Title: An idealization-free experimental test of noncontextuality - Michael Mazurek

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Abstract: <p>To best distinguish between classical and non-classical models of nature requires a good notion of classicality. I will argue that noncontextuality is a good candidate for this notion. Until now, certain theoretical and experimental roadblocks have stood in the way of a test of noncontextuality which is free of unattainable experimental idealizations. I will present solutions to these roadblocks as well as the results of an experimental test.</p>

AN EXPERIMENTAL TEST OF NONCONTEXTUALITY

Michael Mazurek
Institute for Quantum Computing
May 19, 2015

Joint work with Matt Pusey, Ravi Kunjwal, Kevin Resch and Rob Spekkens

Classicality

- How can we distinguish between classical physics and the laws of nature?
- Need a notion of classicality
- Ideally, this notion will be experimentally testable
  - e.g. CHSH inequality violations rule out local realism

Noncontextuality

- What about noncontextuality?
  - Can be defined operationally:
    - *If two experimental procedures are operationally equivalent, then they must be represented equivalently in the underlying model which describes them*
  - Applicable to a wider range of scenarios than local realism
  - Still presents some experimental challenges

[Spekkens, PRA **71**, 052108 (2005)]

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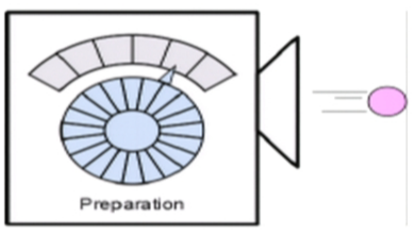
Outline

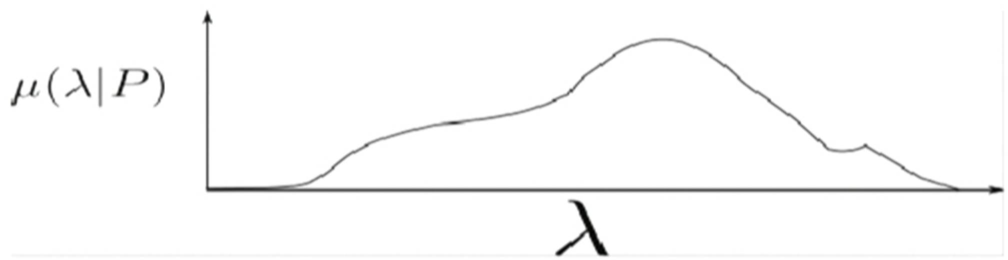
- Ontological models and the assumption of noncontextuality
- A noncontextuality inequality
- The problem of imperfect operational equivalence
- Experiment and results

Ontological models

- Ontological model
  - An attempt to provide a *causal link* between the prepared state of a system and outcomes of future measurements

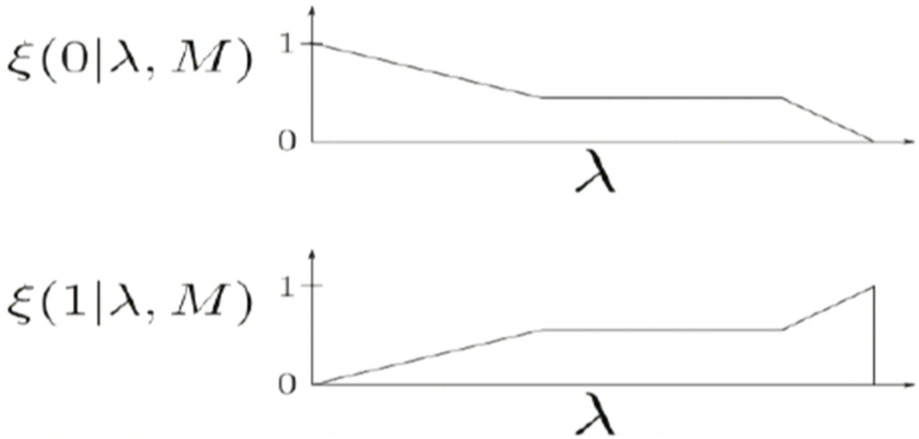
- Preparation procedures prepare the system in an ontic state
  - $\mu(\lambda/P)$: probability that system is in state λ after performing preparation procedure P.

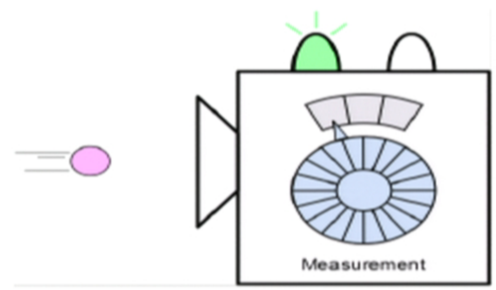




Ontological models

- Measurement procedures return outcomes based on the ontic state, *λ*.
  - ξ(b|λ,M): probability that measurement procedure *M* returns outcome *b*, given an ontic state *λ*.





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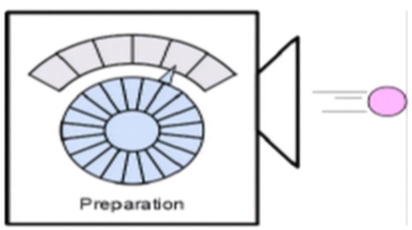

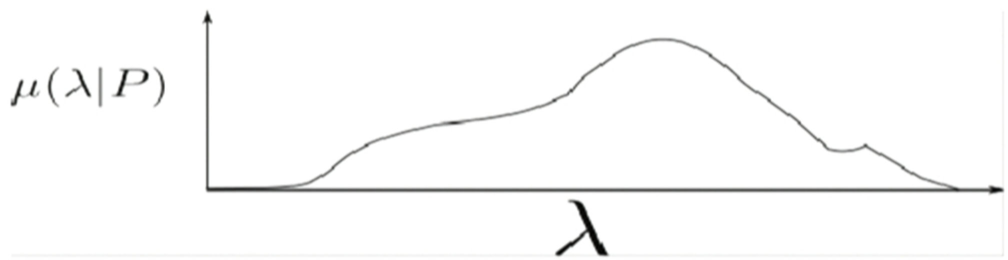

Outcome probabilities

- Probability of a measurement outcome given preparation procedure P and measurement procedure M given by:

$$p(b|M,P) = \int d\lambda \xi(b|\lambda, M)\mu(\lambda|P)$$

- When experimental procedures are mixed, their ontic descriptions are mixed:
  - e.g.

