Title: Gravitational collapse in the ghost-free gravity

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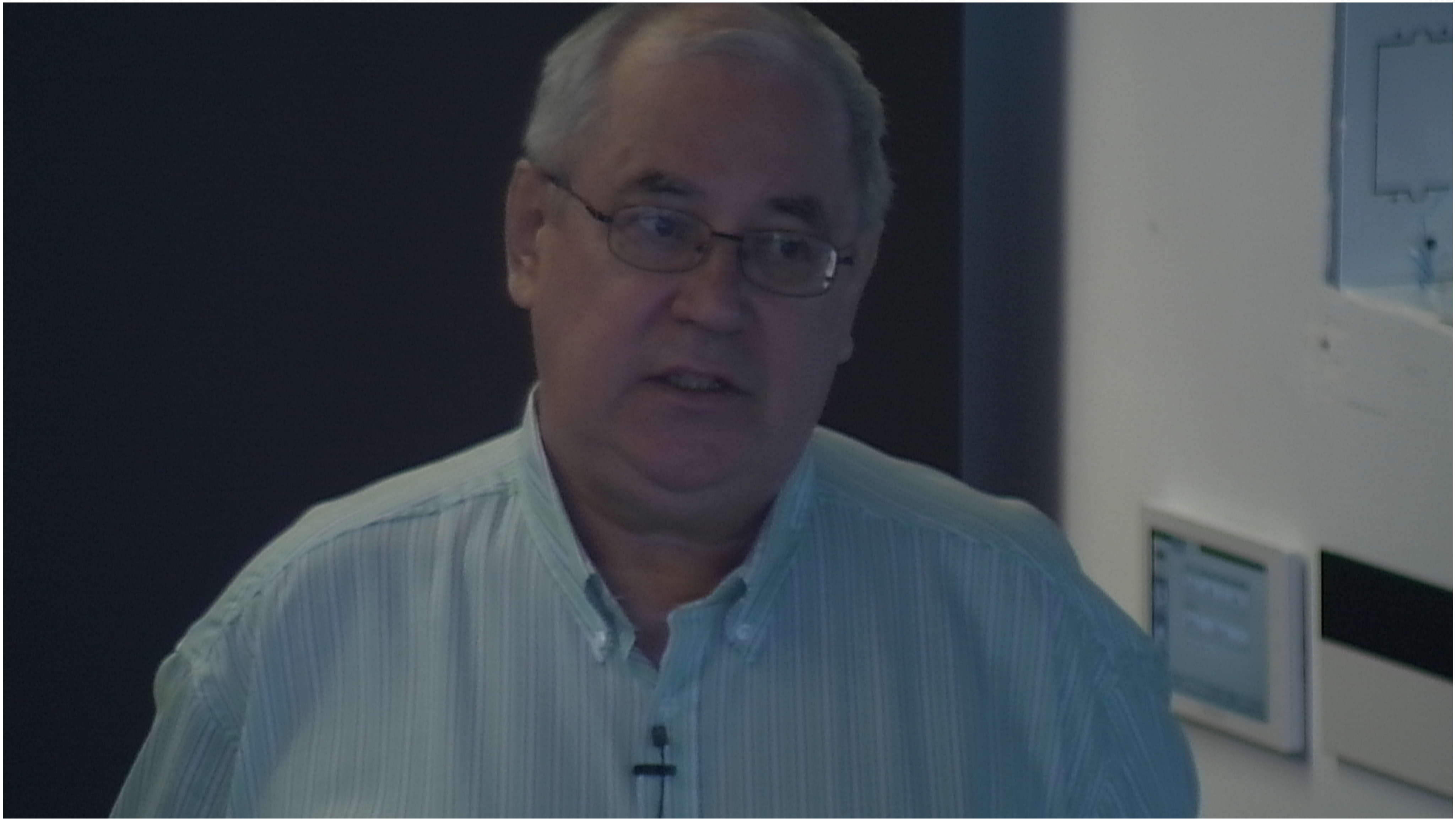
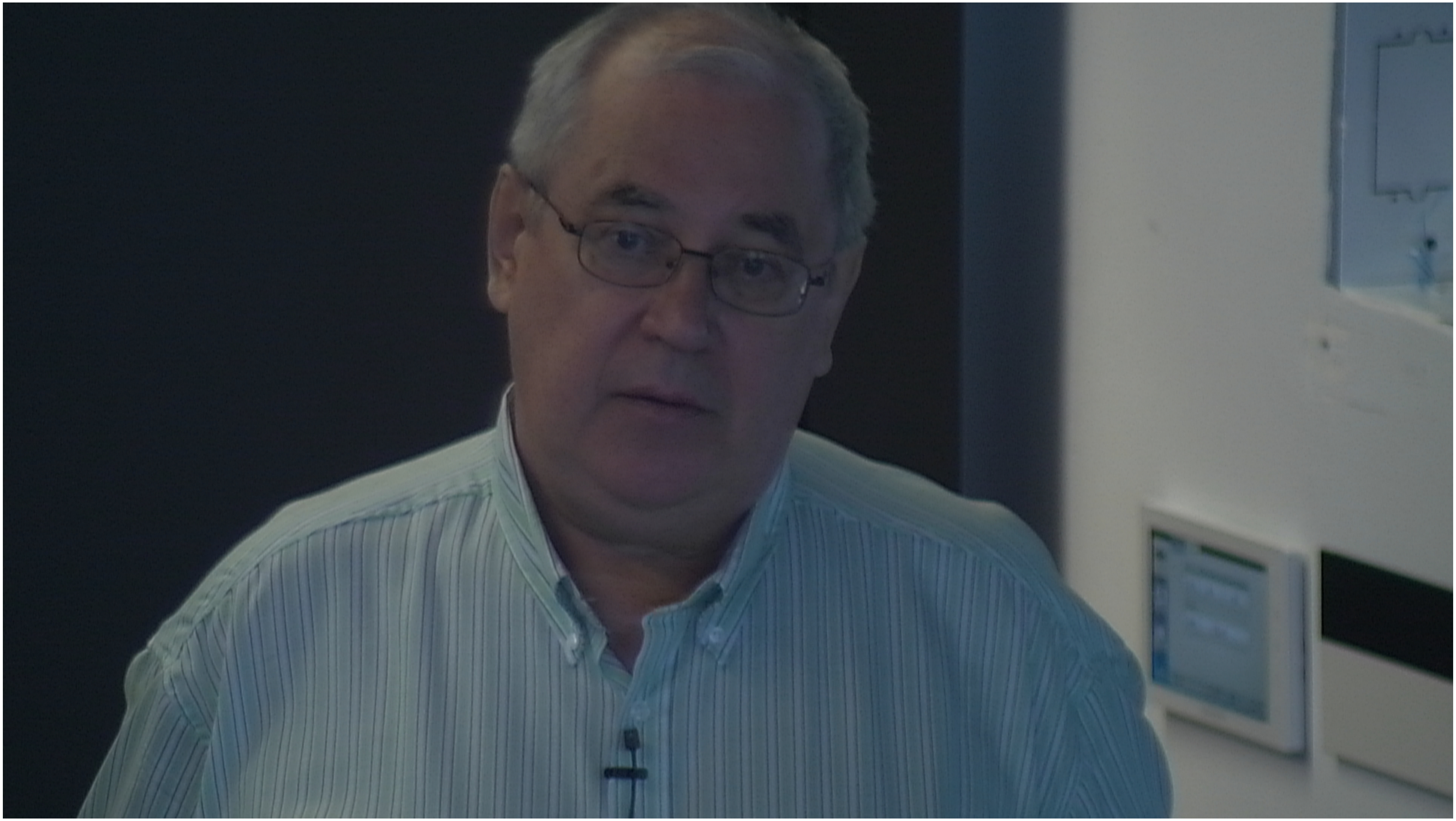
Abstract: <p>We discuss a problem of a black hole formation in the ghost-free gravity. We demonstrate how a non-local modification of gravity equations regularizes static and dynamical solutions. We focus on the problem of a collapse of small masses in the ghost-free gravity, and demonstrate that there exists a mass gap for mini-black-hole formation in this model.</p>

Gravitational collapse in the ghost-free gravity

Valeri P. Frolov,
Univ. of Alberta, Edmonton

Perimeter Institute
Waterloo, May 28, 2015



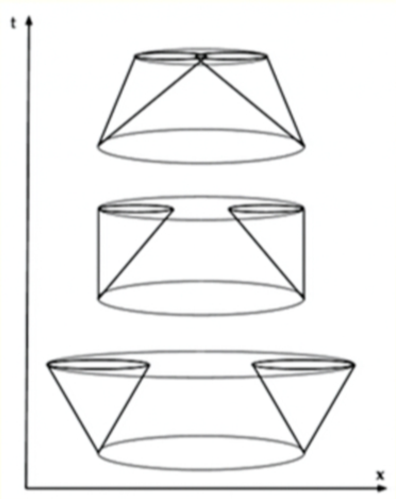


Based on:

"Spherical collapse of small masses in the ghost-free gravity
V.F, A. Zelnikov, T. Netto, e-Print: arXiv:1504.00412 (2015);

"Mass-gap for black hole formation in higher derivative and ghost free gravity", V. F. ,arXiv:1505.00492 (2015);

`Quasi-local definition' of BH: Apparent horizon



A compact smooth surface B is called a trapped surface if both, in- and out-going null surfaces, orthogonal to B, are non-expanding .

