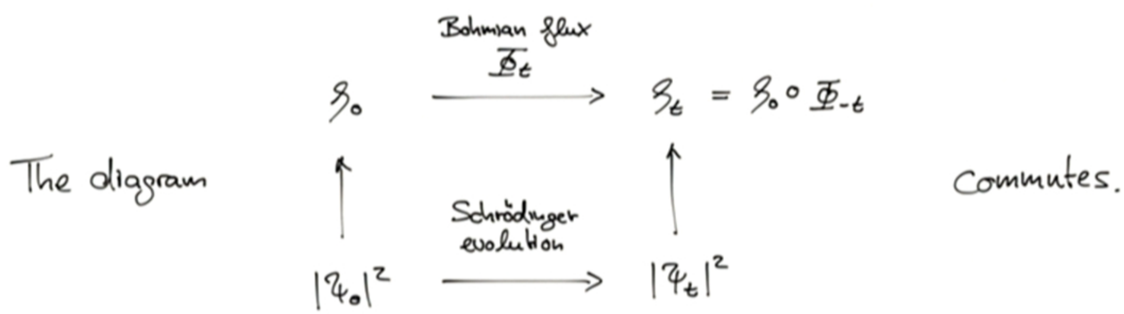

Hence, as best guess we may replace the unknown degrees of freedom in the initial configuration by $|\Psi_t|^2$ distributed random variables to yield an effective description of subsystems.

evolution

$$|\Psi_0|^2 \longrightarrow |\Psi_t|^2$$

Hence, as best guess we may replace the unknown degrees of freedom in the initial configuration by $|\Psi_t|^2$ distributed random variables to yield an effective description of subsystems.

Subsystems:

Configuration $Q_t = (X_t, Y_t)$ and wave function $\Psi_t(x,y)$

(X_t: subsystem; Y_t: rest of the universe)

DEF: The subsystem X_t has effective wave function $\varphi_t(x)$ iff:

i) $\Psi_t(x,y) = \varphi_t(x)\,\xi_t(y) + \Omega_t^{\perp}(x,y)$;

ii) ξ_t and $\Omega_t^{\perp}$ have macroscopically disjoint y-support and $Y_t \in \operatorname{supp} \xi$.

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Based on the typicality measure $|\Psi_t|^2$ one can now prove:

THM: (Dürr, Goldstein, Zanghì)

If the effective wave function φ_t exists, the particles of the x-subsystem are distributed according to $|\varphi_t|^2$.

of the configuration trajectory $t \mapsto Q_t$ are needed to define the motion of the particles.

- What is the meaning of the value of ψ_t in the vast empty regions of configuration space where it has support but the configuration may never roam?
- Feynman's criticism toward electrodynamics

Can we do without ψ_t ?

$Q_t = (q_{1,t}, \ldots, q_{N,t})$

$$\frac{d}{dt}\begin{pmatrix} Q_t \\ \psi_t(X) \end{pmatrix} = \begin{pmatrix} v^{\psi_t}(Q_t) = \operatorname{Im} \frac{\nabla \psi_t}{\psi_t}(Q_t) \\ -i\left(-\sum_{n=1}^{N} \frac{\Delta_{x_n}}{2m} + V(X)\right)\psi_t(X) \end{pmatrix}$$

$X = (x_1, \ldots, x_N)$

Initial data: Q_0, ψ_0

$$\Updownarrow \qquad \psi_t = R_t e^{iS_t}$$

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$$\frac{d}{dt}\begin{pmatrix} Q_t \\ S_t(X) \\ R_t(X) \end{pmatrix} = \begin{pmatrix} \nabla S_t(Q_t) \\ -\frac{1}{2}(\nabla S_t(X))^2 - V(X) + \frac{1}{2}\frac{\Delta R_t(X)}{R_t(X)} \\ -\nabla R_t(X)\cdot\nabla S_t(X) - \frac{1}{2} R_t(X)\, \Delta S_t(X) \end{pmatrix}$$

Initial data: Q_0, S_0, R_0

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$$\frac{d}{dt}\begin{pmatrix} Q_t \\ \psi_t(X) \end{pmatrix} = \begin{pmatrix} \cdots \\ -i\left(-\sum_{n=1}^{N}\frac{\Delta_{x_n}}{2m} + V(X)\right)\psi_t(X) \end{pmatrix}$$

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Initial data: Q_0, S_0, R_0

$$\Updownarrow$$

$$\frac{d^2}{dt^2}Q_t = -\nabla V(Q_t) + \frac{1}{2}\nabla\frac{\Delta R_t(Q_t)}{R_t(Q_t)}$$

depends only on R_t !

