Title: Metal-Insulator Transitions by Holography

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Abstract: <p>Recent progress on the construction of holographic lattices and its applications to AdS/CMT correspondence will be briefly reviewed. Our special interests will focus on the building of bulk geometry of gravity whose holographic duals exhibit metal-insulator transitions (MIT). In particular, the Peierls phase transition induced by charge density waves is implemented in a holographic manner. The holographic entanglement entropy close to quantum critical points will be discussed as well.</p>

Metal-Insulator Transition by Holography

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04/27/2015, Perimeter Institute for Theoretical Physics

References:

- Holographic Entanglement Entropy Close to Quantum Phase Transitions
  Y. Ling, P. Liu, C. Niu, J. Wu, Z. Xian, **arXiv:1502.03661.**

- Holographic fermionic system with dipole coupling on Q-lattice.
  Y. Ling, P. Liu, C. Niu, J. Wu, Z. Xian, JHEP 1412 (2014) 149 , **arXiv:1410.7323**

- Holographic Superconductor on Q-lattice.
  Y. Ling, P. Liu, C. Niu, J. Wu, Z. Xian, JHEP 1502 (2015) 059 , **arXiv:1410.6761**

- Metal-insulator Transition by Holographic Charge Density Waves.
  Y. Ling, C. Niu J. Wu, Z. Xian, H. Zhang, **Phys.Rev.Lett.** 113 (2014) 091602.

Outlines

I. Linking holographic gravity to condensed matter theory

- Preliminary: building holographic lattices in gravity

II. Metal-insulator transition by holography

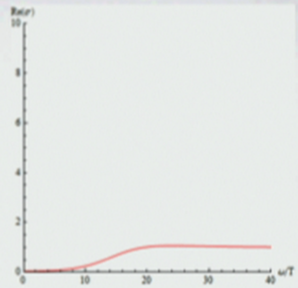
- Scalar lattice and Peierls phase transition (CDW)
- Q-lattice and novel MIT
- HEE close to quantum phase transitions
- Towards a holographic Mott-like insulator

Part I

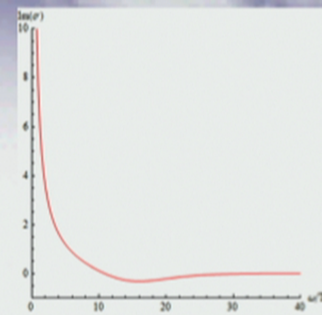
Linking holographic gravity to condensed matter theory

Building a lattice background in holographic gravity

- One issue related to homogeneous gravitational background:

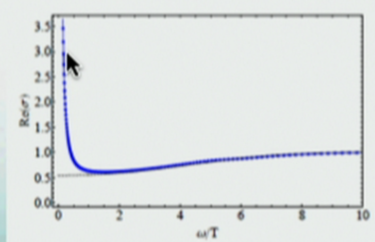


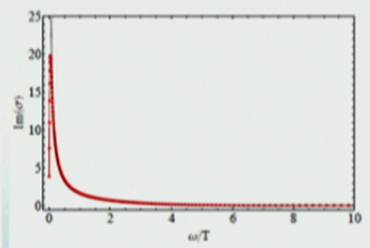
Kramers-Kronig relation ⟷



Lattices lead to the breaking of translational symmetry

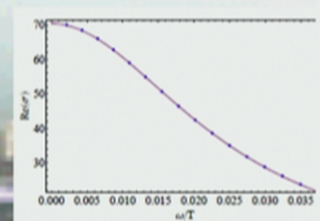
G.Horowitz, J.Santos and D.Tong, JHEP 1207 (2012) 168.

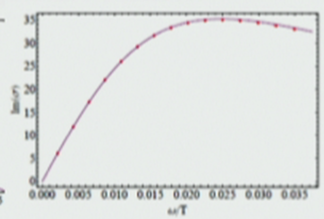




- Drude law for metals

$$\sigma = \frac{\sigma_{DC}}{1 - i\omega\tau}$$





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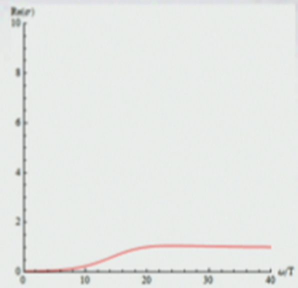

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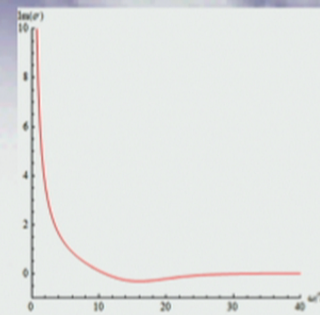

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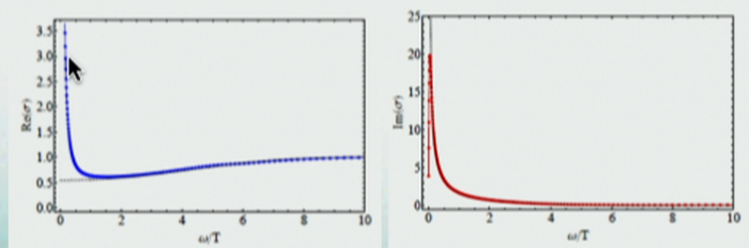

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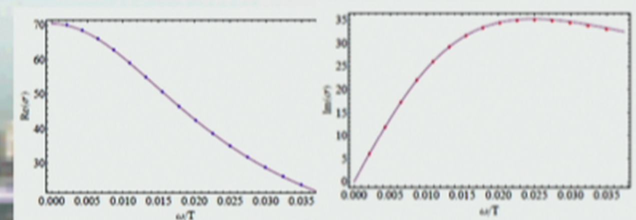

Building a lattice background in holographic gravity

- **Various holographic lattice models:**

1、 Scalar lattice: Simulating lattices with periodic scalar field with potential

$z = \frac{1}{r} \to 0$ Infinity $\quad \Phi \to z\phi_1 + z^2\phi_2 + \dots$

$$dS^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

$$\phi_1(x) = A_0 \cos(k_0 x)$$

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2、 Ionic lattice: Directly introducing a periodic chemical potential

$$\mu(x) = \bar{\mu}[1 + A_0 \cos(k_0 x)] \qquad A_t = \mu - z\rho + \dots$$

3、 Q-lattice: Complex scalar field

$$\Phi = e^{ikx} z^{3-\Delta} \phi(z)$$

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Building a lattice background in holographic gravity

- 4D Framework:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2}F_{ab}F^{ab} - 2\nabla_a\Phi\nabla^a\Phi - 4V(\Phi)\right]$$

Equations of motion:

$$G_{ab} = R_{ab} + \frac{3}{L^2}g_{ab} - .... = 0$$

$$\nabla_a F^a{}_b = 0$$

$$\Box\Phi - V'(\Phi) = 0$$

Building a lattice background in holographic gravity

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Building a lattice background in holographic gravity

