Title: Joint Measurability, Einstein-Podolsky-Rosen Steering, and Bell Nonlocality

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Abstract: <p>In this talk we will discuss the relation between the incompatibility of quantum measurements and quantum nonlocality. We show that any set of measurements that is not jointly measurable (i.e. incompatible) can be used for demonstrating EPR steering, a form of quantum nonlocality. This implies that EPR steering and (non) joint measurability can be viewed as equivalent. Moreover, we discuss the connection between Bell nonlocality and joint measurability, and give evidence that both notions are inequivalent. This suggest the existence of incompatible quantum measurements which are Bell local, similarly to certain entangled states which admit a local hidden variable model. Finally, we discuss applications of these results to problems in joint mesurability, and for EPR steering using randomly chosen measurements.</p>

Joint measurability, EPR steering, and Bell nonlocality

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Phys. Rev. Lett. 113, 160402; Joint with: T. Vértesi, N. Brunner

Joint Measurability

$$\Delta x \, \Delta p \geq \hbar/2$$

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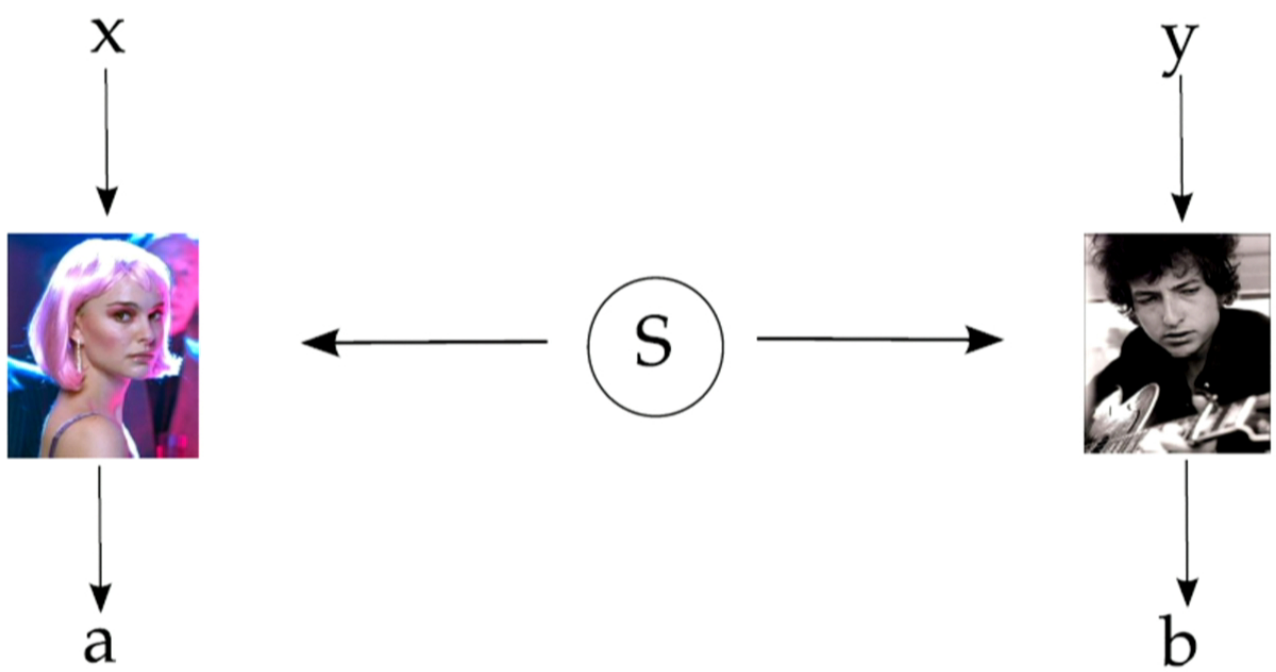
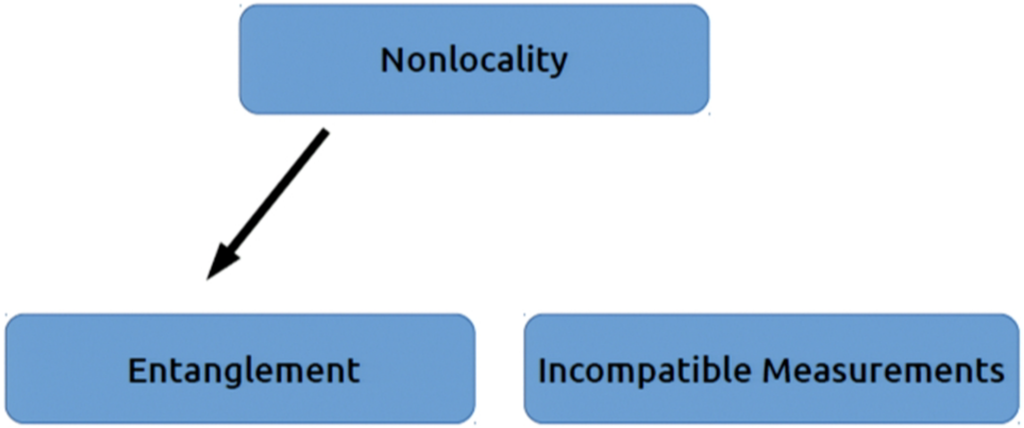
Nonlocality