$$P_{mix} = \mathrm{p} P_0 + (1\text{-p}) P_1$$
$$\Rightarrow \mu(\lambda/P_{mix}) = \mathrm{p}\ \mu(\lambda/P_0) + (1\text{-p})\ \mu(\lambda/P_1)$$

$$M_{mix} = \mathrm{p} M_0 + (1\text{-p}) M_1$$
$$\Rightarrow \xi(b/\lambda, M_{mix}) = \mathrm{p}\ \xi(b/\lambda, M_0) + (1\text{-p})\ \xi(b/\lambda, M_1)$$

Operational equivalence

- Two preparation procedures are operationally equivalent if they cannot be distinguished by any measurement
  - If $p(b|M,P) = p(b|M,P')$ for all measurements $M \in \mathscr{M}$ then $P \approx P'$

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Operational definition of noncontextuality

- The assumption of noncontextuality relates *operational* indistinguishability and *ontic* indistinguishability.

- If two experimental procedures are operationally identical, then their representations in the ontic model are also identical:

$$p(b|M,P) = p(b|M,P') \;\forall\; M \in \mathcal{M} \quad \Rightarrow \quad \mu(\lambda|P) = \mu(\lambda|P')$$

$$p(b|M,P) = p(b|M',P) \;\forall\; P \in \mathcal{P} \quad \Rightarrow \quad \xi(b|\lambda,M) = \xi(b|\lambda,M')$$

- Ontic model does not rely on *context* of experimental procedures

[Spekkens, PRA **71**, 052108 (2005)]

A noncontextuality inequality

- Given six preparation procedures $P_{1,0}$, $P_{1,1}$, $P_{2,0}$, $P_{2,1}$, $P_{3,0}$, $P_{3,1}$ we can define P_t as the *equal mixture* of $P_{t,0}$ and $P_{t,1}$ for $t \in \{1,2,3\}$

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- Given three 2-outcome measurement procedures M_1, M_2, and M_3, we can define M_* as the *equal mixture* of M_1, M_2, and M_3

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- Finally, M_{coin} is the measurement which ignores λ and returns outcomes 0 and 1 with equal probability
    - $\xi(0|\lambda, M_{coin}) = \xi(1|\lambda, M_{coin}) = ½$

A noncontextuality inequality

- Define the *average degree-of-correlation, A,* as:

$$A = \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(b|M_t, P_{t,b})$$

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- If $P_1 \approx P_2 \approx P_3$ *and* $M_* \approx M_{coin}$ then the assumption of noncontextuality implies:

$$A \leq 5/6.$$

Proof (by contradiction) that A ≤ 1

Assume A = 1. $A = \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(b|M_t, P_{t,b})$

Implies $\xi(b/\lambda, M_t) \in \{0,1\}$ for all λ for which $\mu(\lambda/P_{t,b}) \neq 0$

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Preparation noncontextuality implies that:

$$\mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3),$$

so $\xi(b|\lambda, M_t) \in \{0,1\}$ for all λ.

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Measurement noncontextuality implies:

$$(1/3)\ \Sigma_t\ \xi(b|\lambda,M_t) = ½.$$

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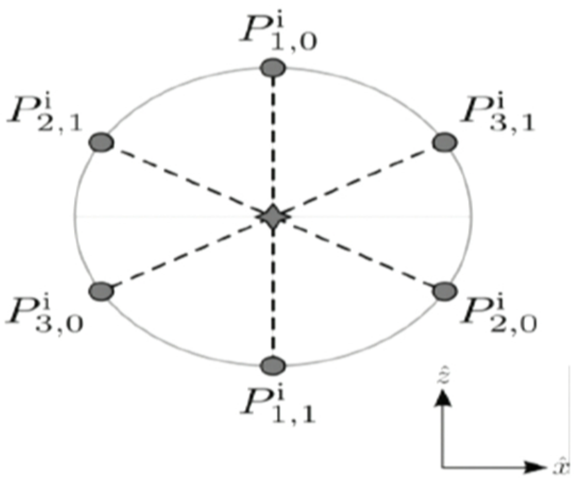
Measurement noncontextuality implies:

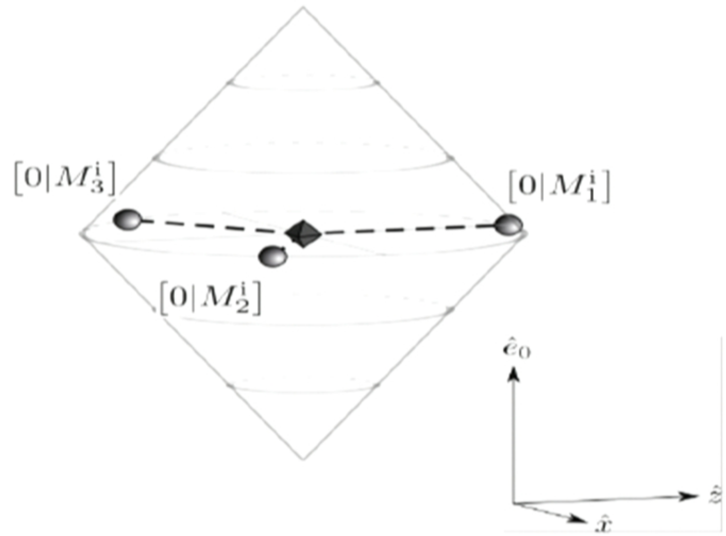
$$(1/3)\ \Sigma_t\ \xi(b|\lambda, M_t) = ½.$$

But $\xi(b|\lambda, M_t) \in \{0,1\}$, which leads to a contradiction.

Quantum violation

- QM can achieve the logical limit $A = 1$.