- 4D Setup :

$$z \to 0, \quad dS_\partial^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dy^2\right) \qquad \Phi \to z\phi_1 + z^2\phi_2 + \ldots$$

$$\phi_1(x) = A_0 \cos(k_0 x)$$

Ansatz of variables

$$ds^2 = \frac{L^2}{z^2}[-(1-z)P(z)Q_{tt}(x,z)dt^2 + \frac{Q_{zz}(x,z)}{P(z)(1-z)}dz^2 + Q_{xx}(x,z)[dx + z^2 Q_{xz}(x,z)dz]^2 + Q_{yy}(x,z)dy^2]$$

$$A = (1-z)\psi(x,z)dt$$

$$\Phi = z\phi(x,z) \qquad P(z) = 1 + z + z^2 - \frac{\mu_1^2 z^3}{2}$$

RN black holes:

$$Q_{tt}(x,z) = Q_{zz}(x,z) = Q_{xx}(x,z) = Q_{yy}(x,z) = 1$$

$$Q_{xz}(x,z) = \phi(x,z) = 0, \quad \psi(x,z) = \mu = \mu_1$$

Temperature:

$$T = \frac{P(1)}{4\pi L} = \frac{6-\mu_1^2}{8\pi}$$

No change!

?

?: $Q_{tt}(x,1) = Q_{zz}(x,1)$

Building a lattice background in holographic gravity

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Building a lattice background in holographic gravity

- **Crucial technical issues in AdS/CMT with lattices:**

  1、Numerically solve the background equations with appropriate boundary and gauge conditions;

  2、Numerically solve the perturbation equations over the background.

- **DeTurck method:**

  1、Einstein-DeTurck equation $$G^H_{ab} \equiv G_{ab} - \nabla_{(a}\xi_{b)} = 0$$

  $$\xi^a := g^{cd}[\Gamma^a_{cd}(g) - \bar{\Gamma}^a_{cd}(\bar{g})]$$

  $\bar{g}$: a reference metric with the same asmptotics and horizon structures

  Here a reference metric is the RN black hole:

  $$Q_{tt}(x,z) = Q_{zz}(x,z) = Q_{xx}(x,z) = Q_{yy}(x,z) = 1, Q_{xz}(x,z) = 0$$

  2、To guarantee the numerical result is a solution to Einstein equation:

  a. The convergence of the solutions

  b. $\xi^a \xi_a < 10^{-10}$

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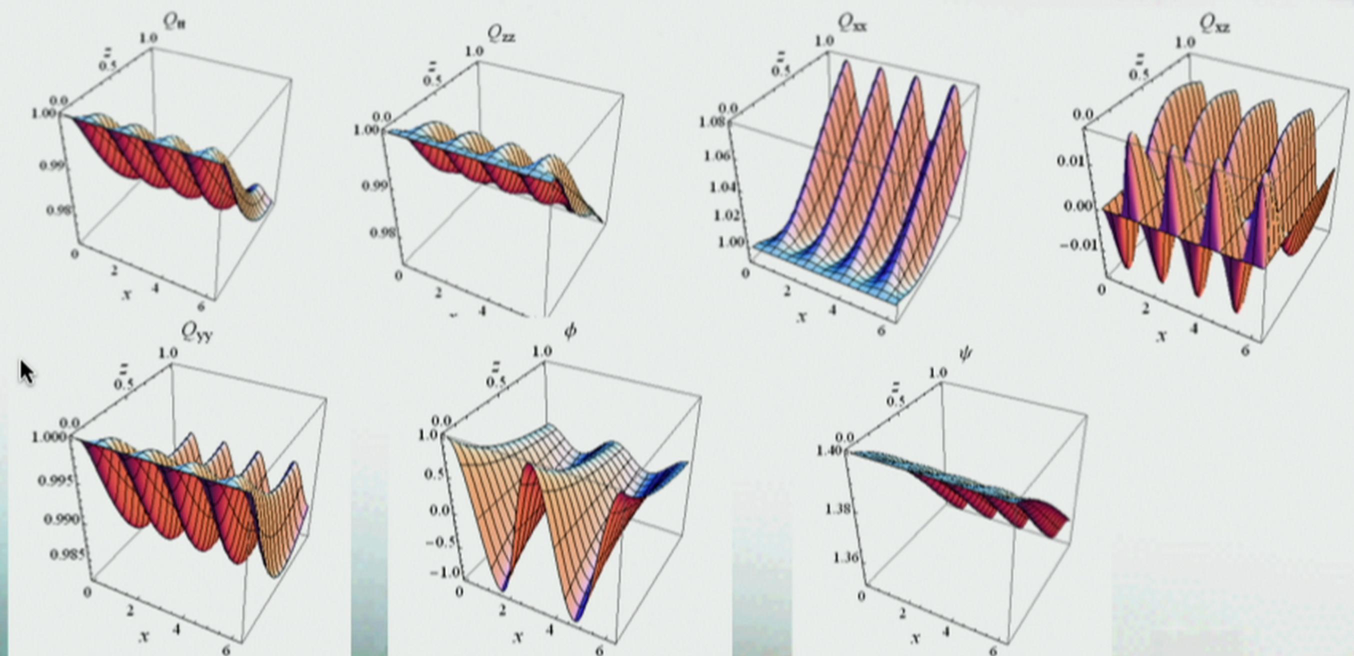
Building a lattice background in holographic gravity