Separable states are local



Compatible Measurements

- Quantum observables:

$$A = A^\dagger, \qquad B = B^\dagger$$

More general measurements

- POVM:

$$A_a \geq 0, \quad \sum_a A_a = I$$

$$B_b \geq 0, \quad \sum_b B_b = I$$

Pauli Measurements

$$\sigma_Z : \{|0\rangle\langle 0|\,,\ |1\rangle\langle 1|\} \quad \sigma_X : \{|+\rangle\langle +|\,,\ |-\rangle\langle -|\}$$

Noise Pauli Measurements

$$\sigma_{Z,\eta} : \left\{ \eta \left|0\right\rangle\left\langle 0\right| + (1-\eta)\frac{I}{2} \; ; \quad \eta \left|1\right\rangle\left\langle 1\right| + (1-\eta)\frac{I}{2} \right\}$$

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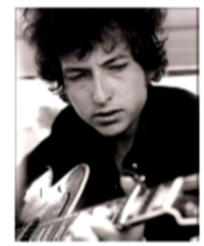
EPR steering

$$\longleftarrow \quad \psi^- \quad \longrightarrow$$



EPR steering

σ_Z: $|0\rangle \xleftrightarrow{p=1/2} |1\rangle$, $|1\rangle \xleftrightarrow{p=1/2} |0\rangle$

σ_X: $|-\rangle \xleftrightarrow{p=1/2} |+\rangle$, $|+\rangle \xleftrightarrow{p=1/2} |-\rangle$

Incompatible measurements + Entangled state –> EPR steering

Assemblage

Bob's system is completely described by an *assemblage*

$$\sigma_{a|x} = \mathrm{tr}_A(\rho_{AB} A_{a|x} \otimes I)$$

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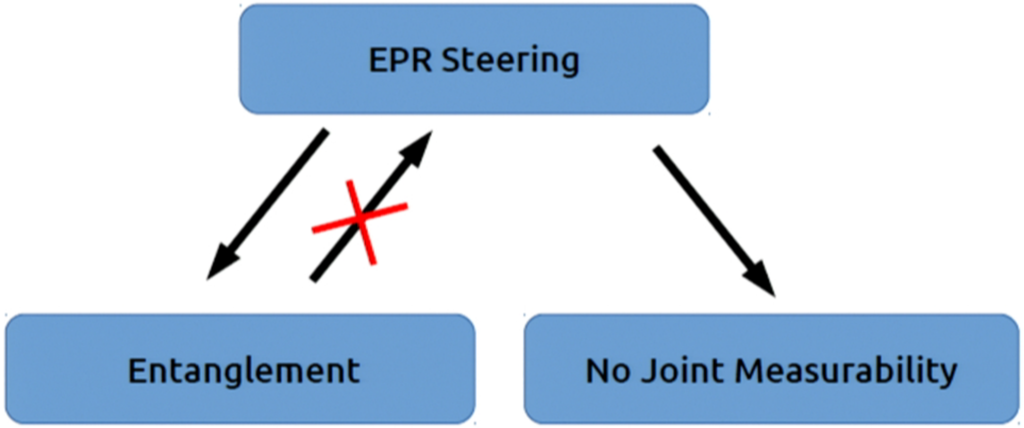
$$\rho_{a|x} = \frac{\sigma_{a|x}}{\mathrm{tr}(\sigma_{a|x})}$$