Many-Interacting-Worlds approach:

- large but finite number of coexisting worlds $Q_t^{(1)}, \dots, Q_t^{(M)} \in \mathbb{R}^{3N}$
- replace R_t^2 by a smoothed version of the empirical density

$$R_t^2(X) := R^2[Q_t^{(1)}, \dots, Q_t^{(M)}](X) := \text{smooth version of } \frac{1}{M}\sum_{m=1}^{M} \delta^{3N}(X - Q_t^{(m)})$$

- equations of motion for M world configurations

$$\frac{d^2}{dt^2} Q_t^{(m)} = -\nabla V(Q_t^{(m)}) + \frac{1}{2}\nabla \frac{\Delta R[Q_t^{(1)}, \dots Q_t^{(M)}](Q_t^{(m)})}{R[Q_t^{(1)}, \dots Q_t^{(M)}](Q_t^{(m)})}, \quad m = 1, 2, \dots, M$$

$$= \underbrace{-\nabla V(Q_t^{(m)})}_{\text{classical interaction between particles world } Q_t^{(m)}} \underbrace{- \nabla U[Q_t^{(1)}, \dots, Q_t^{(M)}](Q_t^{(m)})}_{\text{"quantum" interaction between worlds}}$$

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- Given the initial data $\left(Q_0^{(m)}, \dot{Q}_0^{(m)}\right)_{1\le m\le M}$ the dynamics of $t \mapsto Q_t^{(m)}$ is determined.

- In contrast to the Everett interpretation, in which worlds are orthogonal components of the wave functions, in MIW, there are rather N x M matter points that are differentiated dynamically only.

- While each world evolves deterministically, in which world we are living in is unknown. Hence, assertions about the configuration of matter points in our world naturally become probabilistic in the sense of Laplace:

$$\mathbb{E}_t(f(x)) = \frac{1}{M}\sum_{m=1}^{M} f(Q_t^{(m)})$$

Solutions of Bohmian mechanics can be approximated by MIW solutions:

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Solutions of Bohmian mechanics can be approximated by MIW solutions:

Let Ψ_t be a solution of the Schrödinger equation,

and $t \mapsto Q_t[Q_0]$ the Bohmian trajectory with initial configuration Q_0.

Let MIW solution $t \mapsto (Q_t^{(m)})_{1 \leq m \leq M}$ for initial data $(Q_0^{(m)}, \dot{Q}_0^{(m)})$ such that

i) $R^2[Q_0^{(1)}, \ldots, Q_0^{(M)}](X) \underset{M \gg 1}{\approx} |\Psi_0|^2(X)$

ii) $\dot{Q}_0^{(m)} \underset{M \gg 1}{\approx} \nabla S_0(Q_0^{(m)})$, $\quad m = 1, 2, \ldots, M$, $\quad \Psi_t = R_t e^{i S_t}$

Then:

$$Q_t^m \underset{M \gg 1}{\approx} Q_t[Q_0^m] \qquad (*)$$

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Hence:

m-th MIW world trajectory

$$\mathbb{E}_t(f(X)) := \frac{1}{M}\sum_{m=1}^{M} f(Q_t^{(m)})$$

by $(*)$

Bohmian trajectory for initial configuration $Q_0^{(m)}$

$$\underset{M\gg 1}{\approx} \frac{1}{M}\sum_{m=1}^{M} f(Q_t[Q_0^{(m)}])$$

$$\underset{M\gg 1}{\approx} \int f(Q_t[X])\,|\Psi_0(X)|^2\,dX$$

by equivariance

$$= \int f(X)\,|\Psi_t(X)|^2\,dX = \langle \Psi_t, f(X)\Psi_t\rangle$$

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Then:

$$Q_t^m \underset{M \gg 1}{\approx} Q_t[Q_0^m] \qquad (*)$$

Hence:

find a corresponding wave function.