A trapped region is a region inside B.

A boundary of all trapped regions is called an apparent horizon.

According to GR: Singularity exists inside a black hole.

Theorems on singularities: Penrose and Hawking.

Penrose theorem: Assume

1. The null energy condition holds $T_{\mu\nu} l^{\mu} l^{\nu} \geq 0$;
2. There exists a noncompact connected Cauchy surface.
3. There exist a closed trapped null surface .

Then, we either have null geodesic incompleteness, or closed timelike curves.

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Schwarzschild ST has a spacelike singularity.
RN and Kerr ST have a timelike singularity.
In both cases this is a curvature singularity.

Expectation 1: When curvature becomes high (e.g. reaches the Plancian value) the classical GR must be modified (quantum corrections, it is an emergent theory, etc.).
Expectation 2: Singularities of GR would be resolved.

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Regularity at r=0 and AH

$$ds^2 = -F(t,r)\,dt^2 + \frac{dr^2}{g(t,r)} + r^2 d\omega^2,$$

$$F(t,r) \sim F_0(t) + F_1(t)\,r^2,\; g(t,r) \sim g_0(t) + g_1(t)\,r^2,$$

$$\Re^2 \sim \frac{4(g_0-1)^2}{r^4} = 4\left(\frac{[(\nabla r)^2 - 1]}{r^2}\right)^2.$$

Apparent horizon: g=$(\nabla r)^2 = 0$. If an AH crosses $r=0$, then before this the curvature singularity is developed at $r=0$.

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Schwarzschild metric: $\varphi = -GM / r.$

$$ds^2 = -F dt^2 + \frac{dr^2}{F} + r^2 d\omega^2, \; F = 1 + 2\varphi.$$

Apparent (event) horizon at $F = 0$, r=2GM.

Kretschmann scalar $\mathbb{R}^2 = \dfrac{48\,(GM)^2}{r^3}.$

Linearized version

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\varphi)(dr^2 + r^2 d\omega^2).$$

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Three connected problems:

1. Regularity of potential φ at $r = 0$;

2. Finiteness of the self-energy of a point charge;

3. Existence of AH: $|\varphi| \leq CM$. For $M < C/2,\ F > 0$.

Regularization :

$$\Delta\varphi = 4\pi GM\delta(\vec{r}) \to \varphi = \frac{GM}{r},$$

$$(\Delta + \mu^2)\tilde{\varphi} = 4\pi GM\delta(\vec{r}) \to \tilde{\varphi} = \frac{GMe^{-\mu r}}{r},$$

$$\varphi_{reg}(r) = \varphi(r) - \tilde{\varphi}(r) = \frac{GM(1 - e^{-\mu r})}{r},$$

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$$\Delta G = -I,\ (\Delta + \mu^2)\tilde{G} = -I,$$

$$G_{reg} = G - \tilde{G} = \frac{1}{\Delta + \mu^2} - \frac{1}{\Delta} = -\frac{1}{\Delta(1 + \Delta / \mu^2)};$$

$$\Delta(1 + \Delta / \mu^2)\varphi_{reg} = 4\pi GM\delta(\vec{r}) \rightarrow$$

Higher-derivative theory.

Source-smearing vs non-locality:

$$\Delta\varphi_{reg} = 4\pi G\tilde{\rho},\ \tilde{\rho} = (1 + \Delta / \mu^2)^{-1}\rho = Me^{-\mu r} / r.$$

Brief ("truncated") history of HD gravity :

Weyl (1921); Eddington (1924); Lanczos (1938);

Buchdal (1948); Utiyama and DeWitt (1962); Sakharov (1967);

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Stelle (1978) - Classical theory with higher derivatives;

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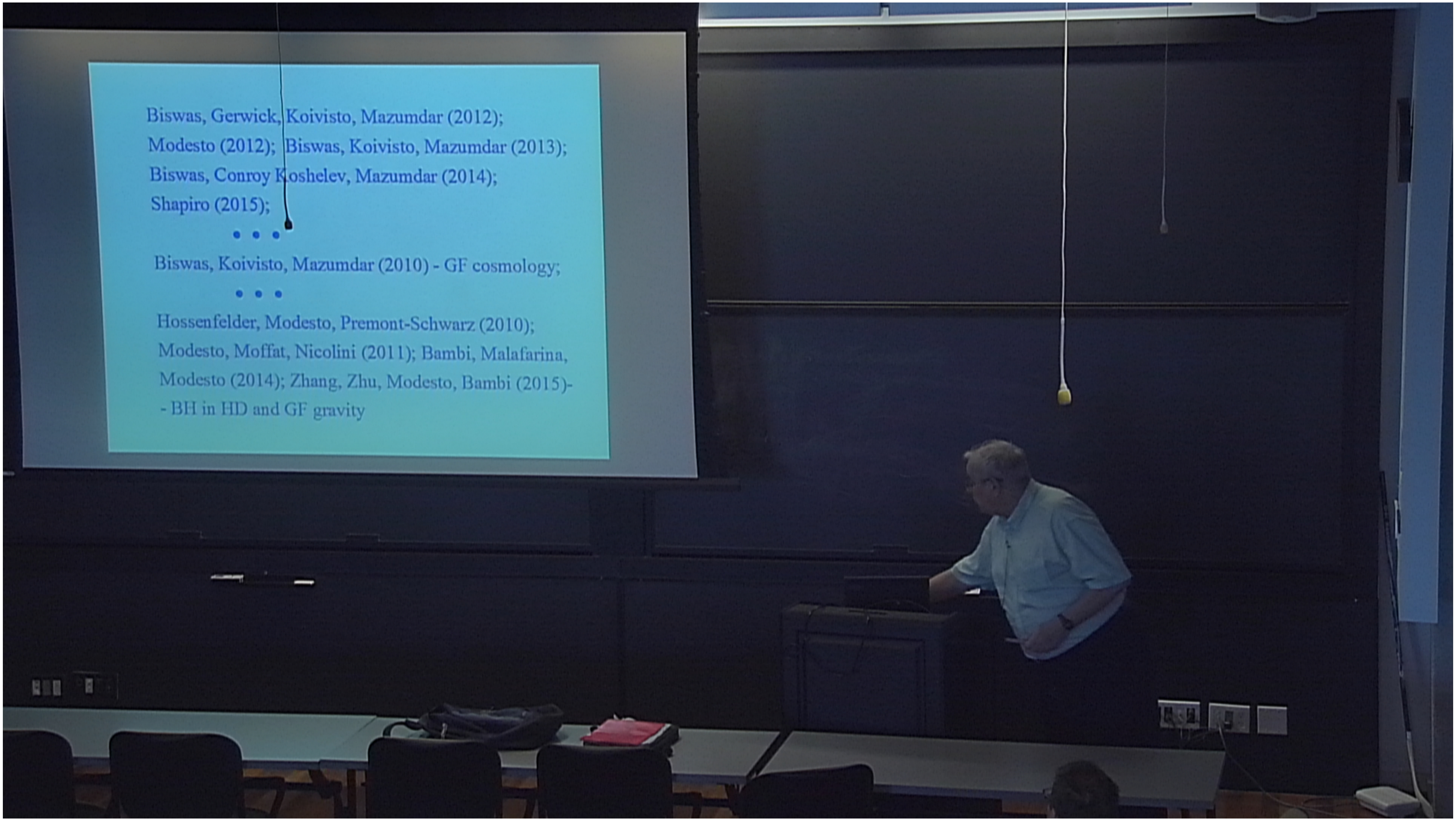
Biswas, Gerwick, Koivisto, Mazumdar (2012);
Modesto (2012); Biswas, Koivisto, Mazumdar (2013);
Biswas, Conroy Koshelev, Mazumdar (2014);
Shapiro (2015);
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Biswas, Koivisto, Mazumdar (2010) - GF cosmology;
• • •
Hossenfelder, Modesto, Premont-Schwarz (2010);
Modesto, Moffat, Nicolini (2011); Bambi, Malafarina,
Modesto (2014); Zhang, Zhu, Modesto, Bambi (2015)-
- BH in HD and GF gravity

Quadratic in Curvature Action

$$S = \int dx \sqrt{-g} \left[R/2 + R_{\bullet} \hat{O}^{\bullet\bullet} R_{\bullet} \right],$$

$\hat{O}^{\bullet\bullet}$ is an operator constructed from ∇ and g.

The number of arbitrary functions of $\Box$ operator (after using the Bianchi identities) is 6.

