

Title: Multiloop SYM and supergravity amplitudes in pure spinor superspace

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Abstract: We present a string-inspired method to construct supersymmetric loop integrands for scattering amplitudes in ten-dimensional SYM and supergravity. The kinematic dependence is described in pure spinor superspace and determined from the BRST-symmetry of the underlying pure spinor superstring. We focus on five-point examples at one- and two-loop order which satisfy the BCJ duality between color and kinematics. Hence, the supergravity integrands follow straightforwardly from a double copy construction of the cubic diagrams in SYM. Preliminary examples and open questions at three-loops are mentioned.

Two-particle superfields

$$A_{12}^\alpha = \frac{1}{2} \left[A_2^a (k_2 \cdot A_1) + A_m^2 (\gamma^m W_1)_\alpha - (1 \leftrightarrow 2) \right]$$

$$A_{12}^m = \frac{1}{2} \left[A_p^1 F_2^{pm} - A_m^1 (k_1 \cdot A_2) - (W_1 \gamma^m W_2) - (1 \leftrightarrow 2) \right]$$

$$W_{12}^\alpha = \frac{1}{4} (\gamma^{mn} W_2)^\alpha F_{mn}^1 + W_2^\alpha (k_2 \cdot A_1) - (1 \leftrightarrow 2)$$

$$F_{12}^{mn} = F_2^{mn} (k_2 \cdot A_1) + \frac{1}{2} F_2^{[m} F_1^{n]p} + k_{[m}^1 (W_1 \gamma_{n]} W_2) - (1 \leftrightarrow 2)$$

Non-linear SYM equations of motion

$$2 D_{(\alpha} A_{\beta)} = \gamma_{\alpha\beta}^m A_m + \{A_\alpha, A_\beta\}$$

$$D_\alpha A_m = \partial_m A_\alpha + (\gamma_m W)_\alpha + [A_\alpha, A_m]$$

$$D_\alpha W^\beta = \frac{1}{4} (\gamma^{mn})_\alpha{}^\beta F_{mn} + \{A_\alpha, W^\beta\}$$

$$D_\alpha F^{mn} = [\partial^{[m} - A^{[m}, (\gamma^{n]} W)_\alpha] + [A_\alpha, F^{mn}]$$

From connections $\nabla^m \equiv \partial^m - A^m$, $\nabla_\alpha \equiv D_\alpha - A_\alpha$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma_{\alpha\beta}^m \nabla_m \quad \& \quad \text{Bianchi's}$$

[Witten 1986]

Linearized SYM equations of motion

$$2 D_{(\alpha} A_{\beta)}^1 = \gamma_{\alpha\beta}^m A_m^1$$

$$D_\alpha A_m^1 = k_m^1 A_\alpha^1 + (\gamma_m W_1)_\alpha$$

$$D_\alpha W_1^\beta = \frac{1}{4} (\gamma^{mn})_\alpha{}^\beta F_{mn}^1$$

$$D_\alpha \overline{F}_1^{mn} = 2 k_1^{[m} (\gamma^{n]} W_1)_\alpha$$

Multiloop SYM & supergravity amplitudes in pure spinor superspace

based on { C. Mafra, DS: 1404.4986, 1410.0668, 1501.05562 & work in progress
Z. Bern, J. Carrasco, H. Johansson: 0805.3993, 1004.0476

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Gauge & gravity amplitudes from cubic graphs T :

$$A_n^{g\text{-loop}} = \int \prod_{j=1}^g d^D l_j \sum_{T \in \mathcal{T}_n} \frac{N_i(l) C_i}{\prod_{h \in T} P_{h,i}(l)}$$

$N_i(l)$: kin. numerator
 C_i : momenta l (super-)polarizations
 $P_{h,i}(l)$: propagators
 f^{abc} : w. Jacobi-identity
 $\sum_{\text{cyc}(a,b,c)} f^{abe} f^{ecd} = 0$
 in m -gon subgraph
 at most $m-4$ powers of l
 no SUSY & tadpoles

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Gauge & gravity amplitudes from cubic graphs T :