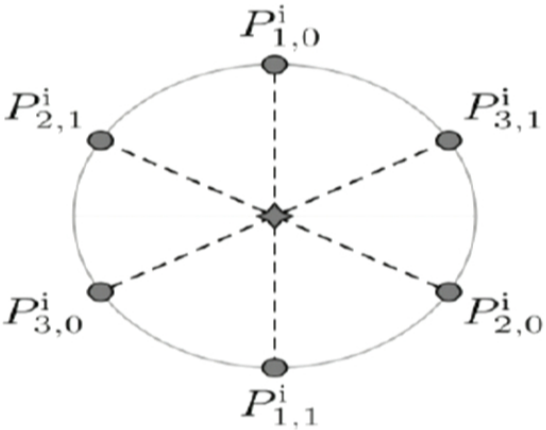
$$\rho = \frac{1}{2}(\mathbb{I} + \vec{r} \cdot \vec{\sigma})$$

$$E = \frac{1}{2}(e_0\mathbb{I} + \vec{e} \cdot \vec{\sigma})$$

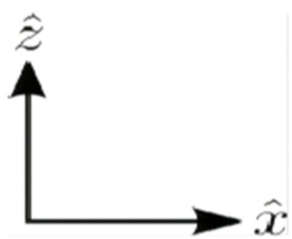
Verifying operational equivalence

- Application of the assumption of noncontextuality requires operational equivalences to be verified.
- Directly measuring exact equality is unrealistic in any experiment.
- We use a general procedure to *infer* statistics of preparation and measurement procedures which are probabilistic mixtures of those performed in the lab

Operational equivalence for preparations



Ideal preparations



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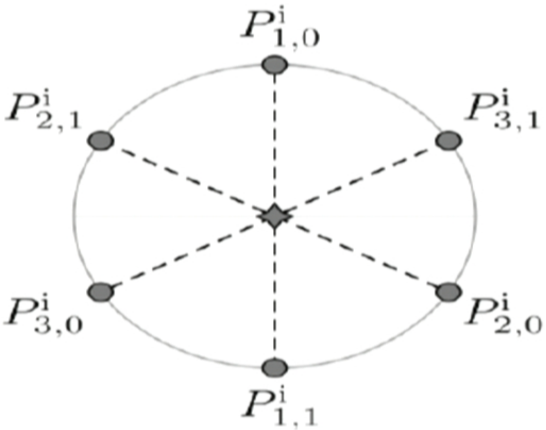

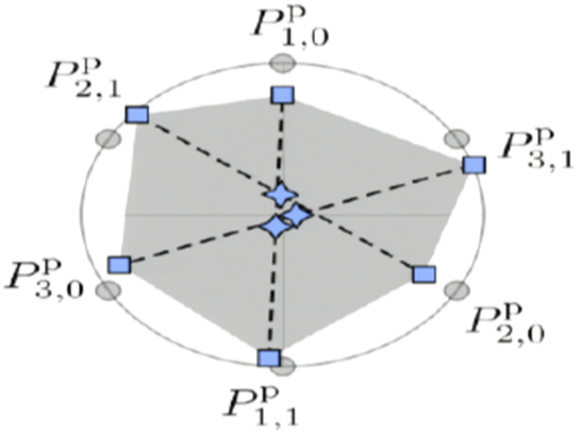

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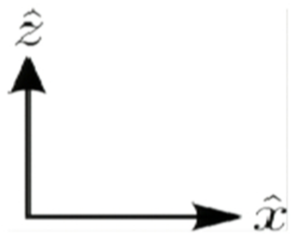

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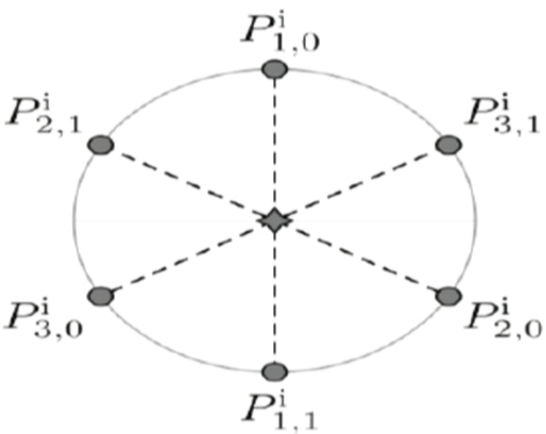

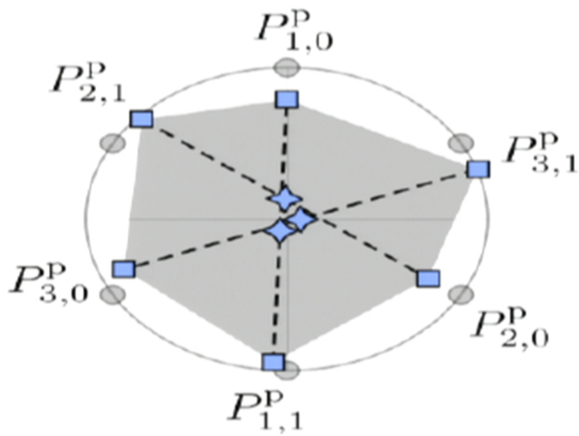

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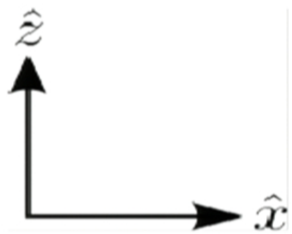