- The numerical results: examples

Scalar lattice $k_0 = 2, A_0 = 1, \mu = 1.4, T/\mu = 0.1$



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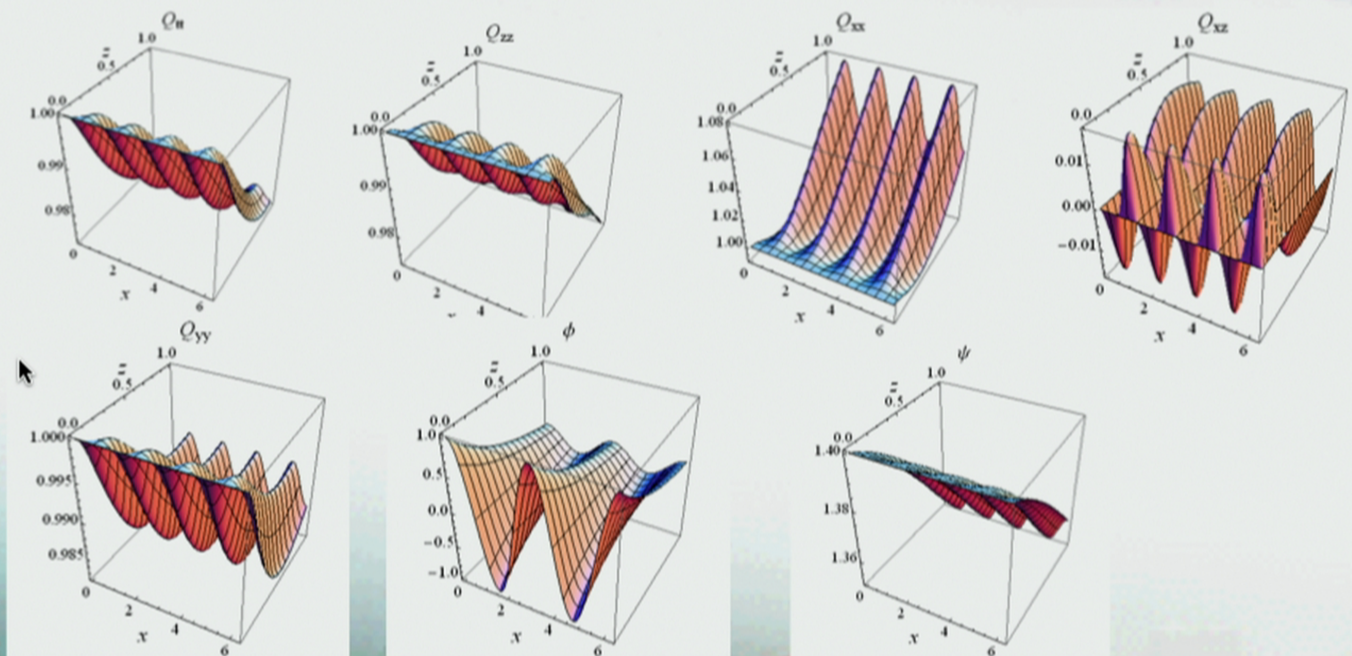

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Part II

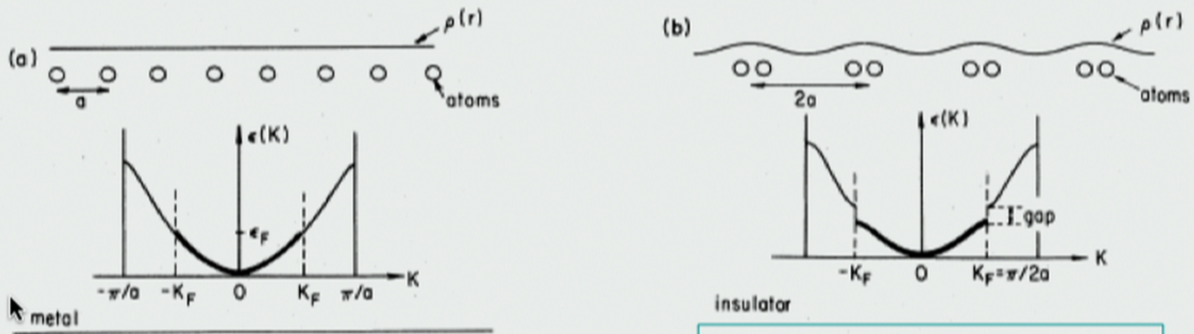
Metal-Insulator Transition by holography

Ionic lattice and Peierls phase transition (CDW)

- **What are charge density waves?**

  1. Basic definition

$$\rho(x) = \rho_0 + \rho_1 \cos(2k_F x + \varphi)$$



$$\left.\begin{aligned} N &= \frac{L}{a} \\ N &= 2\frac{L\cdot 2k_F}{2\pi} \end{aligned}\right\} \Rightarrow k_F = \frac{N\pi}{L2} = \frac{\pi}{2a}$$

Peierls phase transition

$$\vec{F} = \vec{E}q = \frac{d\vec{p}}{dt} = \frac{d\vec{k}}{dt}$$

Gruner, Rev.Mod.Phys.Vol.60(1988),No.4

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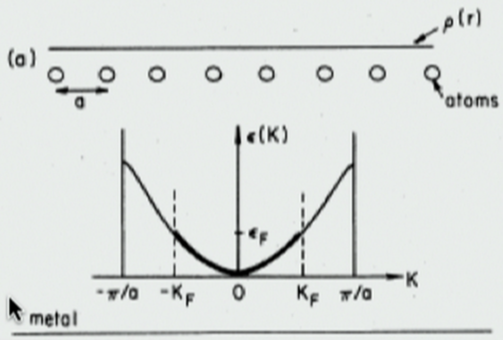

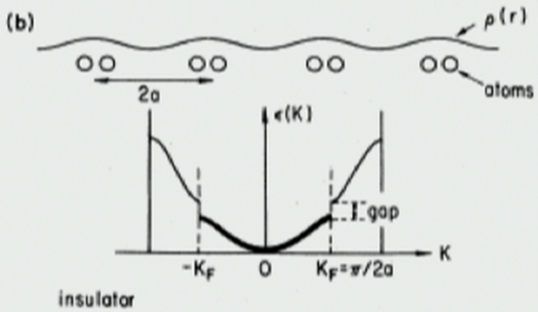

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Ionic lattice and Peierls phase transition (CDW)

- Motivations and recent progress

1. It is essential to introduce some mechanism inducing the instability of the bulk geometry which is usually of spatial homogeneity.

   H. Ooguri and C. -S. Park, Phys. Rev. Lett. 106, 061601 (2011)
   A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)

2. Striped black hole solutions as the examples of spatially modulated unstable modes have been presented.

   M. Rozali, D. Smyth, E. Sorkin and J. B. Stang, Phys. Rev. Lett. 110, 201603 (2013)
   A. Donos, JHEP 1305, 059 (2013)

3. The dynamics of CDW in the holographic approach.

   Y. Ling, C. Niu, J. Wu, Z. Xian and H. Zhang, Phys.Rev.Lett. 113 (2014) 091602.
   Y. Ling, C. Niu, J. Wu, Z. Xian and H. Zhang, To appear.

Ionic lattice and Peierls phase transition (CDW)

- Instability of near horizon geometry which is AdS2

AdS-RN black holes:

A. Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)

$$T=0 \qquad AdS_2\times R^2$$

$$ds^2=-12r^2dt^2+\frac{dr^2}{12r^2}+(dx^2+dy^2)$$

$$F=2\sqrt{3}dr\wedge dt$$

$$S_{top}\sim\int d^4x\frac{c_1\varphi}{16\sqrt{3}}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$\delta g_{ty}=2\sqrt{3}h_{ty}(t,r)\sin(kx)$$

$$\delta g_{xy}=h_{xy}(t,r)\cos(kx)$$

$$\delta A_t=a(t,r)\sin(kx)$$

$$\delta\varphi=w(t,r)\cos(kx)$$

$$\Rightarrow \quad \Box_{AdS2}\vec{V}-M^2\vec{V}=0$$

$$\vec{V}=(\phi_{xy},a,w)$$

$$M^2=\begin{pmatrix} k^2 & \frac{1}{\sqrt{3}}k & 0\\ 24\sqrt{3} & 24+k^2 & -c_1k\\ 0 & -c_1k & k^2+m^2\end{pmatrix}$$

AdS2 BF bound

$$m^2\geq -3$$

Could be violated !