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$\swarrow$ max SUSY
 $\nwarrow$ no triangles & bubbles & tadpoles
 $\nearrow$ propagators
 $\searrow$ in m -gon subgraph at most $m-4$ powers of l

$N_i(l) C_i$ kin. numerator momenta l (super-)polarizations
 $\begin{matrix} & c \\ & \swarrow \quad \searrow \\ a & & b \end{matrix} = f^{abc}$ w. Jacobi-identity
 $\sum_{\text{cyc}(a,b,c)} f^{abe} f^{ecd} = 0$

$$B(\gamma): \nexists C_i + C_j + C_k = 0 \Rightarrow N_i(l) + N_j(l) + N_k(l) = 0$$

$$\text{then } M_n^{g\text{-loop}} = A_n^{g\text{-loop}} \mid C_i \rightarrow \tilde{N}_i(l)$$

This talk: explicit $N_i(l)$ in pure spinor superspace

$$\{x^m, \theta^a, \lambda^a\} \text{ w. } (\lambda \gamma^m \lambda) = 0 \Rightarrow \langle \lambda^3 \theta^5 \rangle = 1$$

$$\text{from } Q A_n^{g\text{-loop}} = 0 \text{ where } Q = \lambda^a D_a$$

$$Q N_i(l) = \sum_k P_{k,i}(l) (\text{stuff})_{k,i} \Rightarrow \text{infer } N_i(l)$$

I) Trees $\leftrightarrow$ multiparticle superfields

Recall superstring vertex op's

$$V_i(z) = \lambda^\alpha A_\alpha^i(x, \theta), \quad U^i(z) = \partial \bar{\theta}^\alpha \underset{\uparrow}{A}_\alpha^i + \bar{\Pi}_m \underset{\uparrow}{A}_i^m + d_\alpha \underset{\uparrow}{W}_i^\alpha + \bar{N}_{mn} \underset{\uparrow}{F}_i^{mn}$$

linearized superfields of D=10 SYM

I) Trees $\leftrightarrow$ multiparticle superfields

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OPE's $\Rightarrow$ poles in $(z_1 - z_2) \Rightarrow \int d\bar{z}$ linearized superfields of D=10 SYM
 $\Rightarrow$ poles in $\boxed{s_{ij} \equiv (k_i \cdot k_j)}$

$$\int U_1(z_1) U_2(z_2) \sim \int (z_1 - z_2)^{-\alpha' k_1 \cdot k_2 - 1} \left[\partial \theta^\alpha A_\alpha^{12} + \Pi_m A_m^{12} + d\alpha W_{12}^\alpha + N^{mn} F_{mn}^{12} \right]$$

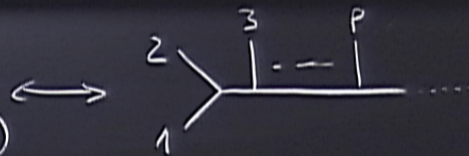
no triangles
& bubbles & tadpoles

in m -gon subgraph at most $m-4$ powers of p

$$\sum_{\text{cyc}(a,b,c)} f_{abe} f_{ecd} + \text{cyc}(a,b,c) = 0$$

$\exists$ p -particle version $\{A_\alpha^{12 \dots p}, \dots\}$

promote $(1 \leftrightarrow 2)$ to $(12 \dots p-1 \leftrightarrow p)$

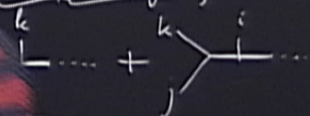


generating series solves non-lin

$$A_\alpha = \underbrace{\sum_i A'_{\alpha i} t^i}_{\text{color generators}} + \sum_{ij} \frac{A'_{\alpha ij}}{s_{ij}} t^i t^j + \dots$$

$M \neq 0, m$

$$\left(\frac{A'_{\alpha ij}}{s_{ij} s_{ijk}} + \frac{A'_{\alpha jk}}{s_{jk} s_{ijk}} \right) t^i t^j t^k$$