(For quantum gravity: Barvinsky and Vilkovisky (1990))

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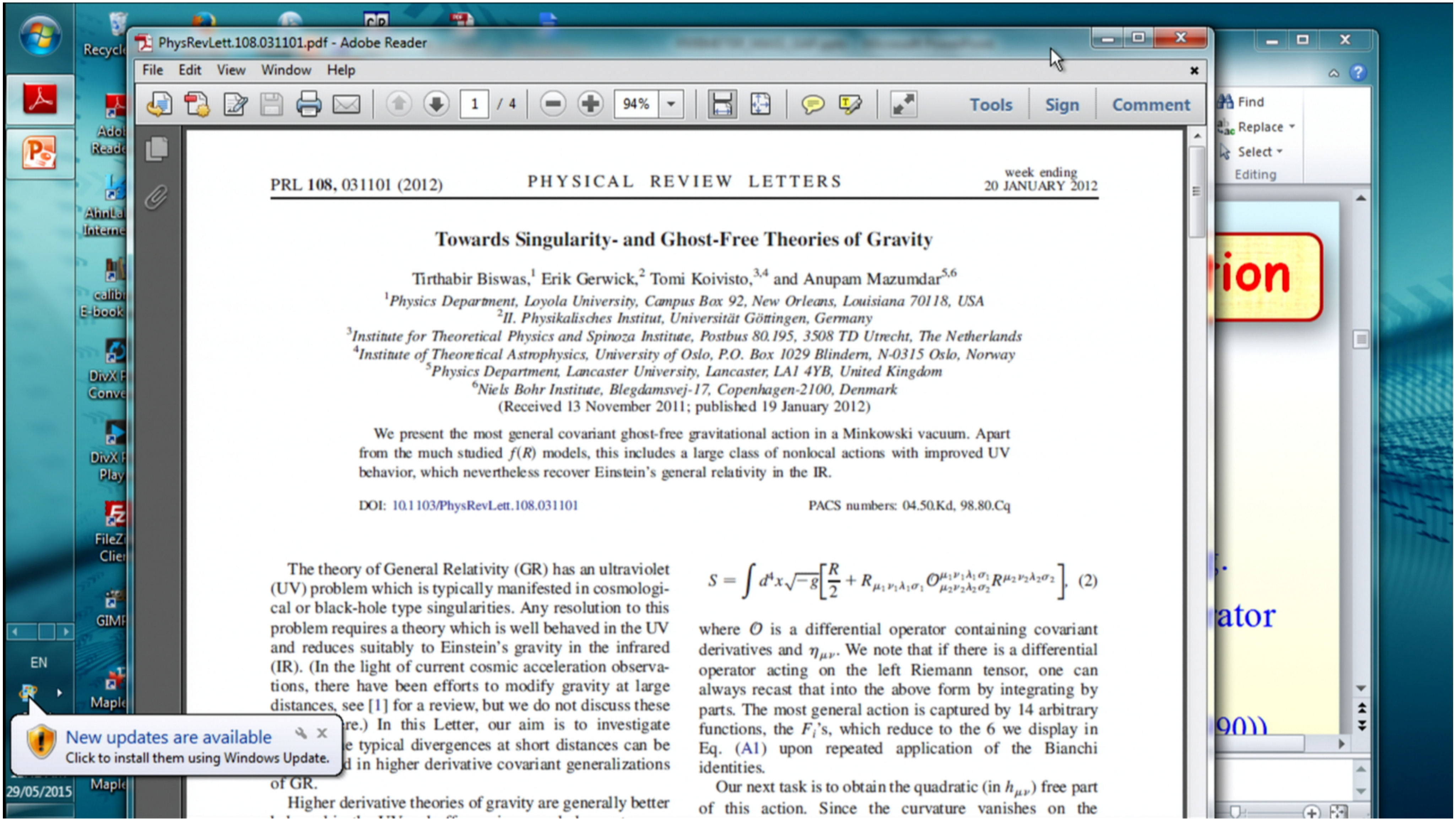


Towards Singularity- and Ghost-Free Theories of Gravity

Tirthabir Biswas,[1] Erik Gerwick,[2] Tomi Koivisto,[3,4] and Anupam Mazumdar[5,6]

[1]*Physics Department, Loyola University, Campus Box 92, New Orleans, Louisiana 70118, USA*
[2]*II. Physikalisches Institut, Universität Göttingen, Germany*
[3]*Institute for Theoretical Physics and Spinoza Institute, Postbus 80.195, 3508 TD Utrecht, The Netherlands*
[4]*Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway*
[5]*Physics Department, Lancaster University, Lancaster, LA1 4YB, United Kingdom*
[6]*Niels Bohr Institute, Blegdamsvej-17, Copenhagen-2100, Denmark*

(Received 13 November 2011; published 19 January 2012)

We present the most general covariant ghost-free gravitational action in a Minkowski vacuum. Apart from the much studied $f(R)$ models, this includes a large class of nonlocal actions with improved UV behavior, which nevertheless recover Einstein's general relativity in the IR.

DOI: 10.1103/PhysRevLett.108.031101 PACS numbers: 04.50.Kd, 98.80.Cq

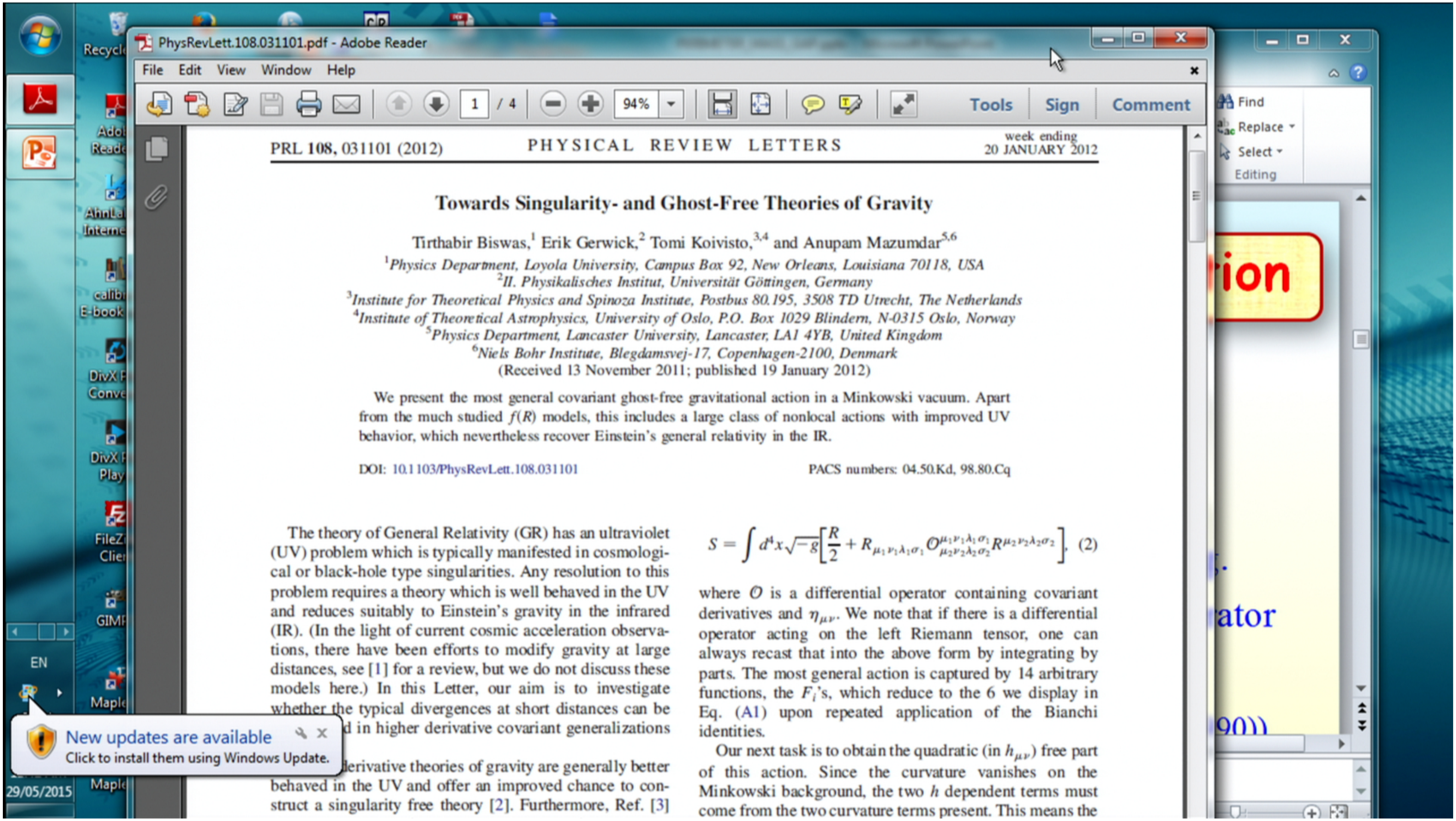
The theory of General Relativity (GR) has an ultraviolet (UV) problem which is typically manifested in cosmological or black-hole type singularities. Any resolution to this problem requires a theory which is well behaved in the UV and reduces suitably to Einstein's gravity in the infrared (IR). (In the light of current cosmic acceleration observations, there have been efforts to modify gravity at large distances, see [1] for a review, but we do not discuss these ...re.) In this Letter, our aim is to investigate ...e typical divergences at short distances can be ...d in higher derivative covariant generalizations of GR.

Higher derivative theories of gravity are generally better

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}\right], \quad (2)$$

where $\mathcal{O}$ is a differential operator containing covariant derivatives and $\eta_{\mu\nu}$. We note that if there is a differential operator acting on the left Riemann tensor, one can always recast that into the above form by integrating by parts. The most general action is captured by 14 arbitrary functions, the F_i's, which reduce to the 6 we display in Eq. (A1) upon repeated application of the Bianchi identities.