Unsteerable Assemblages

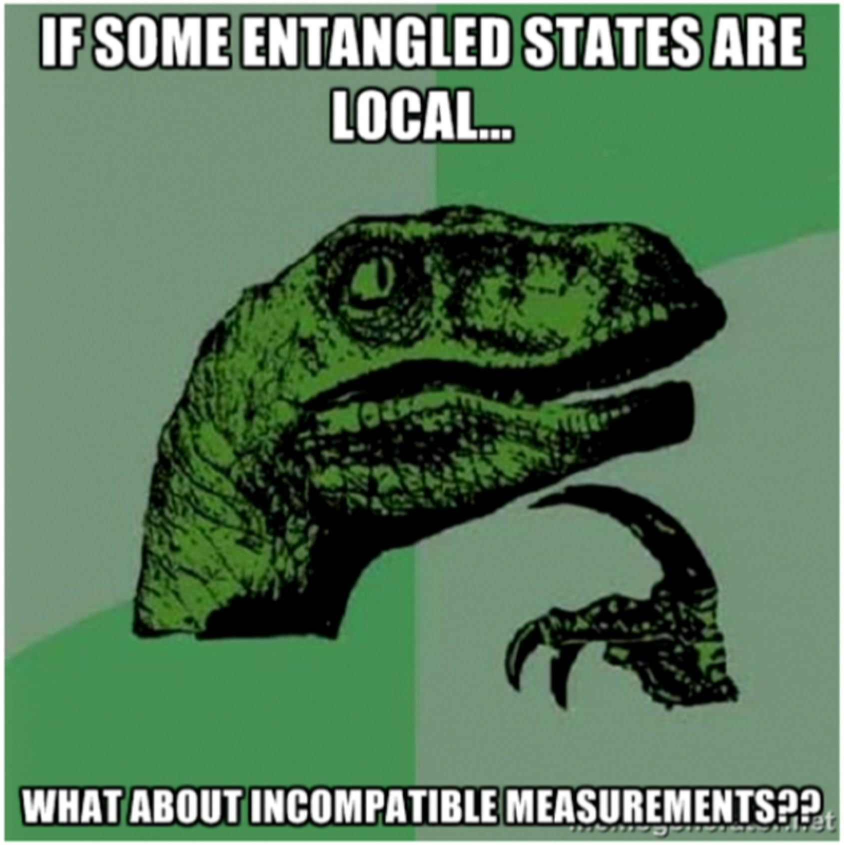
$$\sigma_{a|x} = \sum_{\lambda} \pi(\lambda) p_A(a|x,\lambda) \rho_\lambda$$

Werner states (1989)

Some entangled states are EPR local!



Local Incompatible Measurements??



The precise result

Theorem

Let $\{A_{a|x}\}$ be a set of incompatible measurements. If Alice measures $\{A_{a|x}\}$ on her part of a pure entangled state, the resulting assemblage is steerable.

Applications

- Explore known results from the Steering community to get results for Joint Measurability

Applications

- Imagine that Alice wants to measure ALL projective measurements at the same time.
- Some white noise is accepted (*i.e.* $\eta M + (1-\eta) I$)
- How small should η be?
- Precisely $1/2$!
- Werner's model (1989) + “Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox”: H. M. Wiseman *et al* (2006)

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$$\rho_{UNS} = \frac{1}{2}\Phi_\theta + \frac{1}{2}\frac{I}{2} \otimes \rho_{B,\theta}$$

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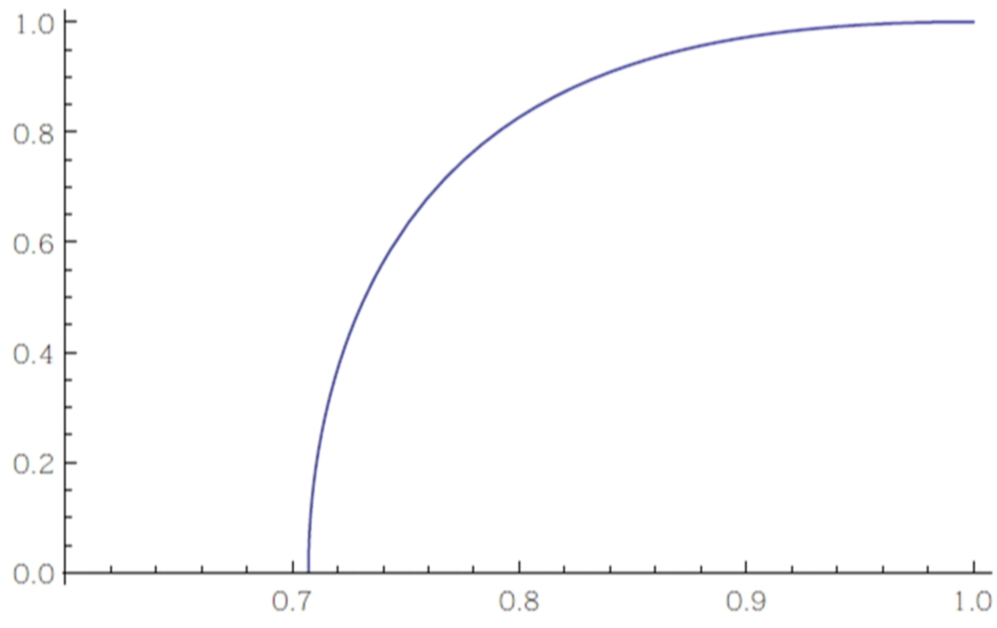
- $\eta \leq 5/12 \implies$ all POVMs can be measured (Barrett 2002 + Quintino *et al* 2015 + this work -> Pusey 2015)