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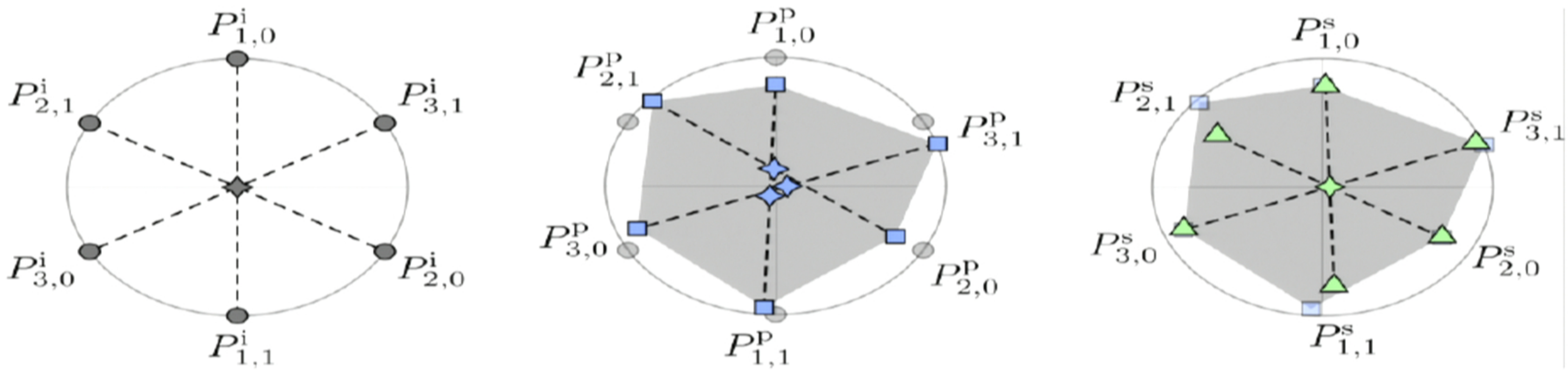
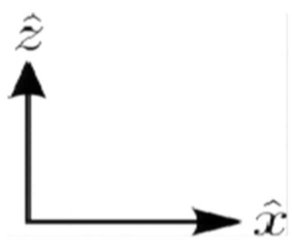


Secondary preparations



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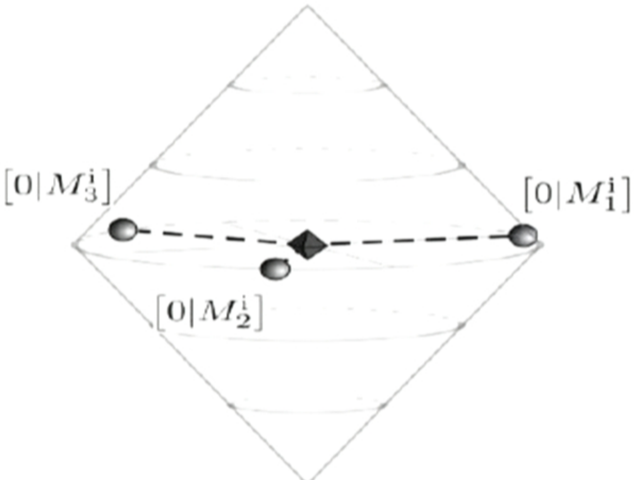
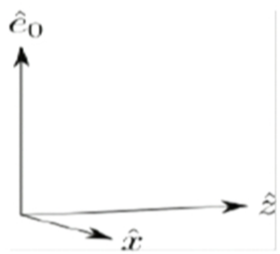
For measurements



Ideal measurements



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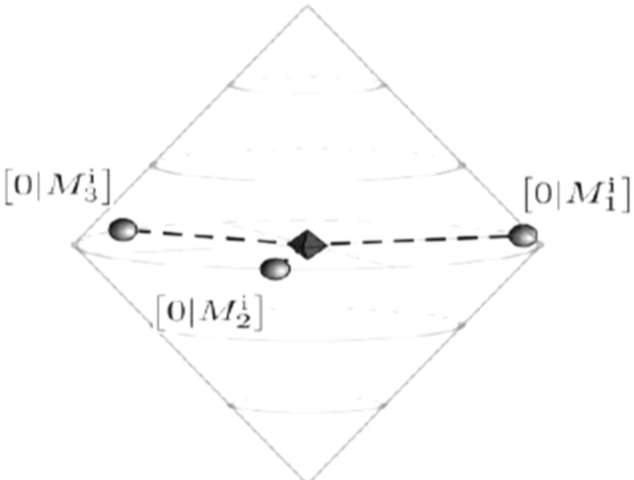

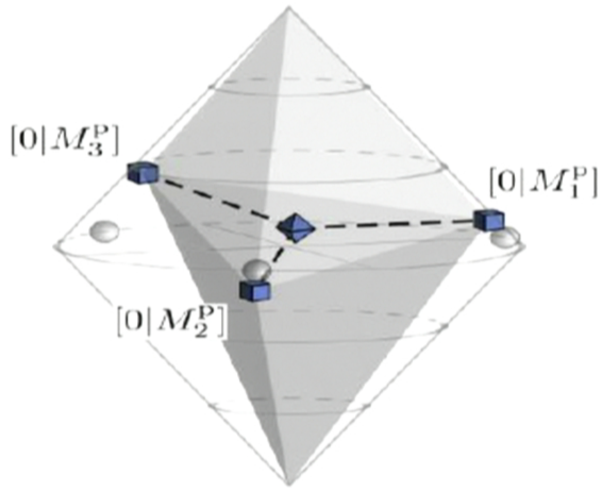

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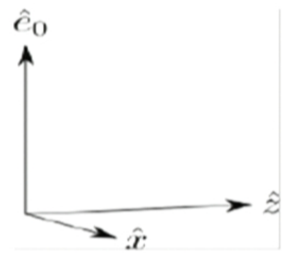