Our next task is to obtain the quadratic (in $h_{\mu\nu}$) free part of this action. Since the curvature vanishes on the



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...derivative theories of gravity are generally better behaved in the UV and offer an improved chance to construct a singularity free theory [2]. Furthermore, Ref. [3]

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R_{\mu_1\nu_1\lambda_1\sigma_1} \mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2} R^{\mu_2\nu_2\lambda_2\sigma_2}\right], \quad (2)$$

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PhysRevLett.108.031101.pdf - Adobe Reader

[15]. Finally, it is know[…] starting from the free quadratic […] consistently coupling to its own stress energy tensor. Similarly, can one obtain *unique* consistent covariant extensions of the higher derivative quadratic actions that we have considered? We leave these questions for future investigations.

We would like to thank Alex Koshelev for pointing out some redundancies in the gravitational action. T. K. is supported by the Research Council of Norway, and A. M. is supported by STFC Grant No. ST/J000418/1. T. B.'s research was supported by the LABoR R&D grant.

Appendix.—The quadratic action in curvature reads

$$S_q = \int d^4x\sqrt{-g}[RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}F_3(\Box)R^{\mu\nu\lambda\sigma} + RF_4(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_\mu^{\nu_1\rho_1\sigma_1}F_5(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_6(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}], \tag{A1}$$

where we have used the Bianchi identities:

$$\nabla_\sigma R_{\mu\nu\lambda\rho} + \nabla_\rho R_{\mu\nu\sigma\lambda} + \nabla_\lambda R_{\mu\nu\rho\sigma} = 0, \tag{A2}$$

to absorb all the other covariant terms into the above six. Further, in the F_4, F_5, and F_6 terms, one ends up with anticommutator of the covariant derivatives due to the antisymmetric properties of the Reimann tensor, but these anticommutators produce a third curvature term, and therefore these terms are at least $\mathcal{O}(h^3)$. Thus, the coefficients of the free theory (3) in terms of the F's are given by

$$a(\Box) = 1 - \frac{1}{2}F_2(\Box)\Box - 2F_3(\Box)\Box, \tag{A3}$$

$$f(\Box) = -2F_1(\Box)\Box - F_2(\Box)\Box - 2F_3(\Box)\Box. \tag{A7}$$

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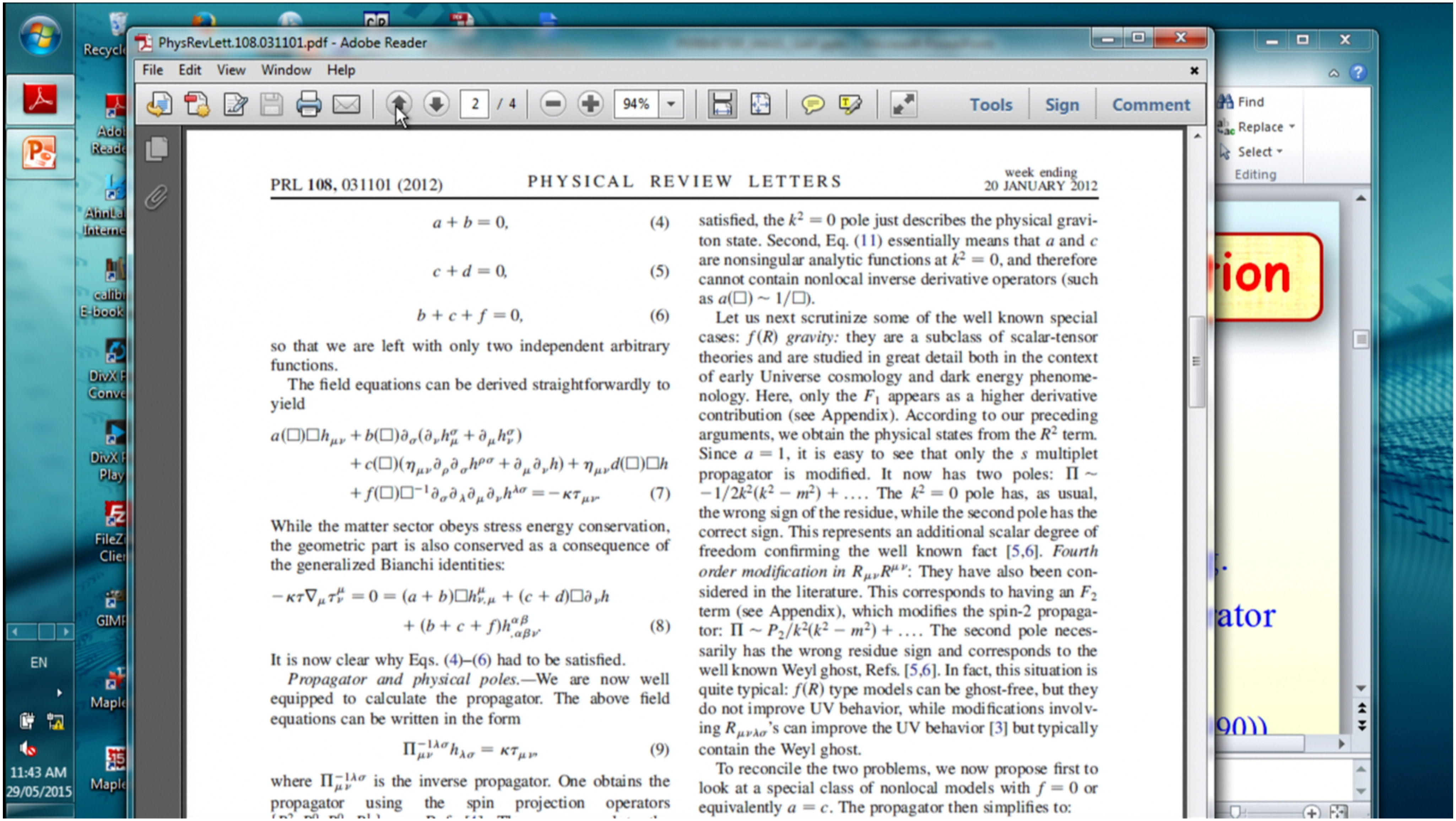


$$a + b = 0, \qquad (4)$$

$$c + d = 0, \qquad (5)$$

$$b + c + f = 0, \qquad (6)$$

so that we are left with only two independent arbitrary functions.

The field equations can be derived straightforwardly to yield

$$a(\Box)\Box h_{\mu\nu} + b(\Box)\partial_\sigma(\partial_\nu h^\sigma_\mu + \partial_\mu h^\sigma_\nu) + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) + \eta_{\mu\nu} d(\Box)\Box h + f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}. \qquad (7)$$

While the matter sector obeys stress energy conservation, the geometric part is also conserved as a consequence of the generalized Bianchi identities:

$$-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (a+b)\Box h^\mu_{\nu,\mu} + (c+d)\Box\partial_\nu h + (b+c+f)h^{\alpha\beta}_{,\alpha\beta\nu}. \qquad (8)$$

It is now clear why Eqs. (4)–(6) had to be satisfied.

Propagator and physical poles.—We are now well equipped to calculate the propagator. The above field equations can be written in the form

$$\Pi^{-1\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu}, \qquad (9)$$

where $\Pi^{-1\lambda\sigma}_{\mu\nu}$ is the inverse propagator. One obtains the propagator using the spin projection operators

satisfied, the $k^2 = 0$ pole just describes the physical graviton state. Second, Eq. (11) essentially means that a and c are nonsingular analytic functions at $k^2 = 0$, and therefore cannot contain nonlocal inverse derivative operators (such as $a(\Box) \sim 1/\Box$).

Let us next scrutinize some of the well known special cases: $f(R)$ *gravity:* they are a subclass of scalar-tensor theories and are studied in great detail both in the context of early Universe cosmology and dark energy phenomenology. Here, only the F_1 appears as a higher derivative contribution (see Appendix). According to our preceding arguments, we obtain the physical states from the R^2 term. Since $a = 1$, it is easy to see that only the s multiplet propagator is modified. It now has two poles: $\Pi \sim -1/2k^2(k^2 - m^2) + \ldots$. The $k^2 = 0$ pole has, as usual, the wrong sign of the residue, while the second pole has the correct sign. This represents an additional scalar degree of freedom confirming the well known fact [5,6]. *Fourth order modification in* $R_{\mu\nu}R^{\mu\nu}$: They have also been considered in the literature. This corresponds to having an F_2 term (see Appendix), which modifies the spin-2 propagator: $\Pi \sim P_2/k^2(k^2 - m^2) + \ldots$. The second pole necessarily has the wrong residue sign and corresponds to the well known Weyl ghost, Refs. [5,6]. In fact, this situation is quite typical: $f(R)$ type models can be ghost-free, but they do not improve UV behavior, while modifications involving $R_{\mu\nu\lambda\sigma}$'s can improve the UV behavior [3] but typically contain the Weyl ghost.