- Similar ideas have been independently developed by
  - P. Holland 2005
  - B. Poirier 2010
  - B. Poirier and J. Schiff 2012
  - C. Sebens 2014

- Connection to "maximal variety principle" by L. Smolin 2015

A simple example for a candidate $R^2[Q^{(1)}, \dots, Q^{(M)}](X)$

- one dimension
- one particle per world
- ordering $Q^{(1)} < Q^{(2)} < \dots < Q^{(M)}$

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$$\mathbb{E}_t(f(X)) = \frac{1}{M} \sum_{m=1}^{M} f(Q^{(m)}) \underset{M \gg 1}{\overset{!}{\approx}} \int f(X)\, R^2[Q^{(1)}, \dots, Q^{(M)}](X)\, dX$$

$$\underset{M \gg 1}{\approx} \sum_{m=1}^{M} \int_{Q^{(m-1)}}^{Q^{(m)}} f(X)\, R^2[Q^{(1)}, \dots, Q^{(M)}](X)\, dX$$

$$\approx \sum^{M} [Q^{(m)} - Q^{(m-1)}]\, f(Q^{(m)})\, R^2[Q^{(1)}, \dots, Q^{(M)}](Q^{(m)})$$

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15

- one particle per world
- ordering $Q^{(1)} < Q^{(2)} < \dots < Q^{(M)}$

$$\mathbb{E}_t\big(f(X)\big) = \frac{1}{M}\sum_{m=1}^{M} f(Q^{(m)}) \underset{M \gg 1}{\overset{?}{\approx}} \int f(X)\, R^2[Q^{(1)},\dots,Q^{(M)}](X)\, dX$$

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$$\underset{M \gg 1}{\approx} \sum_{m=1}^{M} \left[Q^{(m)} - Q^{(m-1)}\right] f(Q^{(m)})\, R^2[Q^{(1)},\dots,Q^{(M)}](Q^{(m)})$$

$$\Longrightarrow \quad R^2[Q^{(1)},\dots,Q^{(M)}](X) \underset{M \gg 1}{\approx} \frac{1}{M\left[Q^{(m)} - Q^{(m-1)}\right]}$$

Using such a linearization also for the derivatives, the equations of motion turn into

$$\underset{M \gg 1}{\;} \sum_{m=1} \int_{Q^{(m-1)}} \ldots$$

$$\underset{M \gg 1}{\approx} \sum_{m=1}^{M} \left[Q^{(m)} - Q^{(m-1)}\right] f(Q^{(m)}) \; R^2\left[Q^{(1)}, \ldots, Q^{(M)}\right](Q^{(m)})$$