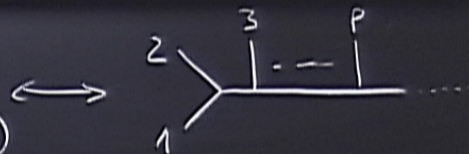


no triangles
& bubbles & tadpoles in m -gon subgraph at most $m-4$ powers of p

$$\sum_{\substack{a,b,c,d \\ \text{of } p}} f_{abe} f_{ecd} + \text{cyc}(a,b,c) = 0$$

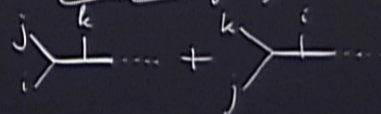
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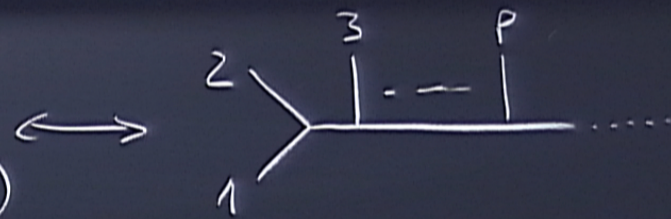
generating series solves non-linear SYM + o.m

$$A_\alpha = \underbrace{\sum_i A'_\alpha t^i}_{\text{color generators}} + \sum_{i,j} \frac{A'_\alpha}{s_{ij}} t^i t^j + \sum_{i,j,k} \left(\frac{A'_\alpha}{s_{ij} s_{jk}} + \frac{A'_\alpha}{s_{jk} s_{ji}} \right) t^i t^j t^k$$



$\exists$ p-particle version $\{A_\alpha^{12 \dots p}, \dots\}$

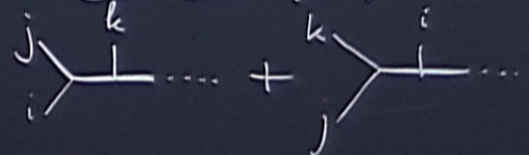
promote $(1 \leftrightarrow 2)$ to $(12 \dots p-1 \leftrightarrow p)$



generating series solves non-linear SYM e.o.m

$$A_\alpha = \sum_i A'_\alpha t^i + \sum_{i,j} \frac{A'_\alpha}{s_{ij}} t^i t^j + \sum_{i,j,k} \left(\frac{A'_\alpha}{s_{ij} s_{jk}} + \frac{A'_\alpha}{s_{jk} s_{ji}} \right) t^i t^j t^k$$

color generator



ditto for $A_m, W^\alpha, F^{\mu\nu}$

II) One loop

4pt superstring kin factor [Berhons 04]

$$\langle \underbrace{V_1 (\lambda \gamma_m W_2) (\lambda \gamma_n W_3)}_{\text{separately BRST-closed}} \underbrace{\overline{F}_4}_{B=12 \dots p} \rangle = s_{12} s_{23} A^{\text{tree}}(1,2,3,4)$$

Multiparticle case $T_{A,B,C} = \frac{1}{3} (\lambda \gamma_m W_A) (\lambda \gamma_n W_B) \overline{F}_C + (C \leftrightarrow A, B)$

$$Q T_{23,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5}) \quad \text{"covariant"}$$

$Q N_i(l) = \sum_k P_{k,i}(l) (\text{stuff})_{k,i} \Rightarrow \text{infer } N_i(l)$
 Specifically: 5pt @ $g=1,2$ in BCJ-form.

II) One loop

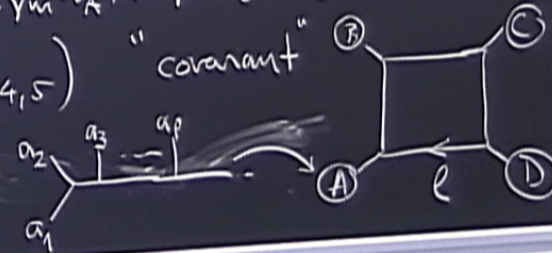
4pt superstring kin factor [Berken's 04]

$$\langle \underbrace{V_1}_{\text{separately BRST-closed}} (\lambda \gamma_m W_2) (\lambda \gamma_n W_3) F_4^{mn} \rangle = s_{12} s_{23} A^{\text{tree}}(1,2,3,4)$$