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Ionic lattice and Peierls phase transition (CDW)

- Two ways leading to spontaneous breaking of translational invariance

1、Topological terms

$$S_{top} \sim \int d^4x \frac{c_1\Phi}{16\sqrt{3}} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

Striped black holes

$$ds^2 = \frac{1}{z^2}[-f(z)Qdt^2 + \frac{S}{f(z)}dz^2 + T[dx + z^2Udz]^2 + V(dy + (1-z)Wdt)^2]$$

$$A = \mu(1-z)\psi dt + \chi dy \qquad \Phi = z\phi$$

2、Non-topological terms

$$S_{non-top} \sim \int d^4x\sqrt{-g}[-\frac{1}{4}t(\Phi)F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}G^{\mu\nu}G_{\mu\nu} - \frac{1}{2}u(\Phi)F^{\mu\nu}G_{\mu\nu}]$$

Striped black holes

$$ds^2 = \frac{1}{z^2}[-f(z)Qdt^2 + \frac{S}{f(z)}dz^2 + T[dx + z^2Udz]^2 + Vdy^2]$$

$$A = \mu(1-z)\psi dt \qquad B = (1-z)\chi dt \qquad \Phi = z\phi$$

$$F = dA, G = dB,$$

$$t(\Phi) = 1 - \frac{\beta}{2}L^2\Phi^2$$

$$u(\Phi) = \frac{\gamma}{\sqrt{2}}L\Phi$$

Ionic lattice and Peierls phase transition (CDW)

- The holographic charge density waves

$$L^2=\frac{1}{24}, m^2=-8$$

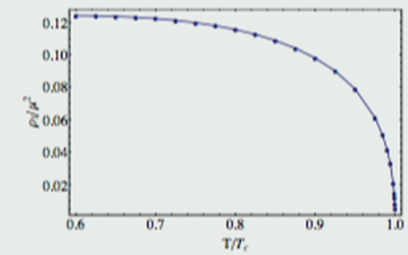
$$B=(1-z)\chi dt$$

$$T_c=0.078\mu, k_c=0.325\mu \Longleftarrow \beta=-138, \gamma=17.1$$

$$\frac{T}{\mu}=\frac{48-\mu^2}{16\pi\mu}$$

Charge density

$$\rho(x)=(1-\left.\frac{\partial\chi}{\partial z}\right|_{z=0}) \Longleftarrow \chi(x,z)=0+z\chi_1(x)+z^2\chi_2(x)+\ldots$$



$$\rho(x)=\rho_1\cos[k_c x]+\rho_3\cos[3k_c x]+\ldots$$

$$\rho_0\cong\rho_2\cong 0\ldots, \quad \rho_1, \quad \rho_3 \quad \rightarrow CDW$$

↓

No Free Electrons

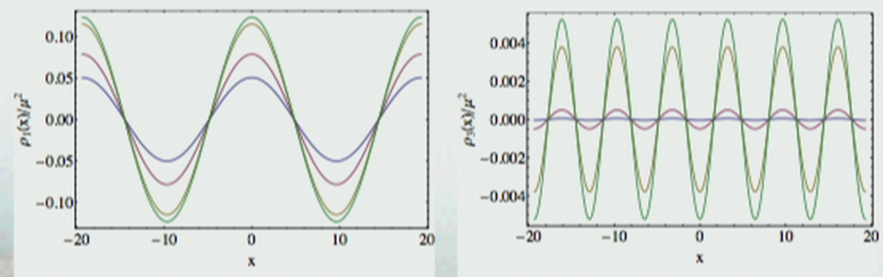


FIG. 2: The first and third modes of CDW for $T/T_c=0.6, 0.8, 0.95, 0.98$ from top to down.

Ionic lattice and Peierls phase transition (CDW)

- The holographic charge density waves

$$L^2 = \frac{1}{24}, m^2 = -8$$

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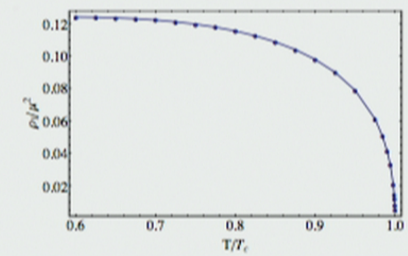
$$\frac{T}{\mu} = \frac{48 - \mu^2}{16\pi\mu}$$

Charge density

$$\rho(x) = (1 - \left.\frac{\partial \chi}{\partial z}\right|_{z=0}) \Leftarrow \chi(x,z) = 0 + z\chi_1(x) + z^2\chi_2(x) + \ldots$$



$$\rho(x) = \rho_1 \cos[k_c x] + \rho_3 \cos[3k_c x] + \ldots$$

$$\rho_0 \cong \rho_2 \cong 0\ldots, \quad \rho_1, \quad \rho_3 \quad \rightarrow CDW$$