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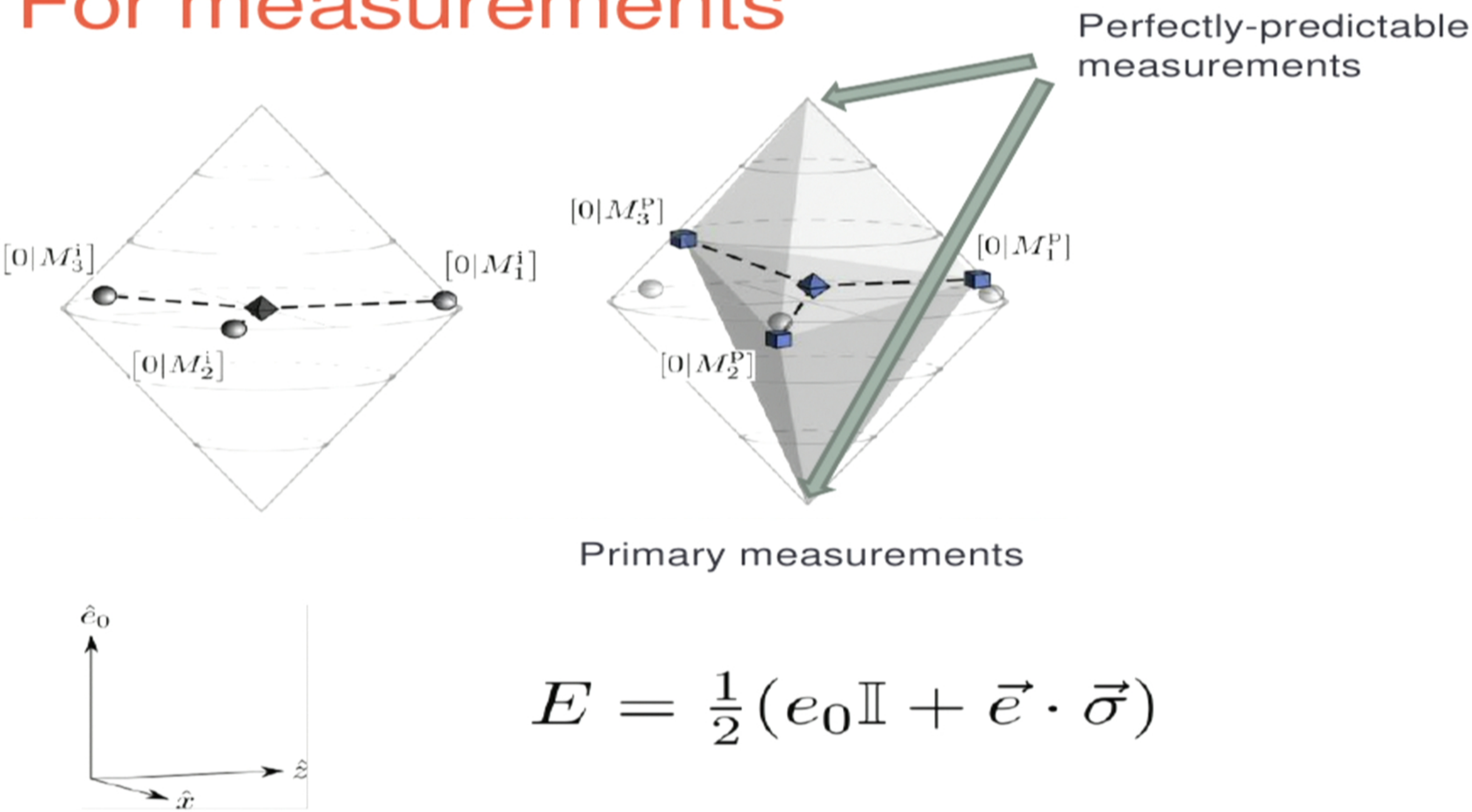

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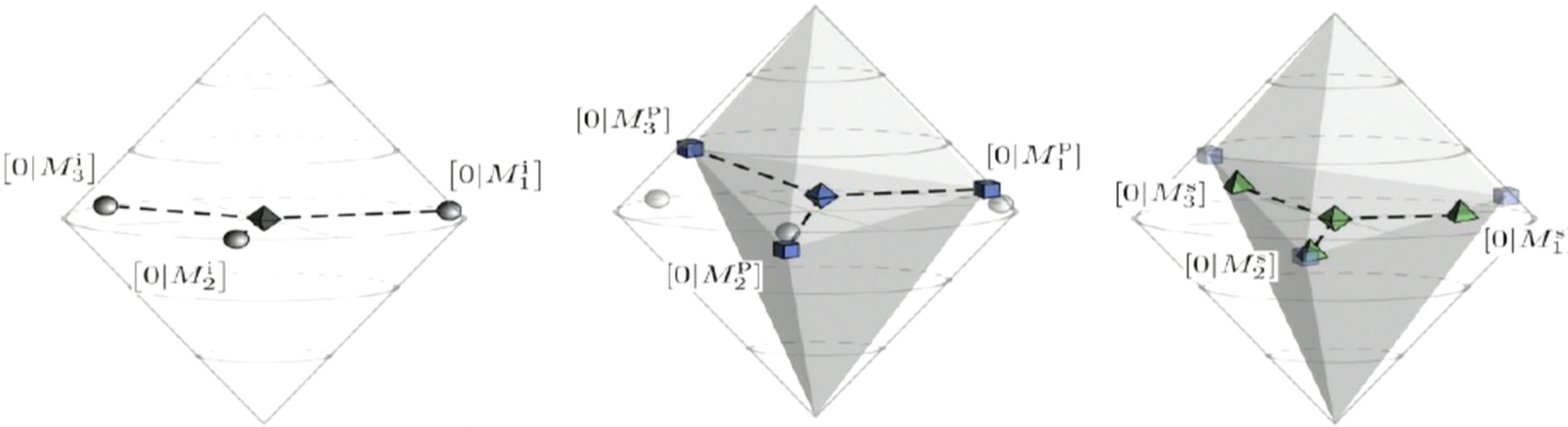
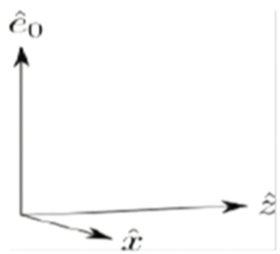