Multiparticle case $T_{A,B,C} = \frac{1}{3} (\lambda \gamma_m W_A) (\lambda \gamma_n W_B) F_C^{mn} + (C \leftrightarrow A, B)$

$$Q T_{23,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5}) \quad \text{"covariant"}$$

Box-numerator $V_A T_{B,C,D}$



Specifically: S_{pt} @ $g=1,2$ in BCJ-form inspired by / valuable for superstring comp.

III) Two loop

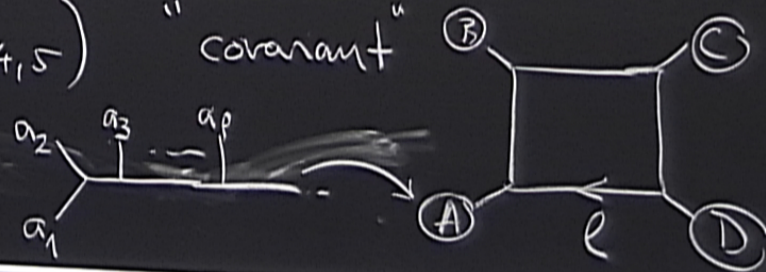
4pt superstring kin factor [Berkenius OS]

$$(\lambda \gamma_{mnpqr} \lambda) F_1^{mn} F_2^{pq} F_3^{rs} (\lambda \gamma_s W_4) \rangle = s_{12} s_{23} A^{tree}(1,2,3,4)$$

3 particle case $T_{A,B,C} = \frac{1}{3} (\lambda \gamma_m W_A) (\lambda \gamma_n W_B) F_C^{mn} + (C \leftrightarrow A, B)$

$T_{23,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5})$ "covariant"

numerator $V_A T_{B,C,D}$



Specifically: 5pt @ $g=1,2$ in BCJ-form inspired by / valuable for superstring comp

III) Two loop

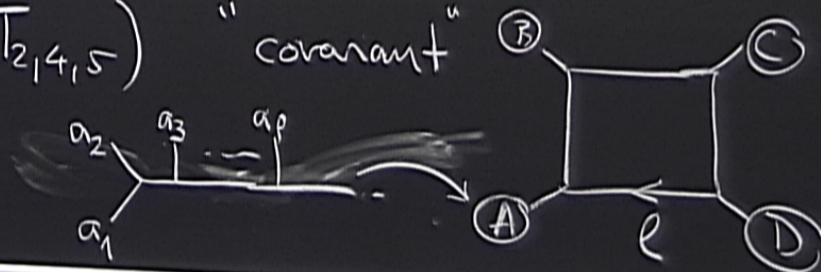
4pt superstring kin factor [Berkenius OS]

$$\langle (\lambda \gamma_{\mu\nu\rho\sigma} \lambda) F_1^{\mu\nu} F_2^{\rho\sigma} F_3^{\mu\nu} (\lambda \gamma_5 W_4) \rangle = s_{12} s_{23} A^{\text{tree}}(1,2,3,4)$$

Multiparticle case $T_{A,B,C}^{\mu\nu} = \frac{1}{3} (\lambda \gamma_{\mu\nu} W_A) (\lambda \gamma_{\mu\nu} W_B) F_C^{\mu\nu} + (C \leftrightarrow A, B)$

Q $T_{23,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5})$ "covariant"

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III) Two loop

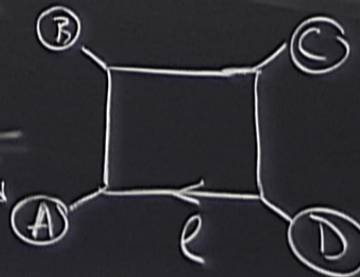
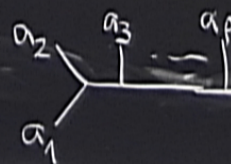
4pt superstring kin factor [Berkovits OS]

$$\langle (\lambda \gamma_{mnpqr} \lambda) \underbrace{F_1^{mn} F_2^{pq} F_3^{rs}}_{\text{BCJ}} (\lambda \gamma_s W_4) \rangle = s_{12} s_{23} A^{\text{tree}}(1,2,3,4)$$