To reconcile the two problems, we now propose first to look at a special class of nonlocal models with $f = 0$ or equivalently $a = c$. The propagator then simplifies to:

1. General Relativity (GR): $L = R, a = c = 1;$

2. Gauss-Bonnet (GB) gravity: $L = R + \alpha(\Box)G;$

$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta};\ a = c = 1;$

3. $L(R)$ gravity: $L(R) = L(0) + L'(0)R + 1/2L''(0)R^2 + \dots\ ;$

$a = 1,\ c = 1 - L''(0);$

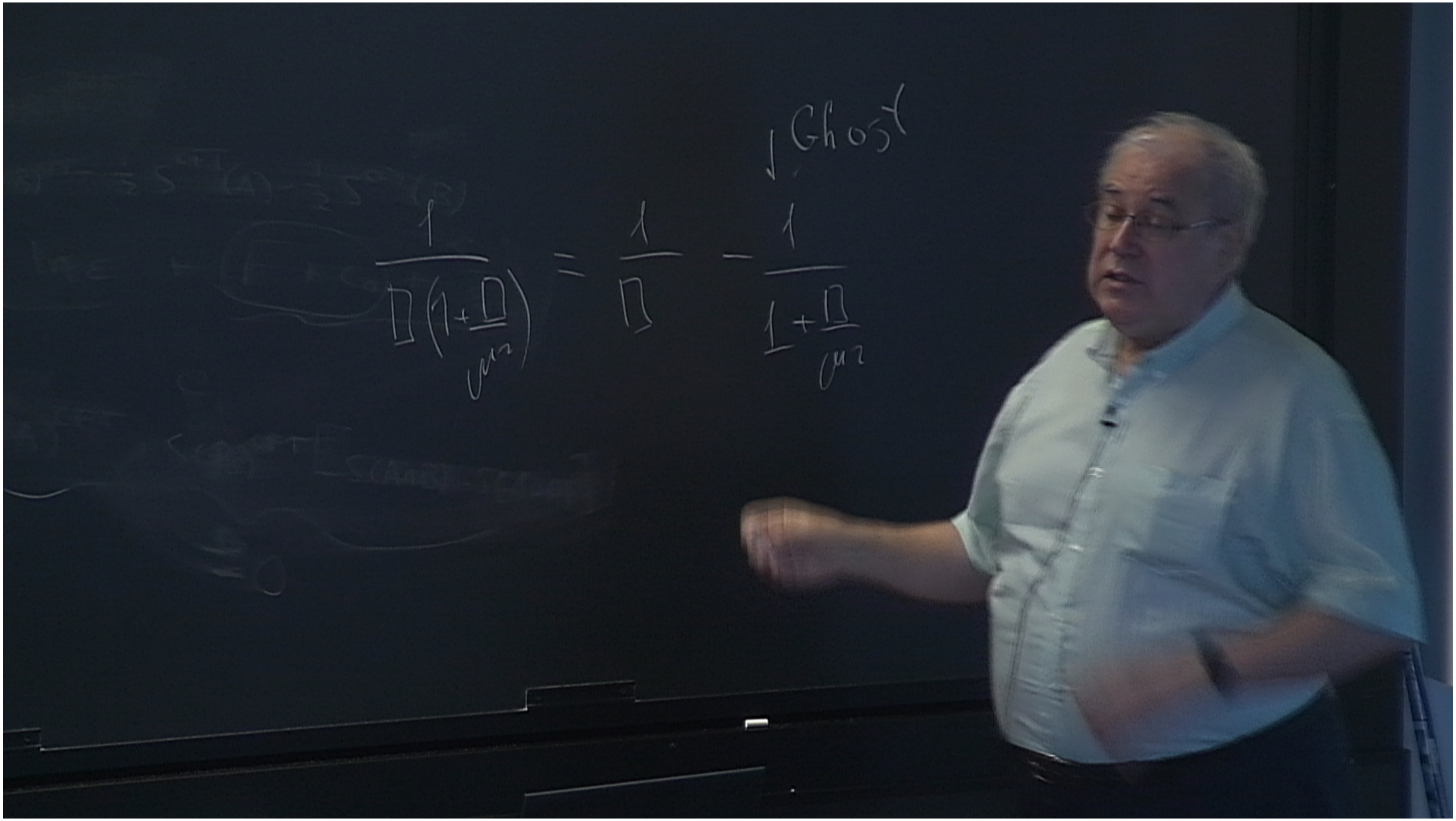
4. Weyl gravity: $L = R - \mu^{-2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta},\ \ a = 1 - \mu^{-2}\Box,$