Secondary measurements



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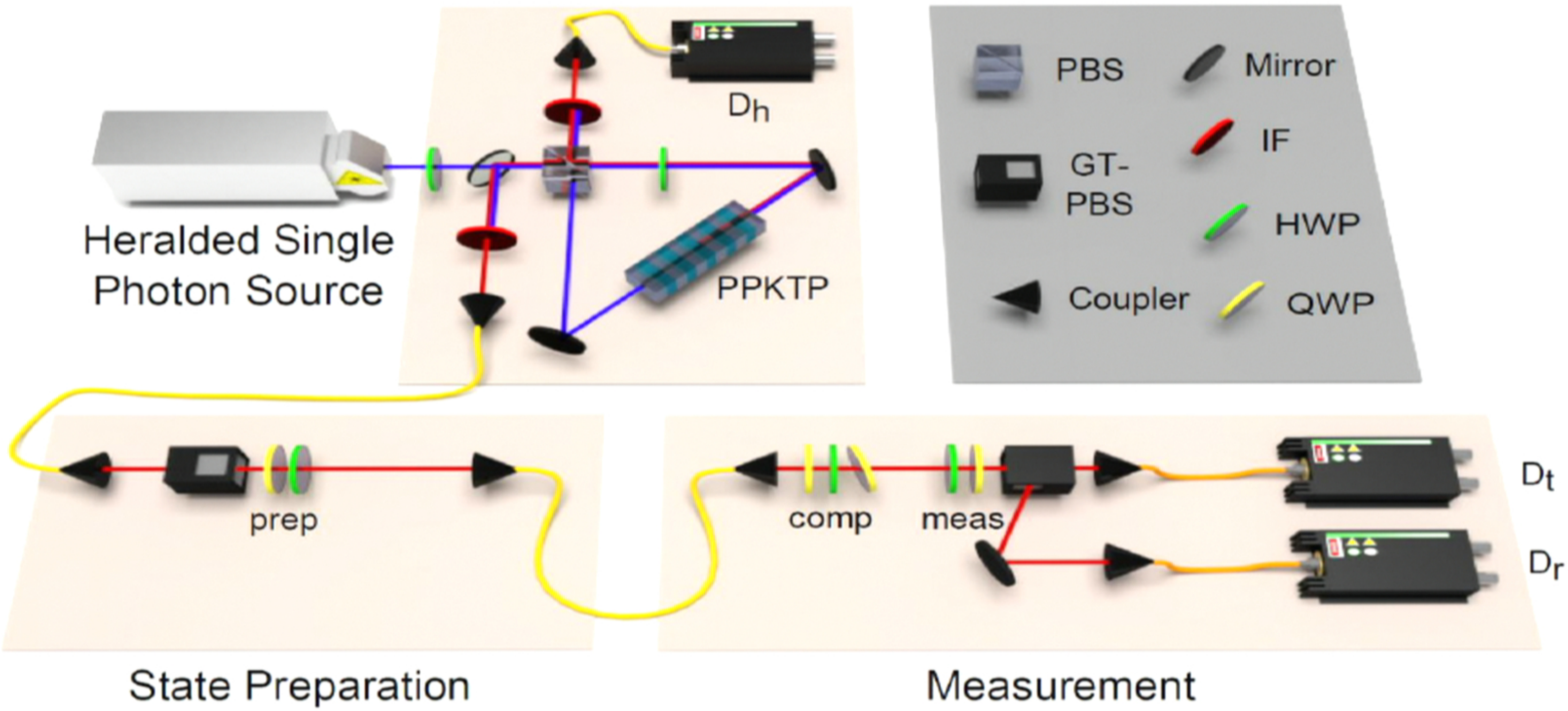
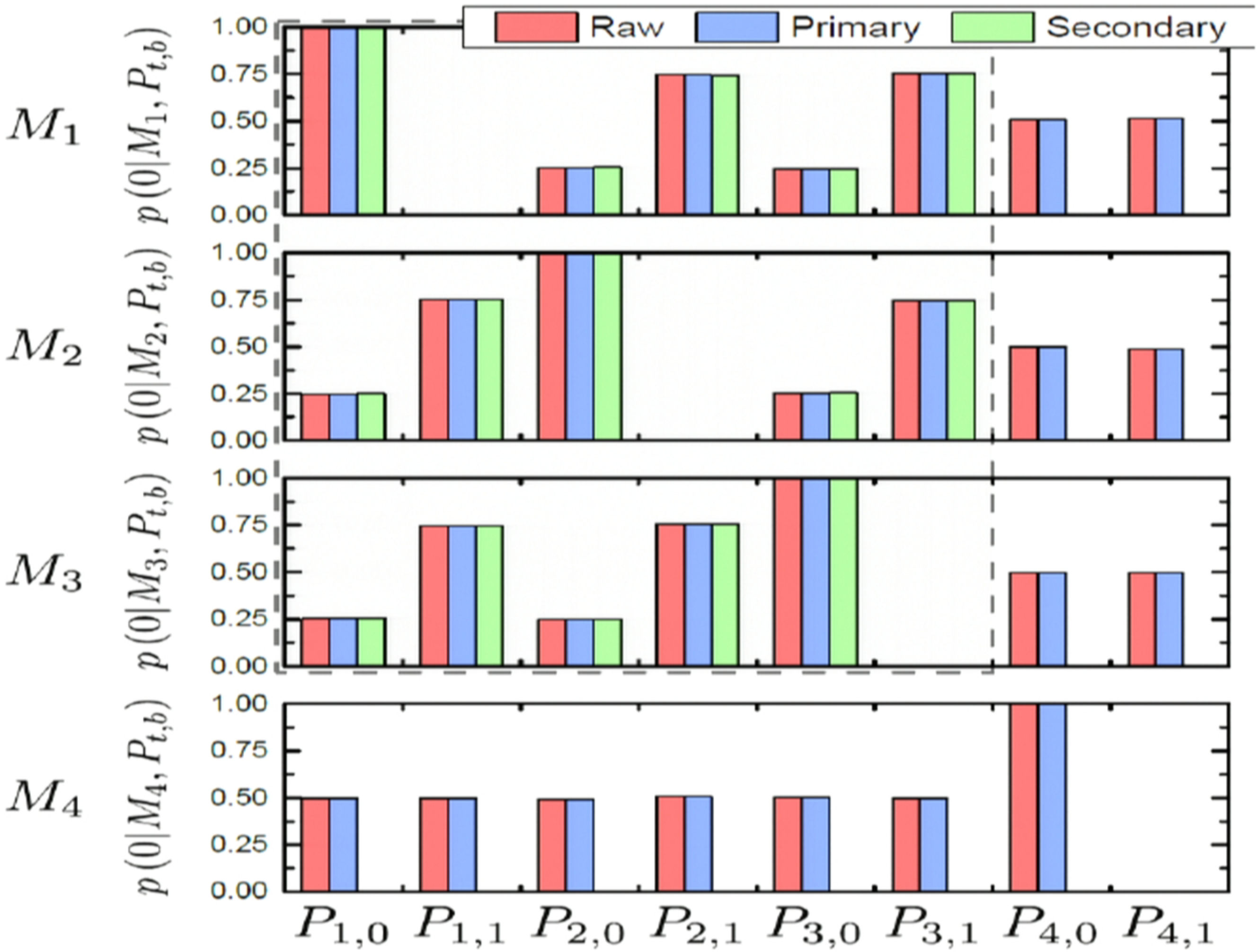
Theory-independence