Multiscale case $T_{A,B|C,D} = \frac{1}{4} (\lambda \gamma_{mnpqr} \lambda) F_A^{mn} F_B^{pq} + \left(\begin{matrix} A, B \\ \updownarrow \\ C, D \end{matrix} \right) F_C^{rs} (\lambda \gamma_s W_D)$

$T_{23,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5})$ "covariant"

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$$\langle (\lambda \gamma_{mpqr} \lambda) \underbrace{F_1^{mn} F_2^{pq} F_3^{rs}}_{\text{BCJ}} (\lambda \gamma_s W_4) \rangle = s_{12}^2 s_{23} A^{\text{tree}}(1,2,3,4)$$

Multiparticle case $T_{A,B|C,D} = \frac{1}{4} (\lambda \gamma_{mpqr} \lambda) F_A^{mn} F_B^{pq} + \left(\begin{matrix} A, B \\ \updownarrow \\ C, D \end{matrix} \right) F_C^{rs} (\lambda \gamma_s W_D)$

$Q T_{2,3,4,5} = s_{23} (V_2 T_{3,4,5} - V_3 T_{2,4,5})$ "covariant"

Box-numerator $V_A T_{B,C,D}$

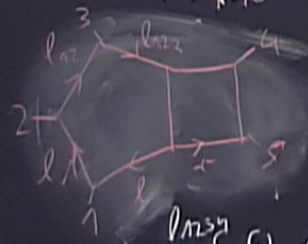


2-particle superfields

Pentagon $\rightarrow$ $\ln T^m$ with $U|_{T^m} \text{ zero}$ b -ghost sector $b \sim T^m dx$

$$T_{A,B,C|D,E}^m = \left[A_{A,B|D,E}^m + (C \leftrightarrow B, A) \right] + W_{A,B,C|D,E}^m$$

$$Q T_{2,3,4|5,6}^m = k_2^m V_2 T_{3,4|5,6} + (2 \leftrightarrow 3, 4)$$



$$N_{12,3,4,5}(l) = (l_m + l_m^1) V_1 T_{2,3,4,5}^m + (V_1 T_{23,4,5} + (23 \leftrightarrow 24, 25, 34, 35, 45))$$

$$Q N_{12,3,4,5}(l) = V_1 \left(V_2 T_{3,4,5}(l_{12}^2 - l_1^2) + V_3 T_{2,4,5}(l_{123}^2 - l_{12}^2) \right. \\ \left. + V_4 T_{2,3,5}(l_{1234}^2 - l_{123}^2) + V_5 T_{2,3,4}(l^2 - l_{1234}^2) \right)$$

$$\Rightarrow Q A_S^{1\text{-loop}} = 0 \text{ \& BCTJ-} N_{12,3,4,5}(l)$$

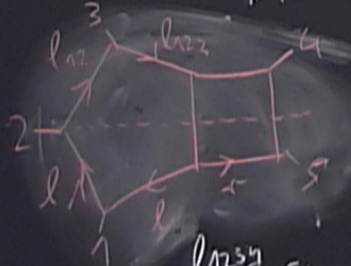
supported by $\alpha' \rightarrow 0$ limit

$$\Rightarrow M_S^{1\text{-loop}} \text{ from } |V_{12} T_{3,4,5}|^2 \text{ \& } |N_{12,3,4,5}(l)|^2 \text{ of } A_S^{1\text{-loop}} \text{ string}$$