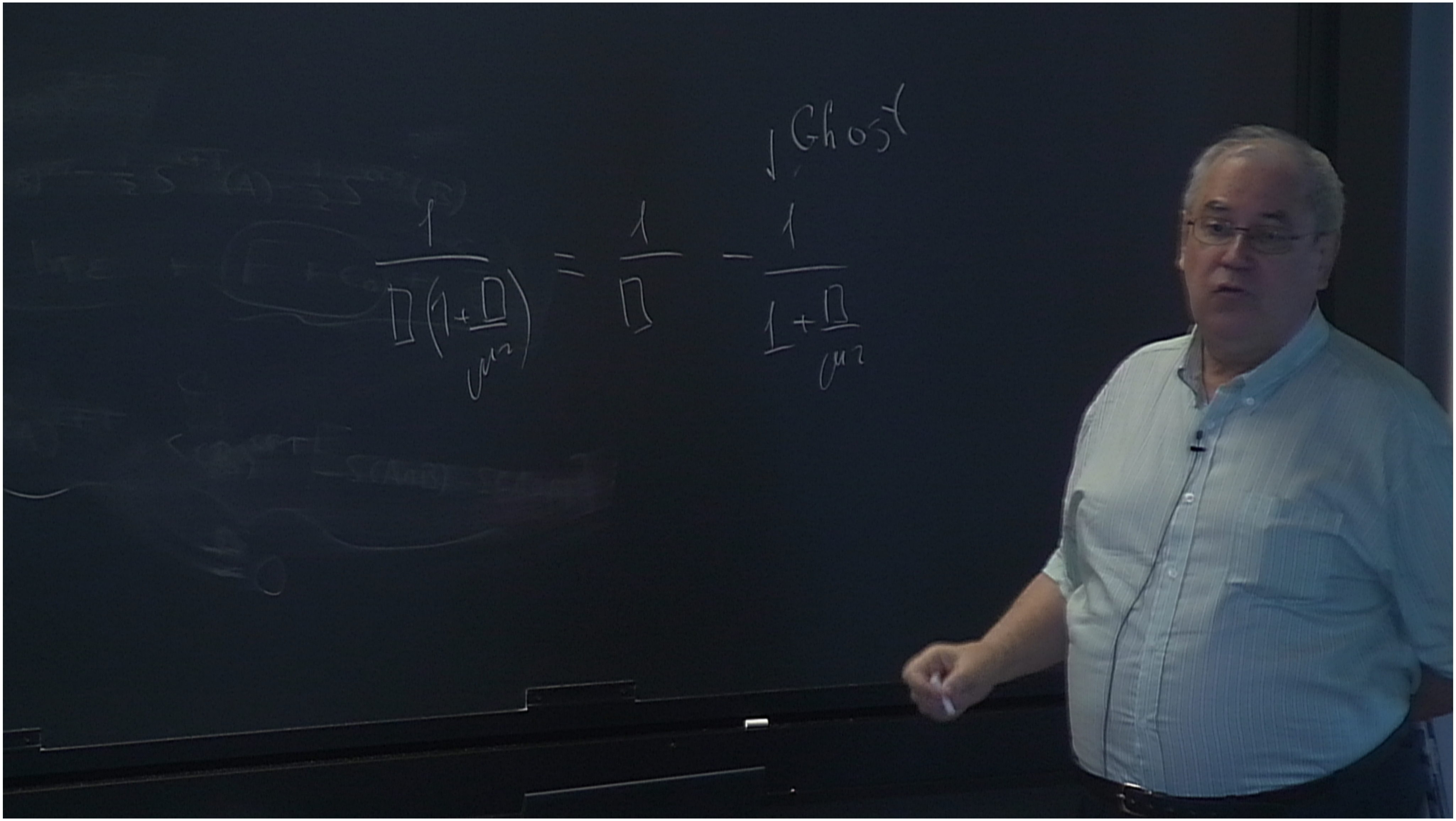
$c = 1 - \frac{1}{3}\mu^{-2}\Box\ ;$

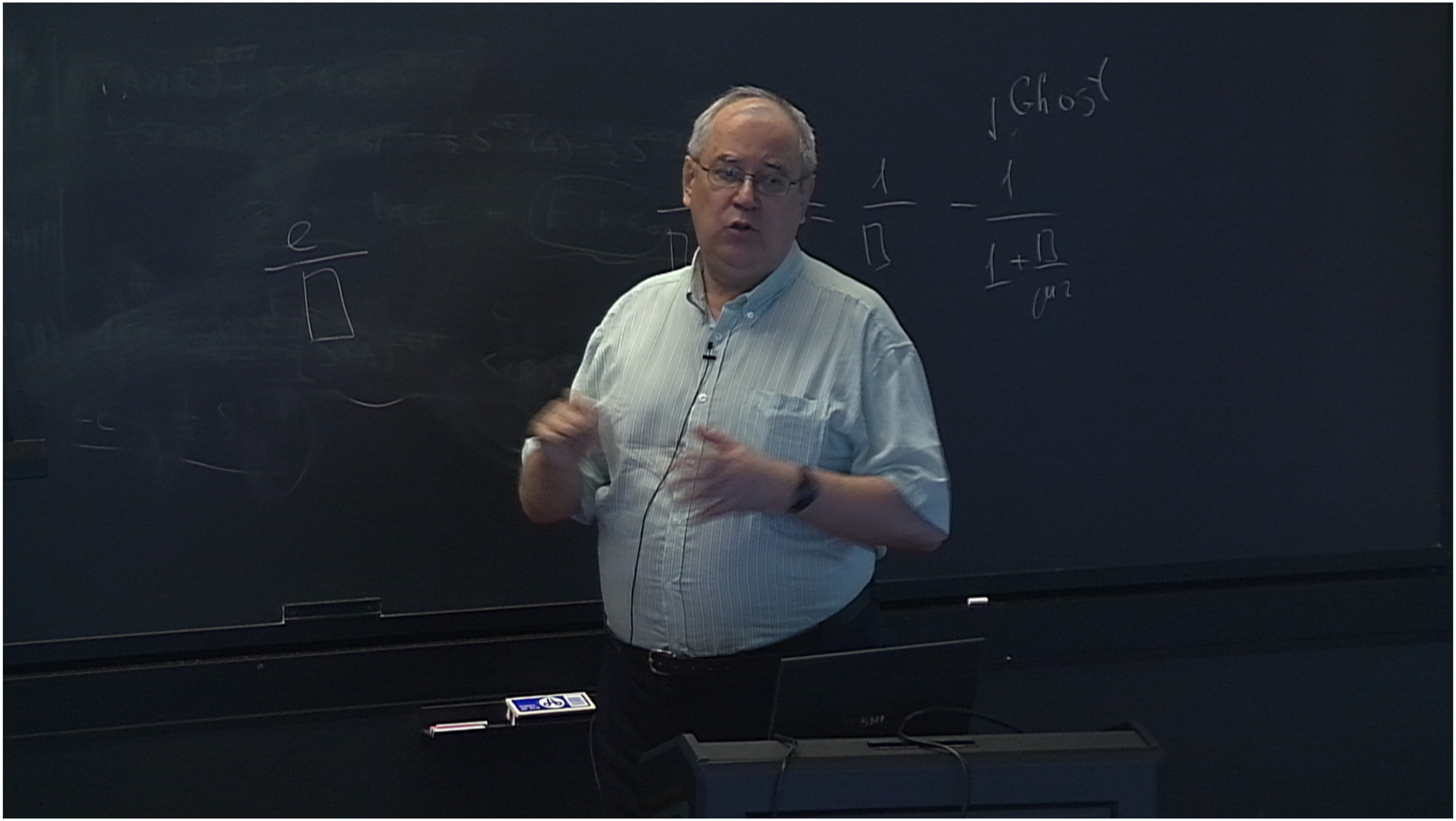
5. Higher derivative (HD) gravity: $a = \prod_{i=1}^{n}(1 - \mu_i^{-2}\Box)\ ,$

$c = \prod_{k=1}^{n_c}(1 - \nu_k^{-2}\Box).$

6. Ghost free (GF) gravity: $a = c = \exp(-\Box/\mu^2).$


Ghost
$\frac{1}{\Box(1+\frac{\Box}{m^2})} = \frac{1}{\Box} - \frac{1}{1+\frac{\Box}{m^2}}$


Ghost
1/(□(1+□/m²)) = 1/□ − 1/(1+□/m²)


Ghost

1. General Relativity (GR): $L = R, a = c = 1;$

2. Gauss-Bonnet (GB) gravity: $L = R + \alpha(\Box)G;$

$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta};\ a = c = 1;$

3. $L(R)$ gravity: $L(R) = L(0) + L'(0)R + 1/2L''(0)R^2 + \ldots\ ;$

$a = 1,\ c = 1 - L''(0);$

4. Weyl gravity: $L = R - \mu^{-2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta},\ \ a = 1 - \mu^{-2}\Box,$

$c = 1 - \frac{1}{3}\mu^{-2}\Box\ ;$

5. Higher derivative (HD) gravity: $a = \prod_{i=1}^{n}(1 - \mu_i^{-2}\Box)\ ,$

$c = \prod_{k=1}^{n_c}(1 - \nu_k^{-2}\Box).$

6. Ghost free (GF) gravity: $a = c = \exp(-\Box/\mu^2).$

Static solutions of linearized gravity equations in the Newtonian limit

Stress-energy tensor: $\tau_{\mu\nu} = \rho(\vec{r})\delta_\mu^0\delta_\nu^0$

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\psi+2\varphi)d\ell^2 .$$

Biswas, Gerwick, Koivisto, Mazumdar (2012):

$$a(\Delta)\Delta\psi = 8\pi G\rho,$$

$$(a(\Delta)-3c(\Delta))(\Delta\varphi - 2\Delta\psi) = 8\pi G\rho$$

Stelle (1978); Modesto, Netto, Shapiro (2014)

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For a point mass $\rho = m\delta(\vec{r})$ the solution is spherically symmetric. We call it finite if near $r = 0$ it is of the form

$$\psi(r) \sim \psi_0 + \psi_1 r + \frac{1}{2}\psi_2 r^2 + O(r^3),$$

$$\varphi(r) \sim \varphi_0 + \varphi_1 r + \frac{1}{2}\varphi_2 r^2 + O(r^3).$$

A finite solution is not necessary regular one.

$$R^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{A_2}{r^2} + \frac{A_1}{r} + O(1),$$

$$A_2 = 8(4\psi_1^2 - 5\psi_1\varphi_1 + 3\varphi_1^2),$$

$$A_1 = 16[\psi_1(5\psi_2 - 4\varphi_2) - 4\varphi_1(\psi_2 - \varphi_2)].$$

The solution is regular if $\psi_1 = \varphi_1 = 0$.

The solution is ψ-regular if $\psi_1 = 0$.

If $a = c$, ψ=2φ, and ψ-regular is regular

$$\hat{O} = a(\Delta)\Delta,$$

$$Q(\xi) = \hat{O}^{-1}(\Delta = -\xi) = -[\xi a(-\xi)]^{-1},$$

$$Q(\xi) = \int_0^\infty ds\, f(s)\, e^{-s\xi},$$

$$f(s) = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} d\xi\, Q(\xi)\, e^{s\xi}$$

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Green function: $\hat{O}\,\hat{G} = -\hat{I}\,.$

$$\hat{G} = -\hat{O}^{-1} = -\int_0^\infty ds\, f(s)\, e^{s\Delta}\,,$$

Heat kernel:

$$< x' \,|\, e^{s\Delta} \,|\, x >= K(|\, x - x' \,|; s) = \frac{e^{-|x-x'|^2/(4s)}}{(4\pi s)^{3/2}}$$

$$\psi(r) = 8\pi G m \int_0^\infty ds\, f(s)\, K(r; s),$$

$$= \frac{Gm}{\pi i r} \int_{\alpha - i\infty}^{\alpha + i\infty} d\xi\, Q(\xi)\, e^{-\sqrt{-\xi}\, r}$$

HD gravity: $Q(\xi) = -[\xi \prod_{i=1}^{n} (1 + \xi/\mu_i^2)]^{-1}$,

The Heaviside expansion theorem:

$$f(s) = -(1 - \sum_{i=1}^{n} P_i^{-1} e^{-\mu_i t}),\ \ P_i = \prod\nolimits_{j=1, j\neq i}^{n} (1 - \mu_j^2/\mu_i^2).$$

$$\psi(r) = -2Gmr^{-1}(1 - \sum_{i=1}^{n} P_i^{-1} e^{-\mu_i r}).$$

General Relativity: $f(s) = 1,\ \ \psi(r) = 2\varphi(r) = -2Gm/r$.

Solution near $r=0$: $\sum_{i=1}^{n} P_i^{-1} = 1.$

$\psi_0 = -2GmS_1, \; \psi_1 = GmS_2, \; S_k = \sum_{i=1}^{n} \mu_i^k P_i^{-1}.$

The solution is ψ-regular if $S_2 = 0$.

For the GF gravity $f(s) = -\vartheta(s - \mu^{-2})$,

$\psi(r) = 2\varphi(r) = -2Gm \; \mathrm{erf}(\mu r/2)/r.$

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4. HD and GF Gyratons

The solution is ψ-regular if $S_2 = 0$.