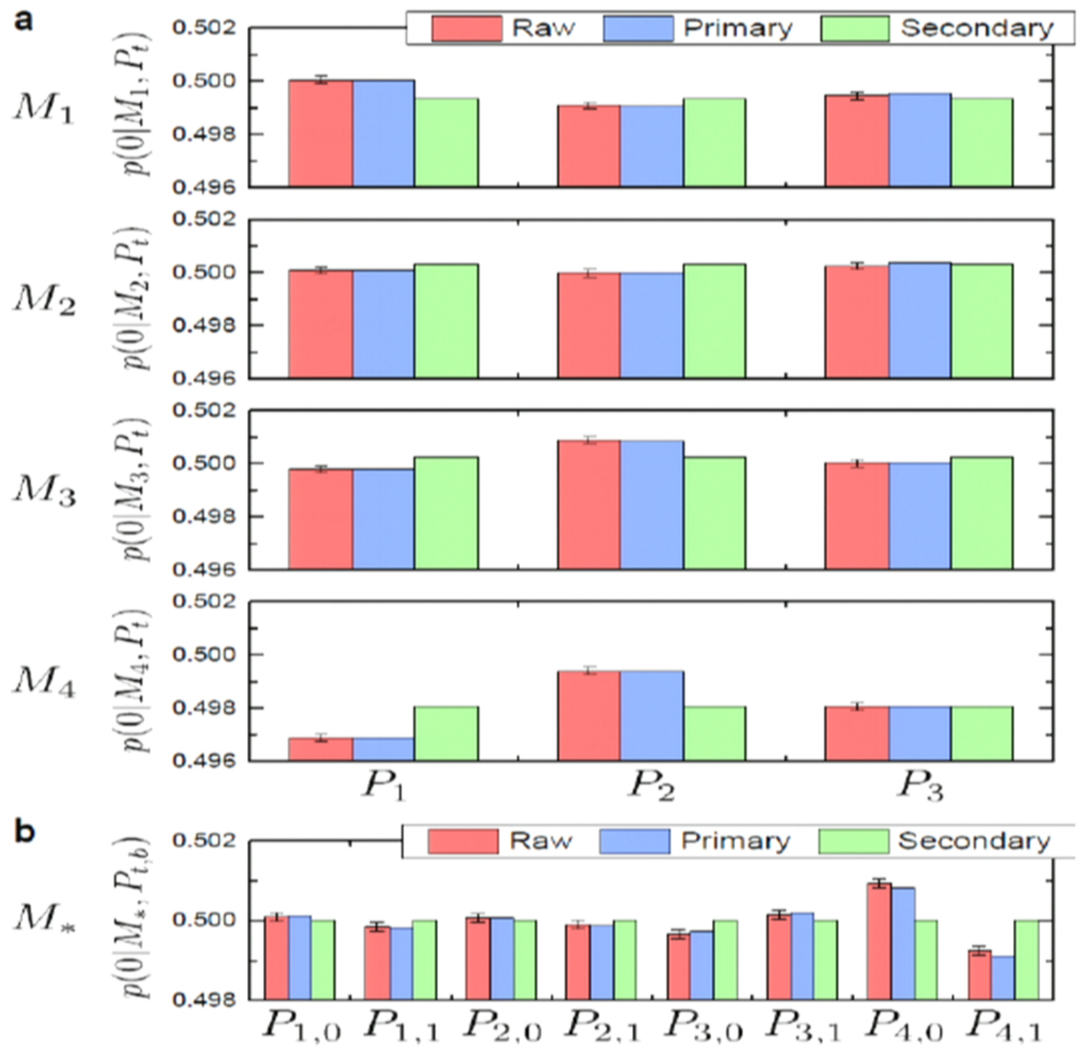
- We *do not* want our analysis to depend on any specific theory
- We fit to a *generalized probabilistic theory* (GPT)
- GPTs are operational theories
  - States and measurement effects are represented by lists of probabilities
- Instead of assuming QT, we reduce our assumptions to the number of tomographically complete measurements in our GPT
- We find the GPT states and effects of best-fit to our data and carry out our analysis from there

[Hardy, quant-ph/0101012 (2001)]
[Barret, PRA **75** 032304 (2007)]