Penta-Box: $\lim T^m$ with $U|_{T^m}$ zero b-ghost sector $b \sim T^m dx$

$$T_{A,B,C|D,E}^m = \left[A_{A,B|D,E}^m + (C \leftrightarrow B, A) \right] + \frac{1}{12} W_{A,B,C|D,E}^m + \dots$$

$$Q T_{2,3,4|5,6}^m = k_2^m V_2 T_{3,4|5,6} + (2 \leftrightarrow 3, 4)$$



$$\longleftrightarrow N_{1|2,3|4,5}^{(2)}(l) = (l_m + l_m^{123}) T_{1|2,3|4,5}^m + \left(T_{1|2,3|4,5} + (12 \leftrightarrow 13, 23) \right)$$

$$Q N_{1|2,3|4,5}^{(2)}(l) = V_1 T_{2,3|4,5} (l^2 - l_1^2) + V_2 T_{1,3|4,5} (l_1^2 - l_{12}^2) + V_3 T_{1,2|4,5} (l_{12}^2 - l_{123}^2)$$

$$\Rightarrow \lim_{s \rightarrow 0} Q A_S^{1-loop} = 0 \text{ \& BCJ-N}(l)$$

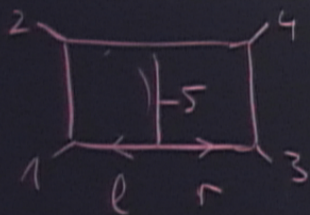
$$\Rightarrow M_S^{1-loop} \text{ from } |V_{12} T_{3,4|5}|^2 \text{ \& } |N_{1|2,3|4,5}(l)|^2$$

supported by $\alpha' \rightarrow 0$ limit of A_S^{1-loop} string

CAUTION
DO NOT USE LENSES OR DISTANT OBJECTS
FROM CENTER OF THE MIRROR AT THE FRONT
DO NOT DISTURB THE LIGHT
DO NOT DISTURB THE LIGHT

& bubbles & tadpoles in m -gon subgraph $\sum_{\substack{e \\ \text{of } \ell}} f_{abe} f_{ecd} + \text{cyc}(a,b,c) = 0$
 at most $m-4$ powers

Non-planar graph via BCJ [1106.4711]



$$\Leftrightarrow N_{1,2,5|3,4}(\ell) + N_{3,4,5|1,2}(r)$$

Multiparticle case $T_{A,B|C,D} = F_{(C}^{\tau s} (\lambda_V^s W_D))$

IV) Three-loop speculations

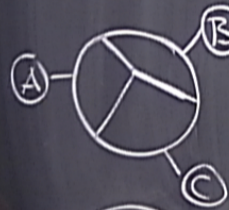
Higher mass-dim. superfields from $[D^m - A^m, W^\alpha]$

$$W_1^{m\alpha} = k_1^m W_1^\alpha, \quad W_{12}^{m\alpha} = k_{12}^m W_{12}^\alpha + s_{12} (W_1^\alpha A_2^m - (1 \leftrightarrow 2))$$

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$$T_{A|B|C} = (\lambda \gamma_m W_A^m) (\lambda \gamma_n W_B^n) (\lambda \gamma_p W_C^p) - (B \leftrightarrow C)$$



$$T_{A|B|C} = (\lambda \gamma_m W_A^m) (\lambda \gamma_n W_B^n) (\lambda \gamma_p W_C^m) - (B \leftrightarrow C)$$

$$[A_1^m T_{2|3|4} + (1 \leftrightarrow 2)] + W_{1,2|3|4}^m$$

$$N_{1,2|3|4}(\ell) = (\ell_1^m + \ell_2^m) T_{1,2|3|4}^m +$$

$$W_{3,4,5|1,2}^m = \frac{4}{3} \left[(\lambda \gamma_{pq} \gamma^m W_4^m) F_{23}^{pq} (\lambda \gamma_t W_5^m) (\lambda \gamma_s W_3^m) F_{41}^{st} \right. \\
\left. - \frac{1}{2} (\lambda \gamma^m W_5^m) (\lambda \gamma_{pq} \gamma^r W_4^r) F_{23}^{pq} (\lambda \gamma_{st} \gamma^m W_3^m) F_{41}^{st} \right. \\
\left. + \frac{1}{6} (W_3 \gamma^{mst} W_4) (\lambda \gamma_{pqrs} \gamma^m) (\lambda \gamma_t W_5^m) F_1^{pq} F_2^{rs} \right. \\
\left. + (5 \leftrightarrow 3, 4) \right]$$

$$W_{1,2|3|4}^m = (\lambda \gamma^m W_4^m) L_{2,3,4} \\
L_{2,3,4} = \frac{1}{3} (\lambda \gamma^m W_2^m) (\lambda \gamma^n W_3^n) F_{41}^{mn}$$