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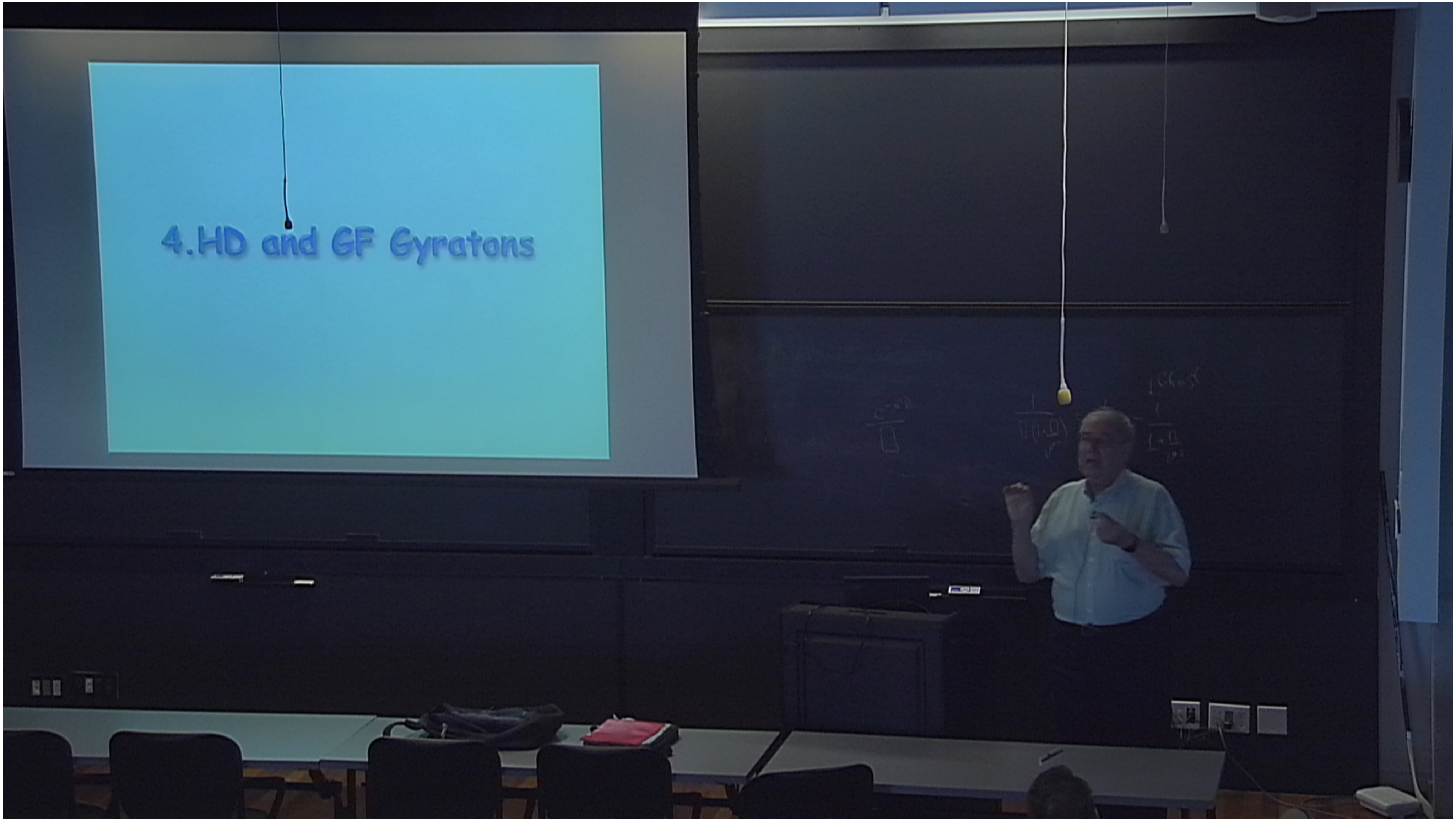
4. HD and GF Gyratons


4.HD and GF Gyratons

In the limit $\gamma \to \infty$ one has

$$y \sim -\gamma u,\ t \sim \gamma u,\ \ell^2 \sim \gamma^2 u^2 + \zeta_\perp^2,$$

$$ds^2 = -du dv + d\zeta_\perp^2 + dh^2,$$

$$dh^2 = \Phi du^2, \quad \Phi = -2 \lim_{\gamma\to\infty} (\gamma^2 \psi).$$

Penrose limit: $M = \gamma m = \text{const};$

$$\lim_{\gamma\to\infty} \gamma \exp(-\gamma^2 u^2/(4s)) = \sqrt{4\pi s}\,\delta(u).$$

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$$\psi(r) = 8\pi G m \int_0^\infty ds\, f(s)\, K(r; s),$$

$$= \frac{Gm}{\pi i r}\int_{\alpha - i\infty}^{\alpha + i\infty} d\xi\, Q(\xi)\, e^{-\sqrt{-\xi}\, r}$$

$$\Phi = -4GMF(\zeta_{\perp}^{2})\delta(u),$$

$$F(z) = \int_0^\infty \frac{ds}{s} f(s) e^{-z/(4s)}.$$

For GR, as well as for GB and $L(R)$ gravity:

$F(z) = \ln(z/\eta^2)$, η is IR cut-off parameter.

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For HD gravity:

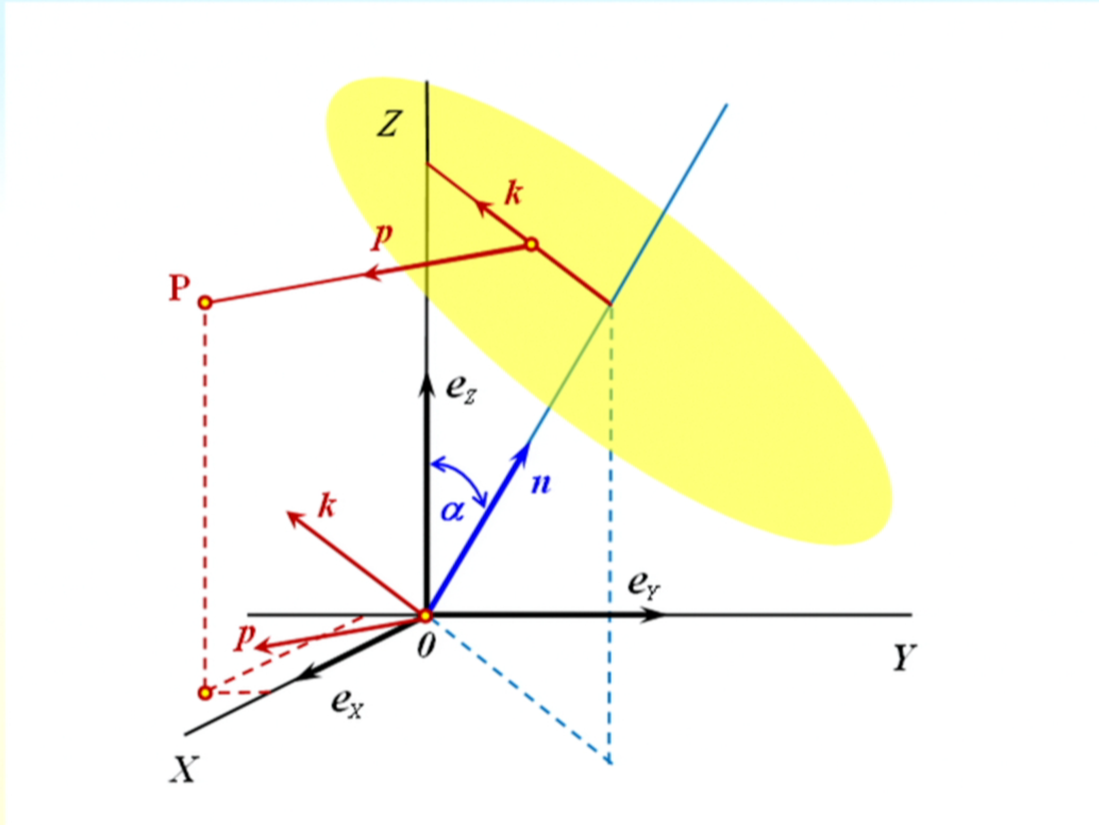
$$F(z) = \ln(z/\eta^2) + 2\sum_{i=1}^{n} P_i^{-1} K_0(\mu_i \sqrt{z}),$$

$$F(z) \sim C - \frac{1}{4} S_2 z(\ln z - 2c) - \frac{1}{4} Sz + O(z^2),$$

$$c = 1 + \ln 2 - \gamma,\ S = \sum\nolimits_{i=1}^{n} \mu_i^2 \ln(\mu_i^2) P_i^{-1}.$$

For ψ-regular theory $S_2 = 0$.

5. Null Shell Collapse in HD and GH Gravity



Gyraton frame vs Minkowski frame

The result of averaging:

$$< dh^2 > = \frac{-2MF(r^2 - t^2)}{r}\left[\left(dt - \frac{t}{r}dr\right)^2 + \frac{r^2 - t^2}{2}d\omega^2\right].$$

$F = 0$ inside the null cones.

For $F =$ const the metric $ds^2 = ds_0^2 + < dh^2 >$ is flat.

$< dh^2 >$ can be "gauged away" .

For $F(z) = \ln z$, $ds^2 = ds_0^2 + \dfrac{2M}{r}(dt^2 + dr^2)$

Apparent Horizon

$$g = (\nabla\rho)^2, \rho^2 \equiv g_{\theta\theta} = r^2 - \frac{GM}{r} zF(z),$$

$$g = 1 - 2GMr^{-1}q(z), \quad q(z) = zF'(z).$$

For GR (as well as for GB and $L(R)$-gravity) $q(z) = 1$, so that an apparent horizon exists for any value of M.