Applications

- Alice and Bob share a two qubit Werner state
- What is the probability of Bob having an steering assemblage when Alice perform random measurements?

Applications

$$p(\eta) = \sqrt{\frac{2\eta^2 - 1}{\eta^4}}$$



Busch, P. (1986) Unsharp reality and joint measurements for spin observables. Phys. Rev. D 33)

Bell Inequalities

Bell Nonlocality can be witnessed by Bell Inequalities

$$CHSH = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

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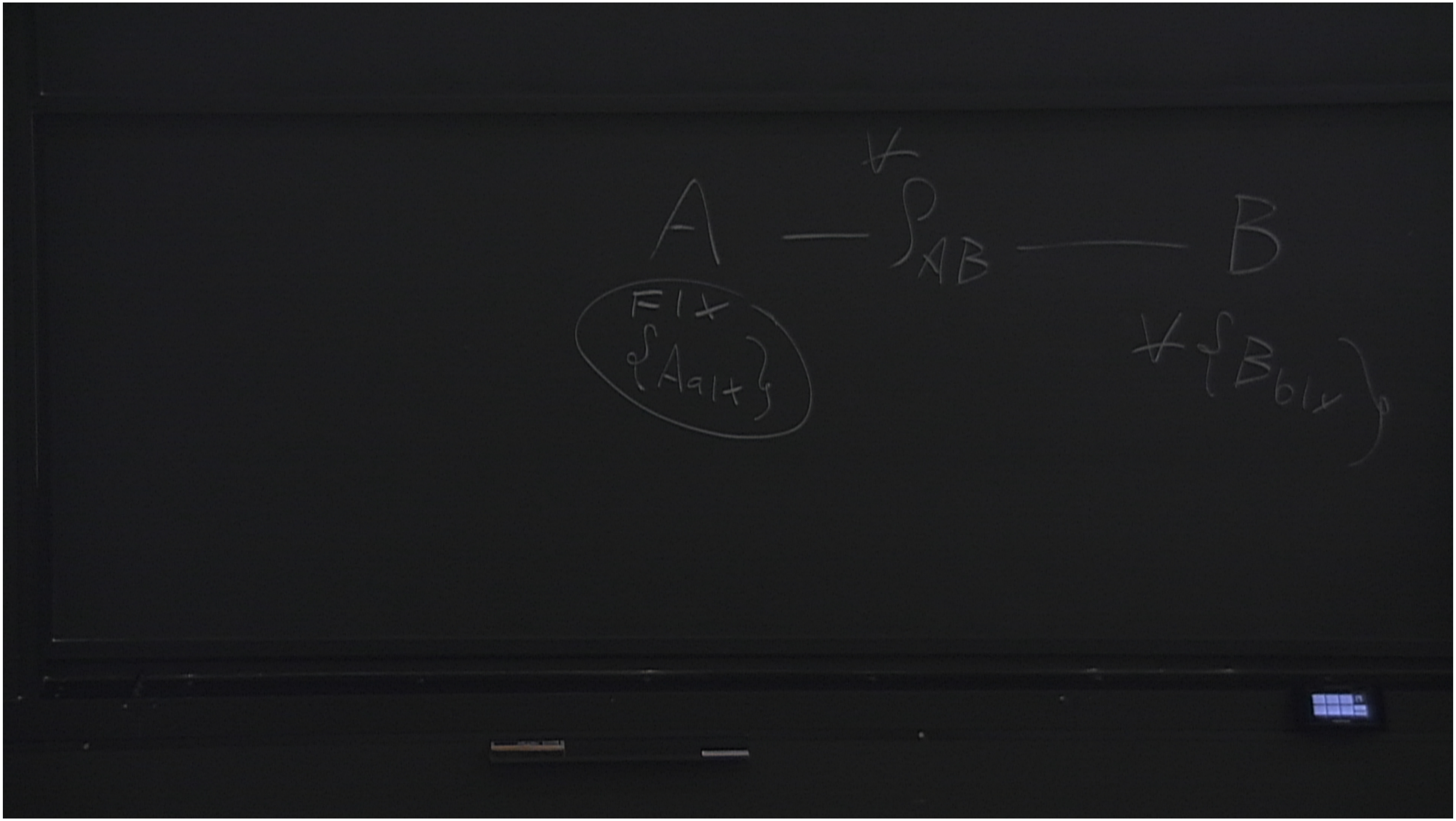
$$\langle A_x B_y \rangle := p(a = b|xy) - p(a \neq b|xy)$$