Experimental set-up




Raw
Primary
Secondary
M_1
M_2
M_3
M_4
$p(0|M_1, P_{t,b})$
$p(0|M_2, P_{t,b})$
$p(0|M_3, P_{t,b})$
$p(0|M_4, P_{t,b})$
1.00
0.75
0.50
0.25
0.00
$P_{1,0}$ $P_{1,1}$ $P_{2,0}$ $P_{2,1}$ $P_{3,0}$ $P_{3,1}$ $P_{4,0}$ $P_{4,1}$


a
Raw
Primary
Secondary
M_1
$p(0|M_1, P_t)$
M_2
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M_3
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M_4
$p(0|M_4, P_t)$
0.502
0.500
0.498
0.496
P_1
P_2
P_3
b
M_*
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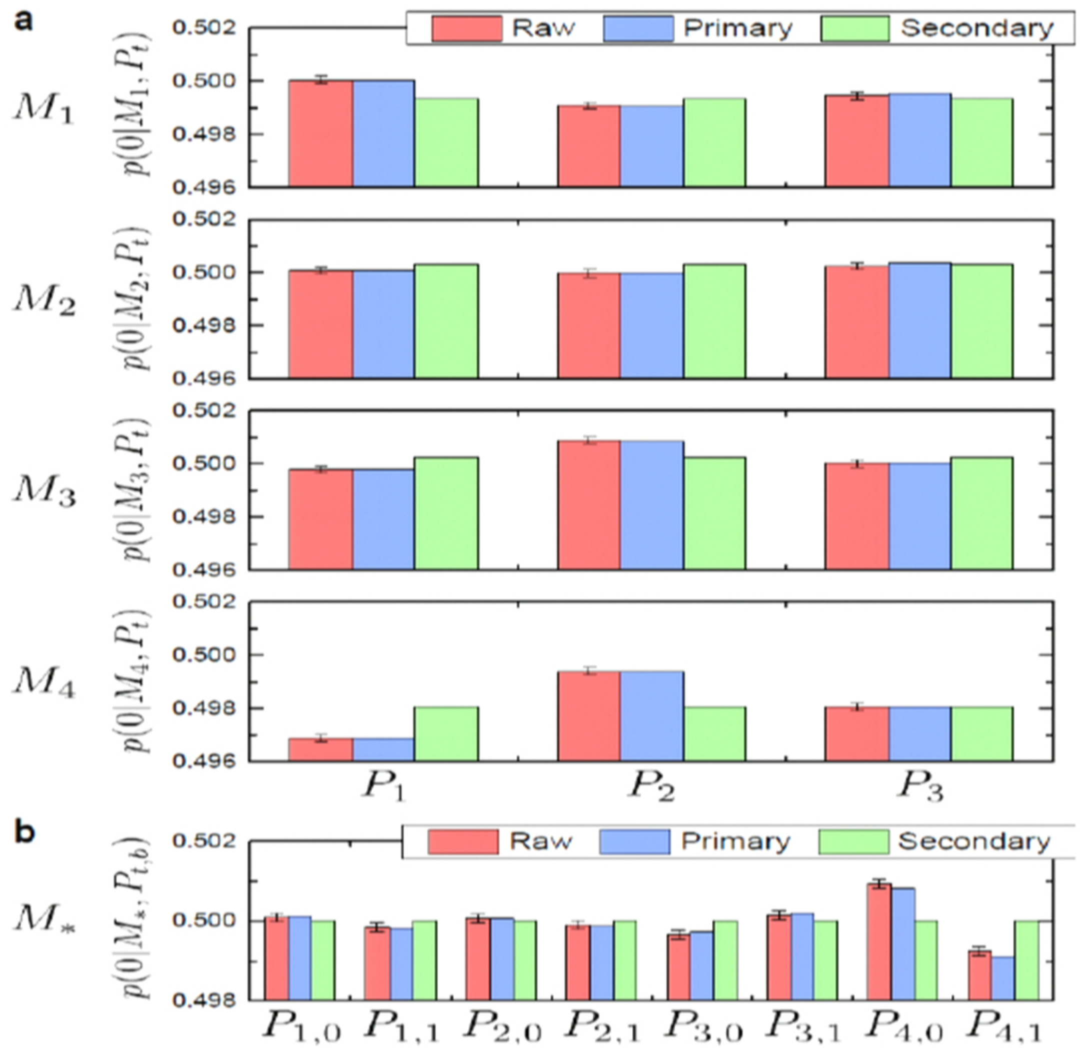
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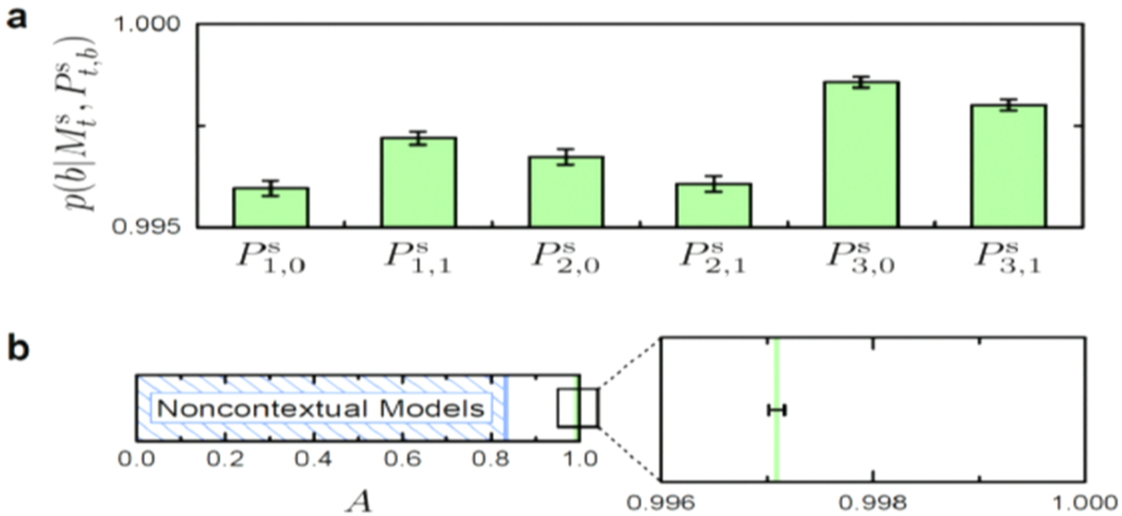
Violation



$$A = \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} p(b|M_t, P_{t,b})$$

Conclusion

- An operational definition of noncontextuality is a good notion of classicality
  - leads to an experimentally-testable inequality
- Solved* the operational equivalence problem
  - *modulo an assumption about tomographic completeness
- We have implemented a test of the assumption of noncontextuality

Thank you!

- Matt Pusey
- Ravi Kunjwal
- Kevin Resch
- Rob Spekkens