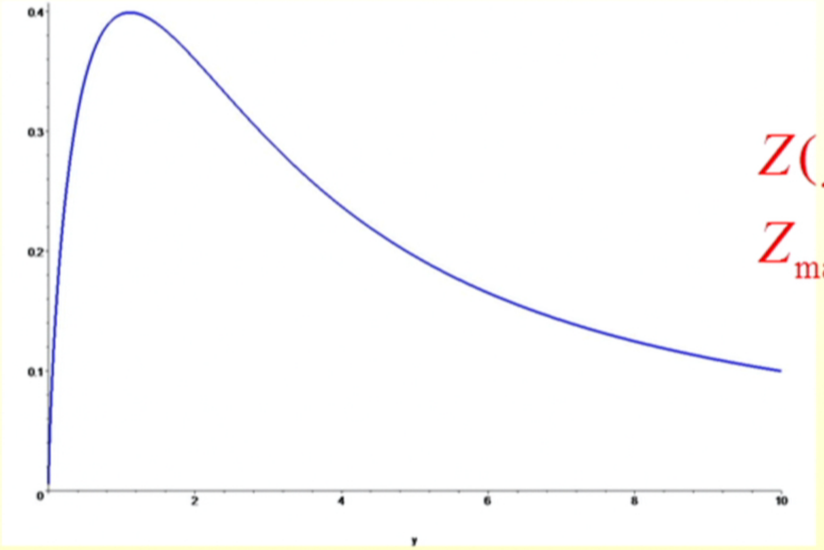
For HD gravity $q(z) = 1 - \sqrt{z}\sum_{i=1}^{n} \mu_i P_i^{-1} K_1(\mu_i \sqrt{z})$,

$$q(z) = -\frac{1}{4} S_2 z(\ln z - 2c + 1) - \frac{1}{4} Sz + O(z^2)$$

Outside the null shell $|t|/r \le 1, t = \pm\sqrt{1-\beta^2}r,$

$$q(z)/r = \beta\sum_{i=1}^{n}\mu_i P_i^{-1}Z(y_i), \quad y_i = \beta\mu_i r,$$

$$Z(y) = \frac{1}{y} - K_1(y).$$



$Z(y)$ is positive,

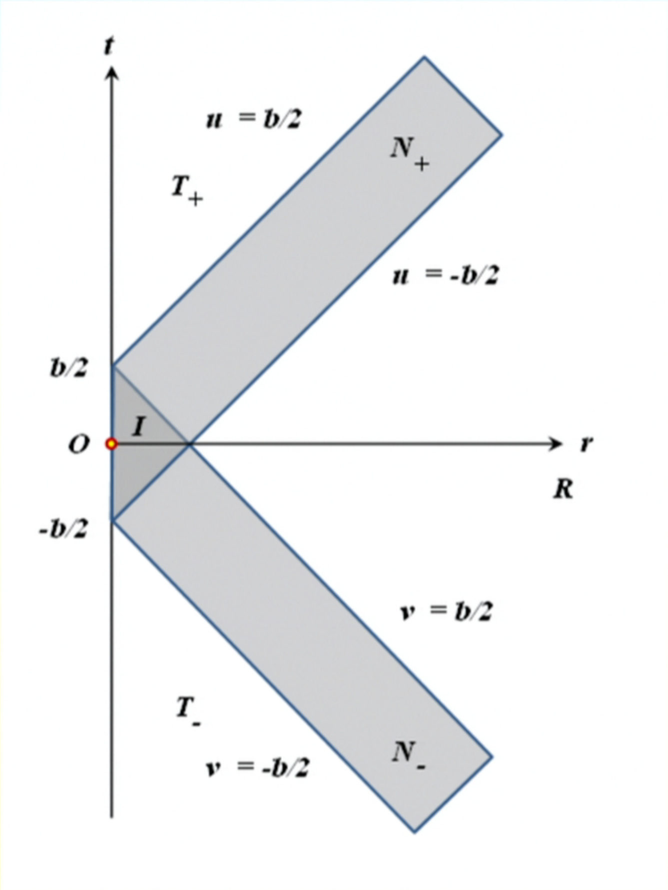
$Z_{\max} = 0.399$ at $y = 1.114$

$$|q(z)|/r < 0.4\sum_{i=1}^{n} \mu_i |P_i|^{-1}$$

If $GM\mu < 1$ there is no apparent horizon.

The same conclusion for the GF gravity.

Metric for Thick Null Shell


t
u = b/2
N+
T+
u = -b/2
b/2
I
O
r
R
-b/2
v = b/2
T-
N-
v = -b/2

Additional averaging results in:

$$<< dh^2 >> = -\frac{2GM}{br}[c_0 dt^2 + c_2 \frac{dr^2}{r^2} + \frac{1}{2}(c_0 r^2 - c_2) d\omega^2],$$

$$c_k = \int_{-r}^{r} dx x^k F(r^2 - x^2),$$

$$c_0 = -\frac{r^3}{9}[(6u-5)S_2 + 3S],\ u = \ln r - c - \ln 2,$$

$$c_2 = -\frac{r^5}{225}[(30u-31)S_2 + 15S].$$

$$R^2 \sim \frac{32}{27} G^2 \dot{M}^2 [(36u^2+5)S_2^2 + 36uS_2 S + 9S^2].$$

For HD gravity R^2 is finite for a ψ-regular theory, that is when $S_2 = 0$.

For ghost free gravity: $R^2 \sim \frac{32}{3} G^2 \dot{M}^2 \mu^4$.

In both cases for small G$M\mu$ the linear perturbarion is uniformly small and higher order corrections can be neglected. This means that a no-apparent-horizon result is robust.

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Remarks on GF gravity:

1. Δ is non-positive definite operator, while $\Box$ does not have this property. As a result, GF theory with $a = \exp(-\mu^{-2}\Box)$ may have growing in time solutions.

2. Heat kernel $K = e^{-s\Box}$ obeys the equation $(\partial_s + \Box)K = 0$, and the initial condition $K(s=0) = I$. If we identify K with $K = (4\pi s)^{-2}\exp[-\frac{(\vec{x}-\vec{x}')^2}{4s} + \frac{(t-t')^2}{4s}]$, we violate the initial condition.

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Summary :

We study solutions of linearized HD and GF theories of gravity;

For a wide class of HD gravity we obtain conditions of regularity of a gravitational potential for a pont mass;

We obtain gravitational field of an ultra-relativistic particle for HD and GF gravity;

A grav. field for a null shell in HD and GF gravity is found;

Main result: For a small mass M collapse in the HD and GF gravity an apparent horizon is not formed. Map-gap for mini BH formation.

Curvature remains finite for the thick null shell collapse, provided a potential for a point mass is (ψ-) regular.

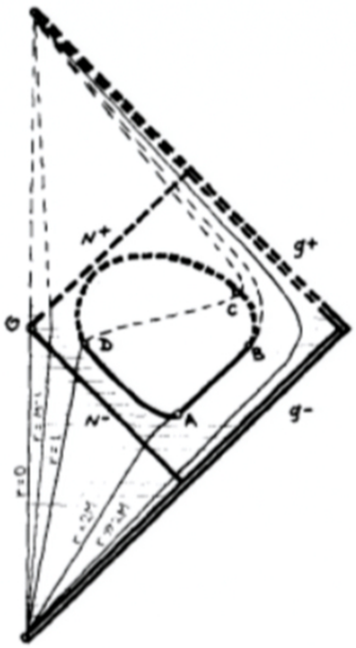

Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

V.F. and G.Vilkovisky, Phys.Lett.B106 (1981)

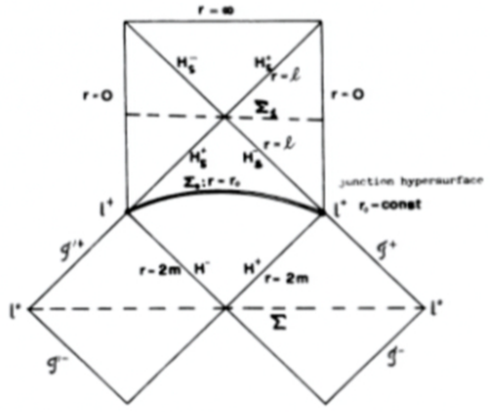

FIG. 3. Conformal diagram for an eternal black hole with the de Sitter-type world in its interior. The surface Σ_0 where $r = r_0 =$ const is a junction surface which represents a short transition layer. After passing this surface the anisotropic Kasner-type contraction of space inside the black hole is changed into the isotropic de Sitter contraction (deflation). The surface Σ_1 corresponds to the moment of "minimal size" of the de Sitter world. After passing this surface the de Sitter world begins its inflationary expansion.

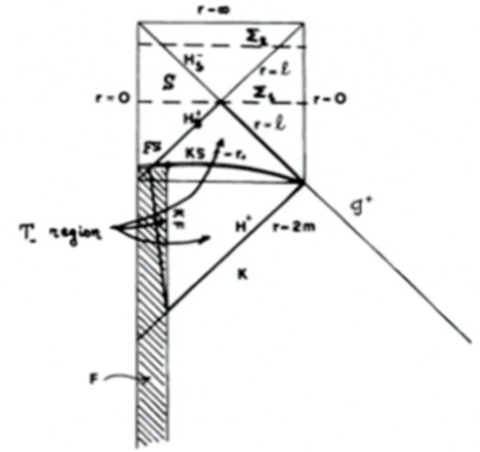

FIG. 6. Conformal diagram for the spacetime of a black hole formed by a collapsing spherically symmetric dust cloud. The domains corresponding to the Schwarzschild-Kruskal, Friedmann, and de Sitter solutions are denoted by *K*, *F*, and *S*, respectively. The boundaries *FK*, *FS*, and *KS* which separate these domains are also shown.

"Black Holes As Possible Sources Of Closed and Semiclosed Worlds" ,V. F., Markov, Mukhanov, IC/88/91. May 1988. **Phys.Rev. D41 (1990) 383;**