↓

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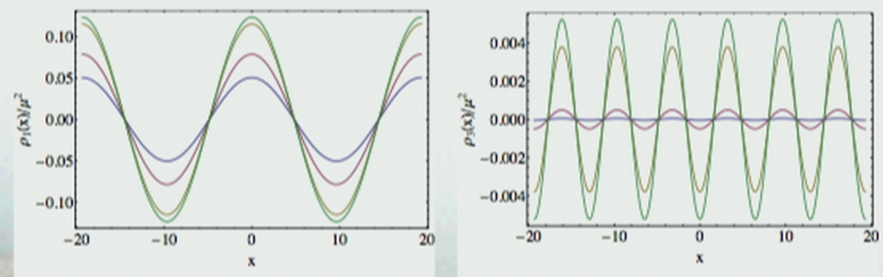
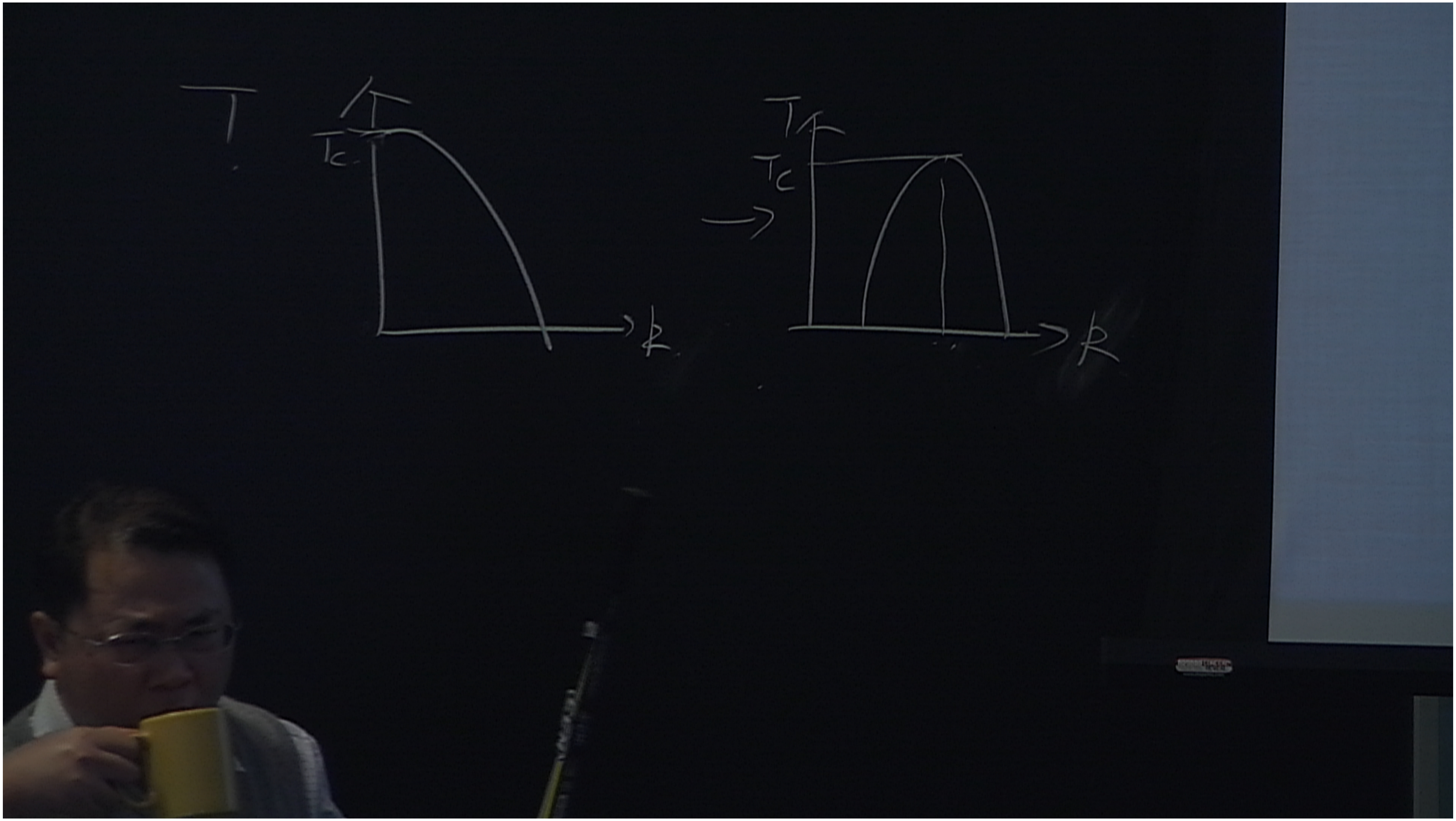
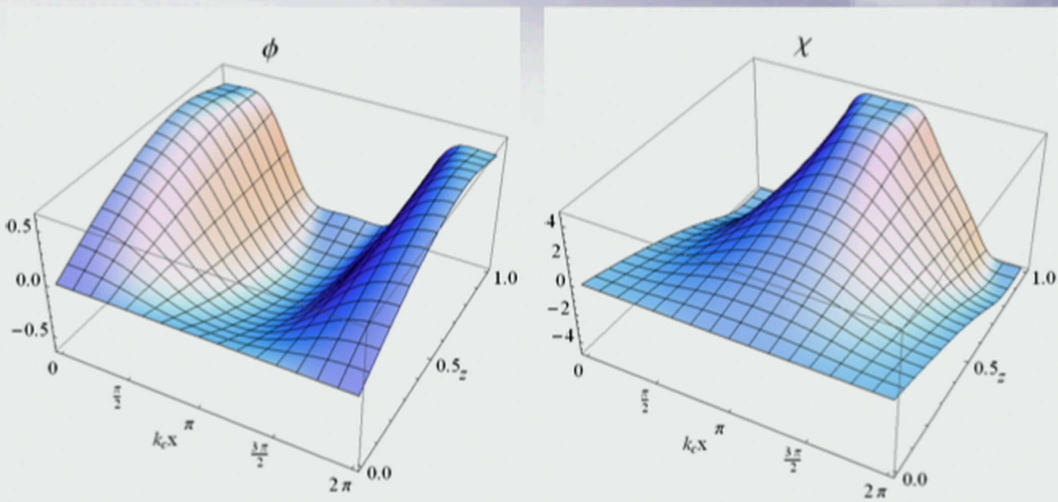


FIG. 2: The first and third modes of CDW for $T/T_c = 0.6, 0.8, 0.95, 0.98$ from top to down.


T
T_c
k
T
T_c
k

Ionic lattice and Peierls phase transition (CDW)

- The examples of background solutions



The solutions of scalar field and the time component of the gauge field at T=0.8Tc

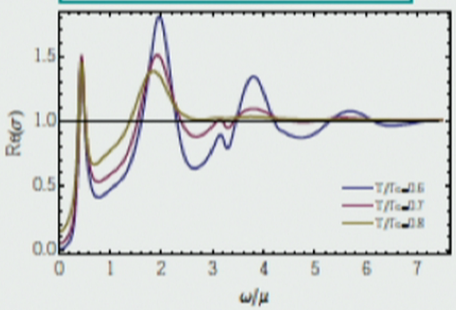
- The optical conductivity

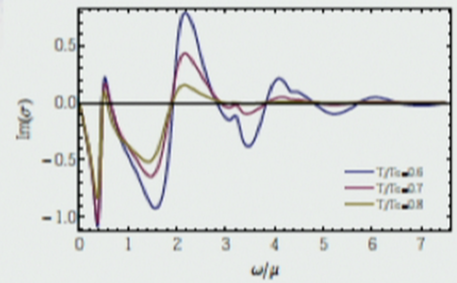
$$b_x = (1 + j_x(x)z + \ldots)e^{-i\omega t}$$

$$\sigma(\omega/\mu) = \frac{4 j_x^{(0)}}{i\omega}$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, A_\mu = \bar{A}_\mu + a_\mu, B_\mu = \bar{B}_\mu + b_\mu, \Phi = \bar{\Phi} + \varphi.$$

$$\nabla^\mu \hat{h}_{\mu\nu} = 0, \nabla^\mu a_\mu = 0, \nabla^\mu b_\mu = 0. \qquad \hat{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2.$$

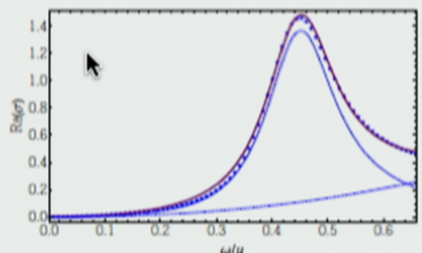


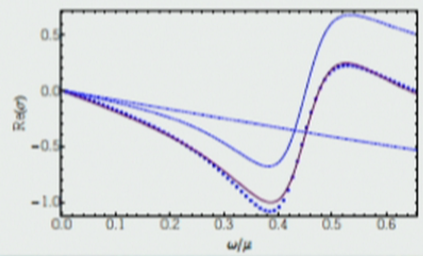


- Two Lorentz formula

$$\sigma_{tot}(\omega) = \sigma_{CDW1}(\omega) + \sigma_{CDW2}(\omega)$$

$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2/\omega^2)}$$





- Single-particle gap:

$$\frac{2\Delta}{T_c} \cong 20.51$$

Remark: metal to insulator phase transition!