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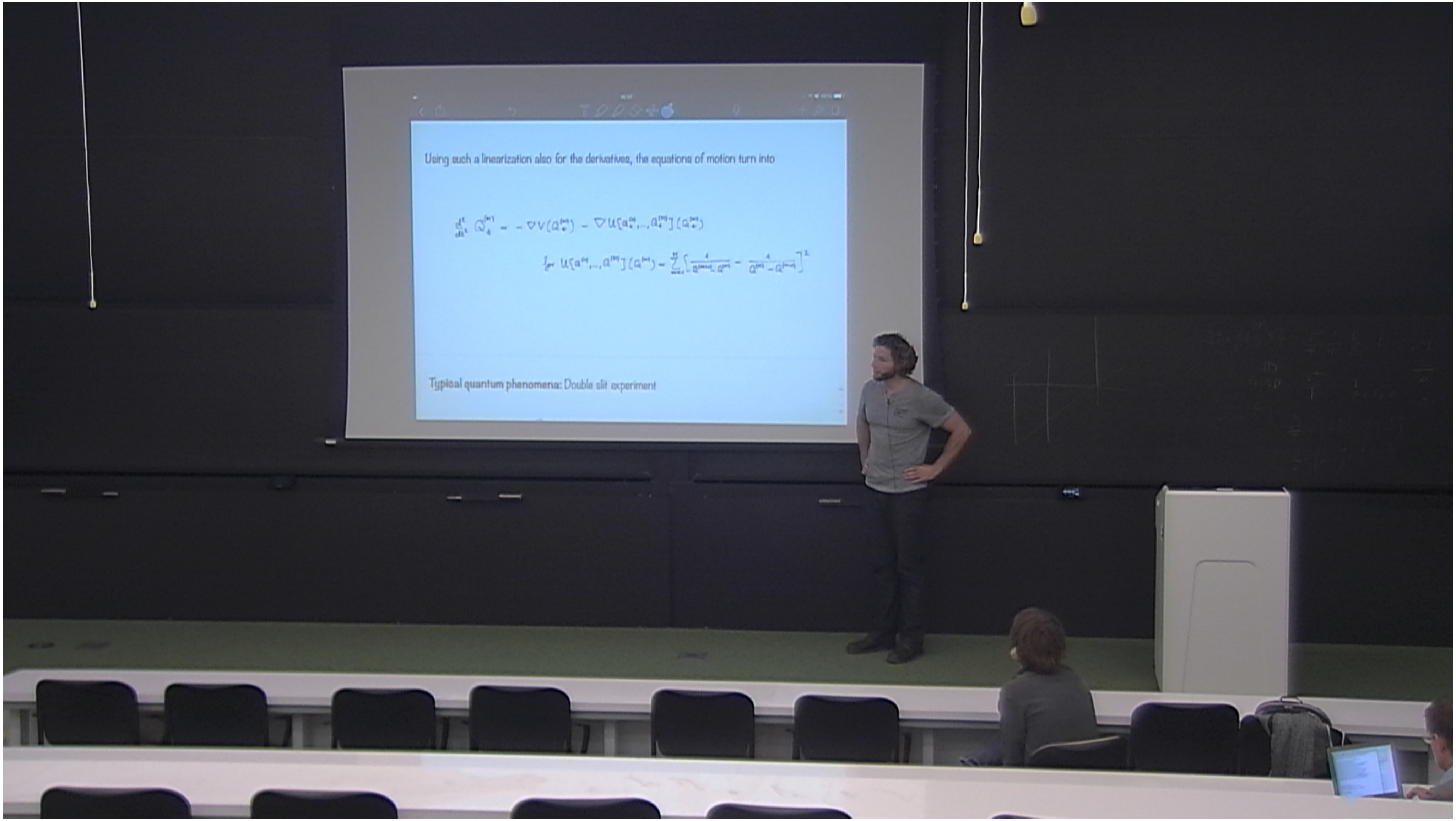
$$\text{for } U\left[Q^{(1)}, \ldots, Q^{(M)}\right](Q^{(m)}) = \sum_{m=1}^{M} \left[\frac{1}{Q^{(m+1)} - Q^{(m)}} - \frac{1}{Q^{(m)} - Q^{(m-1)}}\right]^2$$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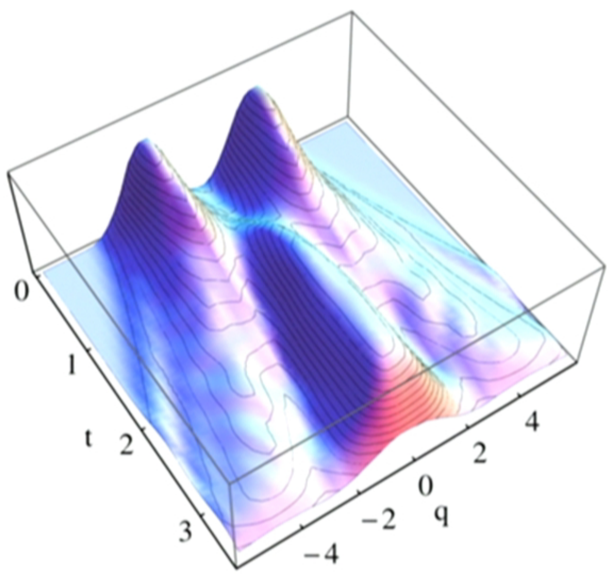