OPE's $\Rightarrow$ pole

$$\int U_1(z_1) U_2(z_2)$$

Pentagon

$$T_{A,B,C|DE}^m =$$

$$Q T_{2,3,4,5,6}^m =$$

$\Rightarrow$

$\Rightarrow M_5$

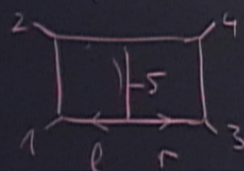
$A_n^{\text{loop}} = \int \prod_{j=1}^n d^D l_j \sum_{\text{propagators}} \frac{N(l) C_i}{\prod_{h \in \text{propagators}} P_{h,i}(l)}$

max SUSY.
 no triangles
 & bubbles & tadpoles

in m -gon subgraph
 at most $m-4$ powers of l

momenta l (super-) polarizations
 w. Jacobi-identity
 $f^{abc} f^{acd} + \text{cyc}(a,b,c) = 0$

Non-planar graph via BCJ [1106.4711]



$$\leftrightarrow N_{1,2,5|3,4}(l) + N_{3,4,5|1,2}(r)$$



support from covariance

$$Q T_{12|3|4} = s_{12} (V_1 T_{2|3|4} - (1 \leftrightarrow 2))$$

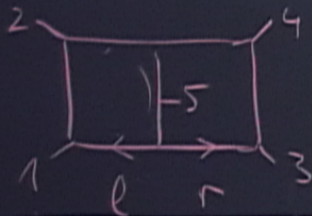
$$Q N_{1,2|3|4}(l) = V_1 T_{2|3|4}(l^2 - l_1^2) + V_2 T_{1|3|4}(l_1^2 - l_2^2)$$

no triangles
& bubbles & tadpoles

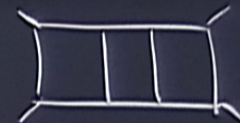
in m -gon subgraph at most $m-4$ powers of ℓ

$$\sum_e f_{abe} f_{ecd} + \text{cyc}(a,b,c) = 0$$

Non-planar graph via BCJ [1106.4711]



$$\Leftrightarrow N_{1,2,5|3,4}(\ell) + N_{3,4,5|1,2}(r)$$



support from
covariance

$$Q T_{12|3|4} = s_{12} (V_1 T_{2|3|4} - (1 \leftrightarrow 2))$$

$$Q N_{1,2|3|4}(\ell) = V_1 T_{2|3|4}(\ell^2 - \ell_1^2) + V_2 T_{1|3|4}(\ell^2 - \ell_{12}^2)$$



$$\Leftrightarrow (\lambda \gamma_{mnpqr} \lambda) (\lambda \gamma_s W_{12}^{mnpq}) F_{34}^{rs} + (12 \leftrightarrow 34)$$

note $(1 \leftrightarrow 2)$ to $(12 \dots p-1 \leftrightarrow p)$

generating series solves non-linear SYM e.o.m

$$A_\alpha = \sum_i A'_\alpha t^i + \sum_{i,j} \frac{A'_{ij}}{s_{ij}} t^i t^j + \sum_{i,j,k} \left(\frac{A'_{ijk}}{s_{ij}s_{jk}} + \frac{A'_{ikj}}{s_{jk}s_{ij}} \right) t^i t^j t^k + \dots$$

color generators

ditto for $A_m, W^\alpha, \mathbb{F}^{nm}$

$$Q(\lambda^\alpha A_\alpha^{12 \dots p}) = \sum_{j=1}^{p-1} \left(\lambda^\alpha A_\alpha^{12 \dots j} \lambda^\beta A_\beta^{j+1 \dots p} \right)$$