15.80 for Single crystalline TbTe3

Y. Ling et.al., Phys.Rev.Lett. 113 (2014) 091602.

Ionic lattice and Peierls phase transition (CDW)

- The holographic charge density waves

$$L^2=\frac{1}{24}, m^2=-8$$

$$B=(1-z)\chi dt$$

$$T_c=0.078\mu, k_c=0.325\mu \Leftarrow \beta=-138, \gamma=17.1$$

$$\frac{T}{\mu}=\frac{48-\mu^2}{16\pi\mu}$$

Charge density

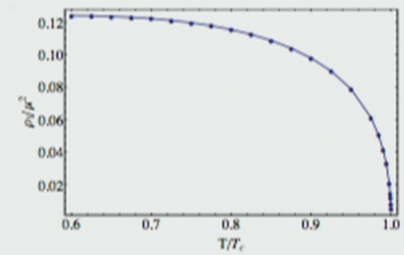
$$\rho(x)=(1-\left.\frac{\partial\chi}{\partial z}\right|_{z=0}) \Leftarrow \chi(x,z)=0+z\chi_1(x)+z^2\chi_2(x)+\dots$$

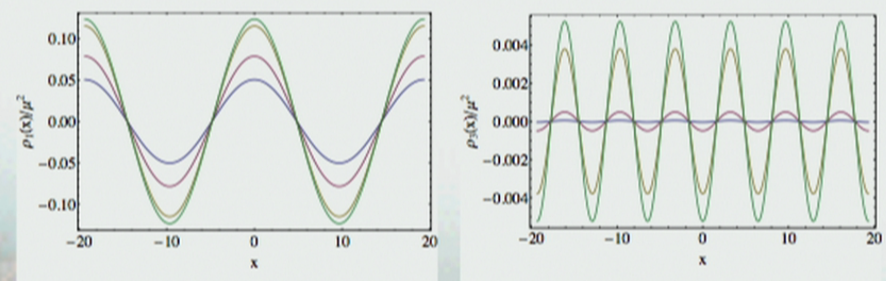
$$\rho(x)=\rho_1\cos[k_c x]+\rho_3\cos[3k_c x]+\dots$$

$$\rho_0\cong\rho_2\cong 0\dots,\quad \rho_1,\quad \rho_3\quad \to CDW$$

⬇

No Free Electrons





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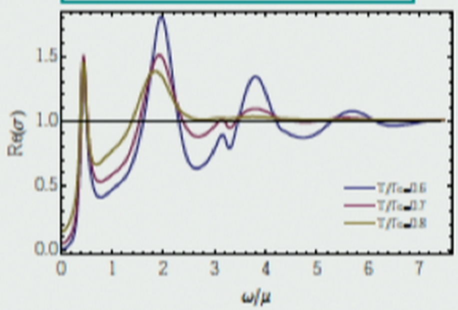

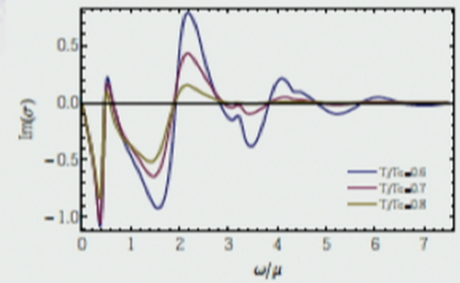

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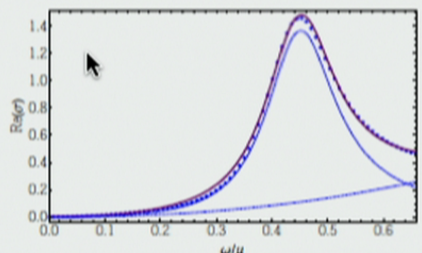

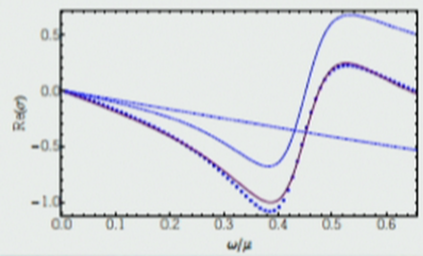

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Y. Ling *et.al.*, Phys.Rev.Lett. 113 (2014) 091602.

Ionic lattice and Peierls phase transition (CDW)

- **Summary:**

1、Superconductivity

$$\sigma(\omega) \propto K\left(\delta(\omega) - \frac{1}{i\omega^{\alpha}}\right)$$

Theory: The breaking of U(1) gauge symmetry
Strategy: Introducing a complex scalar field

2、Drude law for metals

$$\sigma = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

Theory: The breaking of translational symmetry
Strategy: Introducing lattice structure

3、CDW

$$\sigma_{CDW}(\omega) = \frac{K\tau}{1 - i\omega\tau(1 - \omega_0^2 / \omega^2)}$$

Theory: Spontaneously breaking of translational symmetry
Strategy: Introducing a topological term

Q-lattice and novel MIT

- 4D Setup

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R+6-\frac{1}{2}F_{ab}F^{ab}-|\partial\Phi|^2-m^2|\Phi|^2]$$

Φ : Complex scalar field

Equations of motion:

$$G_{ab}=R_{ab}+3g_{ab}-....=0$$

$$\nabla_a F^a{}_b=0$$

$$(\Box-m^2)\Phi=0$$

A. Donos and J. P. Gauntlett, JHEP 1404, 040 (2014).

Q-lattice and novel MIT

- **Ansatz of variables**

$$\Phi = e^{ikx} z^{3-\Delta}\phi(z)$$

$$ds^2 = \frac{1}{z^2}\left[-(1-z)P(z)U(z)dt^2 + \frac{1}{P(z)(1-z)U(z)}dz^2 + V_1(z)dx^2 + V_2(z)dy^2\right]$$

$$A = \mu(1-z)\psi(z)dt \qquad \Delta = 3/2 \pm \left(9/4 + m^2\right)^{1/2} \qquad P(z) = 1 + z + z^2 - \frac{\mu^2 z^3}{2}$$