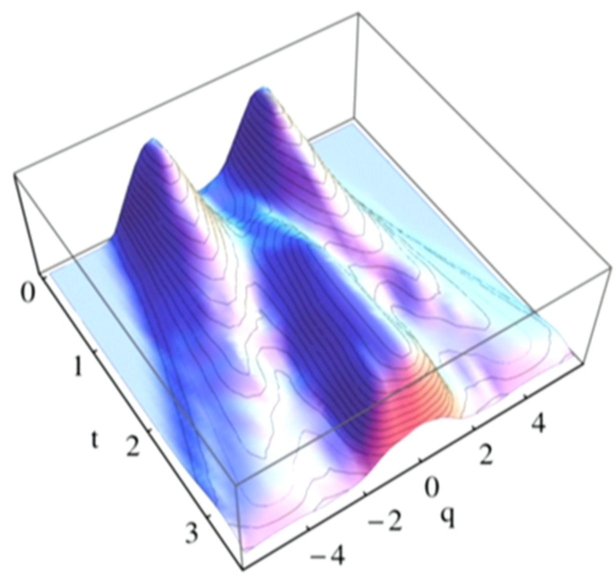

Interpolated histograms over time of MIW and Bohmian trajectories.

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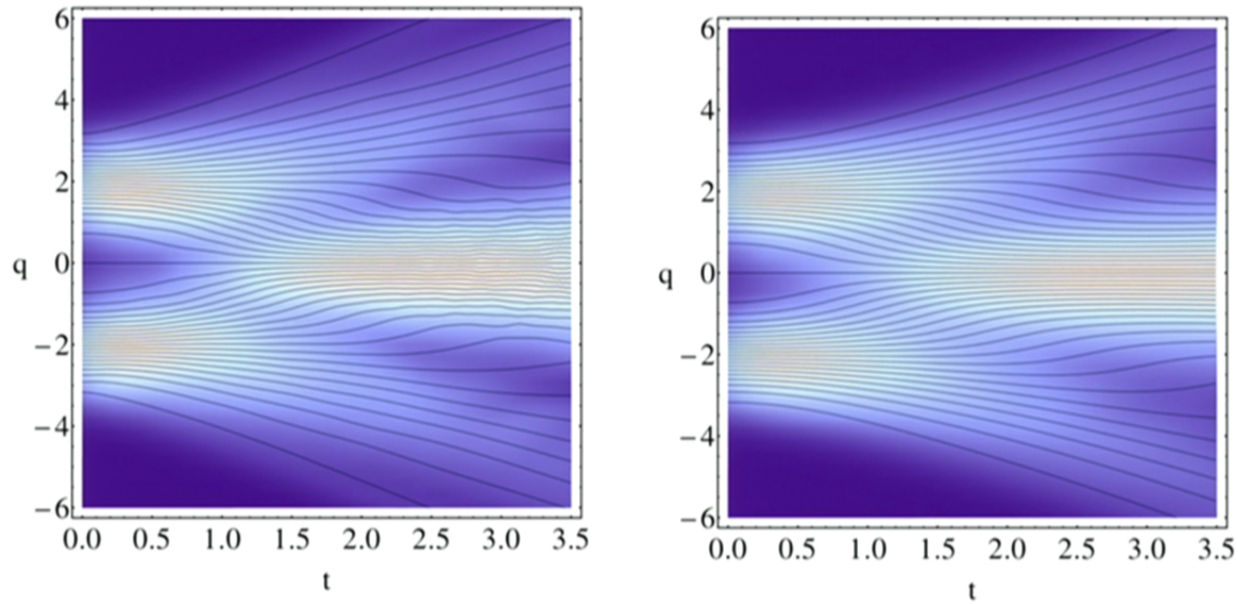

MIW and Bohmian trajectories, interpolated density in background.

However, the cost is high in introducing the many classical worlds.

Open Questions:

- 3d toy model for many worlds
- Description of Spin
- Relativistic equations of motion
- General statistical analysis of subsystems

Thank you!

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Typical quantum phenomena: Double slit experiment

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