A — ∀ ρAB — B
Fix {Aa|x}
∀ {Bb|y}b

Incompatible measurements and Bell Nonlocality

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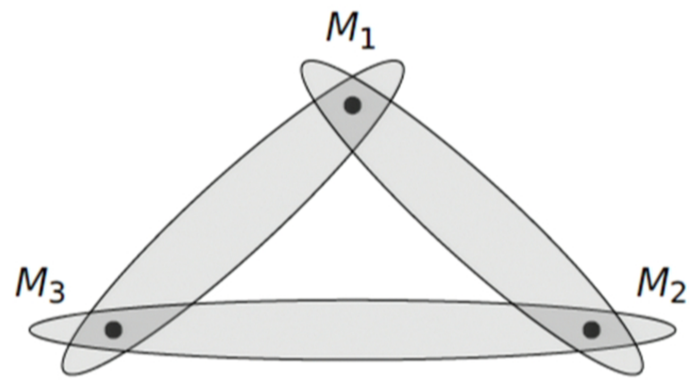
- Evidence 1: Maximally entangled state $\implies$ incompatible Bell local measurements
- Evidence 2: Hollow triangle measurements + full-correlation type cannot show bell nonlocality ($\langle A_x B_y \rangle := p(a = b|xy) - p(a \neq b|xy)$)



Incompatible measurements and Bell Nonlocality

- $$BI = \sum_{xy} \gamma_{xy} \langle A_x B_y \rangle + \sum_x \alpha_x \langle A_x \rangle + \sum_y \beta_y \langle B_y \rangle \leq 1$$

$$\langle A_x B_y \rangle := p(a = b|xy) - p(a \neq b|xy)$$

$$\langle A_x \rangle := p(a = 1|x) - p(a = -1|x)$$

$$\langle B_y \rangle := p(b = 1|y) - p(b = -1|y)$$

- Evidence 3: The Pauli Hollow triangle measurements does not violate many inequalities:

$$\sum_y |\beta_y| \leq \frac{1 - \eta BI_{\mathbb{C}^2}}{1 - \eta} \implies \text{No Bell Violation}$$

More evidences

Evidence 4: SDP optimisations on known Bell inequalities

	$\{M'_{a\|x}(\eta)\}$	$\{M''_{a\|x}(\eta)\}$
Pairwise JM (CHSH violation)	$1/\sqrt{2} \approx 0.7071$	0.5858
Triplewise JM	$1/\sqrt{3} \approx 0.5774$	0.4226
Bell violation ($n=3$): I_{3322}	0.8037	0.6635
Bell violation ($n=4$): I^1_{3422}	0.8522	0.7913
I^2_{3422}	0.8323	0.5636
I^3_{3422}	0.8188	0.6795
Bell violation ($n=5$): I_{3522}	0.7786	0.5636

$$M''_{1|1}(\eta) := \eta\,|0\rangle\langle 0|\,,\; M''_{1|2}(\eta) := \eta\,|+\rangle\langle +|\,,\; M''_{1|3}(\eta) := \eta\,|Y_+\rangle\langle Y_+|$$

Incompatible measurements and Bell Nonlocality

Conjecture

There exists a set of non Jointly Measurable measurements that can never lead to Bell inequality violation.

Conclusions

- Better understanding on the relation between quantum measurements and nonlocality
- Conceptually: How to interpret JM in terms of EPR steering (vice versa!)
- Applications: Some theorems for JM can be translated to Nonlocality (vice versa!)

Thank you!