- **A family of three-parameter black brane solutions:**

Boundary condition $\qquad \phi(0) = \lambda$

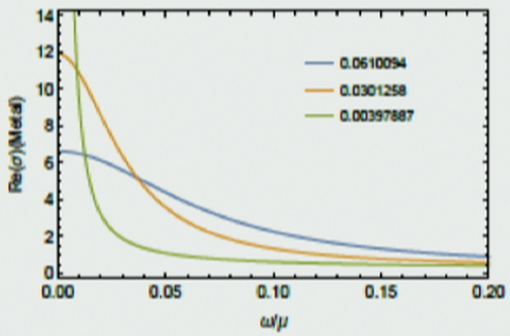
$$T/\mu, \qquad \lambda/\mu^{3-\Delta}, \qquad k/\mu$$

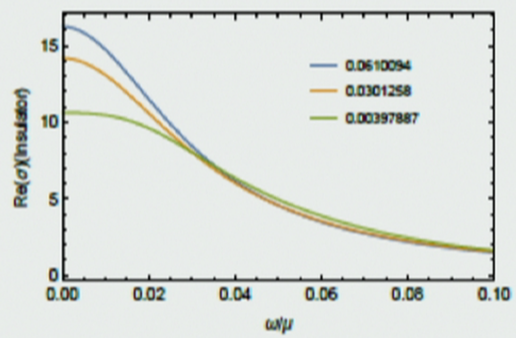
Temperature (T/μ), **Lattice Amplitude** ($\lambda/\mu^{3-\Delta}$), **Wave number** (k/μ)

Q-lattice and novel MIT

- **Novel MIT:**

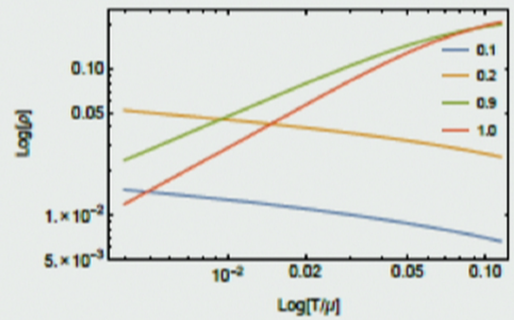
$$\sigma_{DC} = \frac{1}{\rho_{DC}} = 0 \qquad T = 0$$





Metal $\frac{d\rho_{DC}}{dT} > 0$

Insulator $\frac{d\rho_{DC}}{dT} < 0$



Holographic Entanglement Entropy (HEE) close to QPT

- **Holographic description of entanglement entropy:**

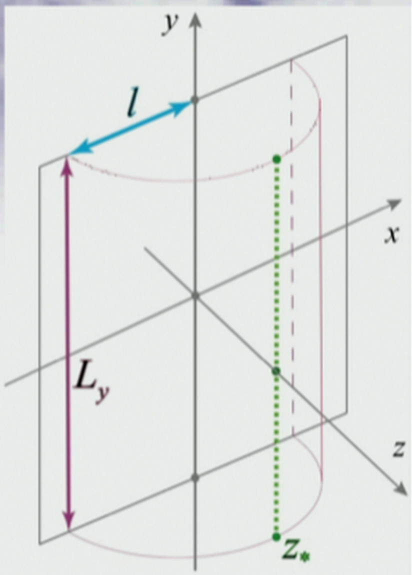
$$S_{full} = \frac{\text{Area}(\gamma_A)}{4G_N}$$

T. Nishioka, S. Ryu and T. Takayanagi, J.Phys.A42:504008,2009

- **The reduced HEE:**

$$l = \mu \int_0^{z_*} dz z^2 \sqrt{\frac{V_1(z_*)V_2(z_*)}{P(z)V_1(z)W(z_*,z)}}$$

$$S = \frac{1}{\mu}\left\{-\frac{1}{z_*} + \int_0^{z_*} \frac{dz}{z^2} dz \left[\frac{z_*^2 V_1(z)V_2(z)}{\sqrt{P(z)V_1(z)W(z_*,z)}} - 1\right]\right\}$$

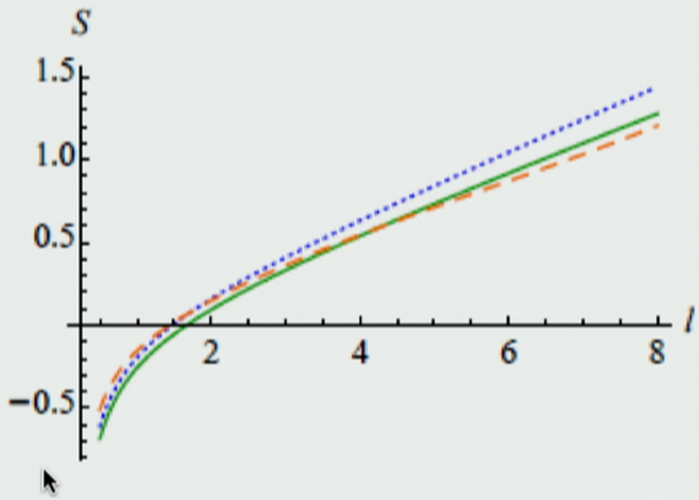


$$4G_N S_{full} = 2L_y(S + 1/\varepsilon)$$

Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Numerical results:**

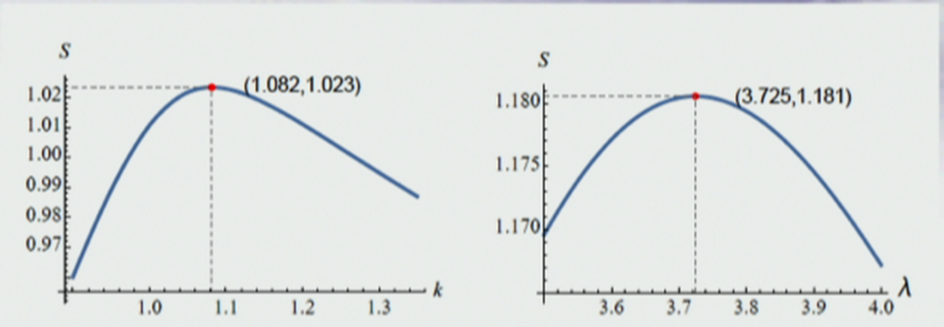


- Q-lattice dual to metallic phase
- AdS-RN black brane
- Q-lattice dual to insulating phase

Y.Ling, et.al.,arXiv:1502.03661

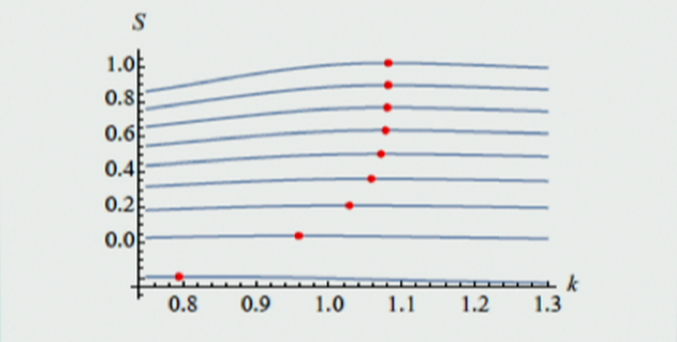
Holographic Entanglement Entropy (HEE) close to QPT

- **Pronounced peaks :**



- **Independence of the width of the strip :**

All the strips have the same width 2l.



The half-width of the strip l is increased from 0.91 to 5.56 uniformly.

Y.Ling, et.al.,arXiv:1502.03661

Holographic Entanglement Entropy (HEE) close to QPT

- **Near horizon analysis:** $z_* \to 1$

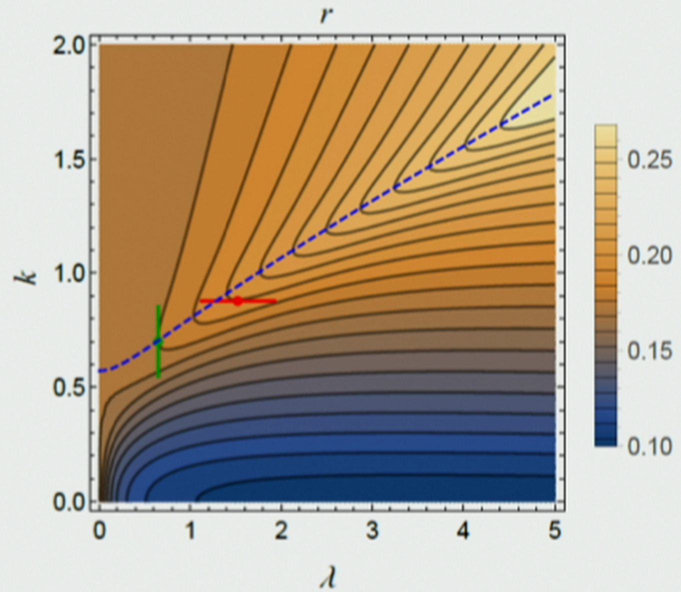
$$S \sim \frac{V_2}{\mu}\sqrt{\frac{V_1}{pUB}}\Bigg|_{z=1} \log\left(\frac{1}{1-z_*}\right)+\ldots,$$

$$l \sim \mu\sqrt{\frac{V_2}{pUB}}\Bigg|_{z=1} \log\left(\frac{1}{1-z_*}\right)+\ldots,$$

$$B = 4V_1V_2 - V_1'V_2 - V_1V_2'$$

$$p = 1 + z + z^2 - \mu^2 z^3/2$$

$$r = \lim_{z_*\to 1}\frac{S}{l} = \frac{\sqrt{V_1V_2}}{\mu^2}\Bigg|_{z=1}$$



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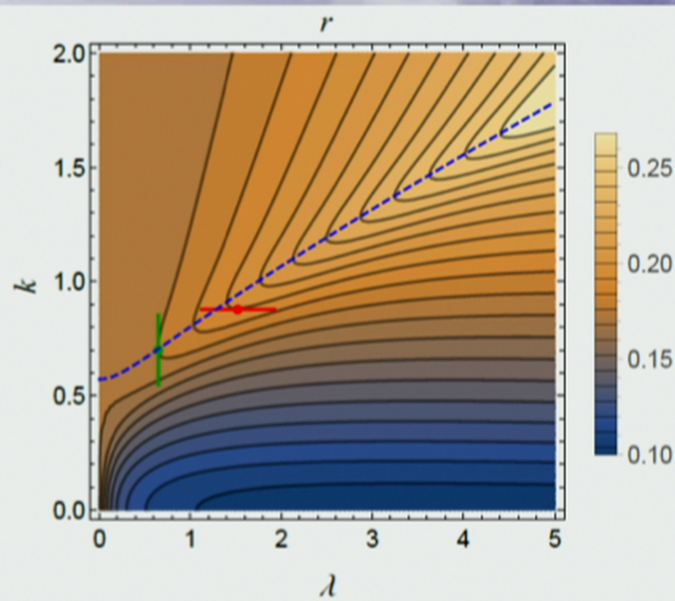
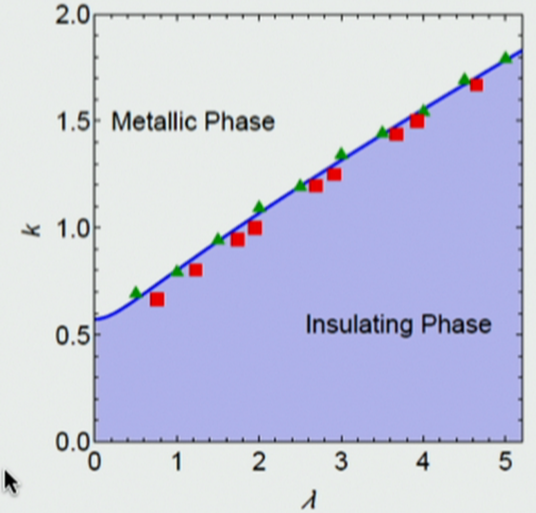
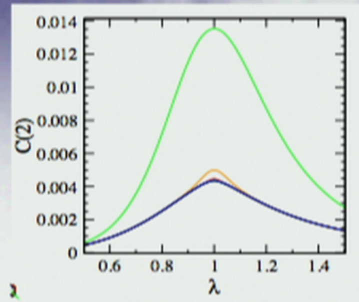


Y.Ling, et.al.,arXiv:1502.03661

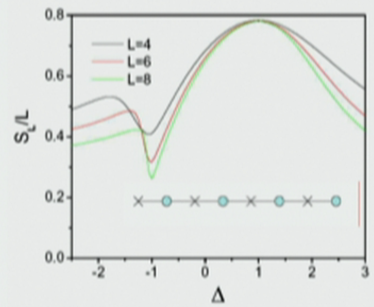
Holographic Entanglement Entropy (HEE) close to QPT

- **HEE close to QPTs :**





Osterloh, *et.al.* Nature 416,608(2002).



Chen, *et.al.* New J. Phys. 8, 97(2006).

Remarkably, we observe that all the turning points of the reduced HEE are distributed in the vicinity of the trajectory of the critical points, clearly indicating that HEE can be used to characterize the occurrence of QPTs.

Y.Ling, et.al.,arXiv:1502.03661

Summary:

Gravity/Gauge duality provides power tools to understand the strongly correlated phenomena, including high temperature superconductivity as well as metal-insulator transitions. We are on the road to link theoretical models to the experiment data in near future!

Thank you !